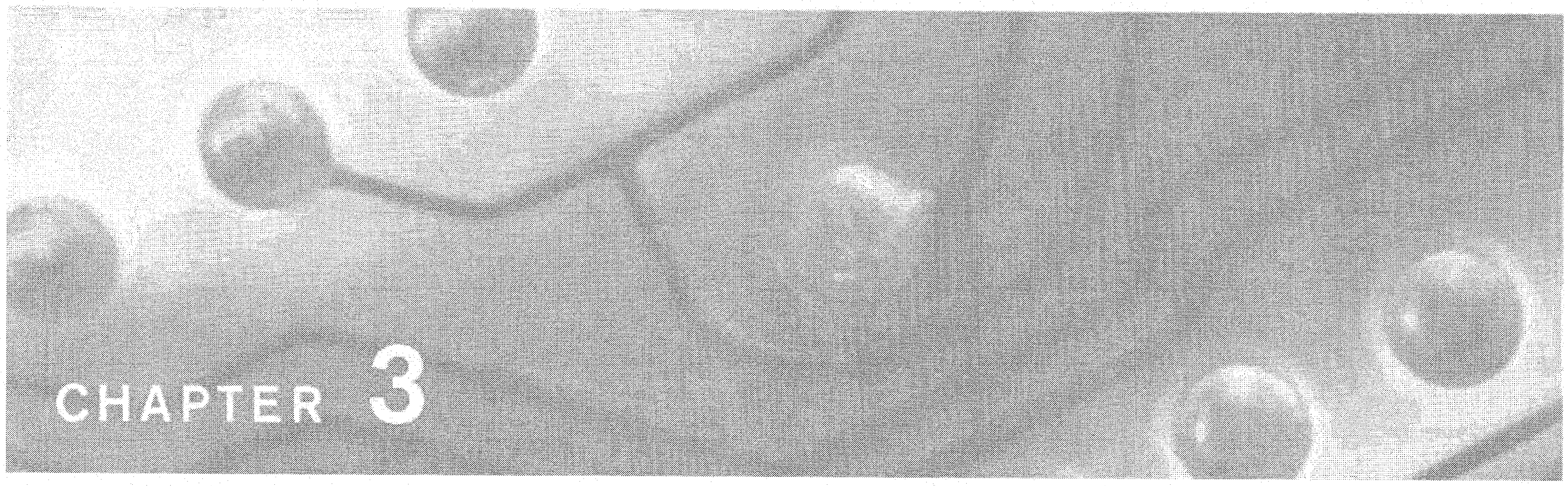
# ECE321 ELECTRONICS I
## FALL 2006

## PROFESSOR JAMES E. MORRIS

Lecture 7

17th October, 2006



# CHAPTER 3

## Diodes

3.7 Diode Physics

(3.8 Other Diode Types)

3.9 SPICE

Completely different
type of material
— not circuits.

Also next lecture:
BJT device physics

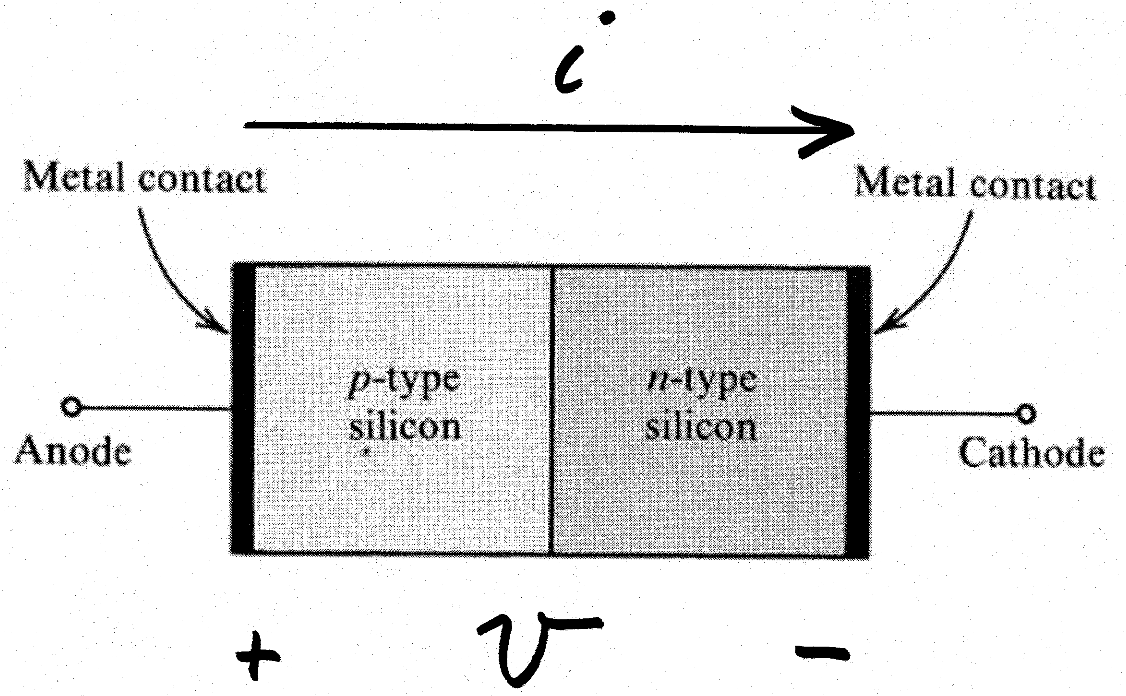
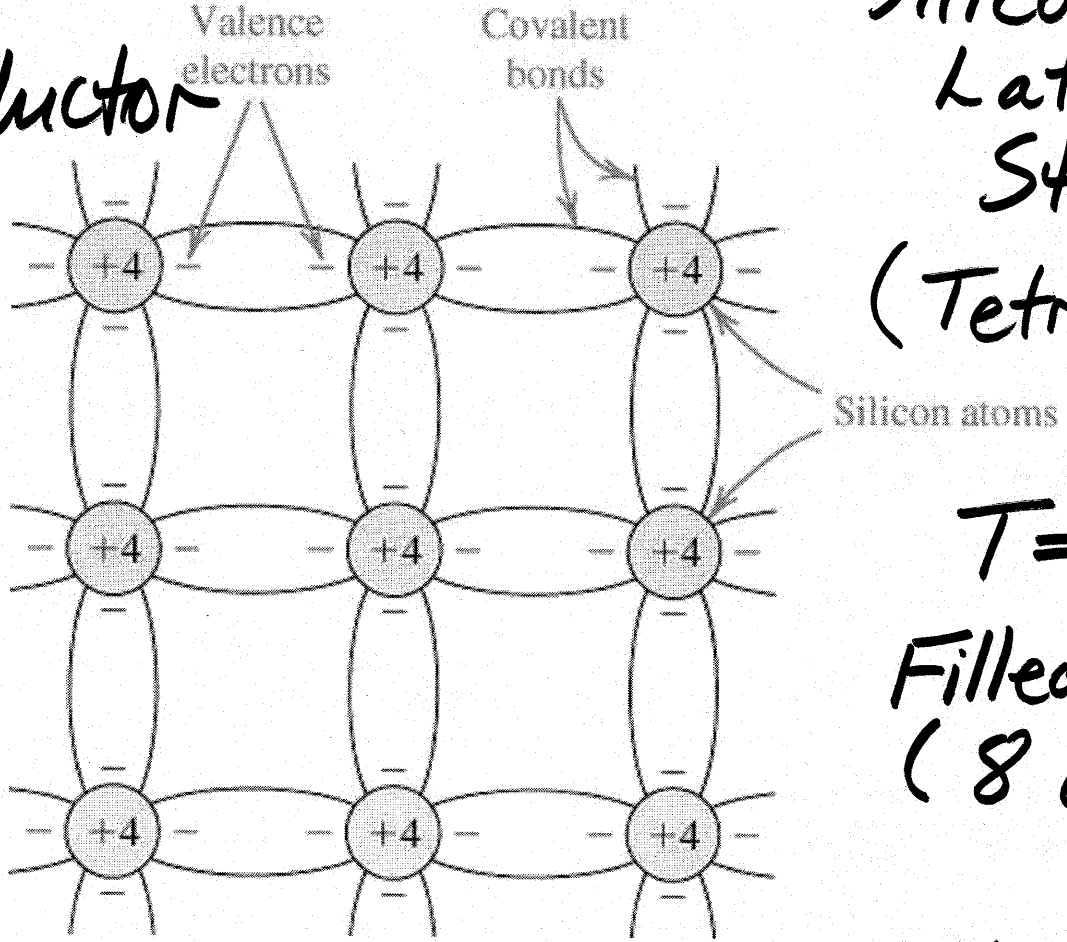


**Figure 3.39** Simplified physical structure of the junction diode. (Actual geometries are given in Appendix A.)

# Intrinsic Semiconductor

Silicon Lattice Structure (Tetrahedral)



T= 0° K

Filled shells (8 electrons)

