10.5.2 The Hybrid or $h$-parameters

The $h$-parameter description is also widely used in electronic circuits and is one convenient model for a current amplifier:

\[
\begin{align*}
\text{v}_1 &= h_{11}\text{i}_1 + h_{12}\text{v}_2 \\
\text{i}_2 &= h_{21}\text{i}_1 + h_{22}\text{v}_2
\end{align*}
\]

Figure 10.7 is the network representation of these equations.

\[
\begin{align*}
\text{v}_1 &= h_{11}\text{i}_1 + h_{12}\text{v}_2 \\
\text{i}_2 &= h_{21}\text{i}_1 + h_{22}\text{v}_2
\end{align*}
\]

As with the $g$-parameters, the $h$-parameters are determined from a given network using a combination of open- and short-circuit measurement conditions:

\[
\begin{align*}
h_{11} &= \left. \frac{\text{v}_1}{\text{i}_1} \right|_{\text{v}_2 = 0} = \text{short-circuit input resistance} \\
h_{12} &= \left. \frac{\text{v}_1}{\text{v}_2} \right|_{\text{i}_1 = 0} = \text{reverse open-circuit voltage gain} \\
h_{21} &= \left. \frac{\text{i}_2}{\text{i}_1} \right|_{\text{v}_2 = 0} = \text{forward short-circuit current gain} \\
h_{22} &= \left. \frac{\text{i}_2}{\text{v}_2} \right|_{\text{i}_1 = 0} = \text{open-circuit output conductance}
\end{align*}
\]

EXAMPLE 10.2 Finding a Set of $h$-parameters

This example provides an example of calculating the $h$-parameters of a network containing a controlled source.

**PROBLEM** Find the $h$-parameters for the circuit in Ex. 10.1.

**SOLUTION** Known Information and Given Data: Circuit given in Ex. 10.1 including specified element values; $h$-parameter definitions in Eq. (10.19)

Unknowns: Values of the four $h$-parameters
Approach: Apply the boundary conditions specified for each \( h \)-parameter, and then use circuit analysis to find the four parameter values. Note that each set of boundary conditions applies to two parameters.

Assumptions: None

Analysis — \( h_{11} \) and \( h_{21} \): Looking at the definitions of the \( g \)-parameters, we see that \( h_{11} \) and \( h_{21} \) use the same boundary condition.

\[
h_{11} = \frac{v_1}{i_1 \mid v_2 = 0} \quad \text{and} \quad h_{21} = \frac{i_2}{i_1 \mid v_2 = 0}
\]

Current source \( i_1 \) is applied to the input port, and the output port is short circuited (i.e., \( v_2 \) is set to zero) as shown in the figure here.

\( h_{11} \): Since \( v_2 = 0 \), input voltage \( v_1 \) is equal to the voltage across the 20-k\( \Omega \) resistor:

\[
v_1 = (20 \text{ k}\Omega)i_1 \quad \text{and} \quad h_{11} = \frac{v_1}{i_1} = 2 \times 10^4 \text{ } \Omega
\]

\( h_{21} \): To find \( i_2 \), we sum the currents at the output node, realizing that the current in the 200-k\( \Omega \) resistor is forced to be zero by the short circuit across the output terminals

\[
i_1 + 50i_1 + i_2 = 0 \quad \text{or} \quad i_2 = -51i_1 \quad \text{and} \quad h_{21} = \frac{i_2}{i_1} = -51
\]

\( h_{12} \) and \( h_{22} \): Looking again at the definitions of the \( h \)-parameters, we see that \( h_{12} \) and \( h_{22} \) use the same boundary condition.

\[
h_{12} = \frac{v_1}{v_2 \mid i_1 = 0} \quad \text{and} \quad h_{22} = \frac{i_2}{v_2 \mid i_1 = 0}
\]

For these parameters, we apply a voltage source \( v_2 \) to the output port, and the input port is open-circuited (i.e., \( i_1 \) is set to zero).

