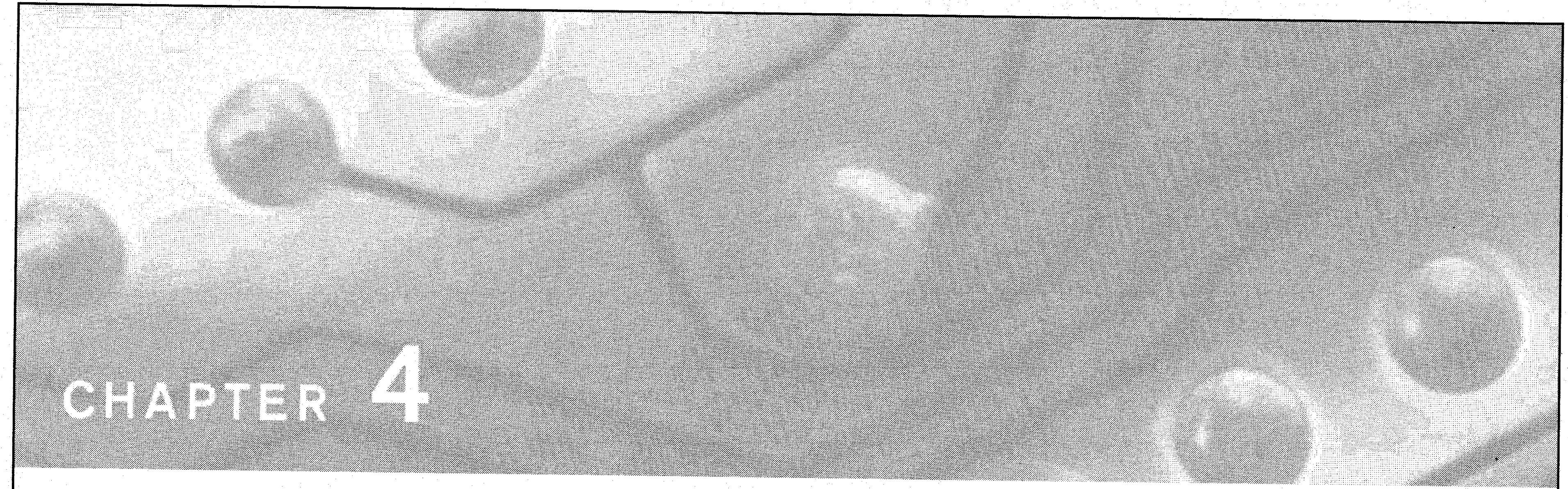
ECE321 ELECTRONICS I
FALL 2006

PROFESSOR JAMES E. MORRIS

Lecture 9

24^{th} October, 2006



MOS Field-Effect Transistors (MOSFETs)

By examples (7 exercises)

4.3 DC Circuits

4.4 MOSFET Amplifier & Switch

4.5 Biasing

Exercises

D4.10

D4.11

4.12

4.13

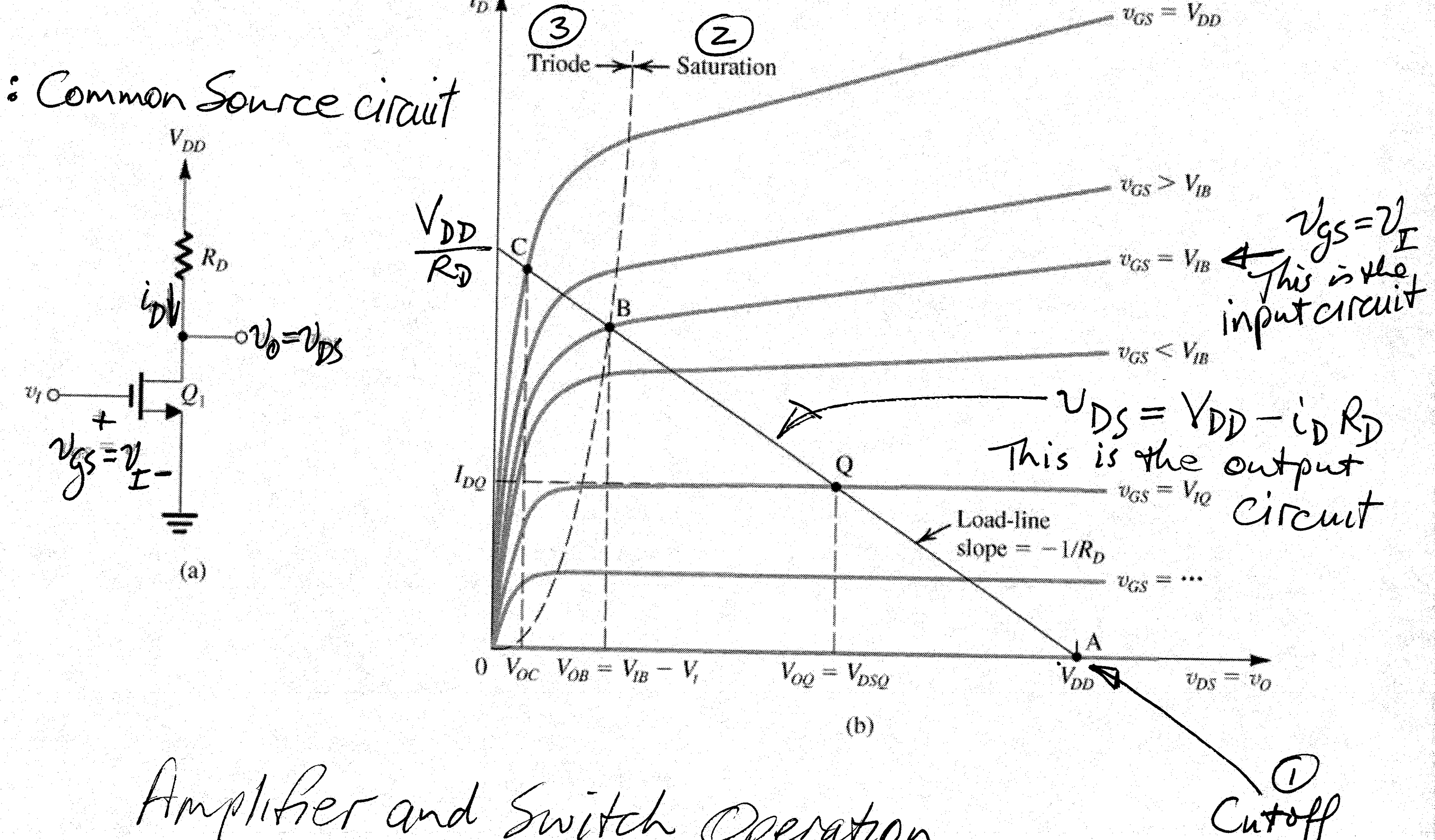
4.14

D4.15

4.16 ← & Example 4.7

LARGE SIGNAL OPERATION: TRANSFER CHARACTERISTIC

CS: Common Source circuit



Amplifier and Switch Operation

Figure 4.26 **(a)** Basic structure of the common-source amplifier. **(b)** Graphical construction to determine the transfer characteristic of the amplifier in (a).

