

Name Master

ECE 321 Electronics I  
Test #1 Winter 2008

Time allowed: 2 hours (including the first 5 minutes for exam review.... no writing.) Calculator permitted.

Closed book; one-side 8½x11 in "crib-sheet" permitted; (turn in the crib sheet with the test paper.)

Answer questions on these pages. If you need more space, use the back of the page.

There are eleven questions at 10 points each. Answer any ten or all eleven. Maximum score 100 points.

In all your answers, state all approximations clearly, and explain each step succinctly.

1 [2.19] InP is composed of equal atoms of In and P in a lattice similar to that of Si. If a Ge atom replaces an In atom in the lattice, Will it act as an acceptor or donor? Explain.

In is group III, Ge is group IV  
∴ Ge will provide one electron more than needed to replace the In bonding electrons. ∴ Donor

2. [2.47] A 5µm-long block of p-type Si has an acceptor doping profile given by:

$$N_A(x) = 10^{14} + 10^{18} \exp(-10^{14}x) \quad \text{where } x \text{ is in cm.}$$

(a) Show that the material must have a non-zero internal electric field, E.

(b) Find E at (i) x=0 and (ii) x=5µm.

Total hole current density  $J_p = q p p E - q D_p \frac{dp}{dx} = 0$  at equilibrium

$$\therefore q p p E = q D_p \frac{dp}{dx}$$

and equilibrium field  $E$  is established

$$E = \frac{D_p}{p p} \frac{1}{p} \frac{dp}{dx} = \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \quad \text{using Einstein's equation}$$

For  $p \approx N_A$   $\frac{dp}{dx} = \frac{dN_A}{dx} = 10^{18} (\exp(-10^{14}x)) (-10^{14}) = -10^{32} \exp(-10^{14}x)$

$$\text{so } E = \frac{-0.025 \times 10^{32} \exp(-10^{14}x)}{10^{14} + 10^{18} \exp(-10^{14}x)} = - \frac{1}{40(10^{-14} \exp(-10^{14}x) + 10^{18})} = \frac{2.5 \times 10^{16}}{1 + 10^{34} \exp(-10^{14}x)}$$

$$E(0) \approx -2.5 \times 10^{12} \text{ V/cm} \quad E(5\mu\text{m}) \approx -2.5 \times 10^{16} \text{ V/cm}$$

Note: There was a typo error on this problem — should have been  $\exp(-10^4 x)$ . Doesn't affect the method, but the results are not practical values.

3. Given the diode equation  $i_D = I_S [\exp(v_D/nV_T) - 1]$ , with reverse saturation current  $I_S$ , derive an expression for the temperature coefficient of  $v_D$  in terms of  $i_D$ ,  $V_T$ ,  $n$ , energy band-gap  $E_G$ , & temperature  $T$ .

$$i_D = I_S (\exp \frac{v_D}{nV_T} - 1) \approx I_S \exp \frac{v_D}{nV_T} \text{ for } v_D \gg nV_T (\sim 0.1V)$$

$$\text{so } v_D = nV_T \ln \frac{i_D}{I_S}$$

For constant  $i_D$  (defined by external circuit)

$$\begin{aligned} \frac{dv_D}{dT} &= nV_T \cdot \frac{I_S}{i_D} \cdot \frac{i_D}{I_S^2} (-1) \frac{dI_S}{dT} + n \frac{dV_T}{dT} \ln \frac{i_D}{I_S} \\ &= -nV_T \frac{1}{I_S} \frac{dI_S}{dT} + \frac{v_D}{V_T} \frac{dV_T}{dT} \end{aligned}$$

