DESIGN NOTE

> Remember, the transfer function analysis in SPICE is a dc analysis and should not be used in circuits containing capacitors and inductors!

10.5.2 The Hybrid or *h*-parameters

The **h-parameter** description is also widely used in electronic circuits and is one convenient model for a current amplifier:

$$\mathbf{v_1} = h_{11}\mathbf{i_1} + h_{12}\mathbf{v_2}$$
$$\mathbf{i_2} = h_{21}\mathbf{i_1} + h_{22}\mathbf{v_2}$$

(10.18)

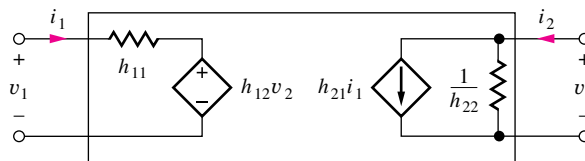Figure 10.7 is the network representation of these equations.

Figure 10.7 Two-port *h*-parameter representation.

As with the *g*-parameters, the *h*-parameters are determined from a given network using a combination of open- and short-circuit measurement conditions:

$$h_{11} = \left.\frac{\mathbf{v_1}}{\mathbf{i_1}}\right|_{\mathbf{v_2}=0} = \textbf{short-circuit input resistance}$$

$$h_{12} = \left.\frac{\mathbf{v_1}}{\mathbf{v_2}}\right|_{\mathbf{i_1}=0} = \text{reverse open-circuit voltage gain}$$

$$h_{21} = \left.\frac{\mathbf{i_2}}{\mathbf{i_1}}\right|_{\mathbf{v_2}=0} = \text{forward short-circuit current gain}$$

$$h_{22} = \left.\frac{\mathbf{i_2}}{\mathbf{v_2}}\right|_{\mathbf{i_1}=0} = \textbf{open-circuit output conductance}$$

(10.19)

EXAMPLE 10.2 FINDING A SET OF *h*-PARAMETERS

This example provides an example of calculating the *h*-parameters of a network containing a controlled source.

PROBLEM Find the *h*-parameters for the circuit in Ex. 10.1.

SOLUTION **Known Information and Given Data:** Circuit given in Ex. 10.1 including specified element values; *h*-parameter definitions in Eq. (10.19)

Unknowns: Values of the four *h*-parameters

**Approach:** Apply the boundary conditions specified for each h-parameter, and then use circuit analysis to find the four parameter values. Note that each set of boundary conditions applies to two parameters.

Assumptions: None

Analysis — h_{11} and h_{21}: Looking at the definitions of the g-parameters, we see that h_{11} and h_{21} use the same boundary condition.

$$h_{11} = \left.\frac{\mathbf{v_1}}{\mathbf{i_1}}\right|_{\mathbf{v_2}=0} \qquad \text{and} \qquad h_{21} = \left.\frac{\mathbf{i_2}}{\mathbf{i_1}}\right|_{\mathbf{v_2}=0}$$

Current source $\mathbf{i_1}$ is applied to the input port, and the output port is short circuited (i.e., $\mathbf{v_2}$ is set to zero) as shown in the figure here.
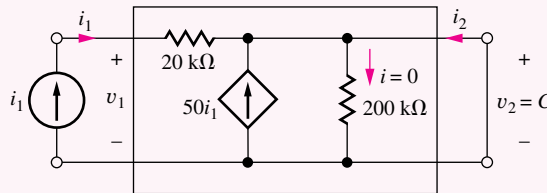
h_{11}: Since $\mathbf{v_2} = 0$, input voltage $\mathbf{v_1}$ is equal to the voltage across the 20-kΩ resistor:

$$\mathbf{v_1} = (20\ \text{k}\Omega)\mathbf{i_1} \qquad \text{and} \qquad h_{11} = \frac{\mathbf{v_1}}{\mathbf{i_1}} = 2 \times 10^4\ \Omega$$

h_{21}: To find $\mathbf{i_2}$, we sum the currents at the output node, realizing that the current in the 200-kΩ resistor is forced to be zero by the short circuit across the output terminals

$$\mathbf{i_1} + 50\mathbf{i_1} + \mathbf{i_2} = 0 \qquad \text{or} \qquad \mathbf{i_2} = -51\mathbf{i_1} \qquad \text{and} \qquad h_{21} = \frac{\mathbf{i_2}}{\mathbf{i_1}} = -51$$

h_{12} and h_{22}: Looking again at the definitions of the h-parameters, we see that h_{12} and h_{22} use the same boundary condition.