Choice of R_D for maximum signal swing

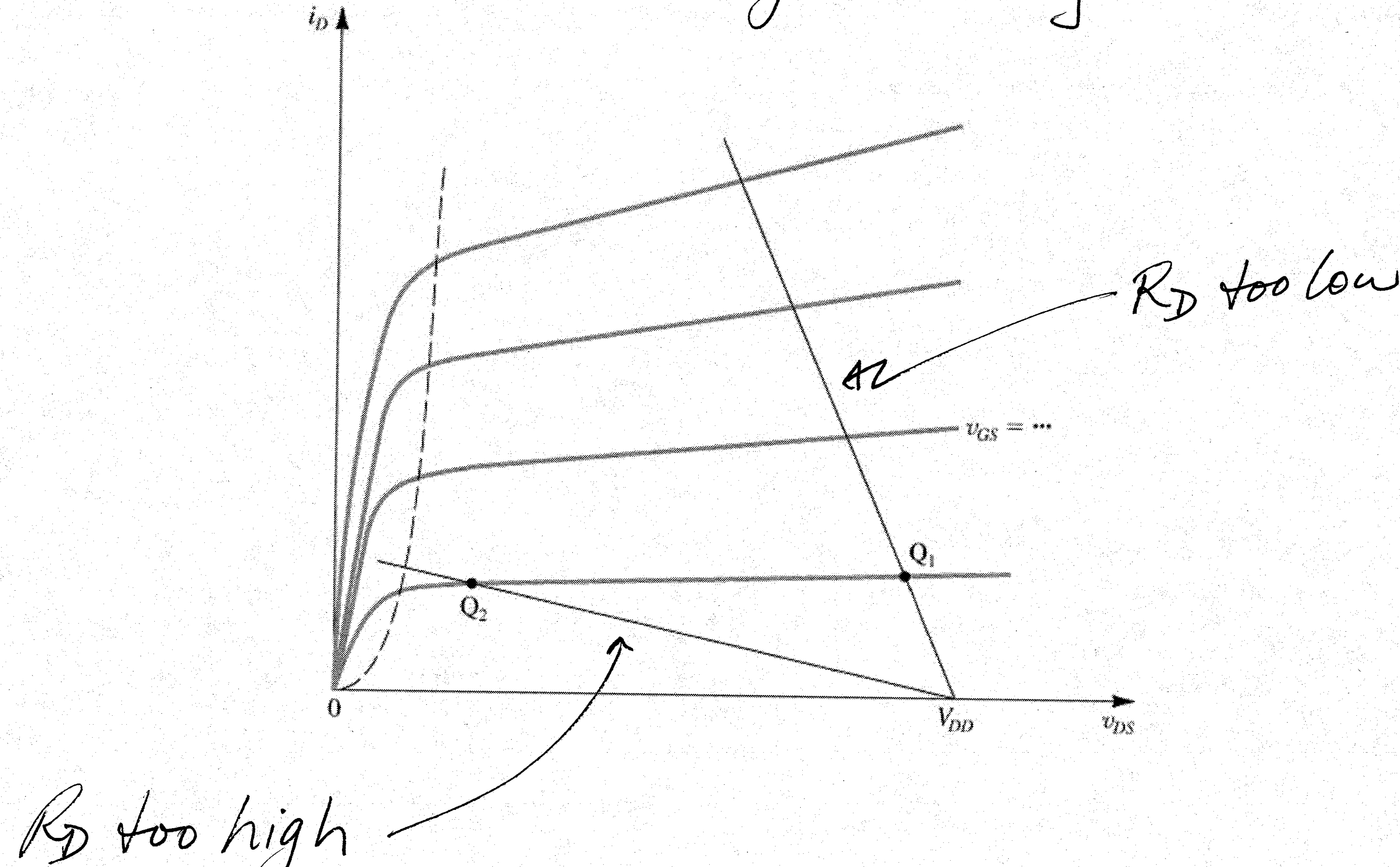
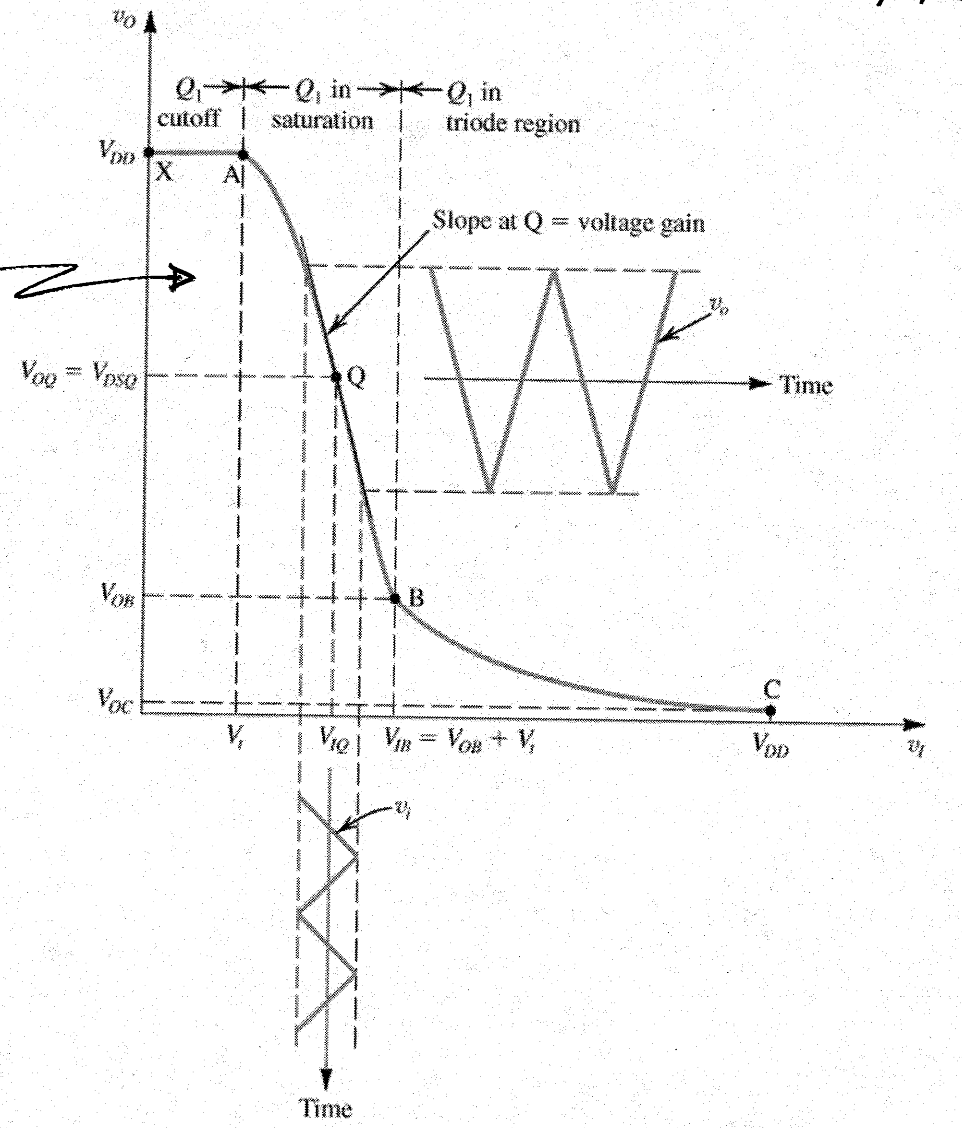


Figure 4.27 Two load lines and corresponding bias points. Bias point Q_1 does not leave sufficient room for positive signal swing at the drain (too close to V_{DD}). Bias point Q_2 is too close to the boundary of the triode region and might not allow for sufficient negative signal swing.

TRANSFER CHARACTERISTIC (Assume $\lambda = 0$)

① Cutoff
where
$v_I < V_t$

$I_D = 0$
& $v_O = V_{DD}$



(c)

Figure 4.26 *(Continued)* **(c)** Transfer characteristic showing operation as an amplifier biased at point Q.

② Saturation:

$$v_I > V_t \qquad v_O > v_I - V_t$$

$$v_{GS} > V_t \qquad v_{DS} > v_{GS} - V_t$$

$$i_D = \tfrac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS}-V_t)^2 = \tfrac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_I - V_t)^2$$

and $v_O = V_{DD} - i_D R_D$

$$\boxed{v_O = V_{DD} - \tfrac{1}{2}\mu_n C_{ox}\frac{W}{L}R_D(v_I - V_t)^2}$$ i.e. clearly non-linear

$$\left(A_v = \frac{dv_O}{dv_I} = -\mu C_{ox}\frac{W}{L}R_D(v_{IQ} - V_t) \quad \text{" " " "}\right)$$

Region's end point is where saturation → triode region

i.e. $v_{DS} < v_{GS} - V_t$ for triode

$$v_O < v_I - V_t \implies v_I > v_O + V_t$$

V_{OB} point defined by $v_O = V_{DD} - R_D \tfrac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_O)^2$ $\quad v_I - V_t$

$$\Rightarrow V_{DD} - \tfrac{1}{\lambda} v_O^2$$

$$\therefore v_{OB}^2 + \lambda v_{OB} - \lambda V_{DD} = 0 \implies v_{OB} = \frac{-\lambda \pm \sqrt{\lambda^2 + 4\lambda V_{DD}}}{2} = \frac{\lambda}{2}\left(\sqrt{1 + \frac{4V_{DD}}{\lambda}} - 1\right)$$

③ Triode

$$v_I > V_t \qquad v_O < v_I - V_t$$
$$v_{DS} < v_{GS} - V_t$$

$$i_D = \mu_n C_{ox} \frac{W}{L}\left((v_I - V_t) v_O - \tfrac{1}{2} v_O^2\right)$$

& $\quad v_O = V_{DD} - i_D R_D = V_{DD} - \mu_n C_{ox} \frac{W}{L} R_D\left((v_I - V_t) v_O - \frac{1}{2} v_O^2\right)$

so $\quad v_O^2 \left[\frac{1}{2}\mu_n C_{ox} \frac{W}{L} R_D\right] + v_O\left[1 + \mu_n C_{ox} \frac{W}{L} R_D (v_I - V_t)\right] - V_{DD} = 0$

Solve for $v_O(v_I) \longrightarrow$ Non-linear

For v_O small, ie. $v_O \ll v_I - V_t$, drop the v_O^2 term

$$v_O \approx V_{DD} - \mu_n C_{ox} \frac{W}{L} R_D (v_I - V_t) v_O$$
$$\Rightarrow \frac{V_{DD}}{1 + \mu_n C_{ox} \frac{W}{L} R_D (v_I - V_t)}$$

& for $r_{DS} = \dfrac{1}{\mu_n C_{ox} \frac{W}{L} (v_I - V_t)}$

$$v_O \longrightarrow \frac{V_{DD}}{1 + R_D/r_{DS}} = V_{DD} \frac{r_{DS}}{R_D + r_{DS}}$$

