Introduction to 2D and 3D Computer Graphics

Realistic Rendering

-- Solids Modeling --
Solid objects can be defined...
  - ...by sweeping an object along a trajectory through space
  - ...this process is called extrusion

Extrusion...
  - ...is a natural way to represent objects made of extruding metal or plastics with cross-sections visible
  - ...can be performed using translational sweeps or rotational sweeps
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- Translational sweeps...
  - ...are a simple sweep defined by a 2D area swept along a linear path normal to the plan of the area, creating a volume

- Rotational sweeps...
  - ...are defined by rotating around a 2D area about an axis
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2D area  
**Translation sweep**  
**Rotational sweep**
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- Quadric surfaces...
  - ...are defined by quadratic equations in two-dimensional space
  - ...for example, can be spheres and cones
  - ...are also called *surfaces of revolution*, since a finite curve in two dimensions is swept in three-dimensional space about one axis to create the surface
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- Quadric surfaces...
  - ...where a circle centered on the origin forms a sphere, otherwise it forms a torus
  - ...where a line with one end on the axis of rotation forms a cone, a line parallel to the axis of rotation forms a cylinder
- RenderMan™ always uses the z axis as the axis of rotation, with a sweep angle...
  - ...sweeping a quadric by less then 360 degrees leaves an open surface
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- Example quadric surfaces...

- Cone
- Sphere
- Torus
- Disk
- Cylinder
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- To make a realistic image...  
  - ...objects must be able to be combined

- One of the most popular ways for combining objects...  
  - ...is with Boolean set operators  
  - ...using union, difference, and intersection

- Boolean set operators are 3D equivalents of simple 2D
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Two intersecting cubes

Union operation

Intersection operation

Difference operation: Bottom-Top

Difference operation: Top-Bottom
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- Using ordinary Boolean set operators, not all intersections form solid objects...
  - ...they may instead form a plane, a line...

Two intersecting cubes producing a solid

Two intersecting cubes producing a plane

Two intersecting cubes producing a line
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- Or...they may instead form a point or be null

Two intersecting cubes producing a point

Two cubes producing a null set
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- Using regularized Boolean set operators, only solid objects or null sets are formed...
  - ...let's look at the same set of examples using regularized operators:

  - Two intersecting cubes producing a solid
  - Two intersecting cubes producing a null set
  - Two intersecting cubes producing a null set
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Two intersecting cubes again producing a null set

Two cubes producing a null set
Solid objects created with sweeps can be manipulated using Boolean set operations...

- ...by first converting the objects into boundary representations, spatial-partitioning representations, or constructive solid geometry.

Two simple sweeps of 2D objects (triangles)

How these objects would look when overlapping

The result of a union operation; it can no longer be thought of as a simple sweep
Boundary representations...
- ...are called *b-reps*
- ...describe objects in terms of their surface boundaries: vertices, edges, and faces
- ...are generally restricted to be planar, with polygonal boundaries and convex faces
- ...are generally restricted to be 2-*manifolds*: this means that support is not provided when there are more than 2 faces sharing an edge and neighboring points on each of the faces are not continuous
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- **Boundary representations...**

  - **Face**
  - **Vertex**
  - **Edge**

*This object has boundaries that are 2-manifold: each point on a face has a neighborhood of surrounding points that is a topological disk*

Notice in this case, there is an edge that has neighboring points from 4 faces that does not create a topological disk.
Boundary representations of simple polyhedra...
- are described by Euler's formula
- ...are solids bounded by a set of polygons, but can be deformed into a sphere and has no holes
- ...follow the rules: $Vertices - Edges + Faces = 2$
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- To guarantee that a solid volume is created...
  - ...each edge must connect to two vertices and be shared by exactly two faces,
  - ...at least three edges must meet at each vertex, and
  - ...faces must not interpenetrate one another
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- Boundary representations of polyhedra with holes...
  - ...are described by a generalization of Euler's formul
  - ...$\text{Vertices} - \text{Edges} + \text{Faces} - \text{Holes} = 2(\text{Separate parts} - \text{Genus})$
  - ...where *Holes* describes the # of holes in the faces
  - ...where *Genus* describes the # of holes that pass through the object
  - ...where *Separate parts* describe the # of separate components in the object
Boundary representations of polyhedra with holes...