$$h_{12} = \left.\frac{\mathbf{v_1}}{\mathbf{v_2}}\right|_{\mathbf{i_1}=0} \qquad \text{and} \qquad h_{22} = \left.\frac{\mathbf{i_2}}{\mathbf{v_2}}\right|_{\mathbf{i_1}=0}$$

For these parameters, we apply a voltage source $\mathbf{v_2}$ to the output port, and the input port is open-circuited (i.e., $\mathbf{i_1}$ is set to zero).
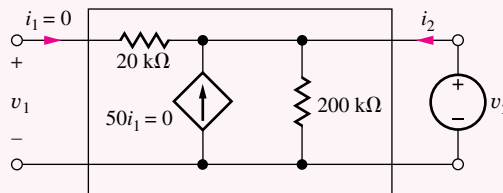
h_{22} and h_{12}: With $\mathbf{i_1} = 0$, the current in the dependent current source is also zero. Therefore,

$$\mathbf{i_2} = \frac{\mathbf{v_2}}{200\ \text{k}\Omega} \qquad \text{and} \qquad h_{22} = \frac{\mathbf{i_2}}{\mathbf{v_2}} = \frac{1}{200\ \text{k}\Omega} = 5 \times 10^{-6}\ \text{S}$$

and

$$\mathbf{v_1} = (20 \text{ k}\Omega)\mathbf{i_1} + \mathbf{v_2} = \mathbf{v_2} \qquad \text{and} \qquad h_{12} = \frac{\mathbf{v_1}}{\mathbf{v_2}} = +1$$

since the boundary sets $\mathbf{i_1} = 0$.

The final h-parameter equations for the network are

$$\mathbf{v_1} = 2 \times 10^4 \mathbf{i_1} + \mathbf{v_2}$$

$$\mathbf{i_2} = -51\mathbf{i_1} + 5 \times 10^{-6}\mathbf{v_2}$$

Check of Results: The results are confirmed below using SPICE.

Discussion: Here again we see that the value of the forward transfer parameter is much larger than that of the reverse transfer parameter $-h_{21} \gg h_{12}$. We will make use of this observation when we study feedback in Chapter 18.

Computer-Aided Analysis: Numerical values for two-port parameters are easily found using the transfer function (TF) analysis capability of SPICE. In order to find the h-parameters for the circuit in Ex. 10.2, we drive the network with current source I1 at the input and voltage source V2 at the output as in the figure here. These choices correspond to the boundary conditions in the definitions of the g-parameters. Zero-value voltage-source VAM_F1 acts as an ammeter and senses the controlling current for CCCS F1. Note that both independent sources are assigned zero values.
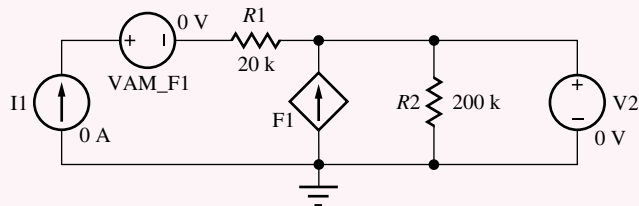
The TF analysis calculates how variables change in response to changes in an independent source, and a starting point of zero is therefore valid. The zero-value sources directly satisfy the boundary conditions required to calculate the h-parameters.

Two TF analyses are used—one to find h_{11} and h_{21} and a second to find h_{12} and h_{22}. The first analysis requests calculation of the transfer function from current source I1 to (the current in) source V2, and SPICE will produce values for the transfer function and the input resistance presented to I1. The SPICE results are transfer function = 51.0 and input resistance = 20.0 kΩ. Note that parameter h_{21} is the negative of the TF value. The sign difference arises from the convention assumed by SPICE in which positive current is directed downward through source V2. Parameter h_{11} equals the input resistance and is in agreement with our hand calculations.

The second analysis requests the transfer function from source V2 to the voltage appearing across I1 (the input node of the two-port). The results are transfer function = 1.00, input resistance = 200 kΩ, and output resistance = 20.0 kΩ. Reverse voltage-gain parameter h_{12} is equal to the transfer function value. Conductance parameter h_{22} is equal to the reciprocal of the 200-kΩ resistance presented to source I2, which is the "input resistance" in this calculation. We find precise in agreement with our hand calculations.