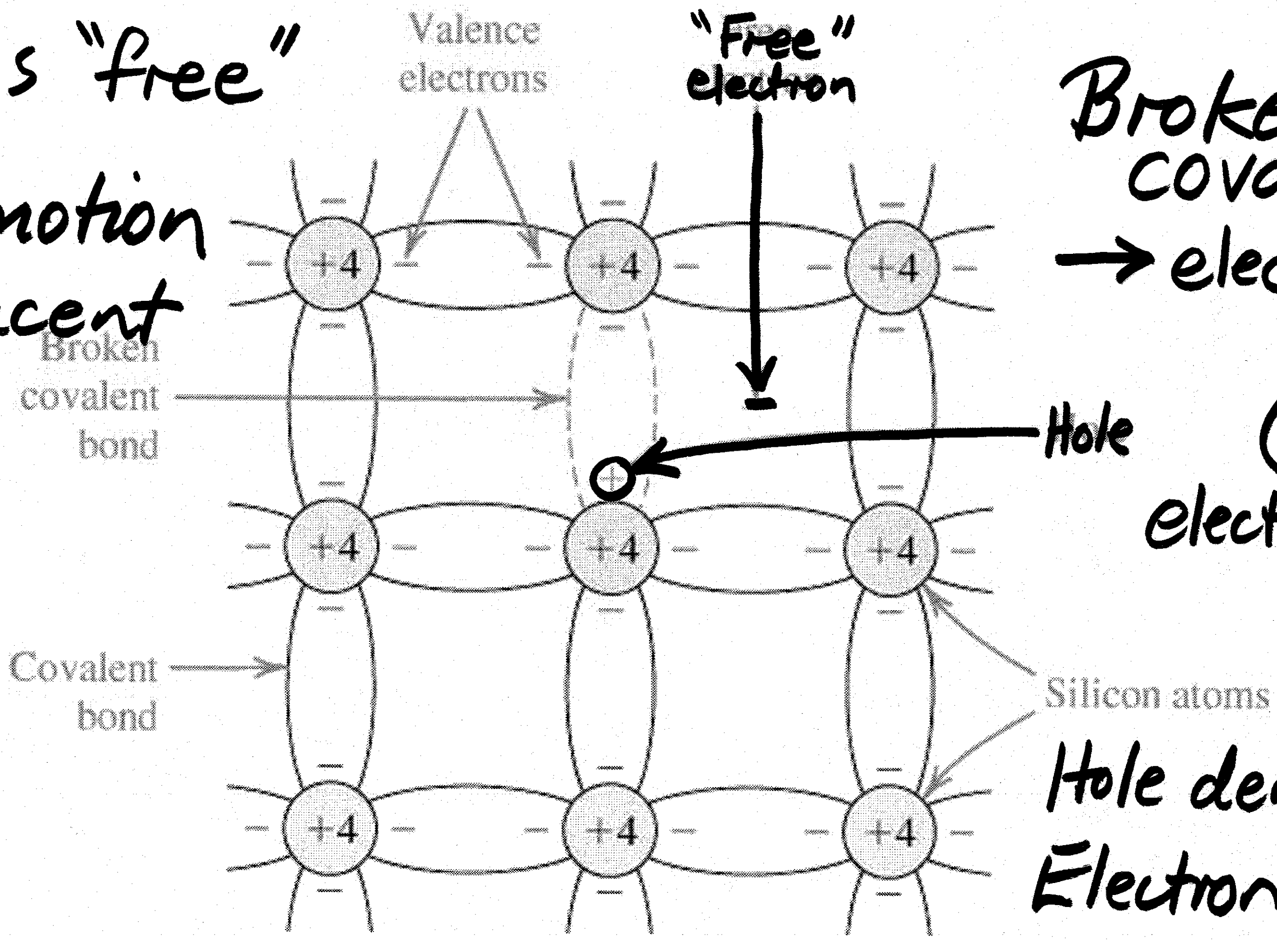
Finite T ⟶

**Figure 3.40** Two-dimensional representation of the silicon crystal. The circles represent the inner core of silicon atoms, with +4 indicating its positive charge of +4q, which is neutralized by the charge of the four valence electrons. Observe how the covalent bonds are formed by sharing of the valence electrons. At 0 K, all bonds are intact and no free electrons are available for current conduction.

Finite T → lattice vibrations (phonons)

Electrons "free"

"Hole" motion by adjacent electron hops.



Broken covalent bonds → electron + "hole"

(EHP electron/hole pair)

Hole density $p$

Electron ($e^-$) density $n$

Intrinsic semiconductor $n = p = n_i$ ← intrinsic density

**Figure 3.41** At room temperature, some of the covalent bonds are broken by thermal ionization. Each broken bond gives rise to a free electron and a hole, both of which become available for current conduction.

General rule: $np = n_i^2$

$$n_i^2 = BT^3 \exp -E_g/kT$$

$n_i$/cc

$5.4 \times 10^{31} / (K.cm^2)^3$

For Si: $n_i = 1.5 \times 10^{10}/cm^3$

at $T = 300K$

(Compare $5 \times 10^{22}$ atoms/$cm^3$)

Absolute T (K)

Boltzmann's constant

$1.38 \times 10^{-23}$ J/K

$= 8.62 \times 10^{-5}$ eV/K

Energy "Band Gap"

1.12eV for Si

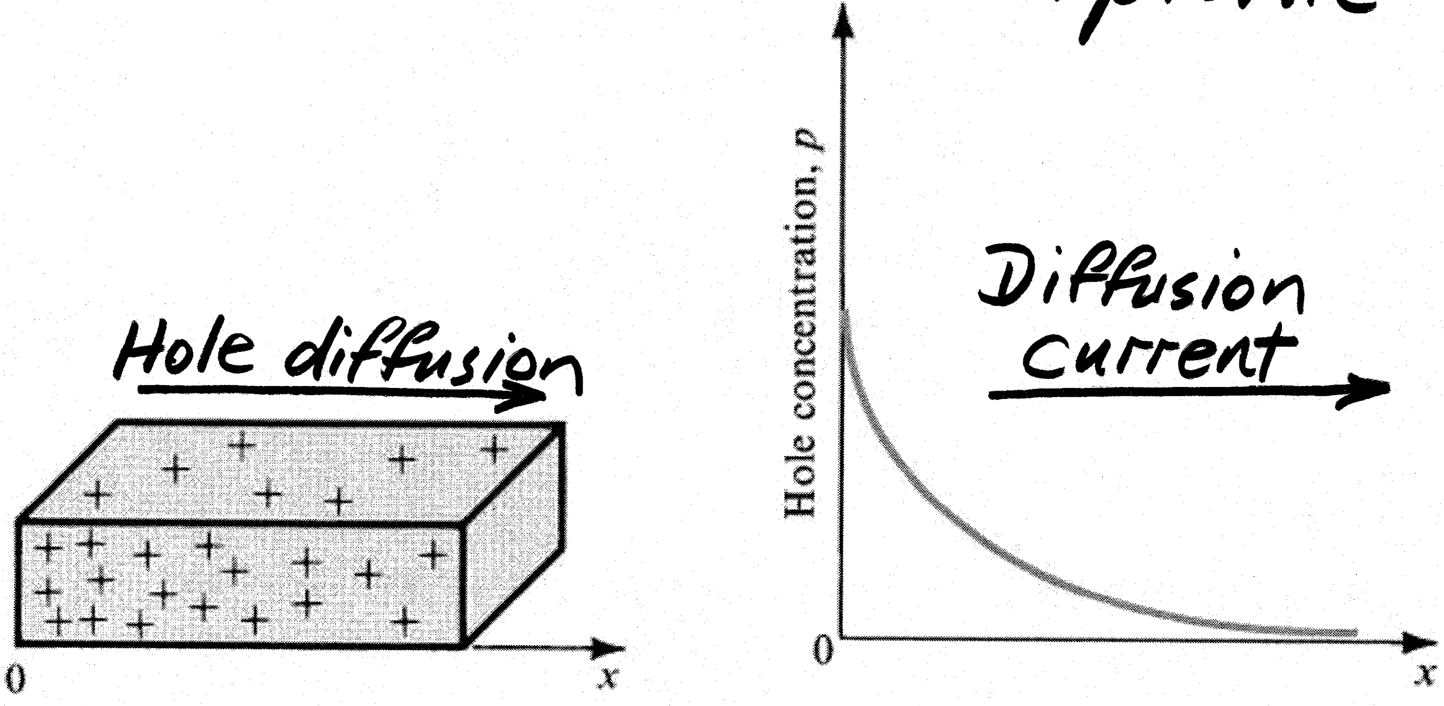
($1.12 \times 1.6 \times 10^{-19}$ J)

This is the energy required to break a covalent bond

→ electron-hole pair

(valence electron → conduction $e^-$)

# Diffusion : Density gradient
Concentration profile



(a)

(b)

$$J_p = -q D_p \frac{dp}{dx} \quad \left( \frac{dp}{dx} < 0, \ \therefore J_p > 0 \right)$$

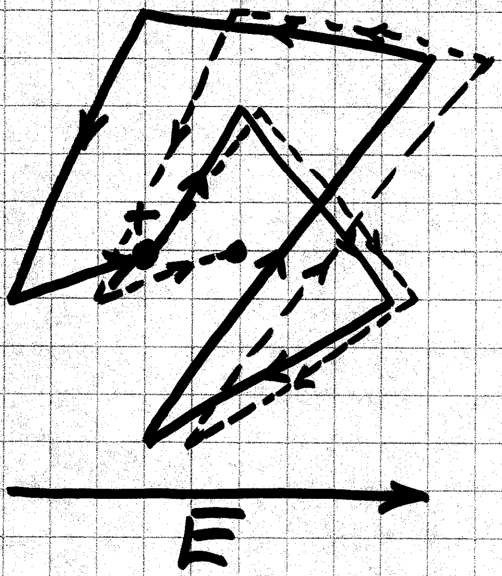
↑ Hole diffusion constant (diffusivity)

**Figure 3.42** A bar of intrinsic silicon **(a)** in which the hole concentration profile shown in **(b)** has been created along the *x*-axis by some unspecified mechanism.

Holes: $J_p = -q D_p \frac{dp}{dx}$ $D_p \sim 12 \text{ cm}^2/\text{sec}$

Electrons: $J_n = q D_n \frac{dn}{dx}$ $D_n \sim 34 \text{ cm}^2/\text{sec}$

# Drift Current (in an electric field)



Random thermal motion: no net effect

Superimpose small deviations due to electric field → small net motion

Small drift velocity << thermal velocities

Holes: $v_{drift} = \mu_p E$ $\therefore J_p = q p \mu_p E$

Electrons: $v_{drift} = -\mu_n E$ $\therefore J_n = q n \mu_n E$

Note: Electrons move opposite to E, but current direction opposite to v.