Example 4.8 Non-linearity

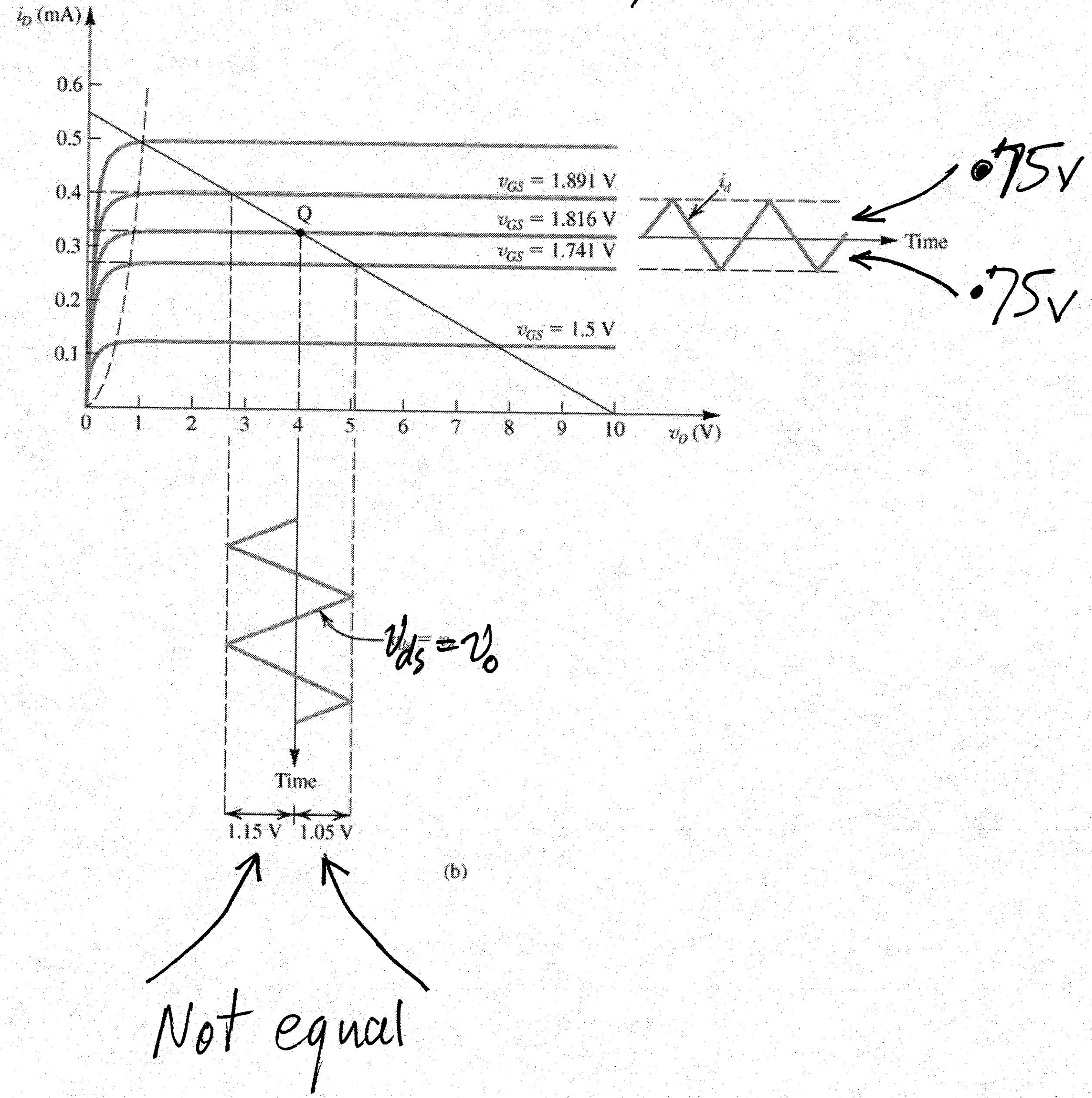


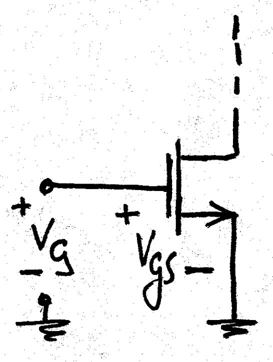
Figure 4.28 *(Continued)*

Exercise 4.17

Fixed Bias

Ex 4.19(a) $V_t = 1V$, $k_n' \frac{W}{L} = 1mA/V^2$

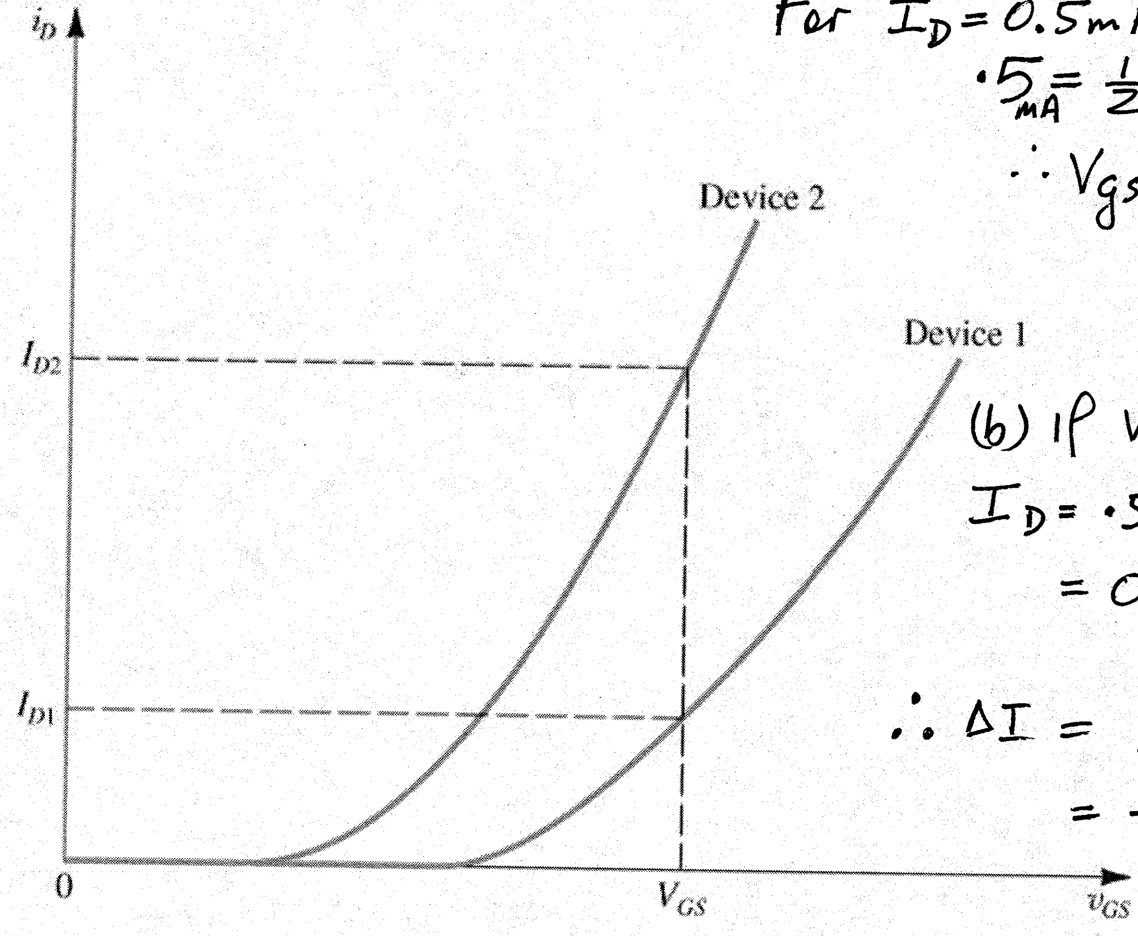
$\lambda = 0$, $V_{DD} = 15V$



For $I_D = 0.5mA$

$$.5_{mA} = \frac{1}{2} 1_{mA}(V_{gs}-1)^2$$

$$\therefore V_{gs} = \sqrt{1}+1 = 2V$$



(b) if $V_t \rightarrow 1.5V$

$$I_D = .5mA(2-1.5)^2$$

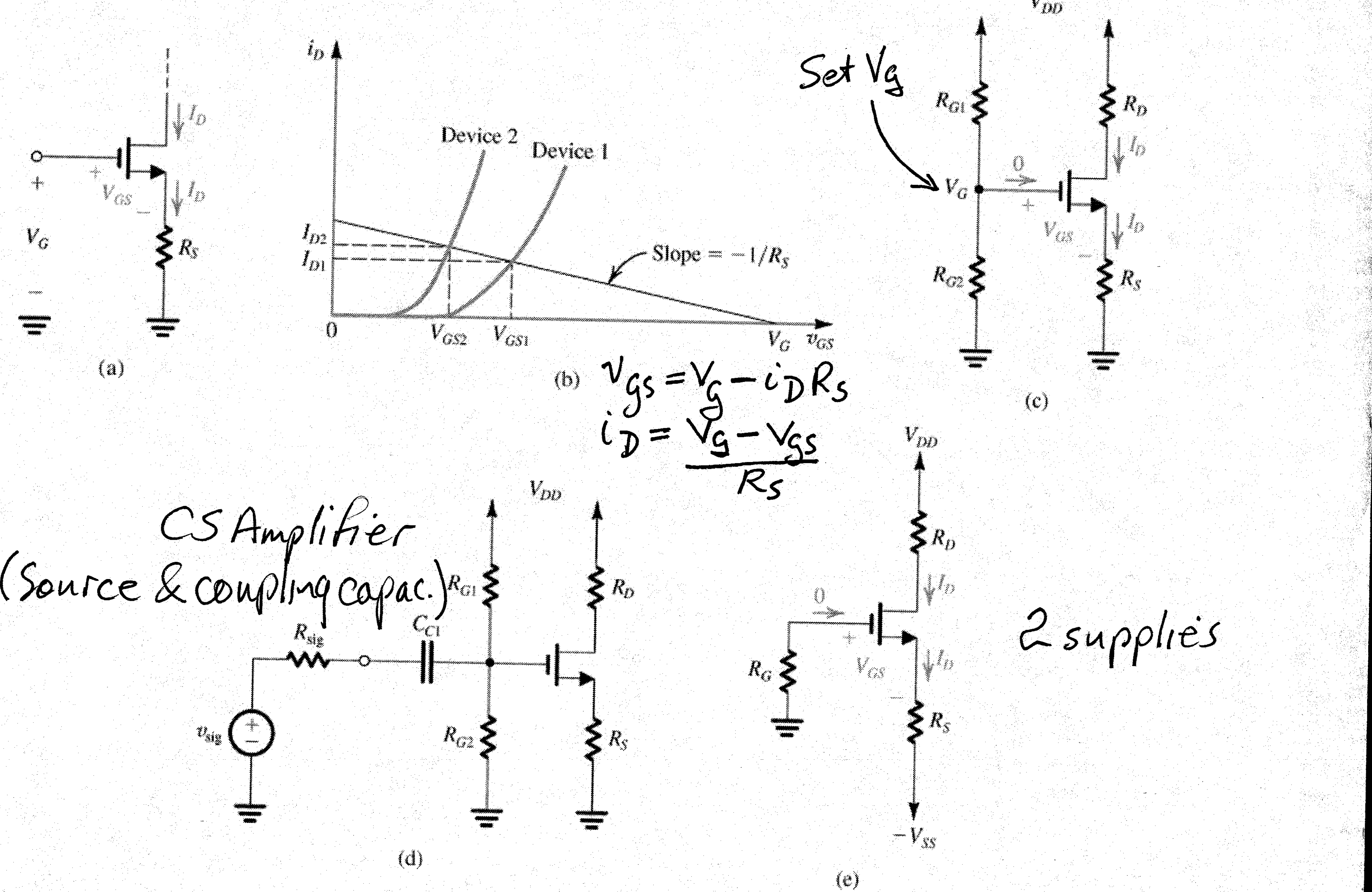
$$= 0.125mA$$

$$\therefore \Delta I = \frac{.125-.5}{.5}100\%$$

$$= -75\%$$

Figure 4.29 The use of fixed bias (constant V_{GS}) can result in a large variability in the value of I_D. Devices 1 and 2 represent extremes among units of the same type.

Self-Bias/Autobias/Classic Bias



Set V_g

$$v_{gs} = V_g - i_D R_s$$

$$i_D = \frac{V_g - V_{gs}}{R_s}$$

CS Amplifier
(Source & coupling capac.)

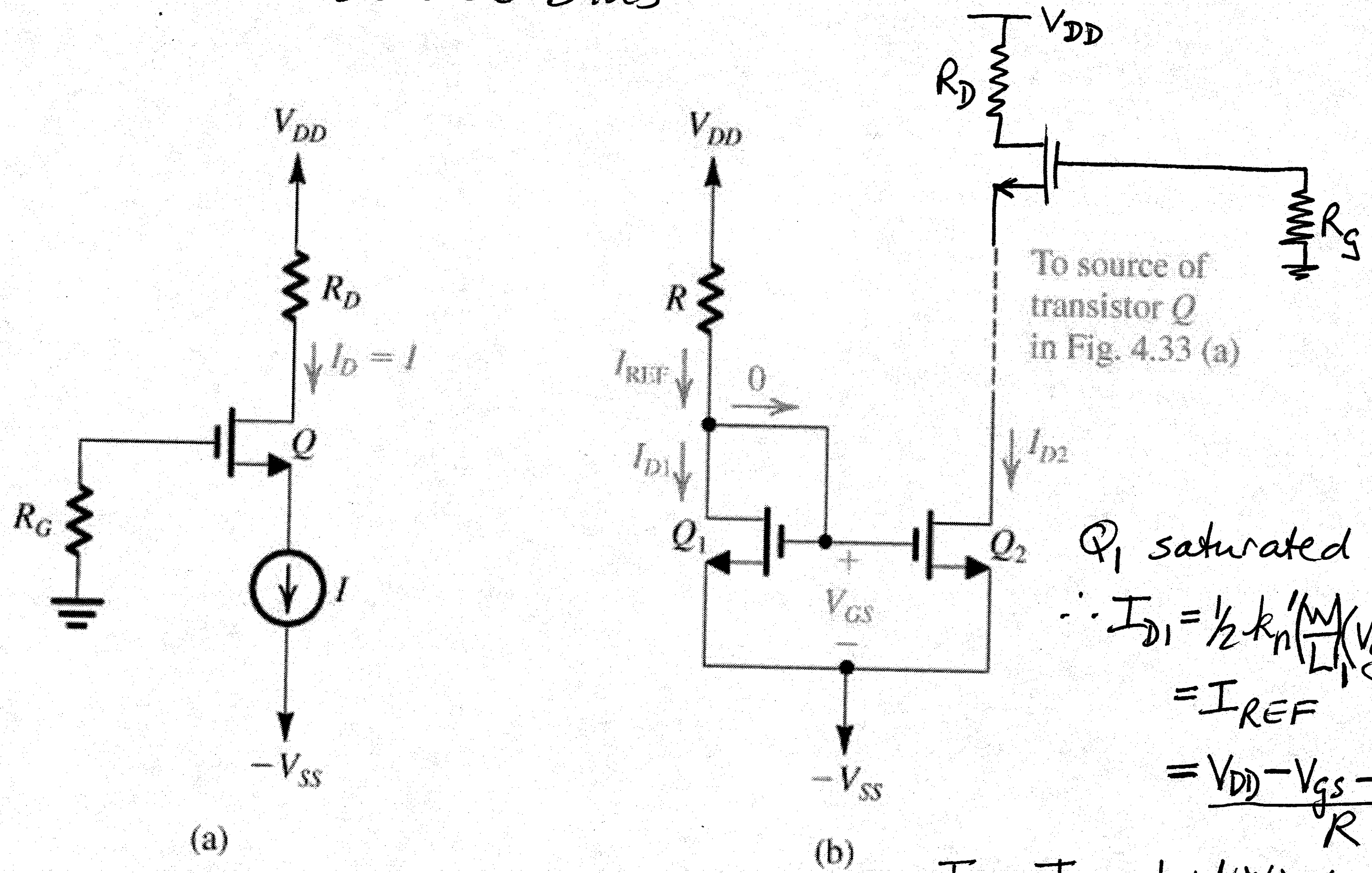
2 supplies

Figure 4.30 Biasing using a fixed voltage at the gate, V_G, and a resistance in the source lead, R_S: **(a)** basic arrangement; **(b)** reduced variability in I_D; **(c)** practical implementation using a single supply; **(d)** coupling of a signal source to the gate using a capacitor C_{C1}; **(e)** practical implementation using two supplies.

Compare Example 4.9 (Selfbias) 9% change as $V_t = 1V \rightarrow 1.5V$
& Exercise 4.19 75% change for fixed bias

Exercise D4.20
Exercise D4.21

Constant Current Source Bias



Q_1 saturated

$$\therefore I_{D1} = \tfrac{1}{2} k_n' \left(\tfrac{W}{L}\right)_1 (V_{GS} - V_t)^2$$

$$= I_{REF}$$

$$= \frac{V_{DD} - V_{GS} - (-V_{SS})}{R}$$

$$I = I_{D2} = \tfrac{1}{2} k_n' \left(\tfrac{W}{L}\right)_2 (V_{GS} - V_t)^2$$

$$\therefore I = I_{REF} \frac{(W/L)_2}{(W/L)_1}$$

Figure 4.33 **(a)** Biasing the MOSFET using a constant-current source *I*. **(b)** Implementation of the constant-current source *I* using a current mirror.

