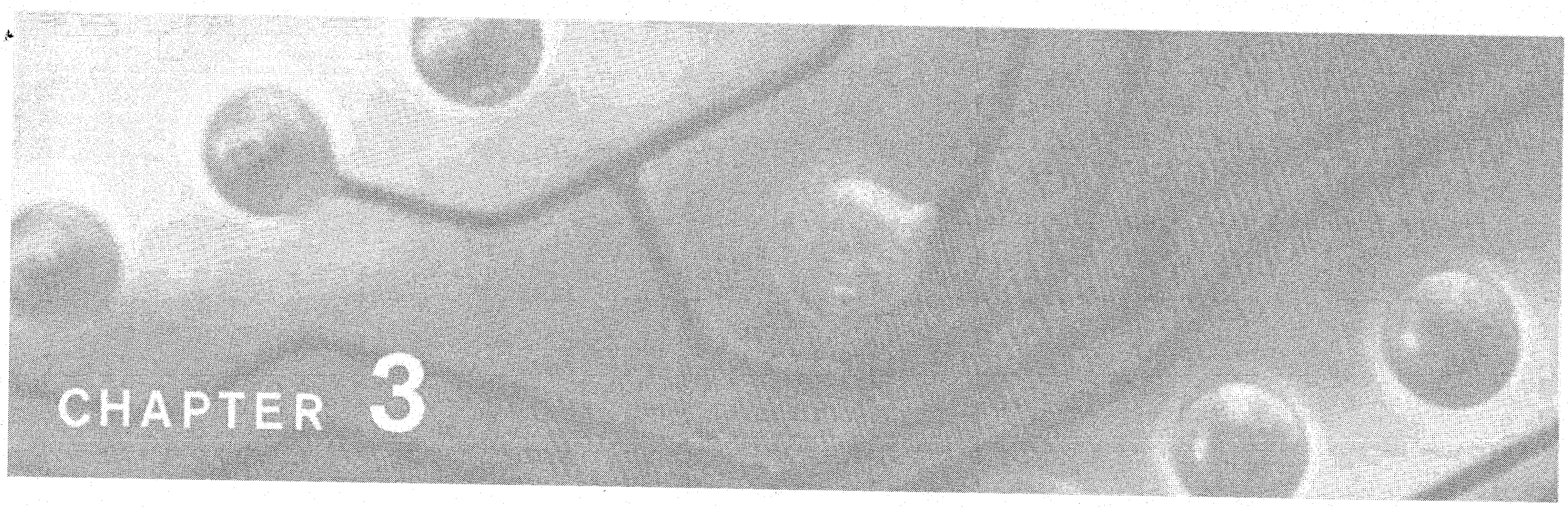
ECE321 Electronics I

Fall 2006

Professor James E. Morris

Lecture 5

10th October, 2006



Diodes ← First non-linear device

MULTIPLE NEW CONCEPTS!!

3.1 Ideal Diodes ← compare introductory ideal op-amp circuits.

3.2 Characteristics

The "real" device, and ----

3.3 Modeling

---- how to deal with it.