$\therefore$ Drift current $J_{drift} = q(n\mu_n + p\mu_p)E = \sigma E$

$$\sigma = q(n\mu_n + p\mu_p) = 1/\rho$$

Intrinsic Si: $\mu_p \sim 480\ \text{cm}^2/\text{v.sec}$ $\mu_n \sim 1350\ \text{cm}^2/\text{v.sec}$

Note Einstein Relation: $D_n/\mu_n = D_p/\mu_p = kT/q = V_T$

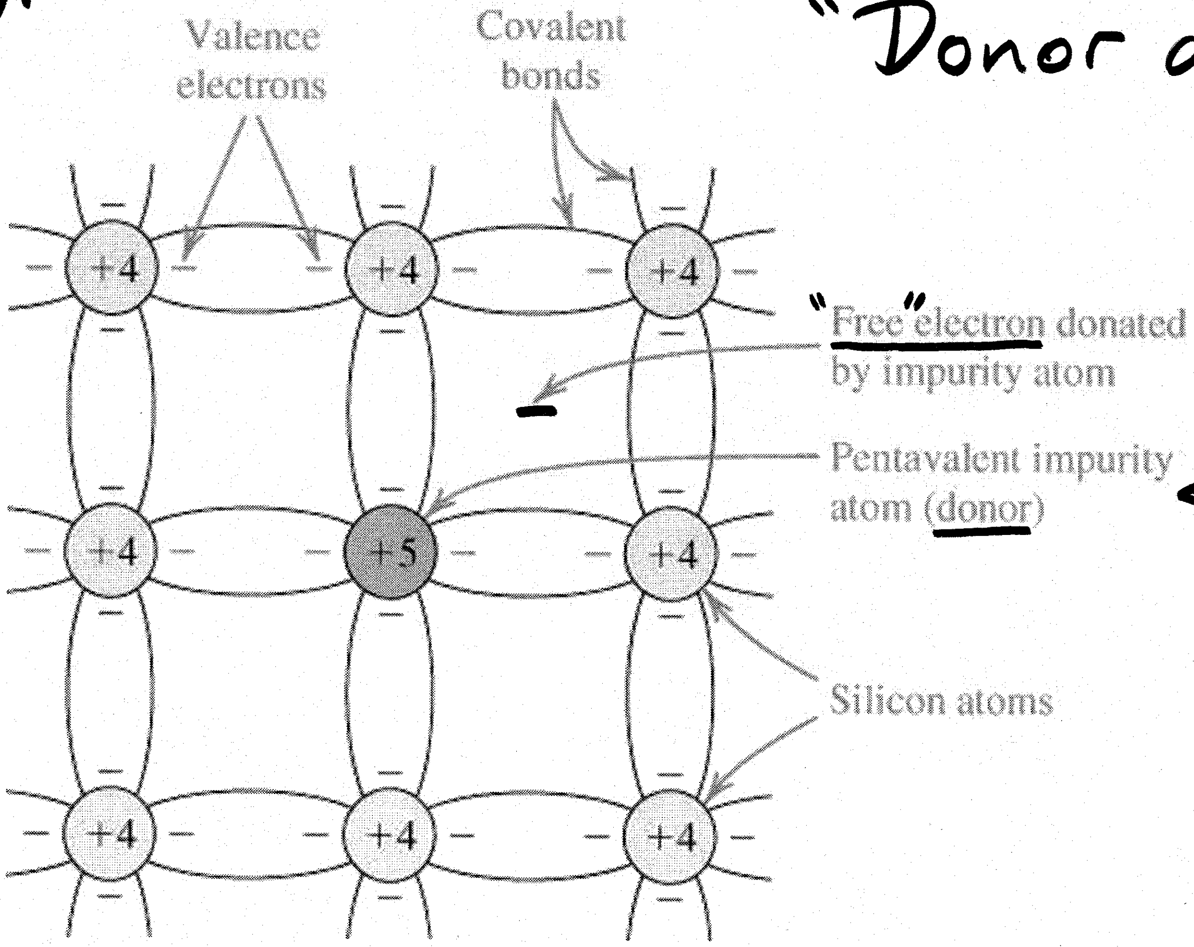
# N-type Extrinsic Semiconductor

"Donor doping"

Majority carriers (electrons)

$n_{NO} \approx N_D$

↑ thermal equilibrium



← e.g. P, As, Sb doping

Minority carriers (holes) $np = n_i^2 \therefore p_{NO} \approx n_i^2 / N_D$

↑ temperature dependent

**Figure 3.43** A silicon crystal doped by a pentavalent element. Each dopant atom donates a free electron and is thus called a donor. The doped semiconductor becomes *n* type.

Note: Electrically neutral (neg charges = pos charges)

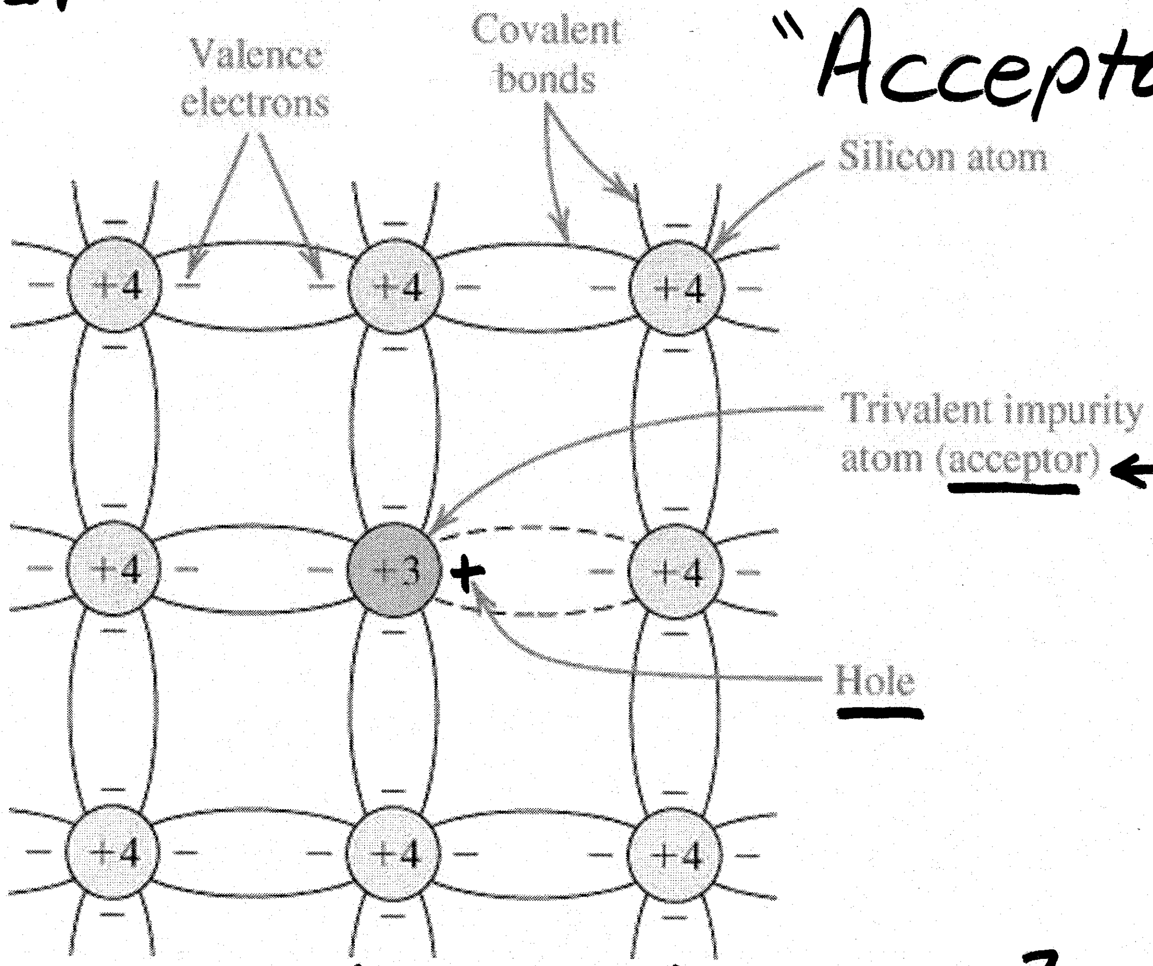
($e^-$s) $n_{NO} = N_D$ (+ve ionized donors) + $n_i^2/N_D$ (+ve carriers)

# P-type Extrinsic Semiconductor

"Acceptor doping"

Majority carriers (holes)

$p_{p0} \approx N_A$



← e.g. Ga, Al doping

Minority carriers (electrons) $np = n_i^2 \therefore n_{p0} \approx n_i^2/N_A$

**Figure 3.44** A silicon crystal doped with a trivalent impurity. Each dopant atom gives rise to a hole, and the semiconductor becomes *p* type.

Electrically neutral → (positive charges = negative charges)