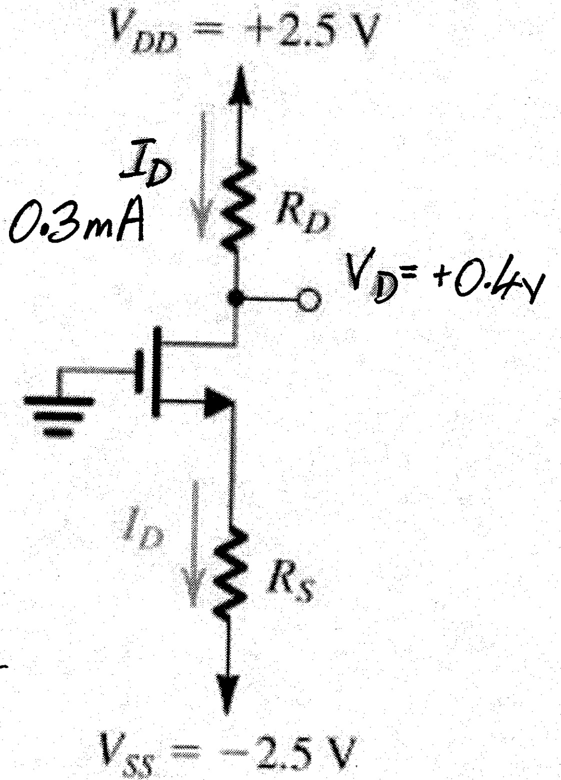
Current Mirror

Exercise D4.22

Ex D4.10 Re-design circuit (Example 4.2) for $V_{DD} = -V_{SS} = 2.5v$
$V_t = 1$ volt $\mu_n C_{ox} = 60 \mu A/v^2$ $W/L = 120\mu m/3\mu m$

$I_D = 0.3mA$ $V_D = +0.4v$

i.e. Find R_S, R_D



$V_D > V_g$
$\therefore$ In saturation mode

$$R_D = \frac{(2.5-0.4)v}{0.3mA} = \frac{2.1}{.3}K = 7K\Omega$$

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{gs}-V_t)^2$$

$$\therefore V_{gs} = V_t + \left(\frac{2I_D}{\mu_n C_{ox}} \frac{L}{W}\right)^{1/2}$$

$$= 1v + \left(\frac{2\times 0.3\times 10^{-3}}{60\times 10^{-6}} \frac{3}{120}\right)^{1/2}$$

$$= 1v + (1/4)^{1/2}_{v} = 1.5v$$

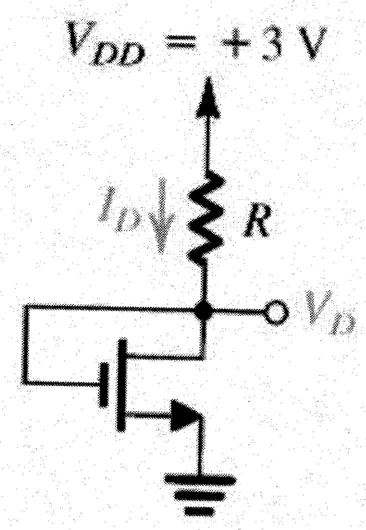
Figure 4.20 Circuit for Example 4.2.

$\therefore V_s = -1.5v$ $\therefore R_s = \frac{-1.5v-(-2.5v)}{0.3mA} = \frac{1v}{.3mA} = 3.3K\Omega$

D4.11 Re-design the circuit (Example 4.3) to double I_D without changing V_D. ie. Find new values of $\frac{W}{L}$ and R.

ie. $I_D = 160\mu A$ and $V_D = 1V$

$\therefore R = \frac{2V}{160\mu A} = 12.5K\Omega$



$V_D = V_g$ $\therefore$ In saturation mode

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs}-V_t)^2$$

so $$\frac{W}{L} = \frac{2I_D}{\mu_n C_{ox}(V_{gs}-V_t)^2} = \frac{320\times10^{-6}}{200\times10^{-6}(1-0.6)^2} = \frac{1.6}{.16} = 10$$

From Example 4.3

$V_{gs} = V_D$

Figure 4.21 Circuit for Example 4.3.

Ex. 4.12 $\quad Q_1$ designed in Example 4.3, i.e. $R = 25K$, $V_D = 1v$

$I_{D1} = 80\mu A \quad V_t = 0.6v$

$\mu_n C_{ox} = 200\mu A/V^2$

$W/L = 5 \quad \lambda = 0$

Q_1, Q_2 same, i.e. V_t's same

$V_{GS2} = V_{GS1} = V_D = 1v$

Q_1 saturated ($V_G = V_D$)

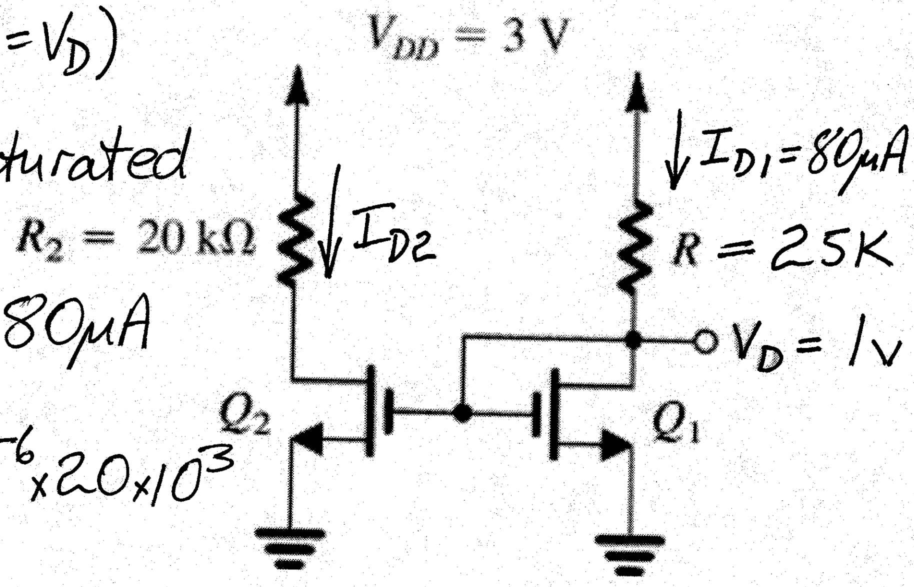
Assume Q_2 also saturated

Then $I_{D2} = I_{D1} = 80\mu A$

$\therefore V_{D2} = 3v - 80\times10^{-6}\times20\times10^{3}$

$= 3 - 1.6\ v$

$= 1.4v$



$\therefore V_{D2} > V_G$ & Q_2 saturation is confirmed

Figure E4.12

Note: If $R_2 = 40K\Omega$, say, then $V_{D2} = 3 - 80\mu A \times 40K = 3 - 3.2v \rightarrow -0.2v$ (Not poss.)

i.e. $V_{D2} \ngtr V_{GS2} - V_t$ & Q_2 NOT saturated

Ex 4.13 Find approx. I_D and V_D if R_D doubled from Example 4.4.

$V_t = 1V$ $\quad k_n' W/L = 1mA/V^2$

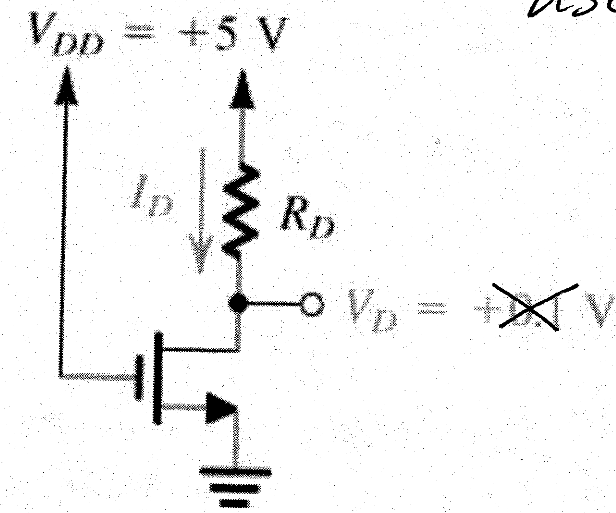
Note: there's a possible ambiguity since calculation in Example 4.4 gives $R_D = 12.4K \longrightarrow 12K$ for nearest 5% value

But the Example 4.4 final result uses 12.4K, so we will use:

$R_D = 24.8K$

$V_G \gg V_D$

$\therefore$ Triode region



$$I_D = k_n' \frac{W}{L}\left[(V_{GS}-V_t)V_{DS} - V_{DS}^2/2\right]$$

$$= 10^{-3}\left((5-1)V_{DS} - 0.5V_{DS}^2\right)$$

$$= 1mA\ (4V_{DS} - .5V_{DS}^2)$$

Figure 4.22 Circuit for Example 4.4.

