EE 510: Mathematical Foundations of Machine Learning **Winter 2020**

Exercises 8

Pages: 6.1-6.37 *Instructor Name: John Lipor*

Exercise 1

Write the term $\|B - A\|_F^2$ in terms of the trace of the matrices A and B.

Exercise 2

A set is called *convex* if for every pair of points in the set, the line connecting those two points also lies in the set. Formally, for $x, y \in S$, all points of the form $(1-t)x + ty$ are also in S, where $t \in [0,1]$. Show that the set of rank-1 matrices in $\mathbb{R}^{2\times 2}$ is not a convex set.

Exercise 3

Convince yourself that $\left\|\hat{B} - A\right\|_F^2 = \sum_{k=K+1}^r \sigma_k^2$ on pg. 6.3.

Exercise 4

Explain the transition from the second to third equality on the bottom of pg. 6.6, i.e., explain why

$$\left\| U^T \sum_{k=K+1}^{r} \sigma_k u_k v_k^T V \right\|_F = \left\| \sum_{k=K+1}^{r} \sigma_k e_k e_k^T \right\|_F .$$

Exercise 5

Work through and understand each line of the proof on pg. 6.15.

Exercise 6

Show that the canonical basis for $\mathbb{F}^{M\times N}$ given on pg. 6.16 is a valid basis.

Exercise 7

Show that $\operatorname{tr}(B^T A)$ is a valid inner product on $\mathbb{R}^{m\times n}$, i.e., that it satisfies

- $\langle x, y\rangle = \langle y, x\rangle$
- $\langle \alpha x, y\rangle = \alpha \langle x, y\rangle$
- $\langle x + y, z\rangle = \langle x, z\rangle + \langle y, z\rangle$
- $\langle x, x\rangle \geq 0$ with equality if and only if $x = 0$,

where $x, y \in \mathbb{R}^{m\times n}$ are matrices (which are referred to as "vectors" in this space).

Exercise 8

Does the canonical basis for $\mathbb{R}^{m\times n}$ form an orthonormal basis for $\mathbb{R}^{m\times n}$ when using the Frobenius inner product?

Exercise 9

Read through the top answer for explaining PCA given here.

Exercise 10

Show that $P^\perp$ defined on 6.33 is a valid projection matrix.

Exercise 11

Verify that $P^\perp 1 = 0$ on pg. 6.33.