$$V_T = \frac{kT}{q} \text{ so } \frac{dV_T}{dT} = \frac{k}{q}$$

$$\begin{aligned} I_S &\propto n_i^2 \text{ since } I_S \text{ due to minority carriers} \\ &= BT^3 \exp -E_G/kT \text{ where } B = \text{constant} \\ \therefore \frac{dI_S}{dT} &= (BT^3 \exp -E_G/kT) \left( -\frac{E_G}{kT} - \left( \frac{1}{T} \right) \cdot 3 \right) \\ &\quad + 3BT^2 \exp -E_G/kT \\ &= I_S \cdot \frac{E_G}{kT} + \frac{3}{T} I_S \end{aligned}$$

$$\begin{aligned} \text{So } \frac{dv_D}{dT} &= \frac{v_D}{V_T} \frac{k}{q} - nV_T \left( \frac{3}{T} + \frac{E_G}{kT^2} \right) \\ &= \frac{v_D - 3nV_T - nE_G/q}{T} \end{aligned}$$

$$\frac{dv_D}{dT} = \frac{v_D - 3nV_T - nE_G/q}{T}$$

4. [3.39] A diode has  $w_{d0} = 1 \mu\text{m}$  and  $\phi_j = 0.6V$ . If the diode breaks down when the internal field exceeds  $300\text{kV/cm}$ , what is the diode breakdown voltage?

$$E_{\text{max}} = \frac{2(\phi_j + V_R)}{w_d} = \frac{2(\phi_j + V_R)}{w_{d0} \sqrt{1 + \frac{V_R}{\phi_j}}} = \frac{2\phi_j}{w_{d0}} \sqrt{1 + \frac{V_R}{\phi_j}}$$

$$\begin{aligned} \therefore V_R)_{\text{breakdown}} &= \phi_j \left[ \left( \frac{E_{\text{max}} w_{d0}}{2\phi_j} \right)^2 - 1 \right] = 0.6 \sqrt{\left[ \frac{(300 \times 10^3 \text{ V/cm} \times 10^{-4} \text{ cm})^2}{2 \times 0.6 \text{ V}} - 1 \right]} \\ &= 0.6 \times 624 \text{ V} = 374.4 \text{ V} \end{aligned}$$

$$V_{BR} = \underline{374.4 \text{ V}}$$

5. [3.46/47] A pn junction diode with  $N_A=10^{19}/\text{cm}^3$  on the p-side and  $N_D=10^{17}/\text{cm}^3$  on the n-side, has cross-sectional area  $10^4 \mu\text{m}^2$ .

- (a) What is the junction capacitance at a reverse bias of 5V?  
 (b) What is the diffusion capacitance at a forward bias current of 100mA for a 10ns diode transit time, and how much charge is stored in the diode?

$$(a) C_j = \frac{C_{j0}}{\sqrt{1 + V_R/\phi_j}}, \quad C_{j0} = \frac{\epsilon_s A}{W_{do}}, \quad W_{do} = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right) \phi_j}, \quad \phi_j = V_T \ln \frac{N_A N_D}{n_i^2}$$

$$\phi_j = 0.025 \ln \frac{10^{19} \times 10^{17}}{10^{20}} = 0.9786 \text{V}, \quad W_{do} = \left[ \frac{2 \times 11.7 \times 8.85 \times 10^{-14} (10^{-19} + 10^{-17}) 0.9786}{1.6 \times 10^{-19}} \right]^{1/2}$$

assuming  $V_T \approx -0.25 \text{V}$   
 $n_i^2 \approx 10^{20}/\text{cm}^6$

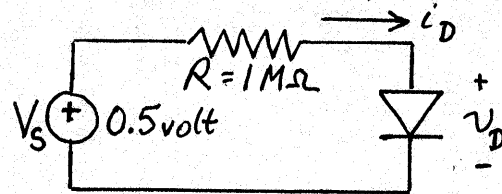
$$= 11.3 \times 10^{-6} \text{cm}$$

$$C_{j0} = \frac{(11.7 \times 8.85 \times 10^{-14}) \times (10^4 \times 10^{-8})}{11.3 \times 10^{-6}} = 9.16 \text{pF}, \quad C_j = \frac{9.16 \text{pF}}{(1 + 5/0.9786)^{1/2}} = 3.71 \text{pF}$$

$$(b) C_D = \frac{I_D \tau_T}{V_T} = \frac{0.1 \times 10^{-8}}{0.025} = 40 \times 10^{-9} \text{F} = 40 \text{nF}, \quad Q = I_D \tau_T = 0.1 \times 10^{-8} = 1 \text{nC}$$

