EE 510: Mathematical Foundations of Machine Learning **Winter 2021**

Exercises 1

Pages: 1.1 - 1.20 *Instructor Name: John Lipor*

The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

Exercise 1

Draw a diagram showing how to vectorize a matrix in order to think of an image as a vector.

Exercise 2

Prove that matrix-vector multiplication is a linear operation, i.e., show that the operation

$$f(x) = Ax$$

where $A \in \mathbb{R}^{m\times n}$ and $x \in \mathbb{R}^n$ satisfies $f(x+y) = f(x) + f(y)$ and $f(\alpha x) = \alpha f(x)$.

Exercise 3

Suppose you are given a set of noisy measurements $y_1, \ldots, y_N$ obtained via the equation $y_i \approx w^T x_i$, where $w, x_i \in \mathbb{R}^n$ for $i = 1, \ldots, N$. Let $y = [y_1 \ \ldots \ y_N]^T$. Define the matrix X such that $y \approx Xw$.

Exercise 4

Work through and understand the math behind the example on pages 1.12-1.13 of the lecture notes.

Exercise 5

Let $A \in \mathbb{R}^{n\times n}$ be diagonal. Write the vector Ax out explicitly.

Exercise 6

For the diagonal matrix $A \in \mathbb{R}^{n\times n}$, write what A^{-1} is explicitly and prove that it is in fact an inverse by showing that $AA^{-1} = A^{-1}A = I_n$.

Exercise 7

Write out an explicit formula (in the form of a sum) for Ax when $A \in \mathbb{R}^{n\times n}$ is tridiagonal.

Exercise 8

Let y_i be a sequence with $y_i = \sum_{j=1}^{i} x_i$, where $x_i \in \mathbb{R}$. Write the vector $y = [y_1 \ \ldots \ y_N]^T$ as a product of $[x_1 \ \ldots \ x_N]^T$ and a lower triangular matrix (that you must define yourself).

Exercise 9

For the matrices $U_1, \dots, U_K \in \mathbb{R}^{n \times n}$, simplify the expressions

- $(U_1 U_2 \dots U_K)^T$
- $(U_1 U_2 \dots U_K)(U_1 U_2 \dots U_K)^T$.

Exercise 10

Use the fact that $(A+B)^T = A^T + B^T$ to show that $(A+B+C)^T = A^T + B^T + C^T$. Next, generalize this to $K \in \mathbb{N}$ matrices via induction.