EXAMPLE 10.3   **NETWORK ANALYSIS USING THE *h*-PARAMETERS**

Now we will use the *h*-parameters to find the voltage gain of a network with a source and load attached.

PROBLEM Find the voltage gain $A_v = \mathbf{v_o}/\mathbf{v_s}$ for the network below with $R_S = 5$ kΩ and $R_L = 1$ kΩ. Assume the two-port can be described by the *h*-parameters found in Ex. 10.2.
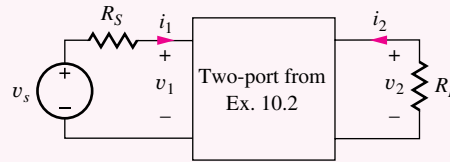
SOLUTION **Known Information and Given Data:** Circuit above with $R_S = 5$ kΩ and $R_L = 1$ kΩ; The two-port is described by

$$\mathbf{v_1} = 2 \times 10^4 \mathbf{i_1} + \mathbf{v_2}$$

$$\mathbf{i_2} = -51\mathbf{i_1} + 5 \times 10^{-6}\mathbf{v_2}$$

Unknowns: Voltage gain $A_v = \mathbf{v_o}/\mathbf{v_s}$

Approach: Relate $\mathbf{v_s}$ to $\mathbf{v_1}$ and $\mathbf{i_1}$ and $\mathbf{v_2}$ to $\mathbf{i_2}$. Substitute the relations into the two-port equations and solve for the voltage ratio $\mathbf{v_o}/\mathbf{v_s}$.

Assumptions: None

Analysis: Let us solve the problem symbolically before substituting the numbers into the equations. The *h*-parameter description for the two-port is

$$\mathbf{v_1} = h_{11}\mathbf{i_1} + h_{12}\mathbf{v_2}$$
$$\mathbf{i_2} = h_{21}\mathbf{i_1} + h_{22}\mathbf{v_2}$$

At the input port, we have to $\mathbf{v_1} = \mathbf{v_s} - \mathbf{i_1}R_S$, and at the output port, $\mathbf{i_2} = -\mathbf{v_2}/R_S$. Substituting these relationships into the *h*-parameter equations and collecting terms yields

$$\mathbf{v_s} = (h_{11} + R_S)\mathbf{i_1} + h_{12}\mathbf{v_2}$$

$$0 = h_{21}\mathbf{i_1} + \left(h_{22} + \frac{1}{R_L}\right)\mathbf{v_2}$$

Using the second equation to eliminate $\mathbf{i_1}$ in the first gives

$$\mathbf{v_s} = -\frac{(h_{11} + R_S)\left(h_{22} + \dfrac{1}{R_L}\right)}{h_{21}}\mathbf{v_2} + h_{12}\mathbf{v_2}$$

and the voltage gain is given by

$$A_v = \frac{\mathbf{v_2}}{\mathbf{v_1}} = \frac{1}{-\dfrac{(h_{11} + R_S)}{h_{21}}\left(h_{22} + \dfrac{1}{R_L}\right) + h_{12}}$$

Substituting the numerical values into the voltage gain expression gives a gain of

$$A_v = \frac{1}{-\dfrac{(2 \times 10^4 + 5 \times 10^3)}{-51}(5 \times 10^{-6} + 10^{-3}) + 1} = 0.670$$

Check of Results: As a check we can also find the voltage gain directly from the original circuit (which is left as an upcoming exercise). This direct analysis yields the same result, so the answer is correct.