\( h_{22} \) and \( h_{12} \): With \( i_1 = 0 \), the current in the dependent current source is also zero. Therefore,

\[
i_2 = \frac{v_2}{200 \text{ k}\Omega} \quad \text{and} \quad h_{22} = \frac{i_2}{v_2} = \frac{1}{200 \text{ k}\Omega} = 5 \times 10^{-6} \text{ S}
\]
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and

\[ v_1 = (20 \text{k}\Omega) i_1 + v_2 = v_2 \]

and

\[ h_{12} = \frac{v_1}{v_2} = +1 \]

since the boundary sets \( i_1 = 0 \).

The final \( h \)-parameter equations for the network are

\[ v_1 = 2 \times 10^4 i_1 + v_2 \]

\[ i_2 = -51 i_1 + 5 \times 10^{-6} v_2 \]

**Check of Results:** The results are confirmed below using SPICE.

**Discussion:** Here again we see that the value of the forward transfer parameter is much larger than that of the reverse transfer parameter \(-h_{21} \gg h_{12}\). We will make use of this observation when we study feedback in Chapter 18.

**Computer-Aided Analysis:** Numerical values for two-port parameters are easily found using the transfer function (TF) analysis capability of SPICE. In order to find the \( h \)-parameters for the circuit in Ex. 10.2, we drive the network with current source \( I_1 \) at the input and voltage source \( V_2 \) at the output as in the figure here. These choices correspond to the boundary conditions in the definitions of the \( g \)-parameters. Zero-value voltage-source \( V_{AM} \) acts as an ammeter and senses the controlling current for CCCS \( F_1 \). Note that both independent sources are assigned zero values.

The TF analysis calculates how variables change in response to changes in an independent source, and a starting point of zero is therefore valid. The zero-value sources directly satisfy the boundary conditions required to calculate the \( h \)-parameters.

Two TF analyses are used—one to find \( h_{11} \) and \( h_{21} \) and a second to find \( h_{12} \) and \( h_{22} \).

The first analysis requests calculation of the transfer function from current source \( I_1 \) to (the current in) source \( V_2 \), and SPICE will produce values for the transfer function and the input resistance presented to \( I_1 \). The SPICE results are transfer function \( = 51.0 \) and input resistance \( = 20.0 \text{k}\Omega \). Note that parameter \( h_{21} \) is the negative of the TF value. The sign difference arises from the convention assumed by SPICE in which positive current is directed downward through source \( V_2 \). Parameter \( h_{11} \) equals the input resistance and is in agreement with our hand calculations.

The second analysis requests the transfer function from source \( V_2 \) to the voltage appearing across \( I_1 \) (the input node of the two-port). The results are transfer function \( = 1.00 \), input resistance \( = 200 \text{k}\Omega \), and output resistance \( = 20.0 \text{k}\Omega \). Reverse voltage-gain parameter \( h_{12} \) is equal to the transfer function value. Conductance parameter \( h_{22} \) is equal to the reciprocal of the 200-k\( \Omega \) resistance presented to source \( I_2 \), which is the “input resistance” in this calculation. We find precise in agreement with our hand calculations.
**Exercise:** Find the $h$-parameters for the circuit in Ex. 10.2 if the 200-kΩ resistor is replaced with a 50-kΩ resistor, and the dependent source is changed to $75i_1$.

**Answers:** $2.00 \times 10^4$ Ω; $-76.0$; $+1.00$; $2.00 \times 10^{-5}$ S

**Exercise:** Confirm your calculations in the previous exercise using SPICE.

**Example 10.3**

**Network Analysis Using the $h$-Parameters**

Now we will use the $h$-parameters to find the voltage gain of a network with a source and load attached.

**Problem**

Find the voltage gain $A_v = v_o/v_s$ for the network below with $R_S = 5$ kΩ and $R_L = 1$ kΩ. Assume the two-port can be described by the $h$-parameters found in Ex. 10.2.