Convention —
compare Ohm's Law

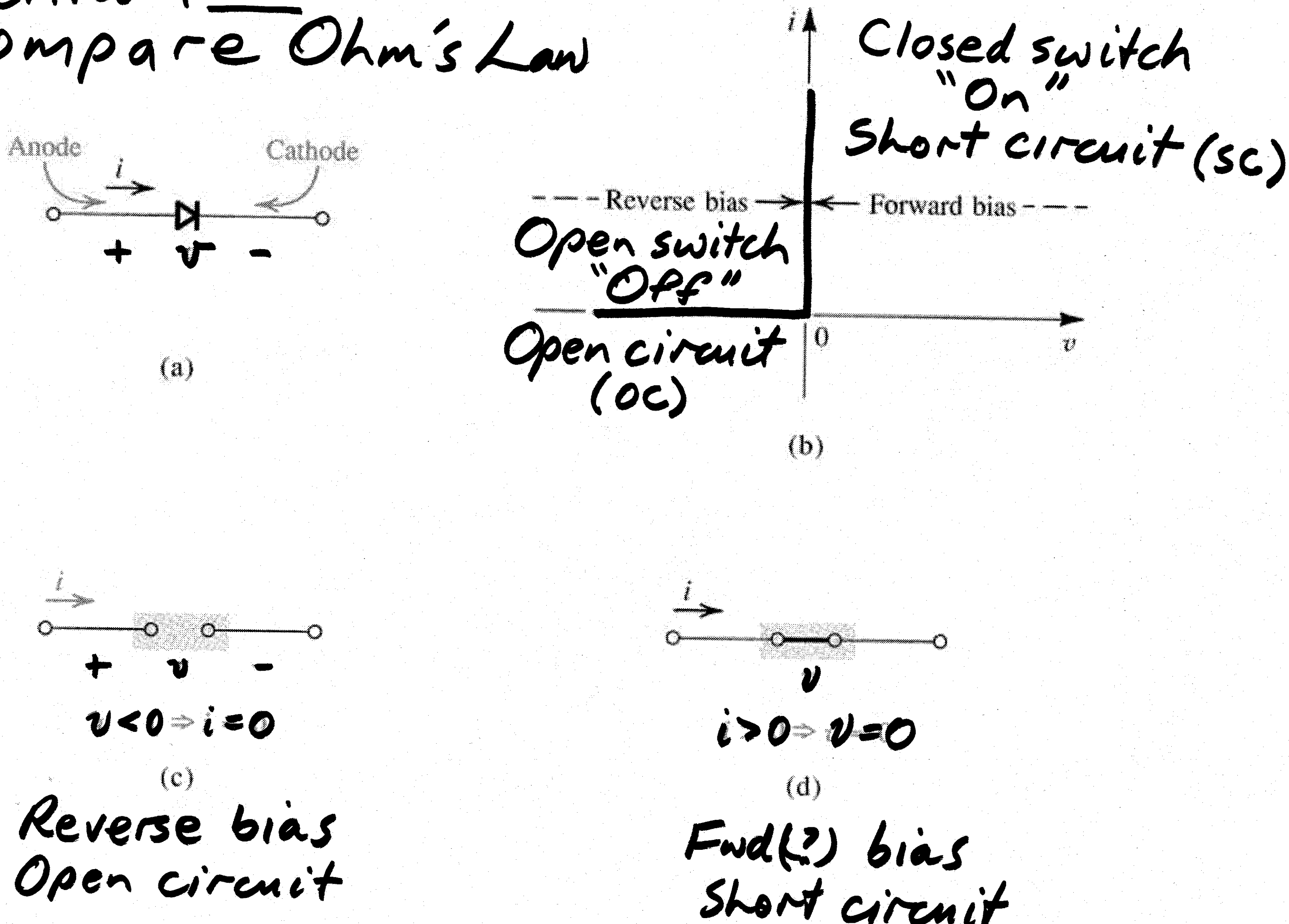
