EE 510: Mathematical Foundations of Machine Learning **Winter 2020**

Exercises 4

Pages: 3.1 - 3.20 *Instructor Name: John Lipor*

Exercise 1

Verify the following are valid vector spaces:

- $\mathbb{R}$
- the $y = 0$ axis in $\mathbb{R}^2$.

Exercise 2

Which view of matrix-vector multiplication is most related to the definition of span? Which view of matrix-matrix multiplication?

Exercise 3

Verify that for $u_1, \ldots, u_n \in \mathcal{V}$, $\text{span}(\{u_1, \ldots, u_n\})$ is a subspace of $\mathcal{V}$.

Exercise 4

Verify that the vectors $[1\ 0\ 1]^T$ and $[1\ 0\ -1]^T$ span the (x, z) plane in $\mathbb{R}^3$.

Exercise 5

BV 5.1

Exercise 6

Show that any two orthogonal vectors are linearly independent.

Exercise 7

Show that the vectors

$$a_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad a_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad a_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

are linearly independent.

Exercise 8

BV 5.4

Exercise 9

- Show that the standard basis vectors $e_1, \ldots, e_n \in \mathbb{R}^n$ are a basis for $\mathbb{R}^n$.
- Show that a_1, a_2, a_3 from Exercise 7 above are a basis for $\mathbb{R}^3$. Are they an orthonormal basis?

Exercise 10

Prove the fact given on pg. 3.14.

Exercise 11

Let

$$\mathcal{S} = \operatorname{span}\left(\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}\right\}\right) \quad \mathcal{T} = \operatorname{span}\left(\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix}\right\}\right).$$

What are

- $\dim(\mathcal{S})$
- $\mathcal{S} + \mathcal{T}$ and its dimension
- $\mathcal{S} \cap \mathcal{T}$ and its dimension.

Exercise 12

Let

$$\mathcal{S} = \operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}\right) \quad \mathcal{T} = \operatorname{span}\left(\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}\right\}\right).$$

- What is $\dim(\mathcal{S} + \mathcal{T})$?
- What is $\operatorname{span}(\mathcal{S} + \mathcal{T})$?
- Is $\mathcal{S} + \mathcal{T}$ a direct sum?

Exercise 13

Draw a picture of the orthogonal complement example given on pg. 3.18.

Exercise 14

Let $A \in \mathbb{R}^{m \times n}$. If $\mathcal{S} = \mathcal{R}(A)$, what dimension are the vectors in $\mathcal{S}^\perp$?

Exercise 15

If $A \in \mathbb{R}^{m \times n}$, what is the maximum dimension of $\mathcal{R}(A)$? What must be true for $\mathcal{R}(A)$ to achieve this dimension?