Ex 1

Use the definition of inner product:

$$x^T y = \sum_{i=1}^{n} x_i y_i$$

$$= \sum_{i=1}^{n} y_i x_i = y^T x$$

Ex 2

Check the two properties:

$$\langle x+y, z \rangle = (x+y)^T z$$

$$= (x^T + y^T) z$$

$$= x^T z + y^T z = \langle x, z \rangle + \langle y, z \rangle$$

$$\langle \alpha x, y \rangle = (\alpha x)^T y$$

$$= \alpha x^T y = \alpha \langle x, y \rangle$$

Ex 3

a) $$\mathbb{1}^T w = [1\ 1\ 1 \cdots 1] \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \sum_{i=1}^{n} 1 \cdot w_i$$

Thus, $\mathbb{1}^T w$ is the number of words in the document.

b) $w_{282} = 0$ means word 282 does not appear in the document.

c) First consider a single element h_i

$$h_i = \frac{\text{count of word } i}{\text{total count}} = \frac{w_i}{\mathbb{1}^T w}$$

Therefore the full histogram vector is

$$h = \frac{w}{\mathbb{1}^T w}$$

Ex 4

a) If $x, y \in \mathbb{R}^n$ are nonnegative, then

$$\langle x, y \rangle = x^T y = \sum_{i=1}^{n} x_i y_i$$

but $x_i, y_i \geq 0$, so $x_i y_i \geq 0$, implying $\langle x, y \rangle \geq 0$.

b) There are two options:

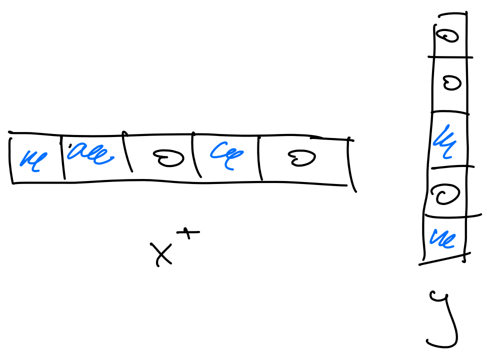
1) Either $x = 0$ and/or $y = 0$

2) The elements for which $x_i > 0$ correspond to elements for which $y_i = 0$. Formally, we write

"set of indices i such that $x_i > 0$"

$$\{i : x_i > 0\} = \{i : y_i = 0\} \quad \text{and}$$

$$\{j : y_j > 0\} = \{j : x_j = 0\}$$



= nonzero element

Ex 5

$A_{17,:}$ is a row of A, having size 1×37

$A_{:,3}$ is a column of A, having size 20×1

AB has size $(20 \times 37)(37 \times 4) = 20 \times 4$

BA is not a valid operation

Ex 6

- Must have $p = q$
- When $C = 0$, $A(B+C) = AB$

  When $A = 0$, $A(B+C) = 0$

Ex 7

ABC has size 3×1. The best way to place parentheses is

$$\underbrace{A}_{3\times 127}(\underbrace{BC}_{127\times 1})$$

← This way only does matrix-vector mult instead of the matrix-matrix mult AB

Ex 8

We used the associative property to do substitution in problem 9.

Ex 9

Sizes are

$A : m \times n$

$v : n \times 1$ $\Rightarrow$ $A v^T$ is <u>not</u> a valid operation but $v^T v$ is

$v^T : 1 \times n$

Therefore we must do $A(v^T v)$ (matlab does this automatically)

Ex 10

For IA, I must have size 200×200. For AI, I must have size 40×40.

Ex 11

$$AB = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} [1 \ 2]\begin{bmatrix} 3 \\ 5 \end{bmatrix} & [1 \ 2]\begin{bmatrix} 2 \\ 7 \end{bmatrix} \\ [4 \ 1]\begin{bmatrix} 3 \\ 5 \end{bmatrix} & [4 \ 1]\begin{bmatrix} 2 \\ 7 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 13 & 16 \\ 17 & 15 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 33 & 17 \end{bmatrix}$$

Takeaway: $AB \neq BA$ in general.

Ex 12

If x, y have angle $0°$ between them we can write $y = \alpha x$ for some α. In this case

$$\begin{aligned}
\|x+y\| &= \|x + \alpha x\| \\
&= \|(1+\alpha)x\| \\
&= (1+\alpha)\|x\| \\
&= \|x\| + \alpha\|x\| \\
&= \|x\| + \|\alpha x\| \\
&= \|x\| + \|y\|
\end{aligned}$$

So the norm adds when vectors are aligned. When $x \perp y$,

$$\begin{aligned}
\|x+y\|^2 &= \|x\|^2 + \|y\|^2 + 2x^T y \\
&= \|x\|^2 + \|y\|^2
\end{aligned}$$

so the norm <u>squared</u> adds when vectors are orthogonal.

Ex 13

- $(x+y)^T(x-y) = x^Tx - y^Ty + x^Ty - x^Ty$

$$= x^Tx - y^Ty = \|x\|^2 - \|y\|^2$$

- $\|x+y\|^2 + \|x-y\|^2 = x^Tx + y^Ty + 2x^Ty + x^Tx + y^Ty - 2x^Ty$

$$= 2x^Tx + 2y^Ty$$

$$= 2(\|x\|^2 + \|y\|^2)$$

Ex 14

The trace of a scalar is the scalar itself, so

$$x^Tx = tr(x^Tx)$$

Now use the cyclic permutation property to see that

$$tr(x^Tx) = tr(xx^T)$$

$$\Rightarrow x^Tx = tr(xx^T)$$