Computer-Aided Analysis: The voltage gain can be found using the transfer function (TF) capability of SPICE. The two-port is modeled in SPICE by its equivalent circuit from Fig. 10.7, as shown in the figure here. Zero-value current source IVM_H12 functions as a voltmeter in parallel with the output and provides the controlling voltage for VCVS EH12. In this circuit, CCCS FH21 depends on the current in source EH12. A zero value input source is used for VS, as discussed in the previous two examples. The TF analysis calculates the transfer function from source VS to the output node voltage appearing across resistor RL.
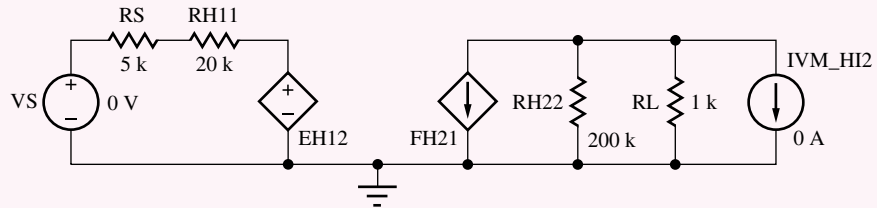
The SPICE results are transfer function $= 0.670$, input resistance $= 75.7$ kΩ, and output resistance $= 328$ Ω.

EXERCISE: Find the voltage gain of the circuit in Ex. 10.3 by direct analysis rather than using the two-port equations.

ANSWER: 0.670

EXERCISE: Find the voltage gain $A_v = \mathbf{v_o}/\mathbf{v_s}$ for the network in Ex. 10.3 if resistor R_L is increased to 5 kΩ.
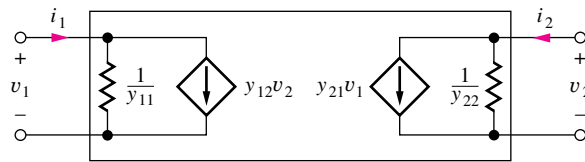
ANSWER: 0.909

10.5.3 THE ADMITTANCE OR *y*-PARAMETERS

The admittance, or **y-parameter,** description will be used during the analysis of feedback amplifiers in Chapter 18,

$$\mathbf{i_1} = y_{11}\mathbf{v_1} + y_{12}\mathbf{v_2}$$
$$\mathbf{i_2} = y_{21}\mathbf{v_1} + y_{22}\mathbf{v_2}$$

(10.20)

Figure 10.8 is a network representation of these equations.

Figure 10.8 Two-port *y*-parameter representation.

The *y*-parameters are often referred to as the short-circuit parameters because they are determined from a given network using only short-circuit terminations:

$$y_{11} = \left.\frac{\mathbf{i_1}}{\mathbf{v_1}}\right|_{\mathbf{v_2}=0} = \textbf{short-circuit input conductance}$$

$$y_{12} = \left.\frac{\mathbf{i_1}}{\mathbf{v_2}}\right|_{\mathbf{v_1}=0} = \text{reverse } \textbf{short-circuit transconductance}$$

(10.21)

$$y_{21} = \left.\frac{\mathbf{i_2}}{\mathbf{v_1}}\right|_{\mathbf{v_2}=0} = \text{forward short-circuit transconductance}$$

$$y_{22} = \left.\frac{\mathbf{i_2}}{\mathbf{v_2}}\right|_{\mathbf{v_1}=0} = \textbf{short-circuit output conductance}$$

EXAMPLE 10.4 **y-PARAMETER ANALYSIS**

This example provides an example of calculating the *y*-parameters of a network containing a controlled source.

PROBLEM Find the *y*-parameters for the circuit in Ex. 10.1

SOLUTION **Known Information and Given Data:** Circuit given in Ex. 10.1 including specified element values; *y*-parameter definitions in Eq. (10.21)

Unknowns: Values of the four *y*-parameters

Approach: Apply the boundary conditions specified for each *h*-parameter, and then use circuit analysis to find the four parameter values. Note that each set of boundary conditions applies to two parameters.

Assumptions: None

Analysis — y_{11} and y_{21}: Looking at the definitions of the *y*-parameters, we see that y_{11} and y_{21} use the same boundary condition.

$$y_{11} = \frac{\mathbf{i_1}}{\mathbf{v_1}}\bigg|_{\mathbf{v_2}=0} \qquad \text{and} \qquad y_{21} = \frac{\mathbf{i_2}}{\mathbf{v_1}}\bigg|_{\mathbf{v_2}=0}$$