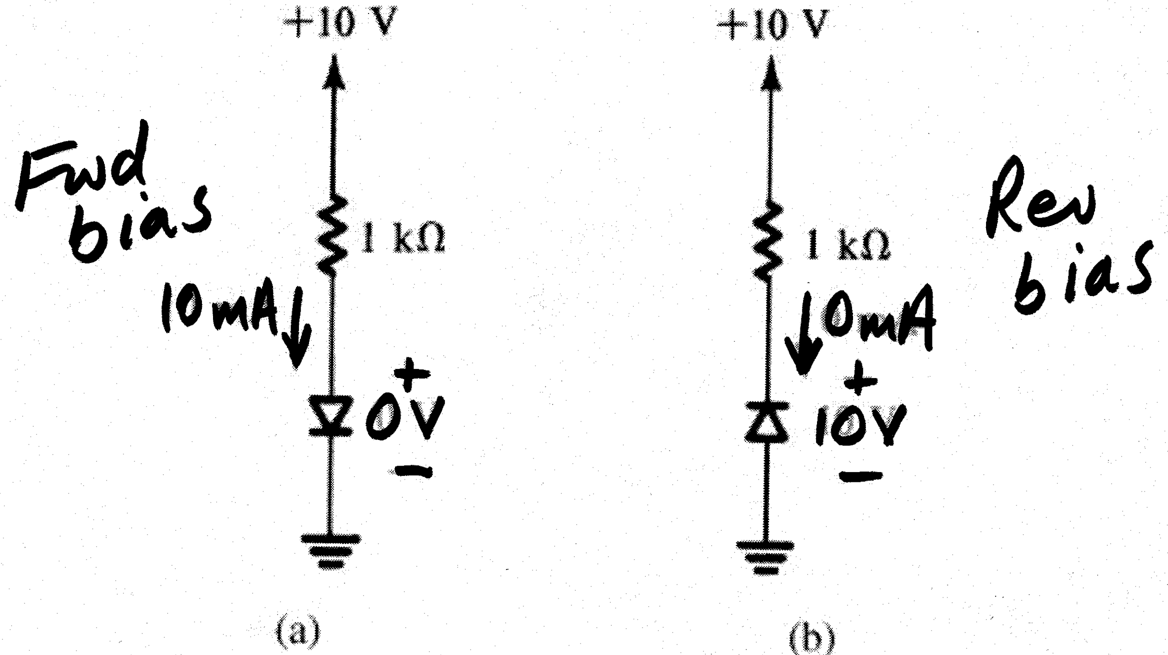