(a)  $C_j = 3.71 \text{pF}$       (b)  $Q = 1 \text{nC}$        $C_D = 0.04 \mu\text{F}$

6. Find the diode operating point ( $i_D$ ,  $v_D$ ) by iteration, for a diode with  $I_S=10^{-12} \text{A}$  and  $n=2$ .



$$0.5 = 10^6 \cdot 10^{-12} [\exp(80v_D) - 1] + v_D$$

$$\therefore f = 0.5 - 10^{-6} (\exp(80v_D) - 1) - v_D$$

$$f' = -8 \times 10^{-5} \exp(80v_D) - 1$$

Assuming  $\frac{v_D}{nV_T} = \frac{v_D}{2 \cdot \frac{1}{40}} = 80v_D$

Note  $v_{D, n+1} = v_{D, n} - \frac{f_n}{f'_n}$

First guess  $v_D < 0.5 \text{V}$   
 say  $v_{D1} = 0.4 \text{V}$

$v_D$	$f_n$	$f'_n$	$f_n/f'_n$
1. 0.4V	$-7.9 \times 10^7$	$-6.3 \times 10^9$	$1.25 \times 10^{-2}$
2. 0.3875	$-2.9 \times 10^7$	$-2.3 \times 10^9$	$1.26 \times 10^{-2}$
3. 0.3749	$-1.06 \times 10^7$	$-8.48 \times 10^8$	$1.25 \times 10^{-2}$
4. 0.3624	$-3.9 \times 10^6$	$-3.12 \times 10^8$	$1.25 \times 10^{-2}$

Taking  $v_D = 0.15924 \text{V}$        $I_D = 10^{-12} (\exp(80v_D) - 1) = 0.34085 \mu\text{A}$

$i_D = 0.341 \mu\text{A}$

$v_D = 0.159 \text{V}$

Note:  $0.34085 \mu\text{A} \times 1 \text{M}\Omega = 0.34085 \text{V}$

$0.34085 + 0.15924 \text{V} = 0.50009 \text{V}$  indicating degree of accuracy

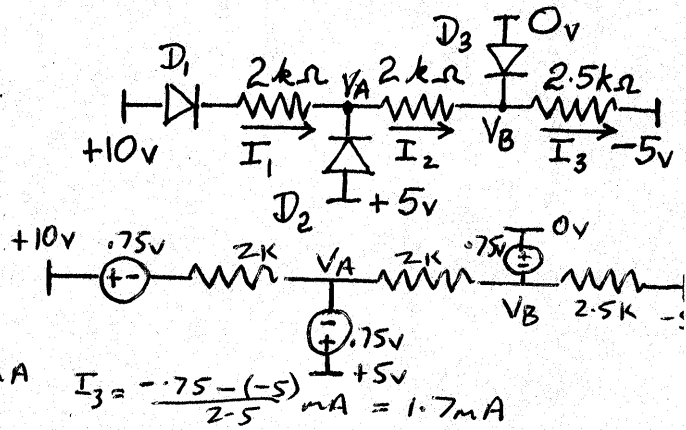
$$f_n = 0.5 - V_D - 10^{-6} (\exp(80V_D) - 1)$$