Voltage source $\mathbf{v_1}$ is applied to the input port, and the output port is short-circuited ($\mathbf{v_2}$ is set to zero), as shown in the figure here.
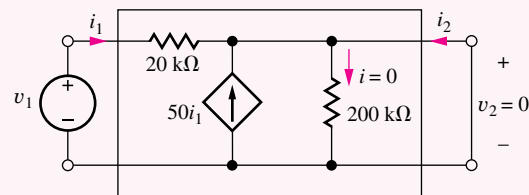
y_{11}: Since $\mathbf{v_2} = 0$, input voltage $\mathbf{v_1}$ appears directly across the 20-kΩ resistor:

$$\mathbf{i_1} = \frac{\mathbf{v_1}}{20\,\text{k}\Omega} \qquad \text{and} \qquad y_{11} = \frac{\mathbf{i_1}}{\mathbf{v_1}} = \frac{1}{20\,\text{k}\Omega} = 5 \times 10^{-5}\,\text{S}$$

y_{21}: To find $\mathbf{i_2}$, we sum the currents at the output node, realizing that the current in the 200-kΩ resistor is forced to be zero by the short circuit across the output terminals:
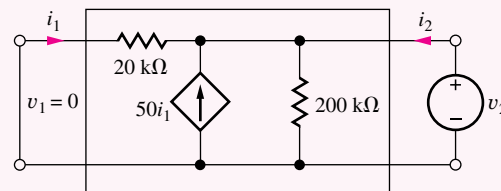
$$\mathbf{i_1} + 50\mathbf{i_1} + \mathbf{i_2} = 0 \qquad \text{or} \qquad \mathbf{i_2} = -51\mathbf{i_1} \quad \text{with} \quad \mathbf{i_1} = \frac{\mathbf{v_1}}{20\,\text{k}\Omega}$$

$$y_{21} = \frac{\mathbf{i_2}}{\mathbf{v_1}} = -\frac{51}{20\,\text{k}\Omega} = -\frac{1}{392\,\Omega} == -2.55 \times 10^{-3}\,\text{S}$$

y_{12} **and** y_{22}: Looking again at the definitions of the *y*-parameters, we see that y_{12} and y_{22} use the same boundary condition.

$$y_{12} = \frac{\mathbf{i_1}}{\mathbf{v_2}}\bigg|_{\mathbf{v_1}=0} \qquad \text{and} \qquad y_{22} = \frac{\mathbf{i_2}}{\mathbf{v_2}}\bigg|_{\mathbf{v_1}=0}$$

For these parameters, we apply a voltage source $\mathbf{v_2}$ to the output port, and the input port is short-circuited ($\mathbf{v_1}$ is set to zero).

y_{12}: With $\mathbf{v_1} = 0$, voltage $\mathbf{v_2}$ appears directly across the 20-kΩ resistor, and

$$\mathbf{i_1} = -\frac{\mathbf{v_2}}{20\,\text{k}\Omega} \qquad \text{and} \qquad y_{12} = \frac{\mathbf{i_1}}{\mathbf{v_2}} = -\frac{1}{20\,\text{k}\Omega} = -5 \times 10^{-5}\,\text{S}$$

y_{22}: To find i_2, we can apply KCL at the output node.

$$i_2 = \frac{v_2}{200\,k\Omega} - 50i_1 + \frac{v_2}{20\,k\Omega} \qquad \text{and we know} \qquad i_1 = -\frac{v_2}{20\,k\Omega}$$

Combining these equations gives the short-circuit output conductance y_{22}:

$$i_2 = \frac{v_2}{200\,k\Omega} + 51\frac{v_2}{20\,k\Omega} \qquad \text{and} \qquad y_{22} = \frac{i_1}{v_2} = \frac{1}{391\,\Omega} = 2.56 \times 10^{-3}\,\text{S}$$

The final y-parameter equations for the network are

$$i_1 = 5 \times 10^{-5}v_1 - 5 \times 10^{-5}v_2$$

$$i_2 = -2.55 \times 10^{-3}v_1 + 2.56 \times 10^{-3}v_2$$

Check of Results: This result appears to be unusual since both equations are nearly the same; i.e., $i_2 \cong -i_1$. We can also observe that the determinant of the system is very close to zero. This is another indication of potential problems using these equations. If we look back at the derivations and keep all the significant digits, we find that

$$i_1 = 5.000 \times 10^{-5}v_1 - 5.000 \times 10^{-5}v_2$$

$$i_2 = -2.550 \times 10^{-3}v_1 + 2.555 \times 10^{-3}v_2$$

Discussion: The determinant of this system of equations is 2.50×10^{-10}, which is much smaller than any of the individual coefficients. This result indicates that analyses using this equation set may be highly sensitive to round-off errors, and therefore the y-parameters are probably not a good parameter set to use to model this network. Note that we still see that the value of the forward transfer parameter is much larger than that of the reverse transfer parameter $-y_{21} \gg y_{12}$.