(Holes) $p_{p0} = N_A$ (−ve ionized acceptors) $+ n_i^2/N_A$

# Doping

Ex. 3.29

Ex. 3.30

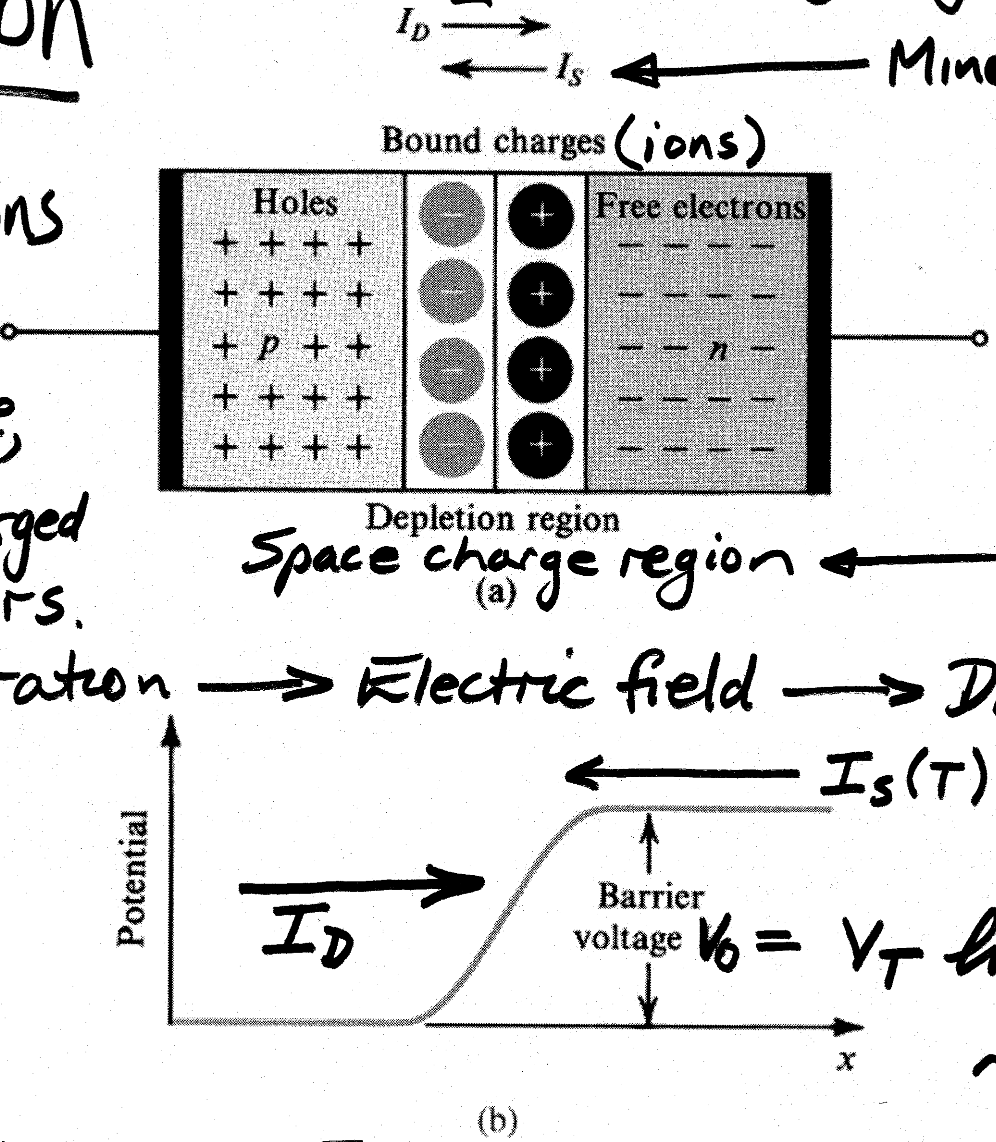
Ex. 3.31

# PN Junction

Majority carriers $I_D$

Minority carriers $I_s$

Holes & electrons in space-charge region recombine, "uncovering" charged donors/acceptors.



Space charge region ← ionized donors & acceptors

Charge separation → Electric field → Diffusion barrier

$I_s(T)$

$I_D$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

~ 0.6 to 0.8 volts

At equilibrium: $I_D = I_S$

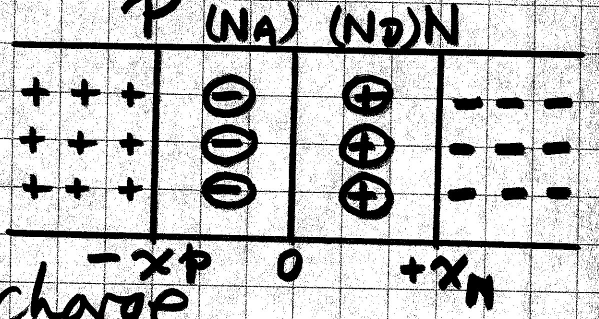
**Figure 3.45** **(a)** The *pn* junction with no applied voltage (open-circuited terminals). **(b)** The potential distribution along an axis perpendicular to the junction.

$I_D$: Diffusion by majority carriers $\underset{e^-s}{\overset{\text{holes}}{\rightleftarrows}}$ impeded by field.

$I_S$: Minority carriers accelerated across region by field $\underset{e^-s \rightarrow}{\overset{\leftarrow \text{holes}}{}}$

[Note: External voltage = 0]

# Space Charge Region Width:
# (Depletion)



Negative charge = Positive charge

$$(q N_A)\, A\, x_p = (q N_D)\, A\, x_n$$

$$\therefore \frac{x_n}{x_p} = \frac{N_A}{N_D}$$

$$W_{depl} = x_n + x_p = \sqrt{\left(\frac{2\varepsilon_{si}}{q}\right)\left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_0}$$

$\sim$ 0.1 to 1 μm

$$= x_n\left(1 + \frac{N_D}{N_A}\right) = x_p\left(1 + \frac{N_A}{N_D}\right)$$

$$\therefore x_n = \frac{N_A}{N_A + N_D} W_{depl} \quad \& \quad x_p = \frac{N_D}{N_A + N_D} W_{depl}$$

$$= \sqrt{(2\varepsilon_{si}/q)\, \frac{N_A/N_D}{N_A + N_D} V_0} \qquad = \sqrt{(2\varepsilon_{si}/q)\, \frac{N_D/N_A}{N_A + N_D} V_0}$$

Note: $\varepsilon_{si} \approx 11.7\, \varepsilon_0 = 1.04 \times 10^{-12}$ F/cm

# Ex. 3.32

## Depletion region

# Reverse Bias

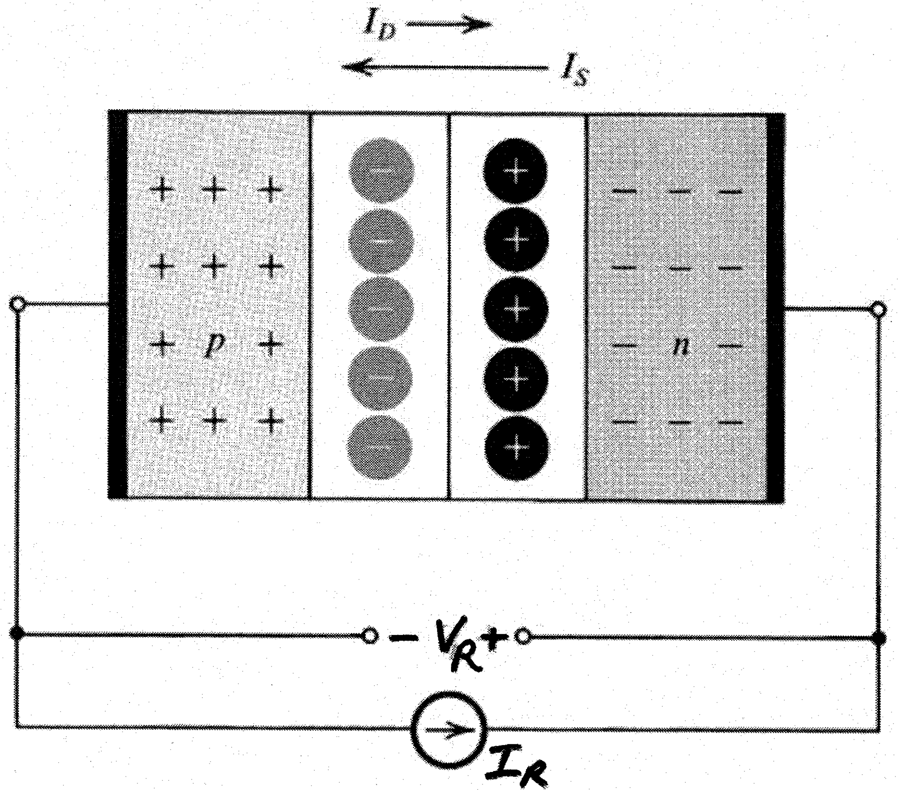
$I_D$ holes → ← electrons

$I_S$ ← holes/electrons →

Either:

Assume current source $I_R \ll I_S$

→ see text treatment



Or: Reverse bias $V_R$ adds to diffusion barrier $V_0$ and decreases equilibrium $I_D$

$$I_R = I_S - I_D$$

**Figure 3.46** The *pn* junction excited by a constant-current source *I* in the reverse direction. To avoid breakdown, *I* is kept smaller than $I_S$. Note that the depletion layer widens and the barrier voltage increases by $V_R$ volts, which appears between the terminals as a reverse voltage.

$$\& \quad W_{depl} = \sqrt{\left(\frac{2\varepsilon_{Si}}{q}\right)\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$$

Eventually $V_R \gg 0$, $I_D \rightarrow 0$, and $I_R \rightarrow I_S$

# Depletion Capacitance → charge separation

$$q_J = q_N = q_P = qN_D x_n A = qN_A x_p A$$

$$\Longrightarrow A\sqrt{(2q\varepsilon_{si})\frac{N_A N_D}{N_A+N_D}(V_0+V_R)}$$

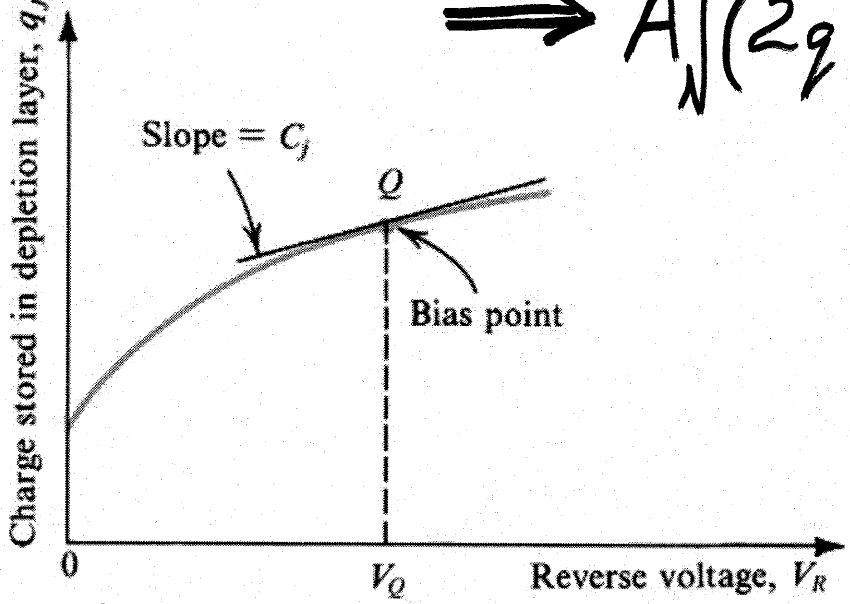


**Figure 3.47** The charge stored on either side of the depletion layer as a function of the reverse voltage $V_R$.

Either: $$C_J = \frac{\varepsilon_{si}A}{W_{depl}} = \frac{\varepsilon_{si}A}{\sqrt{(2\varepsilon_{si}/q)\left(\frac{N_A+N_D}{N_AN_D}\right)(V_0+V_R)}} = A\left(\frac{\varepsilon_{si}q}{2}\cdot\frac{N_AN_D}{N_A+N_D}\right)^{1/2}(V_0+V_R)^{-1/2}$$