$$f_n' = -8 \times 10^{-5} \exp(80V_D) - 1$$

Q6 iteration table continued

	$V_D$	$f_n$	$f_n'$	$f_n/f_n'$
5	0.3499	$-1.435 \times 10^6$	$-1.15 \times 10^8$	$1.25 \times 10^{-2}$
6	0.3374	$-5.3 \times 10^5$	$-4.2 \times 10^7$	$1.26 \times 10^{-2}$
7	0.3248	$-1.93 \times 10^5$	$-1.54 \times 10^7$	$1.25 \times 10^{-2}$
8	0.3123	$-7.1 \times 10^4$	$-5.7 \times 10^6$	$1.25 \times 10^{-2}$
9	0.2998	$-2.6 \times 10^4$	$-2.1 \times 10^6$	$1.24 \times 10^{-2}$
10	0.2874	$-9.7 \times 10^3$	$-7.7 \times 10^5$	$1.26 \times 10^{-2}$
11	0.2748	$-3.53 \times 10^3$	$-2.82 \times 10^5$	$1.25 \times 10^{-2}$
12	0.2623	$-1.3 \times 10^3$	$-1.04 \times 10^5$	$1.25 \times 10^{-2}$
13	0.2498	$-4.8 \times 10^2$	$-3.8 \times 10^4$	$1.26 \times 10^{-2}$
14	0.2372	-174	$-1.4 \times 10^4$	$1.24 \times 10^{-2}$
15	0.2248	-64.37	$-5.2 \times 10^3$	$1.24 \times 10^{-2}$
16	0.2124	-24.25	$-1.9 \times 10^3$	$1.28 \times 10^{-2}$
17	0.1996	-8.907	-688.5	$1.29 \times 10^{-2}$
18	0.1867	-3.38	-246.3	0.0137
19	0.1730	-0.6978	-82.98	0.0084
20	0.1646	+0.28305	-42.86	-0.0066
21	0.1712	-0.55495	-72	+0.0077
22	0.1635	-0.14276	-39.34	+0.0036
23	0.1599	-0.0192	-29.75	+0.00065
24	0.15925	-0.00037	-28.3	+0.000013
25	0.15924	ie. close enough		

7. (a) Determine which diodes are "on" and which are "off" in the circuit shown, if  $V_{on}$  for a conducting diode is 0.75V.  
 (b) Calculate the voltages  $V_A$ ,  $V_B$ , and currents  $I_1$ ,  $I_2$ ,  $I_3$ .  
 (c) Verify your answer, and comment on your result.



1. Assume all diodes ON  $\rightarrow$

Hence  $V_A = 4.25V$ ,  $V_B = -0.75V$

$$I_1 = \frac{9.25 - 4.25}{2} \text{ mA} \quad I_2 = \frac{4.25 + 0.75}{2} \text{ mA} \quad I_3 = \frac{-0.75 - (-5)}{2.5} \text{ mA} = 1.7 \text{ mA}$$

Note  $I_3 < I_2$ , so  $D_3$  cannot be ON,  $I_1 = I_2 \therefore D_2$  just on the ON/OFF edge.

2. Assume  $D_1, D_2$  ON  $D_3$  OFF

$\therefore V_A = 4.25V$ ,  $I_1 = 2.5 \text{ mA}$

$I_2 = I_3 = \frac{4.25 - (-5)}{4.5k} \approx 2 \text{ mA}$

$\therefore D_2$  cannot be ON, Also  $V_B = -5 + \frac{2.5}{4.5} 9.25 = 0.14V$  i.e.  $D_1$  off

3. Assume  $D_1$  ON,  $D_2, D_3$  OFF

$I_1 = I_2 = I_3 = (14.25 / 6.5) \text{ mA} = 2.19 \text{ mA}$

$D_1$  ON  $D_2$  OFF  $D_3$  OFF

$I_1 = 2.19 \text{ mA}$   $I_2 = 2.19 \text{ mA}$   $I_3 = 2.19 \text{ mA}$

Note:  $D_2$  forward biased  $< V_{on}$

$V_A = 4.87V$   $V_B = 0.475V$

8. For the Zener regulator circuit shown,  
 (a) Express  $V_L$  in terms of the circuit parameters, and hence  
 (b) Identify expressions for the line and load regulation.