Computer-Aided Analysis: Numerical values for two-port parameters can be found using the transfer function (TF) analysis of SPICE. In order to find the y-parameters for the circuit in this exercise, we drive the network with voltage source V1 at the input and voltage source V2 at the output, as in the figure here. These choices correspond to the boundary conditions in the definitions of the y-parameters. Zero-value voltage-source VAM_I1 is added to the circuit and functions as an ammeter sensing the controlling current for CCCS F1.
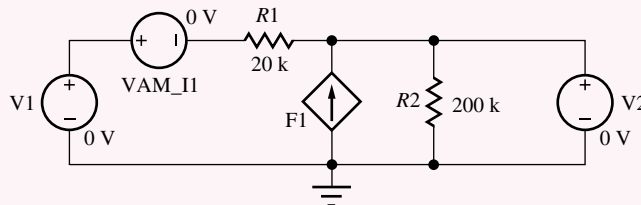
Note that both independent sources are assigned zero values. The TF analysis calculates how variables change in response to changes in an independent source, and a starting point of zero is therefore valid. The zero-value sources directly satisfy the boundary conditions required to calculate the y-parameters.

Two TF analyses are used — the first finds y_{11} and y_{21}, and the second y_{12} and y_{22}. The first analysis requests calculation of the transfer function from voltage source V1 to (the current in) source V2. SPICE will produce values for the transfer function and the input resistance presented to source V1, and the results are transfer function $= 2.550 \times 10^{-3}$, and input resistance $= 20.0$ kΩ. Parameter y_{11} is the input conductance and equals the reciprocal of the 20.0-kΩ value, and parameter y_{21} is the negative of the TF value. The sign difference arises from the convention assumed by SPICE in which positive current is directed downward through source V2.

The second analysis requests the transfer function from source V2 to the current in voltage source V1. The results from SPICE are transfer function $= 50.0 \times 10^{-6}$ and input resistance $= 391.4$ Ω. Reverse transfer conductance y_{12} is equal to the negative of the transfer function value. Conductance parameter y_{22} is equal to the reciprocal of the resistance presented to source V2, which is the "input resistance" in this calculation. Once again we find precise agreement with our hand calculations.

EXERCISE: Find the *y*-parameters for the circuit in Ex. 10.4 if the 200-kΩ resistor is replaced with a 50-kΩ resistor, and the dependent source is changed to 75i_1.

ANSWERS: 5.00×10^{-5} S; 5.00×10^{-5} S; 3.80×10^{-3} S; 3.80×10^{-3} S; Here again we see a problem with the parameter set.

EXERCISE: Confirm your calculations in the previous exercise using SPICE.

10.5.4 THE IMPEDANCE OR *Z*-PARAMETERS

The impedance, or **z-parameters,** will also be used for analysis of feedback amplifiers in Chapter 18,

$$\mathbf{v_1} = z_{11}\mathbf{i_1} + z_{12}\mathbf{i_2}$$
$$\mathbf{v_2} = z_{21}\mathbf{i_1} + z_{22}\mathbf{i_2}$$

(10.22)

Figure 10.9 is a network representation of Eq. (10.22). The *z*-parameters are determined from a given network using open-circuit measurement conditions and are often referred to as the
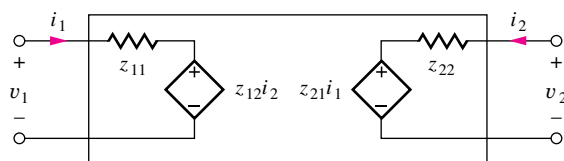
Figure 10.9   Two-port *z*-parameter representation.

open-circuit parameters:

$$z_{11} = \left.\frac{\mathbf{v_1}}{\mathbf{i_1}}\right|_{\mathbf{i_2}=0} = \textbf{open-circuit input resistance}$$

$$z_{12} = \left.\frac{\mathbf{v_1}}{\mathbf{i_2}}\right|_{\mathbf{i_1}=0} = \text{reverse } \textbf{open-circuit transresistance}$$

$$z_{21} = \left.\frac{\mathbf{v_2}}{\mathbf{i_1}}\right|_{\mathbf{i_2}=0} = \text{forward open-circuit transresistance}$$

$$z_{22} = \left.\frac{\mathbf{v_2}}{\mathbf{i_2}}\right|_{\mathbf{i_1}=0} = \textbf{open-circuit output resistance}$$

(10.23)

EXAMPLE 10.5 *z*-PARAMETER ANALYSIS

This example provides an example of calculating the *z*-parameters of a network containing a controlled source.