Or: Small signal Depletion Capacitance

$$C_j = \left.\frac{dq_J}{dV_R}\right|_{\substack{V_R=V_Q \\ \text{Bias point}}} = A\sqrt{(2q\varepsilon_{si})\frac{N_AN_D}{N_A+N_D}}\cdot\frac{1}{2}\cdot(V_0+V_R)^{-1/2}$$

→ same

Write as $$C_j = C_{j0}\frac{(V_0+V_R)^{-1/2}}{V_0^{-1/2}} = \frac{C_{j0}}{(1+V_R/V_0)^{1/2}} \longrightarrow \frac{C_{j0}}{(1+V_R/V_0)^m}$$

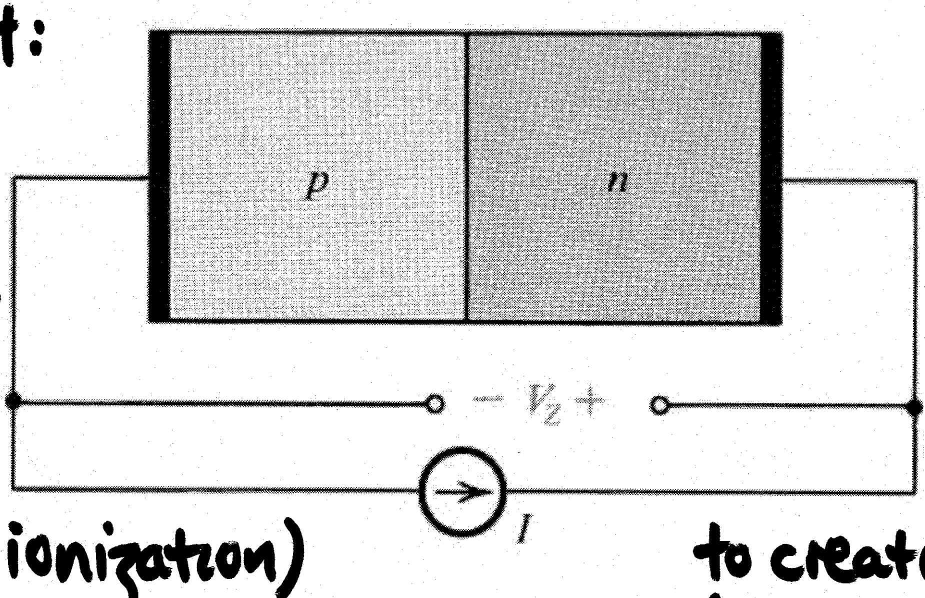
$m = 1/2$ abrupt junction

$m = 1/3$ linearly graded

# Reverse Breakdown: if $V_R \xrightarrow[\text{increase}]{} V_{ZK}$

**Zener Effect:**

Electric field in the depletion region creates electron–hole pairs (by field ionization)



**Avalanche Effect:**

Electrons accelerat in the depletion regio

Electric field gives sufficient energy to create electron-hole pairs (by impact ionization)

→More carriers → current increases

**Figure 3.48** The *pn* junction excited by a reverse-current source $I$, where $I > I_S$. The junction breaks down, and a voltage $V_Z$, with the polarity indicated, develops across the junction.

# Ex. 3.33

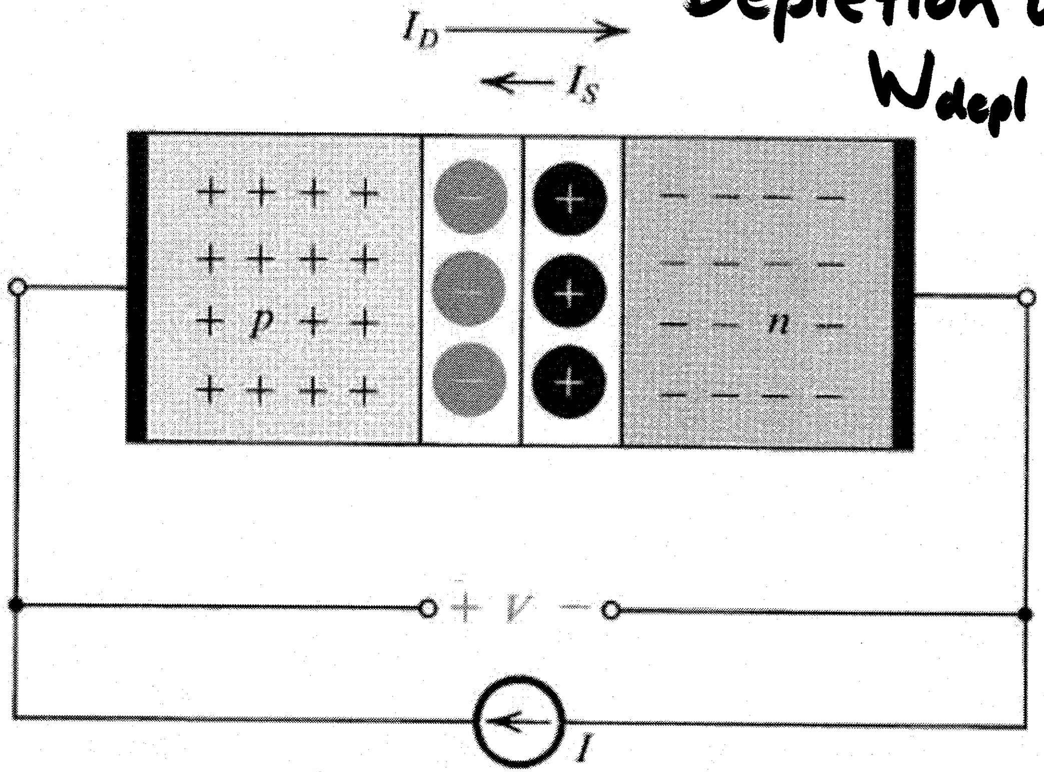
## Junction capacitance

# Forward Bias

$V_0 \longrightarrow V_0 - V$, $I_D$ incr

Depletion width decreases

$W_{depl} \rightarrow (\quad)(V_0 - V)^{1/2}$



Become:

Majority holes from P → cross depletion region → minority in N region

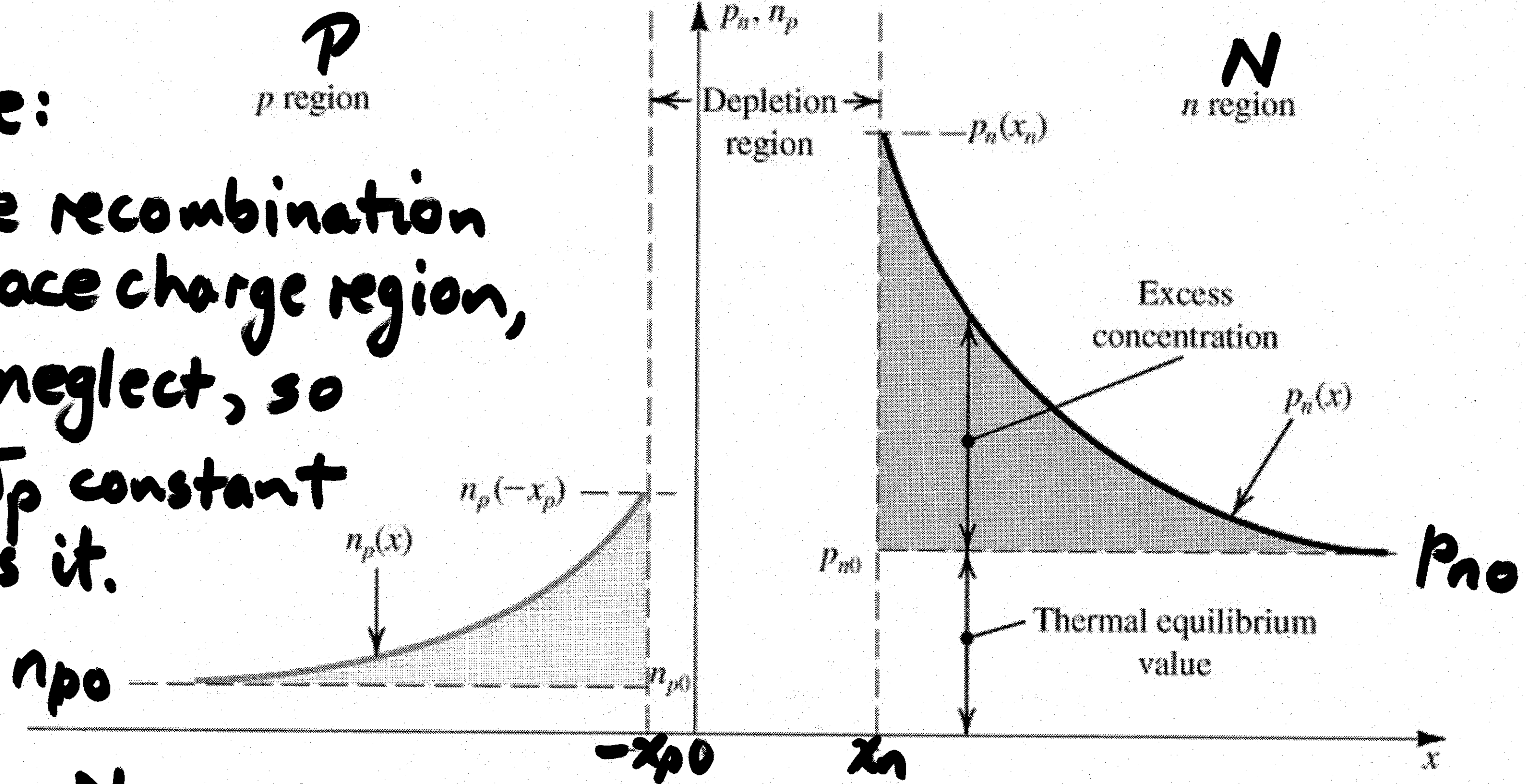
Majority electrons from N → cross depletion region → minority in P region & recombine with majority carriers

**Figure 3.49** The *pn* junction excited by a constant-current source supplying a current *I* in the forward direction. The depletion layer narrows and the barrier voltage decreases by *V* volts, which appears as an external voltage in the forward direction.