Also $V_D = 5 - I_D R_D = 5 - 24.8 I_D \times 10^3$

$$V_D = 5 - 24.8(4V_D - .5V_D^2)$$

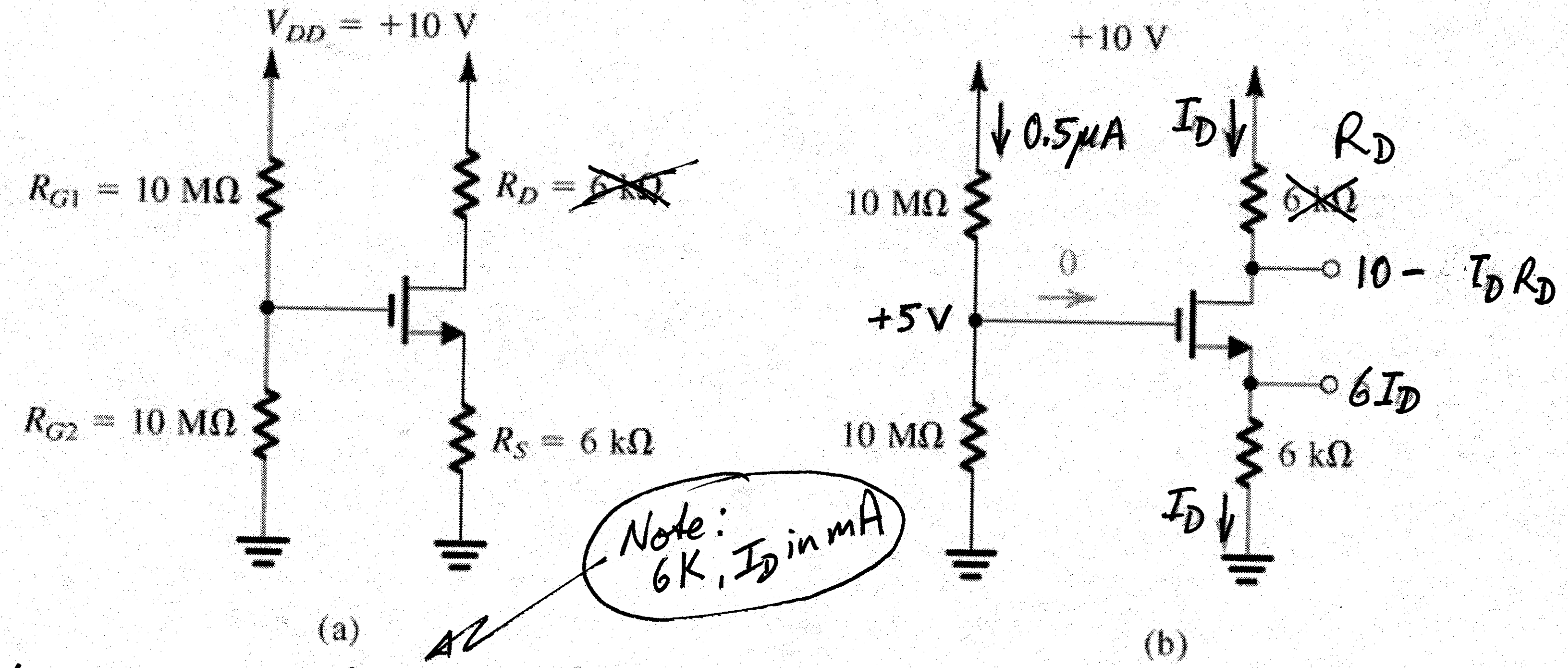
$$12.4V_D^2 - 100.2V_D + 5 = 0$$

$$V_D \approx \frac{1.25}{25} = 0.05V$$

$$\therefore I_D = 1mA\left(4\times 0.05 - \frac{.05^2}{2}\right) \approx 0.2mA$$

Ex. 4.14. $V_t = 1\text{v}$ $\quad k_n' \frac{W}{L} = 1\text{mA/V}^2$ $\quad \lambda = 0$ $\quad$ Max R_D for sat^n?

$I_D = 0.5\text{mA}$ from Example 4.5,
independent of R_D if $\lambda = 0$ as long as saturated.



$V_{gs} = 5 - 6 I_D$

$V_D = 10 - I_D R_D$

For saturation

$V_D \geqslant V_g - V_t = 5 - 1 = 4\text{v}.$

$\therefore \quad 10 - I_D R_D \geqslant 4\text{v}$

$R_D \leqslant \frac{6\text{v}}{0.5} = 12\text{K}\Omega$

in mA

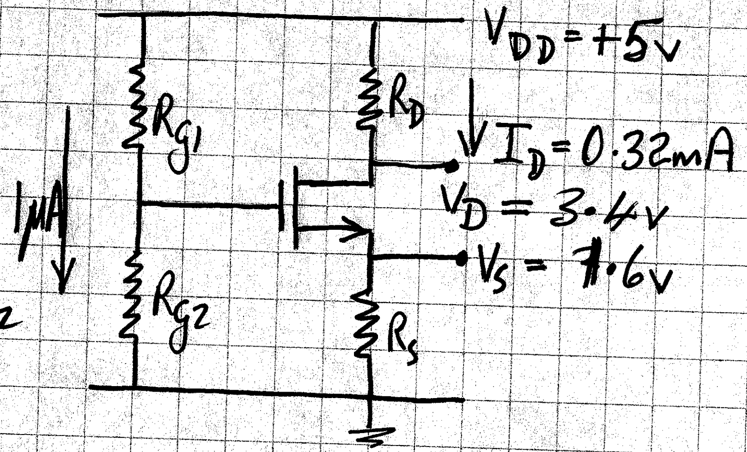
Figure 4.23 **(a)** Circuit for Example 4.5. **(b)** The circuit with some of the analysis details shown.

D4.15

$V_t = 1V$

$k_n' \frac{W}{L} = 1mA/V^2$

$\lambda = 0$



$R_{G1} + R_{G2} = \frac{5v}{1\mu A} = 5M\Omega$

$R_S I_D = 1.6v$

$\therefore R_S = \frac{1.6}{.32} K = 5K$

$5 - I_D R_D = 3.4v$

$\therefore R_D I_D = 1.6v$

$\therefore R_D = 5K$

Assume saturation $\therefore I_D = \frac{1}{2}(V_G - V_S - 1)^2 = .32$

i.e. $V_{GS} = (.64)^{1/2} + 1 = 1.8v$

$\therefore V_G = 1.6 + 1.8 = 3.4v$

& $R_{G2} = 3.4 \; M\Omega$

$R_{G1} = 1.6 \; M\Omega$

Check: $V_D = 3.4v$ $V_G = 3.4v$ $\therefore V_D > V_G - V_t$ Saturation confirmed

Ex 4.16 (Contrast with Example 4.7 → Drains to load)

$k_n' \frac{W_n}{L_n} = k_p' \frac{W_p}{L_p} = 1 mA/V^2$ $\quad V_{tn} = -V_{tp} = 1V \quad \lambda = 0$

Find i_{DN}, i_{DP}, v_O for $v_I = 0, +2.5, -2.5V$

(a) $v_I = v_g = 0$

Symmetrical $\therefore v_O = 0$

$\therefore v_{DG} = 2.5V$

$v_{GS} = 0 < V_t$

$\therefore Q_N, Q_P$ both off

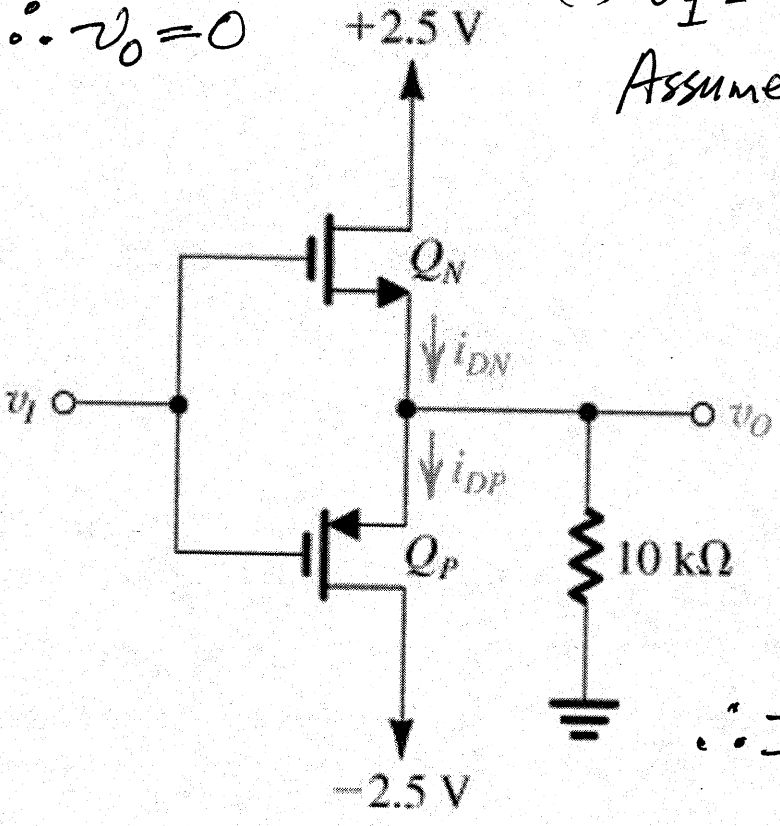
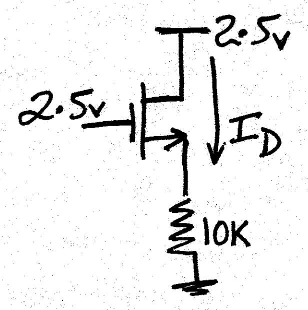