Figure 3.1 The ideal diode: **(a)** diode circuit symbol; **(b)** i–v characteristic; **(c)** equivalent circuit in the reverse direction; **(d)** equivalent circuit in the forward direction.

"Ideal" Model



Non-linear: Can consider operation in 2 regions separately
(linear model for each)

Figure 3.2 The two modes of operation of ideal diodes and the use of an external circuit to limit the forward current **(a)** and the reverse voltage **(b)**.

Application 1. Rectifier

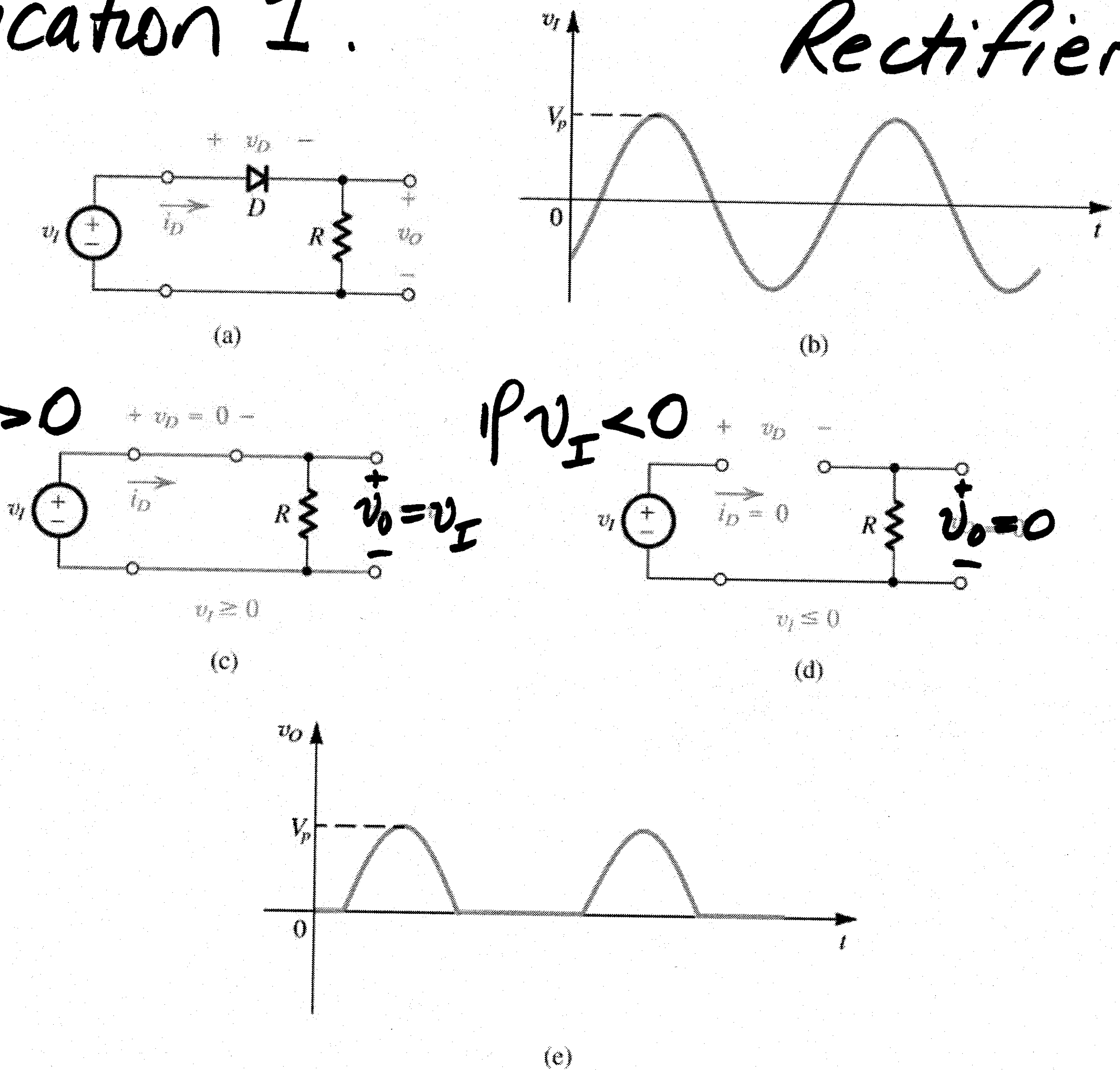
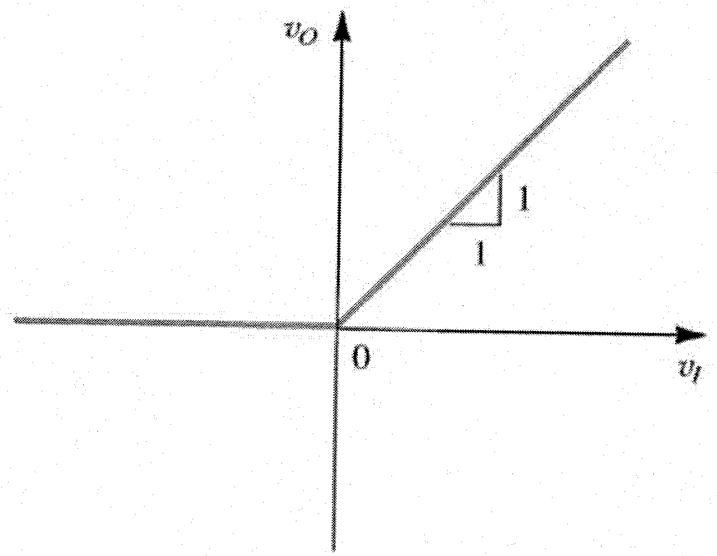


Figure 3.3 **(a)** Rectifier circuit. **(b)** Input waveform. **(c)** Equivalent circuit when $v_I \geq 0$. **(d)** Equivalent circuit when v_I ✧ 0. **(e)** Output waveform.