PROBLEM Find the *z*-parameters for the circuit in Ex. 10.1.

SOLUTION **Known Information and Given Data:** Circuit given in Ex. 10.1 including specified element values; *z*-parameter definitions in Eq. (10.23)

Unknowns: Values of the four *z*-parameters

Approach: Apply the boundary conditions specified for each *z*-parameter, and then use circuit analysis to find the four parameter values. Note that each set of boundary conditions applies to two parameters.

Assumptions: None

Analysis — z_{11} and z_{21}: Looking at the definitions of the *z*-parameters, we see that z_{11} and z_{21} use the same boundary condition.

$$z_{11} = \left.\frac{\mathbf{v_1}}{\mathbf{i_1}}\right|_{\mathbf{i_2}=0} \qquad \text{and} \qquad z_{21} = \left.\frac{\mathbf{v_2}}{\mathbf{i_1}}\right|_{\mathbf{i_2}=0}$$

Current source $\mathbf{i_1}$ is applied to the input port, and the output port is open circuited ($\mathbf{i_2}$ is set to zero) as shown in the figure here.
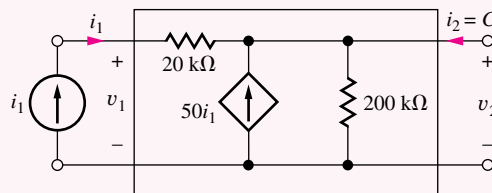
z_{11}**:** We can write a loop equation at the input port to find $\mathbf{v_1}$:

$$\mathbf{v_1} = (20 \text{ k}\Omega)\mathbf{i_1} + (\mathbf{i_1} + 50\mathbf{i_1})(200 \text{ k}\Omega)$$

and

$$z_{11} = \frac{\mathbf{v_1}}{\mathbf{i_1}} = 20 \text{ k}\Omega + 51(200 \text{ k}\Omega) = 10.2 \text{ M}\Omega$$
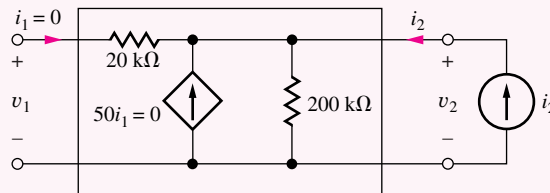
z_{21}: To find $\mathbf{v_2}$, we apply KCL at the output node:

$$\mathbf{v_2} = (\mathbf{i_1} + 50\mathbf{i_1})(200 \text{ k}\Omega) = (51)(200 \text{ k}\Omega)\mathbf{i_1} \qquad \text{and} \qquad z_{21} = \frac{\mathbf{v_2}}{\mathbf{i_1}} = 10.2 \text{ M}\Omega$$

z_{12} **and** z_{22}: Looking again at the definitions of the z-parameters, we see that z_{12} and z_{22} use the same boundary condition.

$$z_{12} = \left.\frac{\mathbf{v_1}}{\mathbf{i_2}}\right|_{\mathbf{i_1}=0} \qquad \text{and} \qquad z_{22} = \left.\frac{\mathbf{v_2}}{\mathbf{i_2}}\right|_{\mathbf{i_1}=0}$$

For these parameters, we apply current source $\mathbf{i_2}$ to the output port, and the input port is open-circuited ($\mathbf{i_1}$ is set to zero).

z_{12} **and** z_{22}: Since $\mathbf{i_1} = 0$, the dependent source is also zero, and

$$\mathbf{v_1} = \mathbf{v_2} = \mathbf{i_2}(200 \text{ k}\Omega)$$

Therefore

$$z_{12} = \frac{\mathbf{v_1}}{\mathbf{i_2}} = 200 \text{ k}\Omega \qquad \text{and} \qquad z_{22} = \frac{\mathbf{v_2}}{\mathbf{i_2}} = 200 \text{ k}\Omega$$