Figure E4.16

(b) $v_I = v_g = +2.5V$

Assume $V_{GSN} > V_{tn} \therefore Q_N$ ON

$V_{GSP} > V_{tp} \therefore Q_P$ OFF



$V_{GS} = V_{DS} \therefore$ Saturated

$\therefore I_D = \frac{1}{2}(2.5 - 10I_D - 1)^2$

$= \frac{1}{2}(1.5 - 10I_D)^2$

$2I_D = 2.25 - 30I_D + 100I_D^2$

$100I_D^2 - 32I_D + 2.25 = 0$

$I_D = \frac{32 \pm \sqrt{32^2 - 900}}{200} \rightsquigarrow$ $.215mA \longrightarrow V_S = 2.15V \therefore V_{GS} = .35V < V_t \therefore$ Impossible

$.105mA \longrightarrow V_S = 1.05V \therefore V_{GS} = 1.45V > V_t \therefore$ OK

Contrast with Exercise 4.16 (Example 4.7 in text)

Q_N, Q_P matched

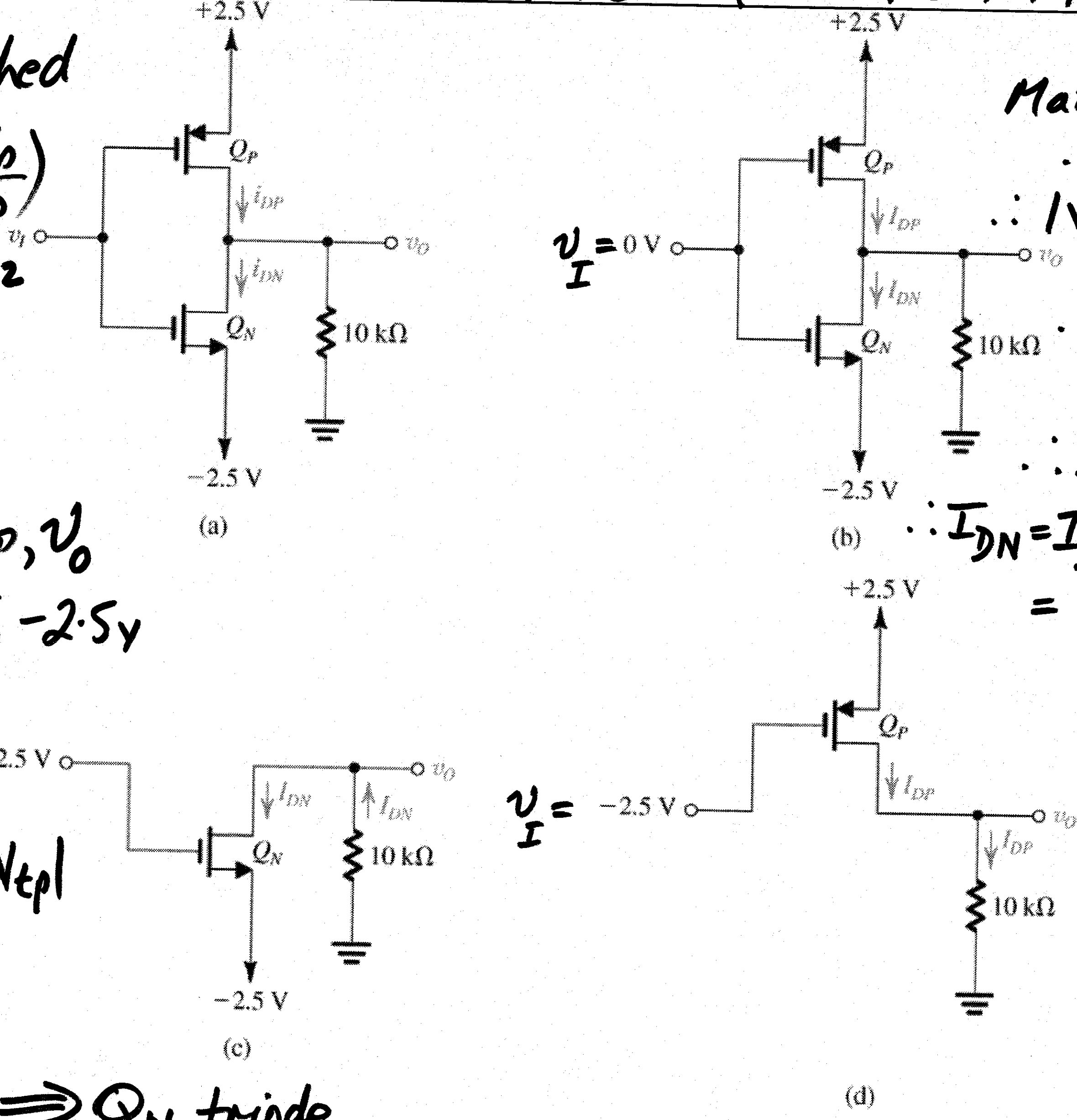
$k_n'\left(\frac{W_n}{L_n}\right) = k_p'\left(\frac{W_p}{L_p}\right)$

$= 1 mA/V^2$

$V_{tn} = -V_{tp} = 1V$

$\lambda = 0$

Find i_{DN} & i_{DP}, v_O for $v_I = 0, +2.5, -2.5V$



Match → symmetry

$\therefore v_O = 0V$

$\therefore |V_{DQ}| = 0V$

$\therefore V_{DS} > V_{GS} - V_t$ (Q_N)

$\therefore$ saturation

$\therefore I_{DN} = I_{DP} = \frac{1}{2} 1mA(2.5-1)^2$

$= 1.125mA$

$V_{GS}]_P = 0 < |V_{tp}|$

$\therefore Q_P$ off.

$v_O = v_D < 0$

$\therefore v_{GD} > V_t \Rightarrow Q_N$ triode

Figure 4.25 Circuits for Example 4.7.

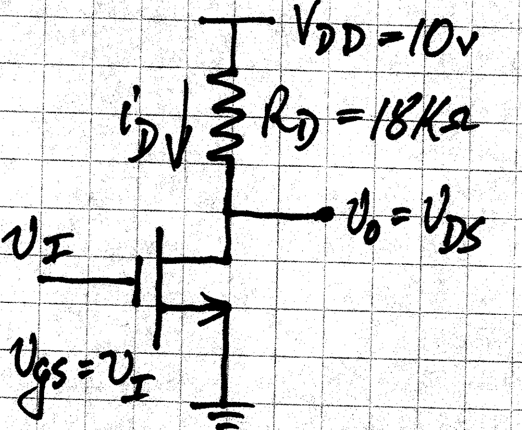
$I_{DN} \approx k_n'(W_n/L_n)(V_{GS} - V_t)V_{DS}$ for v_{DS} small

$= 1mA\,[2.5-(-2.5)-1][v_O-(-2.5)]$ & $I_{DN} = -v_O/10K \Rightarrow v_O = -2.44V$

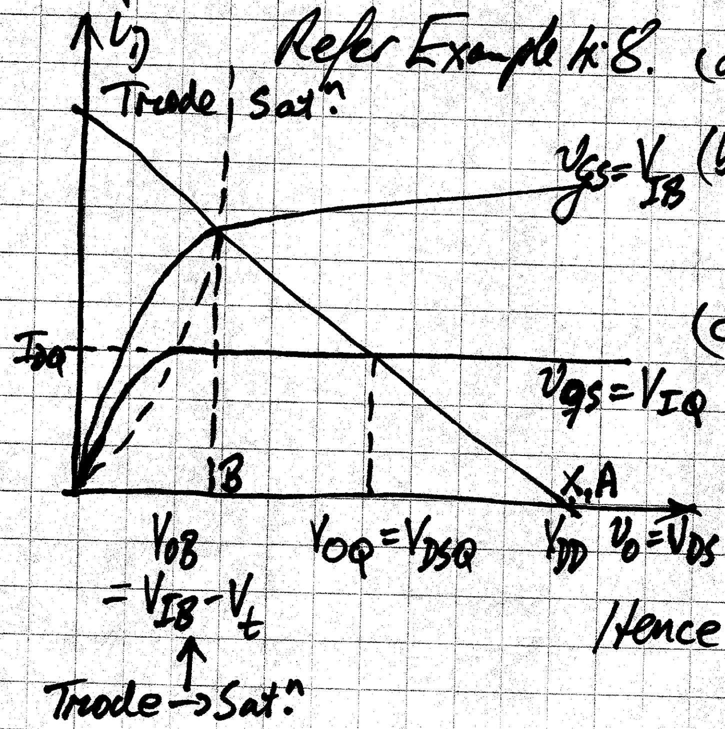
$I_{DN} = 0.244 mA$

Ex 4.17 Fig 4.26 a & b

Refer Example 4.8.



$k_n'(W/L) = 1mA/V^2$
$V_t = 1v$



(a) V_{IQ}, V_{IB}, V_{OQ} & V_{OB} ?

(b) Max neg pk of v_O & hence max pos pk of v_I ?

(c) - - - pos - - - v_O - - -
- - - neg - - - v_I ?

(d) Hence max v_I sinewave amplitude? & v_O ?

Hence A_v ? Why not 14.7v/v as for Example 4.8?