Ex. 3.1 See prior page — sketch transfer characteristic v_O vs. v_I

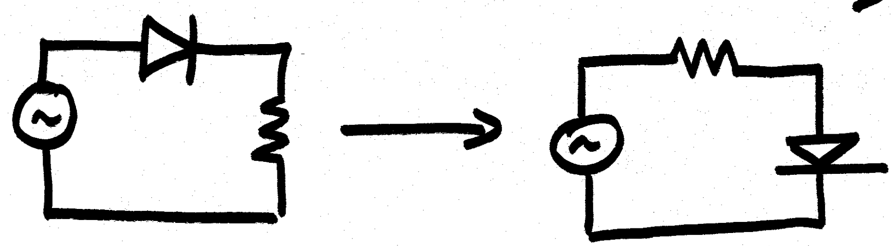
$v_I < 0$
Rev bias
$i = 0$
$\therefore v_O = 0$



$v_I > 0$
Fwd bias
$\therefore v_O = v_I$

Figure E3.1

Ex. 3.2. Prior circuit — sketch $v_D(t)$



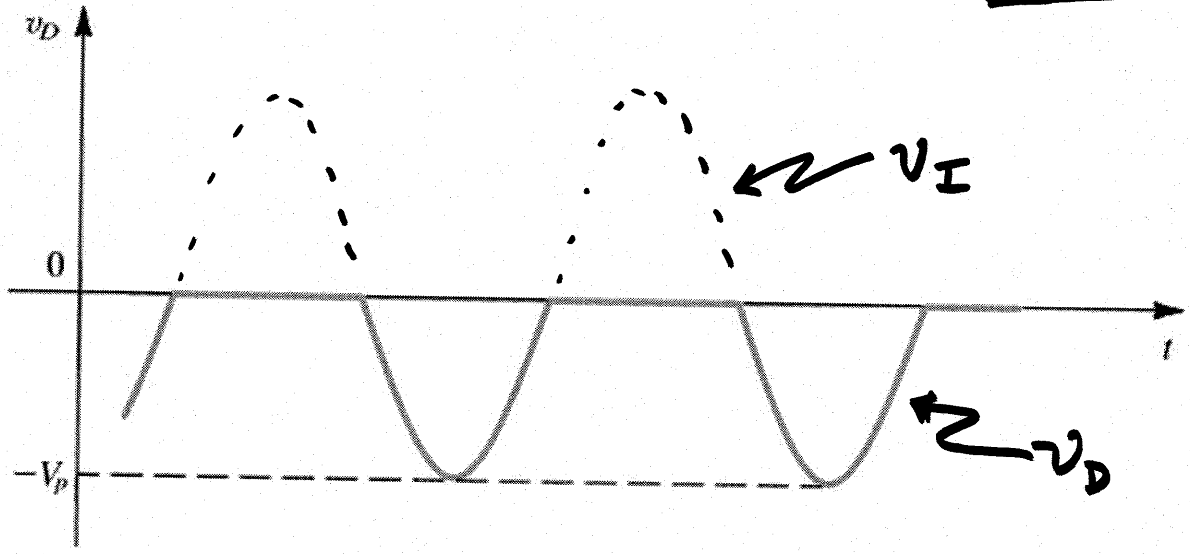
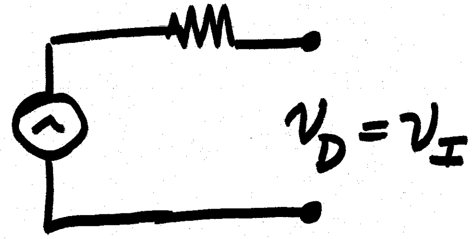
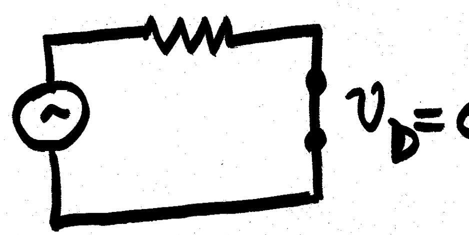