The final z-parameter equations for the network are

$$\mathbf{v_1} = 1.02 \times 10^7 \mathbf{i_1} + 2.00 \times 10^5 \mathbf{i_2}$$

$$\mathbf{v_2} = 1.02 \times 10^7 \mathbf{i_1} + 2.00 \times 10^5 \mathbf{i_2}$$

Check of Results: This result appears to be unusual since both equations are the same, and $\mathbf{v_2} = \mathbf{v_1}$. Observe that the determinant of the system is zero! The equations are not independent and results from a loss of significant digits in the answers. If we look back at the derivations and keep all the significant digits, we find that

$$\mathbf{v_1} = 1.022 \times 10^7 \mathbf{i_1} + 2.000 \times 10^5 \mathbf{i_2}$$

$$\mathbf{v_2} = 1.020 \times 10^7 \mathbf{i_1} + 2.000 \times 10^5 \mathbf{i_2}$$

Discussion: The determinant of this system of equations is 4.0×10^9, which is small relative to each of the cross-product terms. The z-parameters are not a good parameter set to use to model this network. One must be very careful not to lose significance in the calculations. Again we see that the value of the forward transfer parameter is much larger than that of the reverse transfer parameter $- z_{21} \gg z_{12}$.

Computer-Aided Analysis: Numerical values for two-port parameters are found using the transfer function (TF) capability of SPICE. In order to find the z-parameters for the circuit in this example, we drive the network with current source I1 at the input and current source I2 at the output, as in the figure here. These choices correspond to the boundary conditions in the definitions

of the g-parameters. Zero-value voltage-source VAM_I1 functions as an ammeter to sense the controlling current for CCCS F1. Note that both independent sources are assigned zero values. The TF analysis calculates how variables change in response to changes in an independent source, and a starting point of zero is therefore valid. The zero-value sources directly satisfy the boundary conditions required to calculate the z-parameters.
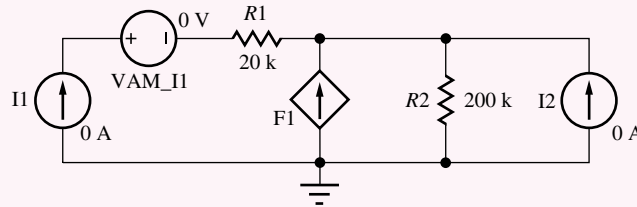
Two TF analyses are used—one to find z_{11} and z_{21} and a second to find z_{12} and z_{22}. The first analysis requests calculation of the transfer function from current source I1 to the voltage at the output node across source I2. SPICE will produce values for the transfer function, the input resistance presented to I1 at the input node, and the resistance presented to I2 at the output node. The SPICE results are transfer function $= 10.20 \times 10^6$, input resistance $= 10.22$ MΩ, and output resistance $= 200$ kΩ. Parameter z_{11} equals the input resistance, and parameter z_{21} equals the TF value.

The second analysis requests the transfer function from source I2 to the voltage appearing across I1 (the input node of the two-port). The results from SPICE are transfer function $= 2.00 \times 10^5$, input resistance $= 200$ kΩ, and output resistance $= 10.22$ MΩ. Reverse transfer parameter z_{12} is equal to the transfer function value, and parameter z_{22} is equal to the 200-kΩ resistance presented to source I2, which is the "input resistance" in this calculation. All parameters agree with our hand calculations. In this particular case, it is interesting to note that each transfer function analysis yields three of the z-parameter values.

EXERCISE: Find the *z*-parameters for the circuit in Ex. 10.5 if the 200-kΩ resistor is replaced with a 50-kΩ resistor, and the dependent source is changed to $75i_1$.

ANSWERS: 3.82 MΩ; 50.0 kΩ; 3.80 MΩ; 50.0 kΩ; Note the nearly identical equations that result from these parameters.

EXERCISE: Confirm your calculations in the previous exercise using SPICE.

EXERCISE: Find the voltage gain $A_v = v_o/v_s$ for the network in Ex. 10.3 assuming that the two-port can be described by the *z*-parameters found in Ex. 10.5. (Be careful not to lose significant digits in the calculations.)

ANSWER: 0.670