- 3 Holes (1 of them is in the bottom face)
- 15 Faces
- 24 Vertices
- 36 Edges
- 1 Separate Parts
- 1 Genus

A polyhedron with a hole passing through the object
Spatial-partitioning representations...
  - ...describe objects as collections of adjoining nonintersecting solids

Spatial-partitioning creates collections of solids that...
  - ...may or may not be the same type as the original object
  - ...are like building blocks
  - ...can vary in type, size, position, parameterization, and orientation
Solid objects can be formed with spatial-partitioning using...
- ...cell decomposition
- ...spatial-occupancy enumeration
- ...octrees and quadtrees, etc.
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- Cell decomposition...
  - ....is a popular form of spatial-partitioning
  - ...composes complex objects from simple primitives in a bottom-up fashion by gluing them together! (like a union but without objects intersecting)
  - ...composes objects from cells, where any two cells must share a single point, edge, or face
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- Cell decomposition...

*Three simple primitives: called cells*

*Keep in mind with this method, that the complex object can be created using cells in more than one way...*
Spatial-occupancy enumeration...

- ...is a special case of cell decomposition
- ...defines objects using identical cells arrayed in a fixed and regular grid (called *voxels*)
- ...most commonly uses a cube cell type
- ...only controls whether or not a cell is present or absent in every cell in a grid
- ...no other controls are defined
- ...creates unique and unambiguous list of occupied cells
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- Spatial-occupancy enumeration...
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- Octrees...
  - ...are a hierarchical way to use voxels
  - ...are designed to reduce the storage requirements of the spatial-occupancy enumeration approach
  - ...are derived from 2D quadtrees...and expanded to 3D
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- Quadtrees...
  - ...successively subdivide a 2D plane in both dimensions
  - ...where each quadrant is full, partly full, or empty depending on how much of the complex object intersects the area
  - ...where partly full quadrants are recursively subdivided
  - ...and subdivision continues until all cells are full or empty
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- Quadtrees...

Cells (voxels) using spatial-occupancy enumeration

Cells using quadtrees
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For example...

```
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
```

```
Empty
Full
Empty
Full
Empty
Full
```
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- Octrees...
  - ...are three dimensional quadtrees
  - ...its three dimensions are recursively subdivided into octants
  - ...have quadrants:
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- Octrees...
- The number of nodes in a quadtree or octree is proportional to the object's perimeter or surface, respectively
  - ...this is because subdivision occurs only from the need to represent an object's boundary
  - ...therefore, subdivision only occurs in those quadrants where a boundary passes
Boolean set operators can also apply to both quadtrees and octrees...

- ...by traversing the two trees in parallel
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- Constructive solid geometry (CSG)...  
  - ...describes objects in terms of regularized Boolean set operators (as part of their representation)  
  - ...stores objects in a tree with operators at the internal nodes and simple primitives at the leaves  
  - ...allows nodes to represent Boolean operators, translation, rotation, or scaling
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- Some implementations limit the primitives to be simple solids -- like cubes or spheres...
  - ...other implementations allow objects that aren't even completely bounded volumes! These are useful for slicing an object by a plane
- CSG does not provide unique representation...
  - ...and therefore can be confusing for systems that allow "leaf" editing (for adding, deleting, replacing, and modifying subtrees)
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• A CSG example:
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Objects using solid modeling: summary

- Accuracy...
  - ...is best achieved by CSG that allows for curved surfaces
  - ...is only approximated using spatial-partitioning and polygonal b-rep methods

- The variety of objects represented...
  - ...is best achieved using spatial-partitioning since they can represent any solid (even if it is an approximation)
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*Objects using solid modeling: summary*

- The variety of objects represented...
  - ...can also be achieved using b-reps, if they provide faces and edges in addition to polygons bounded by straight lines
  - ...is limited using sweeps

- Uniqueness...
  - ...can be guaranteed only using octree and spatial-occupancy enumeration methods
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*Objects using solid modeling: summary*

- Compactness and efficiency...
  - ...are best achieved using CSG, since it is compact and able to record Boolean operations and transformations quickly