Figure E3.2

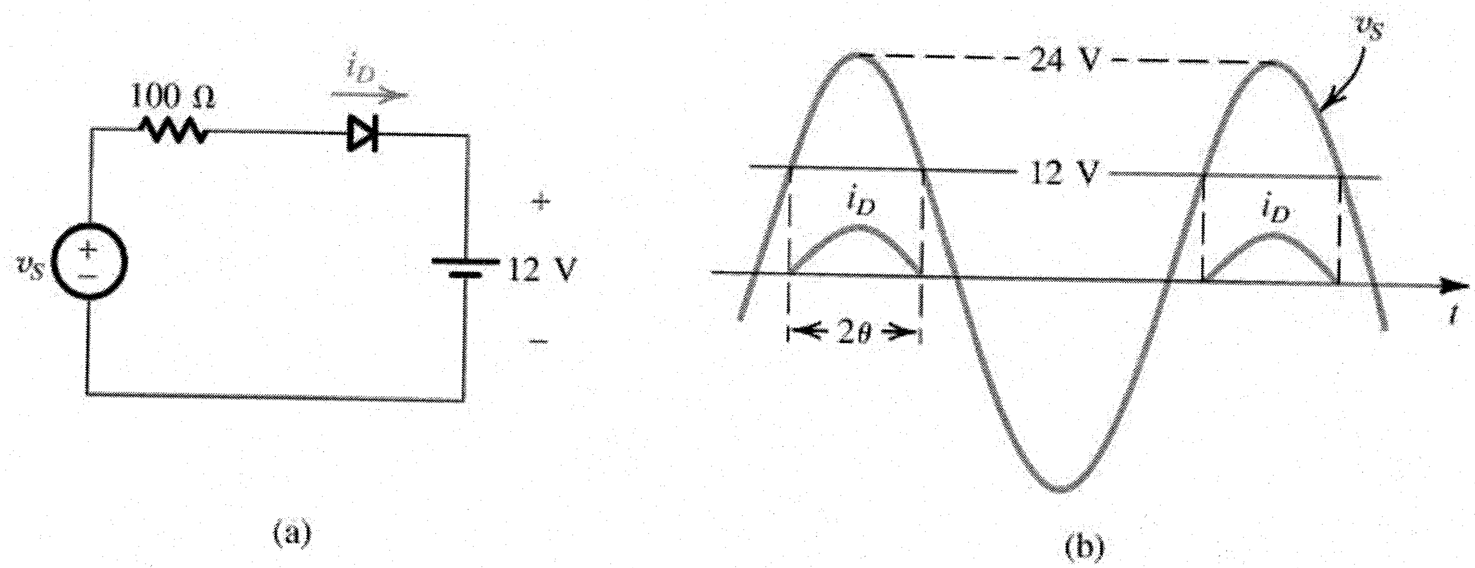
$v_I < 0$ Diode off



$v_I > 0$ Diode on



Example 3.1



D on only when $v_I > 12_V$

& $i_D = \dfrac{v_I - 12_V}{100\Omega}$ for $v_I > 12_V$ ONLY

$= 0$ for $v_I < 12_V$

Figure 3.4 Circuit and waveforms for Example 3.1.

Application 2. Diode Logic

$\sim 0\,V \longrightarrow$ "0"
$\sim 5\,V \longrightarrow$ "1"