Recombination requires replenishment from external circuit

# Minority Carrier Recombination

Note:
Some recombination in space charge region, but neglect, so $J_n, J_p$ constant across it.



Here $N_A > N_D \therefore n_{po} < p_{NO}$ & $x_p < x_n$

$$J_{diffusion} = J_{np}(-x_p) + J_{pn}(x_n)$$

**Figure 3.50** Minority-carrier distribution in a forward-biased *pn* junction. It is assumed that the *p* region is more heavily doped than the *n* region; $N_A$ @ $N_D$.

# Carrier Recombination and Currents

$$p_n(x_n) = p_{no}\exp(V/V_T) \xrightarrow[V=0]{} p_{no}$$

& $$p_n(x) = p_{no} + [p_n(x_n) - p_{no}]\exp - \frac{x - x_n}{L_p}$$

$L_p =$ Minority carrier diffusion length of holes in N-type material

(~ 1 to 100 μm) $= \sqrt{D_p \tau_p}$

Defines Minority Carrier Lifetime $\tau_p = L_p^2/D_p$

(~ 1 to $10^4$ ns)

Diffusion $$J_p = -q D_p \left.\frac{dp_n(x)}{dx}\right]_{x=x_n}$$

$$= -\left[p_{no}\exp\frac{V}{V_T} - p_{no}\right]\left[\exp - \frac{x - x_n}{L_p}\right]\left(-\frac{1}{L_p}\right) q D_p$$

$$J_p(x_n) = (q D_p/L_p)\, p_{no}\left(\exp\frac{V}{V_T} - 1\right)$$

Similarly $$J_n(-x_p) = (q D_n/L_n)\, n_{po}\left(\exp V/V_T - 1\right)$$

& $$J = J_p(x_n) + J_n(-x_p) \quad (= J_p(x) + J_n(x))$$

So $$J = J_n + J_p$$

$$= q\left(\frac{D_n}{L_n} n_{po} + \frac{D_p}{L_p} p_{no}\right)\left(\exp\frac{V}{V_T} - 1\right)$$

$$\rightarrow \quad I = A q n_i^2 \left(\frac{D_n}{L_n}\frac{1}{N_A} + \frac{D_p}{L_p}\frac{1}{N_D}\right)\left(\exp\frac{V}{V_T} - 1\right)$$

$$= I_s\left(\exp\frac{V}{V_T} - 1\right)$$

where $$I_s = A q \underset{\uparrow}{n_i^2} \left(\frac{L_n/\tau_n}{N_A} + \frac{L_p/\tau_p}{N_D}\right)$$

Strong T dependence

# Diffusion Capacitance

$$C_D = dQ/dV = \tau_T (dI/dV)$$

$$I = I_S \left(\exp \frac{V}{V_T} - 1\right)$$

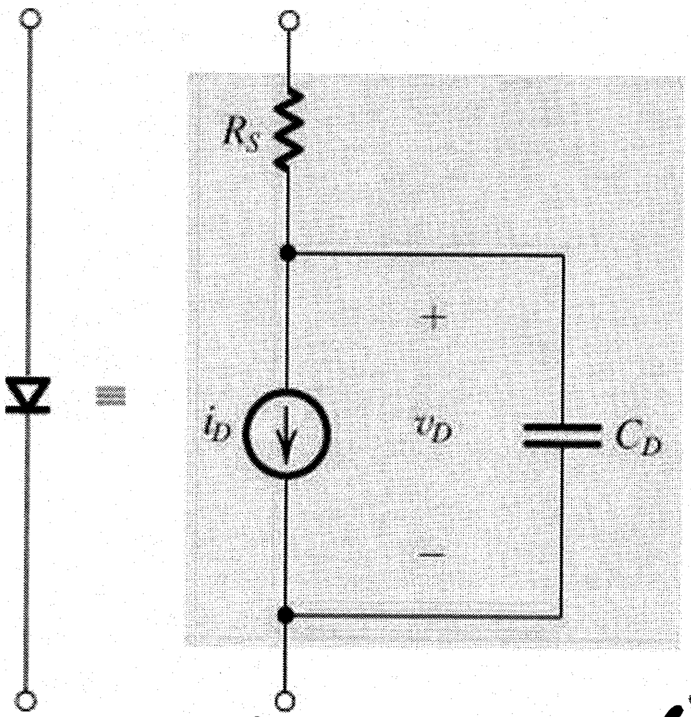
$$\therefore \frac{dI}{dV} = \frac{I_S}{V_T} \exp \frac{V}{V_T} = \frac{I + I_S}{V_T} \approx \frac{I}{V_T}$$

$$\therefore C_D \approx \frac{\tau_T}{V_T} I \longrightarrow 0$$

for reverse bias

$C_j$ depletion capacitance dominates for reverse bias
$C_D$ diffusion " " " forward "

# Diffusion Capacitance & Spice Model



$$i_D = I_S\left(e^{v_D/nV_T} - 1\right)$$

$$C_D = C_d + C_j = \underbrace{\frac{\tau_T}{V_T} I_S e^{v_D/nV_T}}_{\text{Diffusion Capacitance (Excess minority carriers)}} + \overbrace{C_{j0}\Big/\left(1-\frac{v_D}{V_0}\right)^m}^{\text{Depletion Capac}}$$

**Figure 3.51** The SPICE diode model.

Excess stored charge $Q_p = \int_{x_n}^{\infty} \left[p_n(x_n) - p_{no}\right] \exp - \frac{x - x_n}{L_p} . dx$

$$= Aq\, p_{no}\left(\exp\frac{V}{V_T} - 1\right)(-L_p)(0-1)$$

$I_p = qA\frac{D_p}{L_p}p_{no}(\quad) \rightarrow$ $= L_p^2/D_p \cdot I_p = \tau_p I_p$

$\& \; Q_n = \tau_n I_n$

$\therefore Q = \tau_p I_p + \tau_n I_n \Rightarrow \tau_T I$ where $\tau_T$ = mean transit time

# Spice Model : Zener Diode

Ideal : n very small makes $V_{D0}$ small

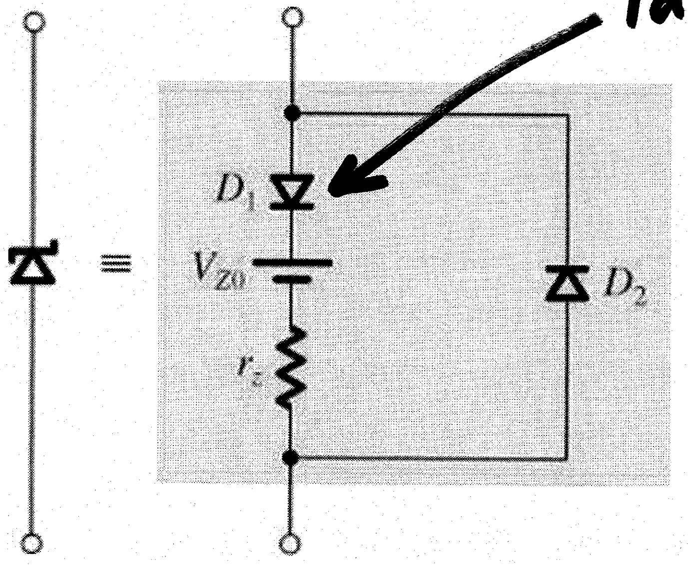
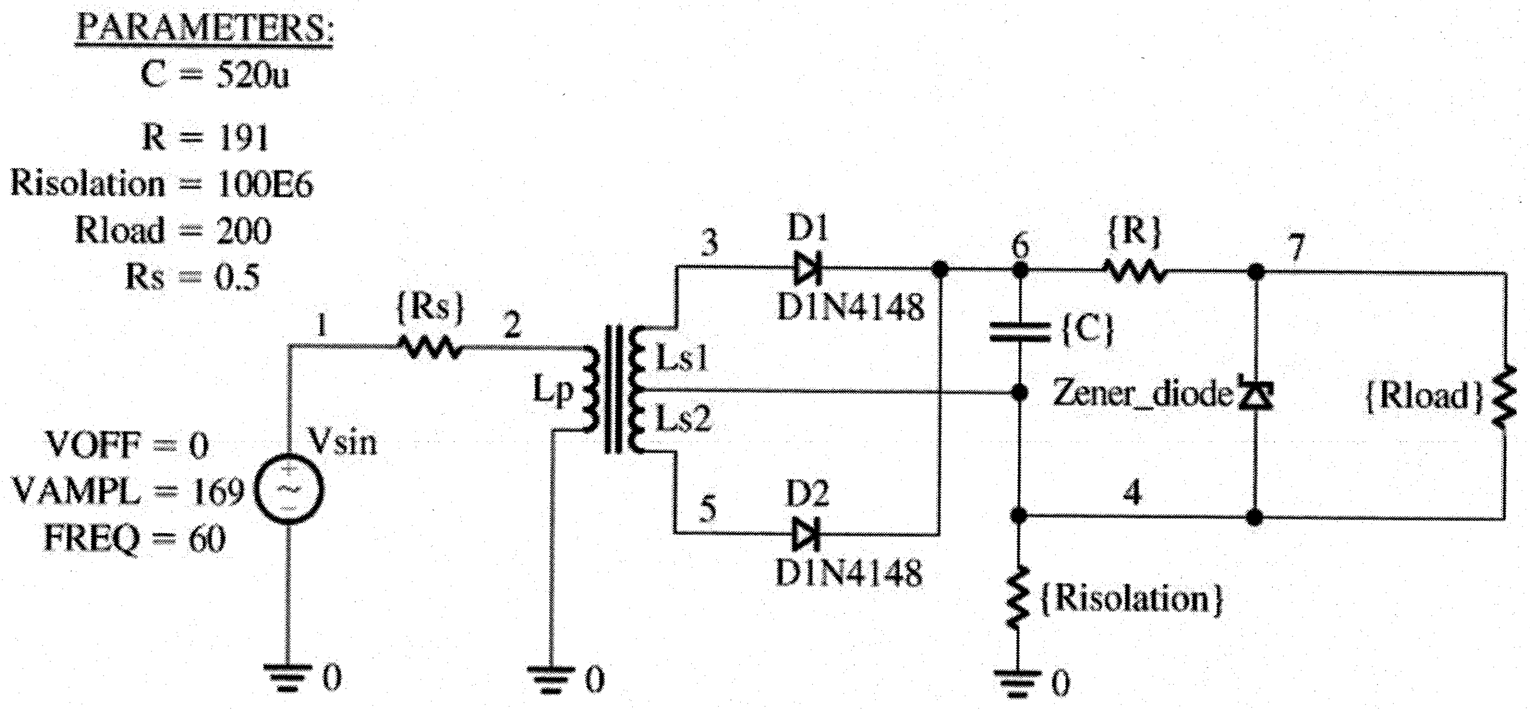


**Figure 3.52** Equivalent-circuit model used to simulate the zener diode in SPICE. Diode $D_1$ is ideal and can be approximated in SPICE by using a very small value for $n$ (say $n = 0.01$).

