





























































• Differentiating (5), we have
$$\frac{dQ}{dx} = k_{S} \left(\pi r^{2}\right) \frac{d^{2}T}{dx^{2}} + \left(\pi r^{2}\right) \frac{dT}{dx} \frac{dk_{S}}{dx} \cong k_{S} \left(\pi r^{2}\right) \frac{d^{2}T}{dx^{2}} \quad (6)$$
• Substituting (6) into (4), we have
$$\frac{d^{2}T}{dx^{2}} - \frac{2\sigma\varepsilon}{k_{S}r}T^{4} = 0 \quad (7)$$
• k_{S} varies roughly as 1/T, so if k_{M} is the thermal conductivity at the melting point, $k_{S} = k_{M} \frac{T_{M}}{T} \quad (8)$
 $\therefore \frac{d^{2}T}{dx^{2}} - \frac{2\sigma\varepsilon}{k_{M}rT_{M}}T^{5} = 0 \quad (9)$
• Solving this differential equation, evaluating it at $x = 0$ and substituting the result into (3), we obtain (see text):
$$v_{PMAX} = \frac{1}{LN} \sqrt{\frac{2\sigma\varepsilon k_{M}T_{M}^{5}}{3r}} \quad (10)$$
• This gives a max pull rate of ≈ 24 cm hr⁻¹ for a 6" crystal (see text). Actual values are $\approx 2X$ less than this.







Impurity Doping: Rapid Stirring Approx

(Solute Concentration uniform in melt) Initially: weight of melt = W _m		$\int_{CmWm}^{S} dS/S = -k \int_{0}^{W} dW / (W_{m}-W)$	
solute conc'n in melt = C_m (by weight) After time t: Crystal weight grown = W solute conc'n in crystal = C_s solute conc'n in melt = C_L weight of solute in melt = S		$ \ln S \left[\begin{matrix} S \\ CmWm \end{matrix} \right]_{CmWm}^{S} = -k \left[-\ln \left(W_m W \right) \right]_{0}^{W} $ $ \ln S / C_m W_m = \ln \left[\left(W_m W \right) / W_m \right]^{k} $ $ Therefore: $ $ S = C_m W_m \left[1 - \left(W / W_m \right) \right]^{k} $	
(Freeze an element of crystal, weight dW)		But S = (W_m - W) C _L by definition	
Solute loses $C_s dW = -dS$ and $C_L = S/(W_m-W)$		$= W_{m} (1 - W / W_{m}) C_{s} / k$	
Therefore:		= W _m (1-W / W _m) ^k C _m	
dS/S	= - (C_{s} / C_{L}) (dW / (W_{m} -W))	Therefore:	
	= - k (dW / (W _m -W))	$C_{s} = kC_{m} (1-W / W_{m})^{k-1}$	
		$C_s = kC_m / (1-W / W_m)^{1-k}$	
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Impurity Doping: Partial Stirring Approx #2				
Define R = crystal growth rat	e	$\therefore \text{ dc/dx }]_{x=0} = -AR/D = -R/D (C_L'-C_s)$		
Define D = diffusion constan	t of solute atoms	$\therefore A = (C_1' - C_s) \therefore B = C_s$		
(typ: 5x10 ⁻⁵ cm²/s Diffusion equation at any poi	in liquid) nt x	and $C(x) = C_s + (C_L'-C_s) \exp - (R/D) x$ (for 0 <x <="" <math="">\delta)</x>		
$D (d^2c/dx^2) = dc/dt \& (dc/dx)$	$\cdot dx/dt$) = R(dc/dx)	$:: C(\delta) = C_{L} \to [(C_{L} - C_{s}) / (C_{L} ' - C_{s})] = exp - R\delta / D$		
Therefore: D (d^2c / dx^2) = R (dc/dx) If we define an effective $k_e = C_s/C_1$ and the true $k = C_s/C_1$				
Solution: C = A exp - (R / D)	(+B	then ($\rm C_S$ / $\rm k_e$ - $\rm C_S$) / ($\rm C_S$ / $\rm k$ - $\rm C_S$) = exp - R\delta/D		
Boundary Conditions:		i.e. k _e = k/[k +(1-k)exp -Rδ/D] (Campbell 2.14)		
1. $C(O) = C_{L'} \therefore A + B = C_{L'}$		and $C_s = k_e C_m (1 - W / W_m)^{k_e - 1}$		
2. Zero net flux at x=0 (neglected diffusion in solid)		$k_e \rightarrow k$ if R δ /D is large		
\therefore - D dc/dx] =	(C _L '-C _s) R	i.e. for high pull rate (R large, limits diameter)		
Diffusion away from	Rate of rejection of	and for low spin rate (a $\delta^{\text{-1})}$		
interface into liquid	solute at interface	δ = 1.8 D ^{1/3} R ^{1/6} - ^{1/2} (from Sze)		
1/27/00		Can vary R, to compensate for C 38		















