EE 510: Mathematical Foundations of Machine Learning

Winter 2020

Exercises 7

Pages: 4.30-4.52

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Exercise 1

 $\mathrm{BV}\ 12.2$

Exercise 2

 $\mathrm{BV}\ 12.3$

Exercise 3

What is the difference between an over-determined system and an under-determined system for least squares problems?

Exercise 4

Let have $A \in \mathbb{R}^{m \times n}$ have rank r. In this case, what is $\Sigma \Sigma^+$, where $A = U \Sigma V^T$?

Exercise 5

Why do we refer to the estimate A_K as "low rank" on pg. 4.36?

Exercise 6

Verify the equivalence between Eq. (4.19) and the term $\left\|\tilde{A}x - \tilde{y}\right\|_{2}^{2}$ on pg. 4.39.

Exercise 7

Show that every square, diagonalizable matrix with eigenvalues that are all either 0 or 1 is a valid projection matrix.

Exercise 8

Show that every projection matrix P with rank at least one (i.e., not all zeros) can be written of the form QQ^{T} , where Q is a matrix with orthonormal columns.