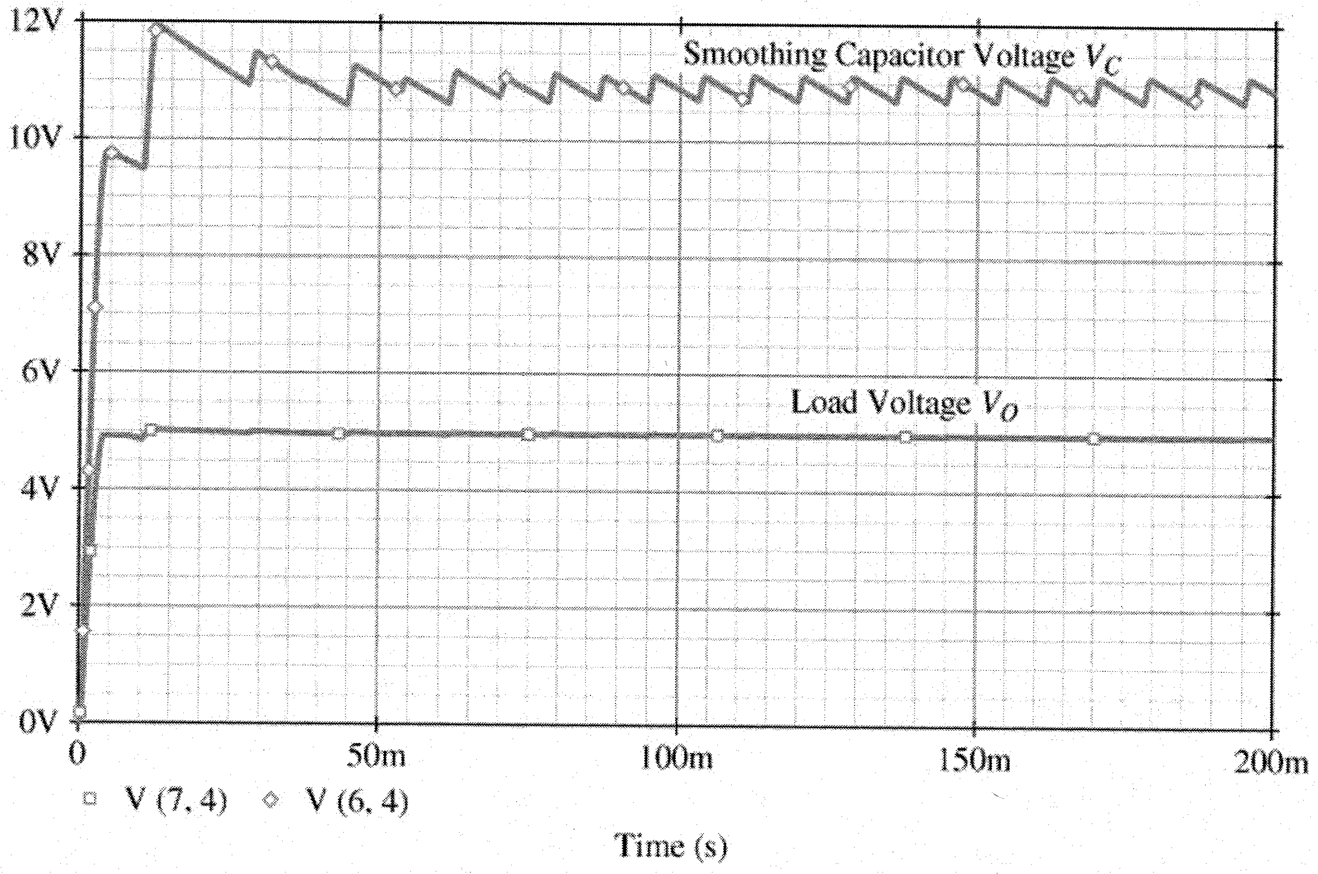
# Check the Spice examples



Full wave rectification + smoothing filter

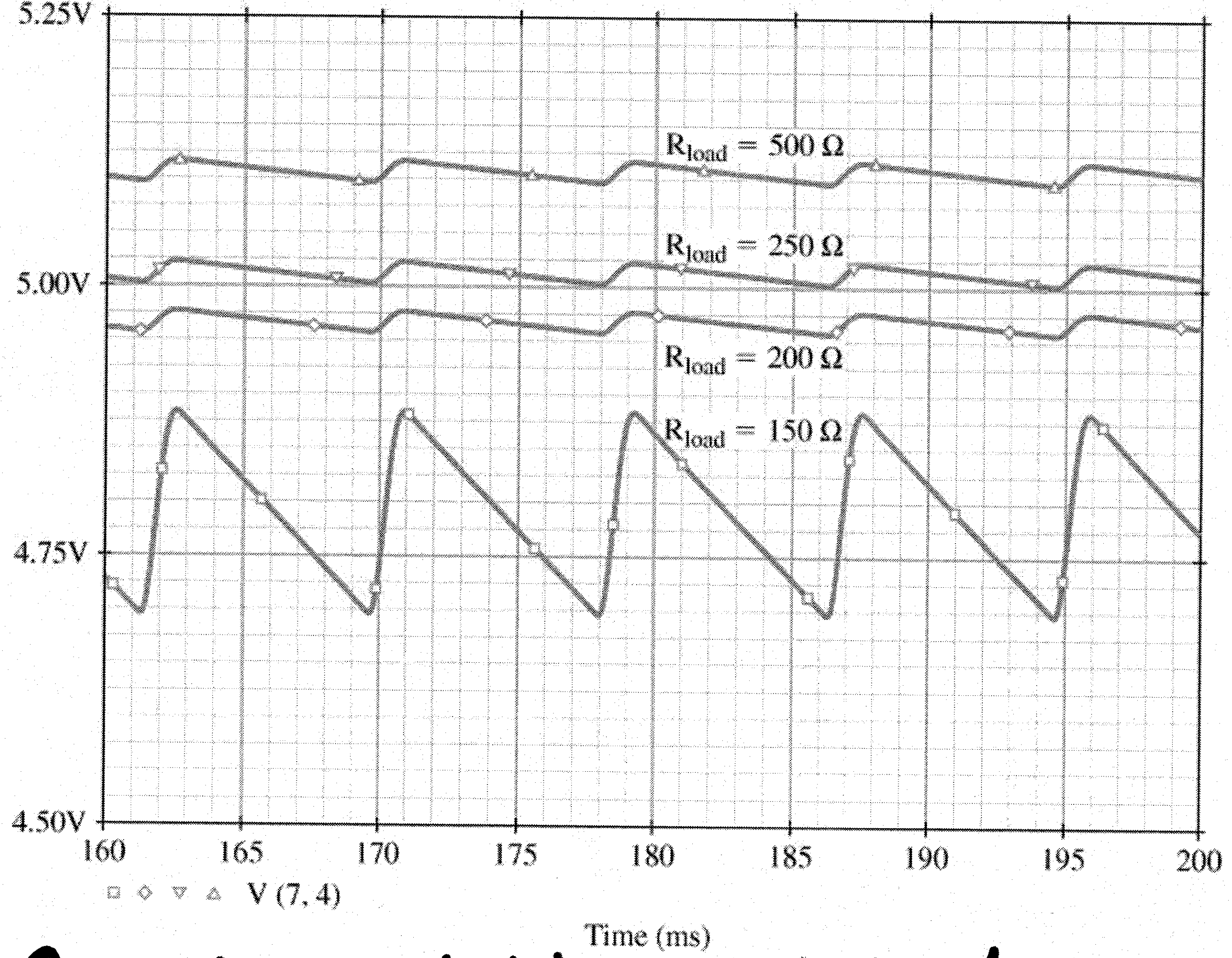
+ zener regulator

**Figure 3.53** Capture schematic of the 5-V dc power supply in Example 3.10.



Transient power supply response

**Figure 3.54** The voltage $v_C$ across the smoothing capacitor $C$ and the voltage $v_O$ across the load resistor $R_{\text{load}} = 200\ \Omega$ in the 5-V power supply of Example 3.10.



Ripple variation with load

**Figure 3.55** The output-voltage waveform from the 5-V power supply (in Example 3.10) for various load resistances: $R_{\text{load}}$ = 500 Ω, 250 Ω, 200 Ω, and 150 Ω. The voltage regulation is lost at a load resistance of 150 Ω.