**Solution**

**Known Information and Given Data:** Circuit above with $R_S = 5$ kΩ and $R_L = 1$ kΩ; The two-port is described by

$$v_1 = 2 \times 10^4 i_1 + v_2$$

$$i_2 = -51i_1 + 5 \times 10^{-6} v_2$$

**Unknowns:** Voltage gain $A_v = v_o/v_s$

**Approach:** Relate $v_o$ to $v_1$ and $i_1$ and $v_2$ to $i_2$. Substitute the relations into the two-port equations and solve for the voltage ratio $v_o/v_s$.

**Assumptions:** None

**Analysis:** Let us solve the problem symbolically before substituting the numbers into the equations. The $h$-parameter description for the two-port is

$$v_1 = h_{11} i_1 + h_{12} v_2$$

$$i_2 = h_{21} i_1 + h_{22} v_2$$

At the input port, we have to $v_1 = v_s - i_1 R_S$, and at the output port, $i_2 = -v_2/R_S$. Substituting these relationships into the $h$-parameter equations and collecting terms yields

$$v_s = (h_{11} + R_S) i_1 + h_{12} v_2$$

$$0 = h_{21} i_1 + \left(h_{22} + \frac{1}{R_L}\right) v_2$$
Using the second equation to eliminate $i_1$ in the first gives

$$v_s = -\frac{(h_{11} + R_S) \left(h_{22} + \frac{1}{R_L}\right)}{h_{21}} v_2 + h_{12} v_2$$

and the voltage gain is given by

$$A_v = \frac{v_2}{v_1} = \frac{1}{-\frac{(h_{11} + R_S)}{h_{21}} \left(h_{22} + \frac{1}{R_L}\right) + h_{12}}$$

Substituting the numerical values into the voltage gain expression gives a gain of

$$A_v = 0.670$$

**Check of Results:** As a check we can also find the voltage gain directly from the original circuit (which is left as an upcoming exercise). This direct analysis yields the same result, so the answer is correct.

**Computer-Aided Analysis:** The voltage gain can be found using the transfer function (TF) capability of SPICE. The two-port is modeled in SPICE by its equivalent circuit from Fig. 10.7, as shown in the figure here. Zero-value current source IVM_H12 functions as a voltmeter in parallel with the output and provides the controlling voltage for VCVS EH12. In this circuit, CCCS FH21 depends on the current in source EH12. A zero value input source is used for VS, as discussed in the previous two examples. The TF analysis calculates the transfer function from source VS to the output node voltage appearing across resistor RL.

The SPICE results are transfer function $= 0.670$, input resistance $= 75.7 \, \text{k\Omega}$, and output resistance $= 328 \, \Omega$.

**Exercise:** Find the voltage gain of the circuit in Ex. 10.3 by direct analysis rather than using the two-port equations.

**Answer:** 0.670

**Exercise:** Find the voltage gain $A_v = v_o/v_s$ for the network in Ex. 10.3 if resistor $R_L$ is increased to 5 kΩ.

**Answer:** 0.909
**Exercise:** Use the two-port equations to verify the input and output resistance values calculated by SPICE.

**Answers:** 75.8 kΩ; 328 Ω

### 10.5.3 The Admittance or \( y \)-parameters

The admittance, or \( y \)-parameter, description will be used during the analysis of feedback amplifiers in Chapter 18,

\[
\begin{align*}
i_1 &= y_{11} v_1 + y_{12} v_2 \\
i_2 &= y_{21} v_1 + y_{22} v_2
\end{align*}
\]

(10.20)

Figure 10.8 is a network representation of these equations.