(a) Bias point: Output swing in sat^n $V_{OB} \rightarrow V_{DD}$
pt B X & A
$v_I = 0$ $v_I = V_t = 1v$

(b) Neg v_o max $= V_{OQ} - V_{OB} = 4 - 1 = 3v$
$\therefore$ Pos v_I max $= v_{IB} - V_{IQ} = 2 - 1.816 = 0.184v$

(c) Pos v_o max $= V_{DD} - V_{OQ} = 10 - 4 = 6v$
$\therefore$ Neg v_I max $= v_{IQ} - V_t = 1.816 - 1 = 0.816v$

(d) $\therefore$ Max v_I input amplitude $= 0.184v \longrightarrow 3v$ output
gives $A_v = 3v/0.184v = 16.3 v/V$ (cf. 14.7v/v in Example 4.8 from differential at Q due to non-linearity)

Pt B: Saturation / Triode

$i_D = \frac{1}{2} k_n' \frac{W}{L} (v_{gs} - V_t)^2$

$V_{OB} = V_{IB} - V_t = V_{DD} - i_D R_D$

i.e. $V_{OB} = V_{DD} - \frac{R_D}{2}\left(k_n' \frac{W}{L}\right) V_{OB}^2$

$= 10v - 9 \times 10^3 \times 10^{-3} V_{OB}^2$

$9V_{OB}^2 + V_{OB} - 10 = 0$

$(V_{OB} - 1)(V_{OB} + 10) = 0$

$\therefore V_{OB} = 1v$ & $V_{IB} = 2v$

Choose $V_{OQ} = 4v$ (See Example 4.8)

$\therefore I_Q = (V_{DD} - V_{OQ})/R_D$
$= (10 - 4)/18K = 1/3 mA$

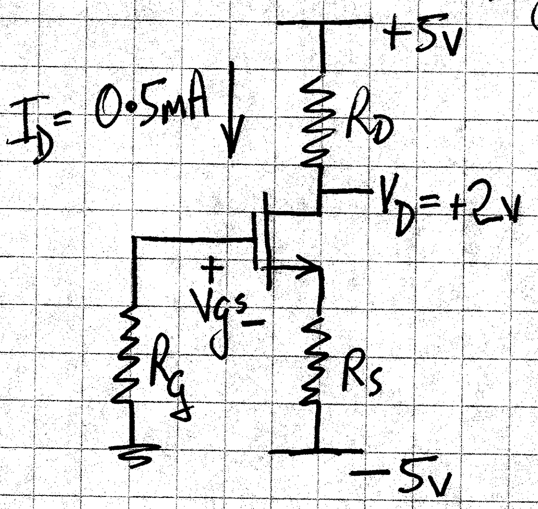
$I_Q = \frac{1}{2} k_n'(W/L)(V_{IQ} - V_t)^2$

$\therefore V_{IQ} \rightarrow \frac{1}{3} \times 10^{-3} = \frac{1}{2}(10^{-3})(V_{IQ} - 1)^2$

$V_{IQ} = 1 + \sqrt{2/3} = 1 + \frac{1.414}{1.732} = 1.816v$

Ex D4.20 (Fig 4.30(e))

Design for $I_D = .5mA$ $V_D = 2v$ with standard 5% resistors.



$V_t = 1v$

$\lambda = 0$

$k_n' \frac{W}{L} = 1mA/v^2$

$R_D = 3v/.5mA = 6K$

$\longrightarrow 6.2K$ (5% preferred value)

$0.5mA = \frac{1}{2} 1mA (V_{gs} - V_t)^2$

$\therefore V_{gs} = \sqrt{1} + V_t = 2v \quad \therefore V_s = -2v$

$\therefore R_s = 3v/.5mA = 6K \longrightarrow 6.2K$

Re-calculate for $R_s = R_D = 6.2K$

$I_D = \frac{1}{2} mA \left(\underset{\uparrow V_g}{0} - (\underset{\uparrow V_s}{-5 + I_D 6.2K}) - \underset{\uparrow V_t}{1} \right)^2$

$= \frac{1}{2} (4 - 6.2 I_D)^2$

$2 I_D = 16 - 49.6 I_D + 38.44 I_D^2$

$I_D = \frac{+51.6 \pm \sqrt{51.6^2 - 2460.16}}{76.88} = 0.856mA \longrightarrow I_D R_s = 5.3v \therefore V_{gs} < 0$ Inconsistent

or $0.486mA \longrightarrow = 3.013v \therefore V_s = -1.987$

$V_{gs} \approx 2v > V_t$

Ex. D4.21 $k_n' \frac{W}{L} = 1mA/V^2$ $V_t = 1V$ $\lambda = 0$ 5% Standard resistors

$V_{DS} = V_{GS}$ ∴ Saturation

$I_D = \frac{1}{2} 1mA (V_{GS} - V_t)^2 = 0.5mA$

∴ $V_{GS} = V_t + \sqrt{1} = 2V$

∴ $R_D = \frac{5-2}{.5} K\Omega = 6K$

$\longrightarrow 6.2K$ preferred value

∴ $I_D = \frac{1}{2} mA (5 - I_D 6.2K - 1)^2$

$2I_D = 16 - 24.8 I_D + 38.44 I_D^2$

$\Longrightarrow .49mA$

& $V_D = 5 - 6.2 \times .49$

$= 1.962V$

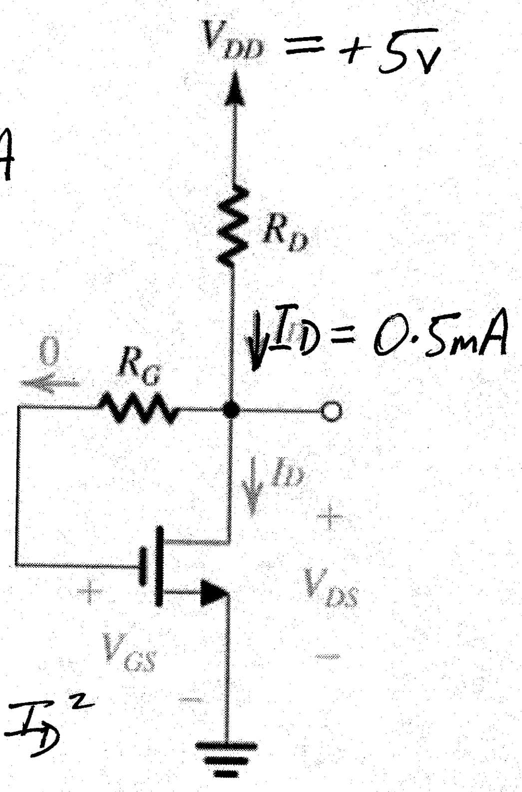
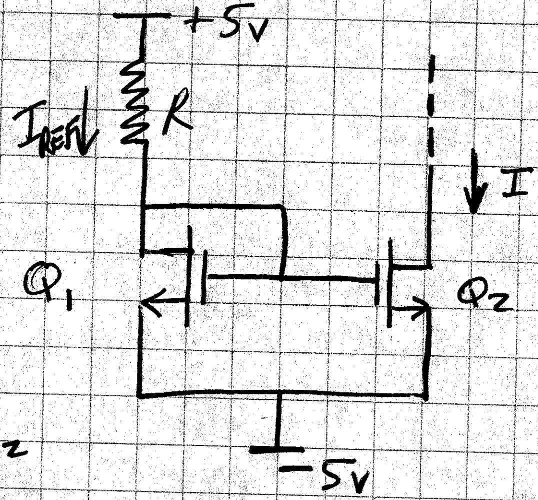


Figure 4.32 Biasing the MOSFET using a large drain-to-gate feedback resistance, R_G.

Ex D4.22



$L_1 = L_2$

$W_2 = 5 W_1$

$I = .5mA$

$k_n'\left(\frac{W}{L}\right)_1 = 0.8\,mA/V^2$

$\lambda = 0 \quad V_t = 1V$

R = ? V_{G1}, V_{G2} = ?

Minimum V_{D2} for Q_2 in saturation

Note: $I = I_{D2} = .5mA$

$k_n'\left(\frac{W}{L}\right)_2 = 5\,k_n'\left(\frac{W}{L}\right)_1 = 4mA/V^2$

$\therefore I_{D1} = I_{REF} = .1mA$

$$R = \frac{V_{DD} - V_{GS1} - (-V_{SS})}{I_{D1}} = \frac{10 - V_{GS}}{.1mA}$$

$V_{GS1} = V_{DS1}$ ∴ in saturation

and $I_{D1} = 1/2\, k_n'\left(\frac{W}{L}\right)_1 (V_{GS} - V_t)^2$

$\therefore .1mA = .4mA\,(V_{GS} - 1)^2$

$V_{GS} = 1 + (0.25)^{1/2} = 1.5V$

$\therefore V_{G1} = V_{G2} = -5 + 1.5 = -3.5V$

$$R = \frac{10 - 1.5}{.1mA} = 85K\Omega$$

Lowest V_{D2} for Q_2 saturated?

$V_{D2} > V_{G2} - V_t$

$> -3.5V - 1V$

$V_{D2} > -4.5V$