At the node:  $\frac{V_s - V_L}{R_s} = \frac{V_L - V_z}{r_z} + \frac{V_L}{R_L}$

Rearrange:  $V_L = \frac{V_s/R_s + V_z/r_z}{1/R_s + 1/r_z + 1/R_L} = \frac{V_s R_L r_z + V_z R_L R_s}{R_L R_s + R_s r_z + r_z R_L}$

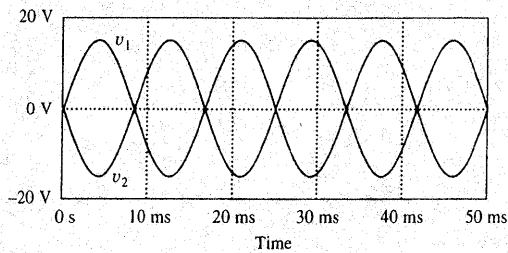
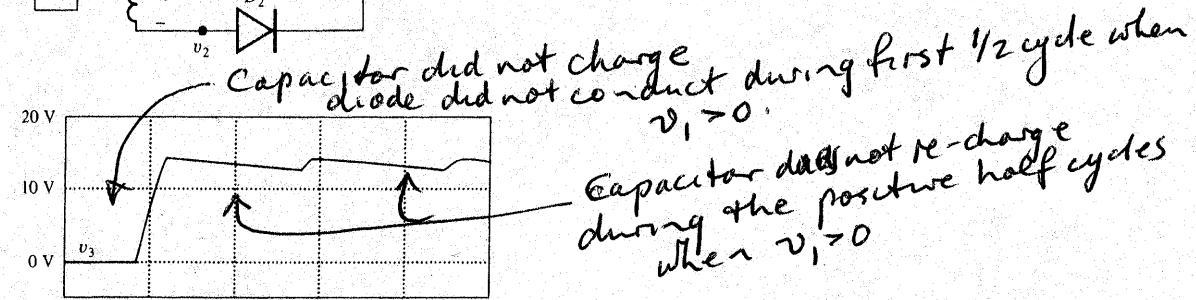
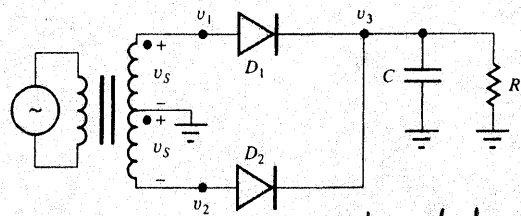
For load regulation,  $\frac{dV_L}{dI_L}$ , need  $I_L$  as the variable parameter, rather than  $R_L = \frac{V_L}{I_L}$

$\therefore \frac{V_s - V_L}{R_s} = \frac{V_L - V_z}{r_z} + I_L$   $V_L = \frac{V_s/R_s + V_z/r_z - I_L}{1/R_s + 1/r_z} = V_s \frac{r_z}{R_s + r_z} + V_z \frac{R_s}{R_s + r_z} - I_L \frac{R_s r_z}{R_s + r_z}$

(a)  $V_L =$  either of equations shown

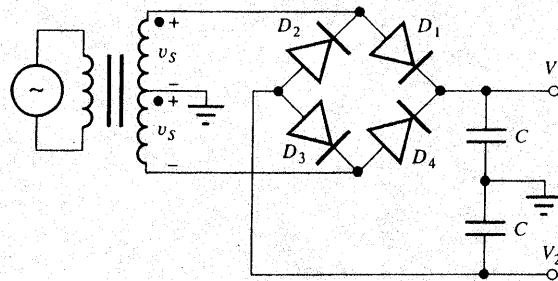
(b) Line regulation =  $\frac{dV_L}{dV_s} = \frac{r_z}{R_s + r_z}$  Load regulation =  $\frac{dV_L}{dI_L} = -\left(\frac{r_z R_s}{R_s + r_z}\right)$

9. [3.105] The full-wave rectifier circuit was designed to have a maximum ripple of approximately 1V, but is not operating properly. The measured waveforms at the 3 nodes identified are shown. What is wrong with the circuit?