![Figure 10.8 Two-port \( y \)-parameter representation.](image)

The \( y \)-parameters are often referred to as the short-circuit parameters because they are determined from a given network using only short-circuit terminations:

\[
\begin{align*}
ym_{11} &= \left. \frac{i_1}{v_1} \right|_{v_2=0} = \text{short-circuit input conductance} \\
ym_{12} &= \left. \frac{i_1}{v_2} \right|_{v_1=0} = \text{reverse short-circuit transconductance} \\
ym_{21} &= \left. \frac{i_2}{v_1} \right|_{v_2=0} = \text{forward short-circuit transconductance} \\
ym_{22} &= \left. \frac{i_2}{v_2} \right|_{v_1=0} = \text{short-circuit output conductance}
\end{align*}
\]

(10.21)

### Example 10.4 \( y \)-Parameter Analysis

This example provides an example of calculating the \( y \)-parameters of a network containing a controlled source.

**Problem** Find the \( y \)-parameters for the circuit in Ex. 10.1

**Solution** **Known Information and Given Data:** Circuit given in Ex. 10.1 including specified element values; \( y \)-parameter definitions in Eq. (10.21)

**Unknowns:** Values of the four \( y \)-parameters
**Approach:** Apply the boundary conditions specified for each $h$-parameter, and then use circuit analysis to find the four parameter values. Note that each set of boundary conditions applies to two parameters.

**Assumptions:** None

**Analysis — $y_{11}$ and $y_{21}$:** Looking at the definitions of the $y$-parameters, we see that $y_{11}$ and $y_{21}$ use the same boundary condition.

\[
y_{11} = \left. \frac{i_1}{v_1} \right|_{v_2=0} \quad \text{and} \quad y_{21} = \left. \frac{i_2}{v_1} \right|_{v_2=0}
\]

Voltage source $v_1$ is applied to the input port, and the output port is short-circuited ($v_2$ is set to zero), as shown in the figure here.

![Circuit Diagram](image)

$y_{11}$: Since $v_2 = 0$, input voltage $v_1$ appears directly across the 20-kΩ resistor:

\[
i_1 = \frac{v_1}{20 \text{ kΩ}} \quad \text{and} \quad y_{11} = \left. \frac{i_1}{v_1} \right|_{v_2=0} = \frac{1}{20 \text{ kΩ}} = 5 \times 10^{-5} \text{ S}
\]

$y_{21}$: To find $i_2$, we sum the currents at the output node, realizing that the current in the 200-kΩ resistor is forced to be zero by the short circuit across the output terminals:

\[
i_1 + 50i_1 + i_2 = 0 \quad \text{or} \quad i_2 = -51i_1 \quad \text{with} \quad i_1 = \frac{v_1}{20 \text{ kΩ}}
\]

\[
y_{21} = \left. \frac{i_2}{v_1} \right|_{v_1=0} = -\frac{51}{20 \text{ kΩ}} = -\frac{1}{392 \text{ Ω}} = -2.55 \times 10^{-3} \text{ S}
\]

**$y_{12}$ and $y_{22}$:** Looking again at the definitions of the $y$-parameters, we see that $y_{12}$ and $y_{22}$ use the same boundary condition.

\[
y_{12} = \left. \frac{i_1}{v_2} \right|_{v_1=0} \quad \text{and} \quad y_{22} = \left. \frac{i_2}{v_2} \right|_{v_1=0}
\]

For these parameters, we apply a voltage source $v_2$ to the output port, and the input port is short-circuited ($v_1$ is set to zero).

![Circuit Diagram](image)

$y_{12}$: With $v_1 = 0$, voltage $v_2$ appears directly across the 20-kΩ resistor, and

\[
i_1 = -\frac{v_2}{20 \text{ kΩ}} \quad \text{and} \quad y_{12} = \left. \frac{i_1}{v_2} \right|_{v_1=0} = -\frac{1}{20 \text{ kΩ}} = -5 \times 10^{-5} \text{ S}
\]

\[
y_{22} = \left. \frac{i_2}{v_2} \right|_{v_1=0}
\]

\[
y_{22} = -\frac{1}{392 \text{ Ω}} = -2.55 \times 10^{-3} \text{ S}
\]
$y_{22}$: To find $i_2$, we can apply KCL at the output node.