# Voltage doubler

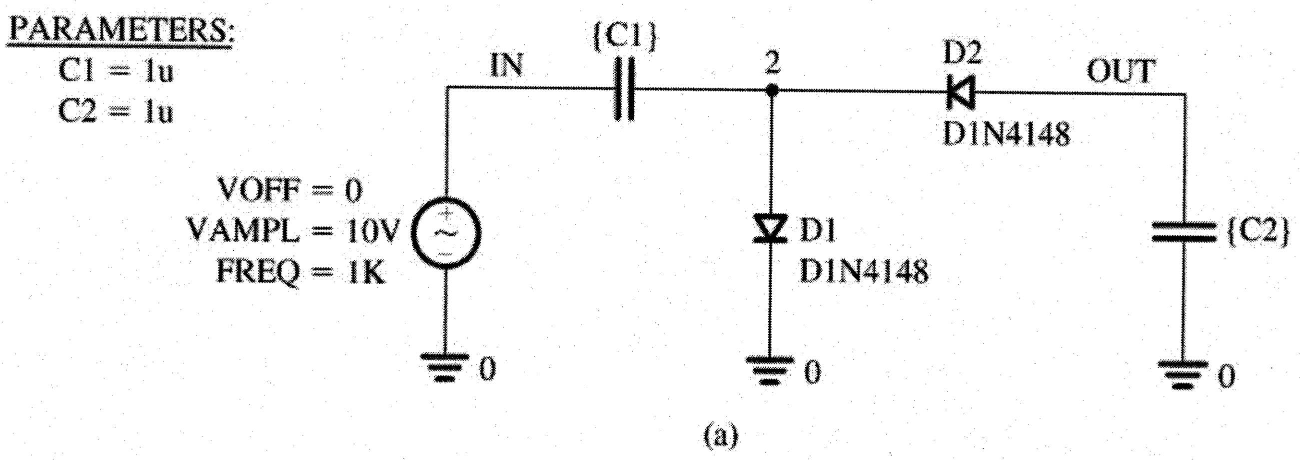
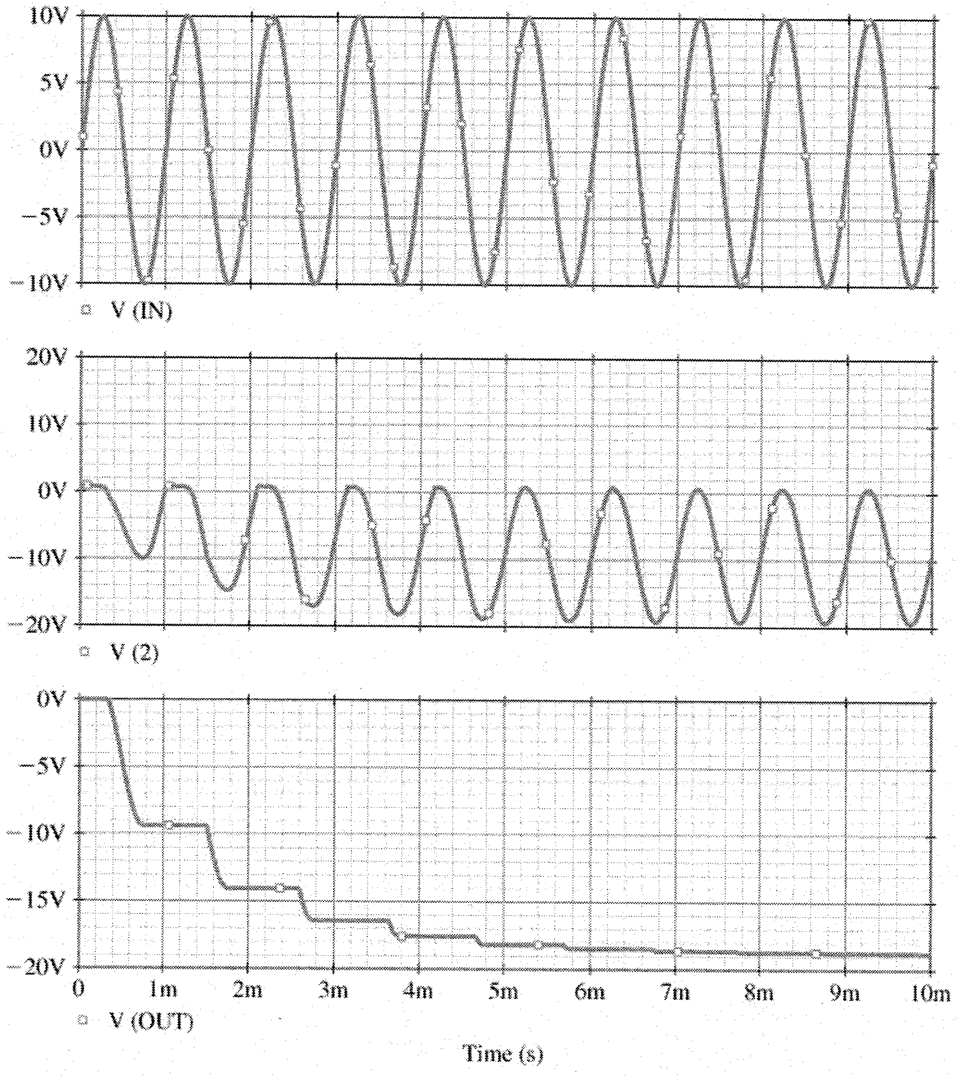


**Figure E3.35** **(a)** Capture schematic of the voltage-doubler circuit (in Exercise 3.35).

Doubler
transient

$V_p = 10v$



(b)

$V_{out} = -20v$

**Figure E3.35** *(Continued)* **(b)** Various voltage waveforms in the voltage-doubler circuit. The top graph displays the input sine-wave voltage signal, the middle graph displays the voltage across diode $D_1$, and the bottom graph displays the voltage that appears at the output.

Ex 3.29 Calculate $n_i$ for Si at 250K, 300K, 350K.

Either $n_i/cc = \sqrt{5.4\times10^{31} \cdot T^3 \cdot \exp - \frac{1.12\times1.6\times10^{-19}}{1.38\times10^{-23}\,T}}$

OR $n_i(300K) = 1.5\times10^{10}/cc \;\therefore\; n_i(T) = (1.5\times10^{10}/cc)\left(\frac{T}{300}\right)^{3/2} \exp\left[-\frac{6493}{300}\left(1-\frac{300}{T}\right)\right]$

gives $n_i(350K) = 41.6\times10^{10}/cc$

and $n_i(250K) = 0.015\times10^{10}/cc$

Ex 3.30 N type Si, $N_D = 10^{17}/cc$. Find $n$ & $p$ at 250K, 300K, 350K.

| Temperature | $n_{NO} \simeq N_D$ | $n_i$ (above) | $n_i^2$ | $p_{NO} = n_i^2/N_D$ |
|---|---|---|---|---|
| 250K | $10^{17}/cc$ | $1.5\times10^8$ | $2.25\times10^{16}$ | $2.25\times10^{-1}/cc$ |
| 300K | $10^{17}/cc$ | $1.5\times10^{10}$ | $2.25\times10^{20}$ | $2.25\times10^{3}/cc$ |
| 350K | $10^{17}/cc$ | $4.2\times10^{11}$ | $17.64\times10^{22}$ | $1.76\times10^{6}/cc$ |

Ex 3.31 Find resistivities $\rho$ of (a) intrinsic & (b) extrinsic Si (P type)

$n_i = 1.5\times10^{10}/cc$ $\mu_n = 1350\,cm^2/Vs$ $\mu_p = 480\,cm^2/Vs$ for intrinsic

$N_A = 10^{16}/cc$ $\mu_n = 1110\,cm^2/Vs$ $\mu_p = 400\,cm^2/Vs$ for extrinsic

(a) Intrinsic $\rho = [1.6\times10^{-19} \cdot 1.5\times10^{10}(1350+480)]^{-1} = 2.28\times10^5\,\Omega.cm$

(b) Extrinsic

$\rho = \frac{1}{q(n_p\mu_n + p_p\mu_p)}$ $n_p = n_i^2/N_A = (1.5\times10^{10})^2/10^{16}$ $= 2.25\times10^4$ & $p_p = N_A$

$\therefore \rho = [1.6\times10^{-19}(2.25\times10^4\times1110 + 10^{16}\times400)]^{-1}$ $= 1.56\,\Omega.cm$

Ex 3.32 PN junction $N_A = 10^{17}/cm^3$ $N_D = 10^{16}/cm^3$

$T = 300K$. Find $V_0$, $W_{depl}$, $x_p$, $x_n$.

$n_i = 1.5\times10^{10}/cc$

$$V_0 = V_T \ln \frac{N_A N_D}{n_i^2} = 25mV \ln \frac{10^{17}\,10^{16}}{2.25\times10^{20}} = 0.728v$$

$$x_n = \left(\frac{2\epsilon_{Si}}{q}\,\frac{N_A/N_D}{N_A+N_D}\,V_0\right)^{1/2}$$

$$= \left[\frac{2\times1.04\times10^{-12}\times0.728}{1.6\times10^{-19}\times1.1\times10^{17}}\right]^{1/2}\sqrt{10}$$

$$= 0.86\sqrt{10}\times10^{-5}cm$$

$$= 0.27\mu m$$

$$x_p = \left(\frac{2\epsilon_{Si}}{q}\,\frac{N_D/N_A}{N_A+N_D}\,V_0\right)^{1/2}$$

$$= \left[\qquad\qquad\right]^{1/2}\sqrt{0.1}$$

$$= 1.087\sqrt{.1}\times10^{-5}cm$$

$$= 0.027\mu m$$

$\therefore W_{depl} = 0.27 + 0.027 = 0.30\mu m$

OR find $W_{depl}$, then $x_n$ & $x_p$

## Ex 3.33 PN junction $N_A = 10^{17}/cm^3$ $N_D = 10^{16}/cm^3$ at 300K

Find (a) $C_{jo}/\mu m^2$ (b) $C_j$ at rev bias $V_R = 2v$ for junction area $2500 \mu m^2$

Assume $n_i \simeq 1.5 \times 10^{10}/cc$ $m = 1/2$ & $V_0 = 0.728v$ (Ex 3.32)

(a) $$C_{jo} = A \left( \frac{\epsilon_{si}\, q}{2} \cdot \frac{N_A N_D}{N_A + N_D} \right)^{1/2} V_0^{-1/2} \rightarrow \underset{\overset{\uparrow}{\mu m^2}}{C_{jo}/A} = 10^{-8} \left\{ \frac{1.6 \times 10^{-19} \; 1.04 \times 10^{-12} \; 10^{17} \times 10^{16}}{2 \times .728 \times 1.1 \times 10^{17}} \right\}^{1/2}$$

$$= 3.22 \times 10^{-16}/\mu m^2$$

$$= 0.322\ fF/\mu m^2$$

(b) $$\therefore C_j = \frac{C_{jo}}{\left(1 + \frac{2}{0.728}\right)^{1/2}} \times 2500\ fF = 415.85\ fF$$

$$= 0.42\ pF$$