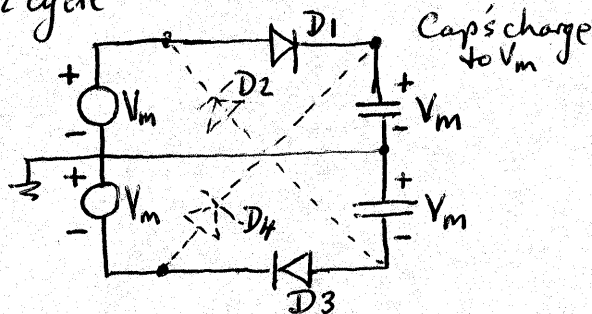
$\therefore D_1$  is open circuit  
(or connections to  $D_1$ , bad)

10. [3.109] What are the dc output voltages  $V_1$  and  $V_2$  for this circuit with  $v_s = 35\sin 377t$  and  $C = 10,000\mu\text{F}$ ?



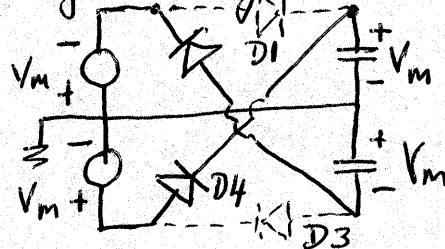
Assume ideal diodes.  
Consider each  $\frac{1}{2}$  cycle in turn,  
and re-draw showing the sources  
at maximum amplitudes  $\pm V_m$   
Note: No RC ripple — peak detectors

Positive  $\frac{1}{2}$  cycle



$V_1 = \underline{\quad + 35 \text{ V} \quad}$

Negative  $\frac{1}{2}$  cycle



$V_2 = \underline{\quad - 35 \text{ V} \quad}$

11. [3.107] A 3000V, 1A dc power supply is to be designed with a ripple voltage  $\leq 1\%$  at 60Hz input, using a full-wave bridge rectification circuit with capacitor filter.

- What is the size of the filter capacitor C?
- What is the minimum PIV rating for the diodes?
- What is the rms value of the transformer voltage?
- What is the peak value of the repetitive current in the diode?
- What is the surge current at  $t=0$ ?

(a)  $V_r \approx V_m \frac{T/2}{RC}$  assuming: ripple small (linear approx?)  
 conduction angle small  
 $\therefore C = \frac{V_m/R}{V_r} \cdot \frac{T}{2} = \frac{I_m}{V_r} \frac{T}{2}$  ripple small (DC voltage  $\approx$  peak voltage)  
 $= \frac{1A}{0.01 \times 3000} \frac{1}{120} = 277.8 \mu F$

(b) Bridge  $\rightarrow$  PIV =  $V_m = 3000V$

(c)  $V_{RMS} = \frac{V_m}{\sqrt{2}} = \frac{3000}{\sqrt{2}} = 2121.3V$

(d)  $I_p = I_{dc} \frac{T}{\Delta T}$  where  $\Delta T = \frac{1}{\omega N} \sqrt{\frac{2V_r}{V_p}}$   $\therefore I_p = I_{dc} T \omega \sqrt{V_p/2V_r}$   
 $= 1A \cdot 2\pi \cdot (100/2)^{1/2}$   
 (since  $2\pi f T = 2\pi$   
 and  $V_p/V_r = 100$ )  
 $= 44.42 A$

(e)  $I_{surge} = \omega C V_m = (2\pi \cdot 60) \cdot 277.8 \times 10^{-6} \times 3000 = 314.2 A$

(a) C = 277.8  $\mu F$       (b) PIV = 3000V      (c)  $V_{rms} =$  2121.3V  
 (d)  $I_p =$  44.42 A       $I_{sc} =$  314.2 A