$$i_2 = \frac{v_2}{200 \text{k}\Omega} - 50i_1 + \frac{v_2}{20 \text{k}\Omega}$$

and we know $i_1 = -\frac{v_2}{20 \text{k}\Omega}$

Combining these equations gives the short-circuit output conductance $y_{22}$:

$$i_2 = \frac{v_2}{200 \text{k}\Omega} + 51 \frac{v_2}{20 \text{k}\Omega}$$

and

$$y_{22} = \frac{i_1}{v_2} = \frac{1}{391 \text{k}\Omega} = 2.56 \times 10^{-3} \text{S}$$

The final $y$-parameter equations for the network are

$$i_1 = 5 \times 10^{-5}v_1 - 5 \times 10^{-5}v_2$$

$$i_2 = -2.55 \times 10^{-3}v_1 + 2.56 \times 10^{-3}v_2$$

**Check of Results:** This result appears to be unusual since both equations are nearly the same; i.e., $i_2 \approx -i_1$. We can also observe that the determinant of the system is very close to zero. This is another indication of potential problems using these equations. If we look back at the derivations and keep all the significant digits, we find that

$$i_1 = 5.000 \times 10^{-5}v_1 - 5.000 \times 10^{-5}v_2$$

$$i_2 = -2.550 \times 10^{-3}v_1 + 2.555 \times 10^{-3}v_2$$

**Discussion:** The determinant of this system of equations is $2.50 \times 10^{-10}$, which is much smaller than any of the individual coefficients. This result indicates that analyses using this equation set may be highly sensitive to round-off errors, and therefore the $y$-parameters are probably not a good parameter set to use to model this network. Note that we still see that the value of the forward transfer parameter is much larger than that of the reverse transfer parameter $-y_{21} \gg y_{12}$.

**Computer-Aided Analysis:** Numerical values for two-port parameters can be found using the transfer function (TF) analysis of SPICE. In order to find the $y$-parameters for the circuit in this exercise, we drive the network with voltage source $V_1$ at the input and voltage source $V_2$ at the output, as in the figure here. These choices correspond to the boundary conditions in the definitions of the $y$-parameters. Zero-value voltage-source VAM_I1 is added to the circuit and functions as an ammeter sensing the controlling current for CCCS F1.

Note that both independent sources are assigned zero values. The TF analysis calculates how variables change in response to changes in an independent source, and a starting point of zero is therefore valid. The zero-value sources directly satisfy the boundary conditions required to calculate the $y$-parameters.
Two TF analyses are used — the first finds $y_{11}$ and $y_{21}$, and the second $y_{12}$ and $y_{22}$. The first analysis requests calculation of the transfer function from voltage source $V_1$ to (the current in) source $V_2$. SPICE will produce values for the transfer function and the input resistance presented to source $V_1$, and the results are transfer function $= 2.550 \times 10^{-3}$, and input resistance $= 20.0 \, \text{k}\Omega$. Parameter $y_{11}$ is the input conductance and equals the reciprocal of the $20.0\, \text{k}\Omega$ value, and parameter $y_{21}$ is the negative of the TF value. The sign difference arises from the convention assumed by SPICE in which positive current is directed downward through source $V_2$.

The second analysis requests the transfer function from source $V_2$ to the current in voltage source $V_1$. The results from SPICE are transfer function $= 50.0 \times 10^{-6}$ and input resistance $= 391.4 \, \Omega$. Reverse transfer conductance $y_{12}$ is equal to the negative of the transfer function value. Conductance parameter $y_{22}$ is equal to the reciprocal of the resistance presented to source $V_2$, which is the “input resistance” in this calculation. Once again we find precise agreement with our hand calculations.

**Exercise:** Find the $y$-parameters for the circuit in Ex. 10.4 if the 200-k$\Omega$ resistor is replaced with a 50-k$\Omega$ resistor, and the dependent source is changed to $75i_1$.

**Answers:** $5.00 \times 10^{-5} \, \text{S}; 5.00 \times 10^{-5} \, \text{S}; 3.80 \times 10^{-3} \, \text{S}; 3.80 \times 10^{-3} \, \text{S}$; Here again we see a problem with the parameter set.

**Exercise:** Confirm your calculations in the previous exercise using SPICE.

### 10.5.4 The Impedance or $z$-parameters

The impedance, or $z$-parameters, will also be used for analysis of feedback amplifiers in Chapter 18,

\[
\begin{align*}
v_1 &= z_{11}i_1 + z_{12}i_2 \\
v_2 &= z_{21}i_1 + z_{22}i_2
\end{align*}
\]

(10.22)

Figure 10.9 is a network representation of Eq. (10.22). The $z$-parameters are determined from a given network using open-circuit measurement conditions and are often referred to as the

![Figure 10.9](image-url)
open-circuit parameters:

\[ z_{11} = \frac{v_1}{i_1} \bigg|_{i_2=0} = \text{open-circuit input resistance} \]

\[ z_{12} = \frac{v_1}{i_2} \bigg|_{i_1=0} = \text{reverse open-circuit transresistance} \]

\[ z_{21} = \frac{v_2}{i_1} \bigg|_{i_2=0} = \text{forward open-circuit transresistance} \]

\[ z_{22} = \frac{v_2}{i_2} \bigg|_{i_1=0} = \text{open-circuit output resistance} \]

**EXAMPLE 10.5  z-PARAMETER ANALYSIS**

This example provides an example of calculating the z-parameters of a network containing a controlled source.

**PROBLEM**
Find the z-parameters for the circuit in Ex. 10.1.

**SOLUTION**

**Known Information and Given Data:** Circuit given in Ex. 10.1 including specified element values; z-parameter definitions in Eq. (10.23)

**Unknowns:** Values of the four z-parameters

**Approach:** Apply the boundary conditions specified for each z-parameter, and then use circuit analysis to find the four parameter values. Note that each set of boundary conditions applies to two parameters.

**Assumptions:** None

**Analysis — \(z_{11}\) and \(z_{21}\):** Looking at the definitions of the z-parameters, we see that \(z_{11}\) and \(z_{21}\) use the same boundary condition.

\[ z_{11} = \frac{v_1}{i_1} \bigg|_{i_2=0} \quad \text{and} \quad z_{21} = \frac{v_2}{i_1} \bigg|_{i_2=0} \]

Current source \(i_1\) is applied to the input port, and the output port is open circuited (\(i_2\) is set to zero) as shown in the figure here.

\(z_{11}\): We can write a loop equation at the input port to find \(v_1\):

\[ v_1 = (20 \, \text{k}\Omega)i_1 + (i_1 + 50i_1)(200 \, \text{k}\Omega) \]

and

\[ z_{11} = \frac{v_1}{i_1} = 20 \, \text{k}\Omega + 51(200 \, \text{k}\Omega) = 10.2 \, \text{M}\Omega \]
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To find $v_2$, we apply KCL at the output node:

$$v_2 = (i_1 + 50i_1)(200 \text{ k\,\Omega}) = (51)(200 \text{ k\,\Omega})i_1 \quad \text{and} \quad z_{21} = \frac{v_2}{i_1} = 10.2 \text{ M\,\Omega}$$

$z_{12}$ and $z_{22}$: Looking again at the definitions of the $z$-parameters, we see that $z_{12}$ and $z_{22}$ use the same boundary condition.

$$z_{12} = \left. \frac{v_1}{i_2} \right|_{i_1=0} \quad \text{and} \quad z_{22} = \left. \frac{v_2}{i_2} \right|_{i_1=0}$$

For these parameters, we apply current source $i_2$ to the output port, and the input port is open-circuited ($i_1$ is set to zero).

$z_{12}$ and $z_{22}$: Since $i_1 = 0$, the dependent source is also zero, and

$$v_1 = v_2 = i_2(200 \text{ k\,\Omega})$$

Therefore

$$z_{12} = \frac{v_1}{i_2} = 200 \text{ k\,\Omega} \quad \text{and} \quad z_{22} = \frac{v_2}{i_2} = 200 \text{ k\,\Omega}$$

The final $z$-parameter equations for the network are

$$v_1 = 1.02 \times 10^7 i_1 + 2.00 \times 10^5 i_2$$

$$v_2 = 1.02 \times 10^7 i_1 + 2.00 \times 10^5 i_2$$

Check of Results: This result appears to be unusual since both equations are the same, and $v_2 = v_1$. Observe that the determinant of the system is zero! The equations are not independent and results from a loss of significant digits in the answers. If we look back at the derivations and keep all the significant digits, we find that

$$v_1 = 1.022 \times 10^7 i_1 + 2.000 \times 10^5 i_2$$

$$v_2 = 1.020 \times 10^7 i_1 + 2.000 \times 10^5 i_2$$

Discussion: The determinant of this system of equations is $4.0 \times 10^9$, which is small relative to each of the cross-product terms. The $z$-parameters are not a good parameter set to use to model this network. One must be very careful not to lose significance in the calculations. Again we see that the value of the forward transfer parameter is much larger than that of the reverse transfer parameter $- z_{21} \gg z_{12}$.

Computer-Aided Analysis: Numerical values for two-port parameters are found using the transfer function (TF) capability of SPICE. In order to find the $z$-parameters for the circuit in this example, we drive the network with current source $I_1$ at the input and current source $I_2$ at the output, as in the figure here. These choices correspond to the boundary conditions in the definitions.
of the \( g \)-parameters. Zero-value voltage-source VAM\(_{I1} \) functions as an ammeter to sense the controlling current for CCCS F1. Note that both independent sources are assigned zero values. The TF analysis calculates how variables change in response to changes in an independent source, and a starting point of zero is therefore valid. The zero-value sources directly satisfy the boundary conditions required to calculate the \( z \)-parameters.

Two TF analyses are used — one to find \( z_{11} \) and \( z_{21} \) and a second to find \( z_{12} \) and \( z_{22} \). The first analysis requests calculation of the transfer function from current source \( I_1 \) to the voltage at the output node across source \( I_2 \). SPICE will produce values for the transfer function, the input resistance presented to \( I_1 \) at the input node, and the resistance presented to \( I_2 \) at the output node. The SPICE results are transfer function \( = 10.20 \times 10^6 \), input resistance \( = 10.22 \text{ M} \Omega \), and output resistance \( = 200 \text{ k} \Omega \). Parameter \( z_{11} \) equals the input resistance, and parameter \( z_{21} \) equals the TF value.

The second analysis requests the transfer function from source \( I_2 \) to the voltage appearing across \( I_1 \) (the input node of the two-port). The results from SPICE are transfer function \( = 2.00 \times 10^5 \), input resistance \( = 200 \text{ k} \Omega \), and output resistance \( = 10.22 \text{ M} \Omega \). Reverse transfer parameter \( z_{12} \) is equal to the transfer function value, and parameter \( z_{22} \) is equal to the 200-k\( \Omega \) resistance presented to source \( I_2 \), which is the “input resistance” in this calculation. All parameters agree with our hand calculations. In this particular case, it is interesting to note that each transfer function analysis yields three of the \( z \)-parameter values.

**Exercise:** Find the \( z \)-parameters for the circuit in Ex. 10.5 if the 200-k\( \Omega \) resistor is replaced with a 50-k\( \Omega \) resistor, and the dependent source is changed to 75\( i \).

**Answers:** 3.82 M\( \Omega \); 50.0 k\( \Omega \); 3.80 M\( \Omega \); 50.0 k\( \Omega \); Note the nearly identical equations that result from these parameters.

**Exercise:** Confirm your calculations in the previous exercise using SPICE.

**Exercise:** Find the voltage gain \( A_v = v_o/v_s \) for the network in Ex. 10.3 assuming that the two-port can be described by the \( z \)-parameters found in Ex. 10.5. (Be careful not to lose significant digits in the calculations.)

**Answer:** 0.670