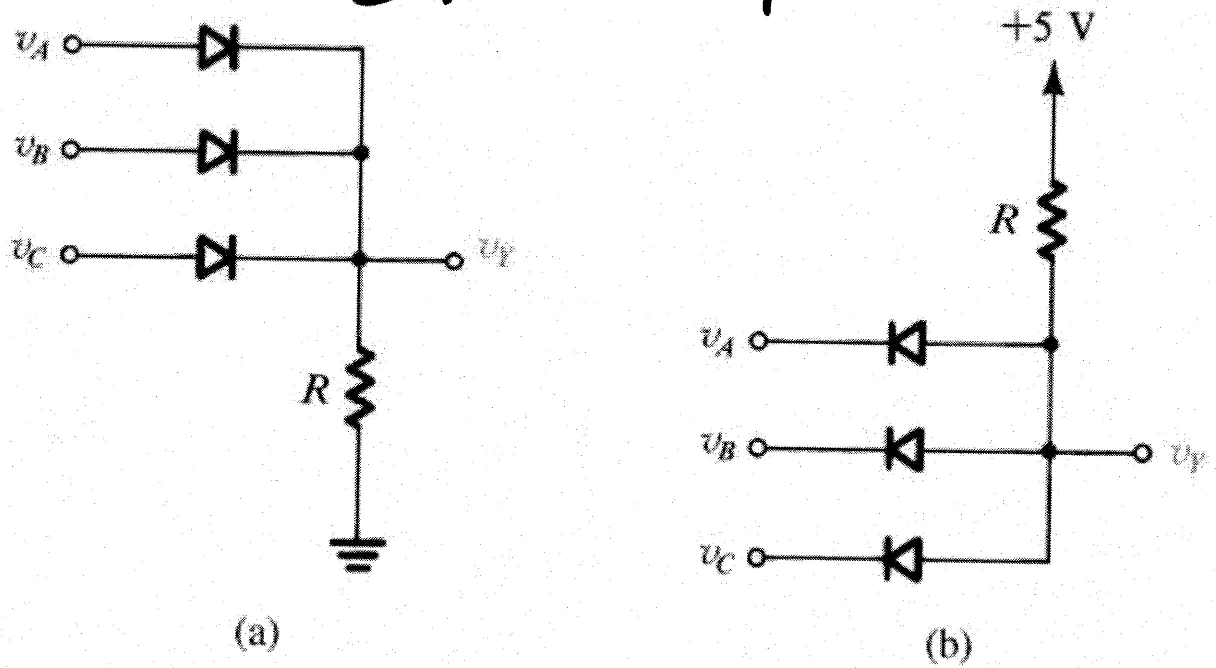


Figure 3.5 Diode logic gates: **(a)** OR gate; **(b)** AND gate (in a positive-logic system).

See Table

(a)

v_A	v_B	v_C	D_A	D_B	D_C	v_y
5v	5v	5v	ON	ON	ON	5v
		0v			OFF	5v
	0v	5v		OFF	ON	5v
		0v			OFF	5v
0v	5v	5v	OFF	ON	ON	5v
		0v			OFF	5v
	0v	5v		OFF	ON	5v
		0v			OFF	0v

$$Y = A + B + C$$

Any input 5v biases diode ON; current flows & v_y = 5v

(b)

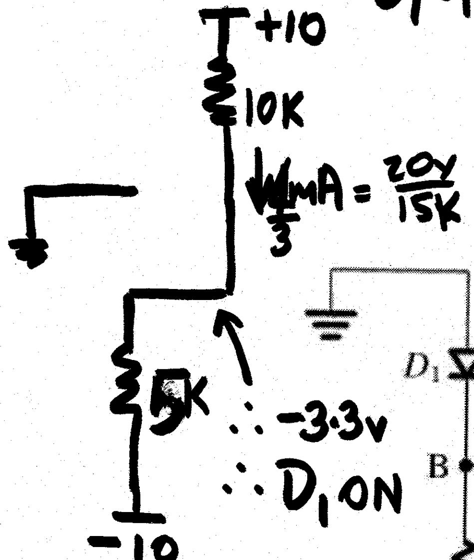
v_A	v_B	v_C	D_A	D_B	D_C	v_y
5v	5v	5v	OFF	OFF	OFF	5v
		0v			ON	0v
	0v	5v		ON	OFF	0v
		0v			ON	0v
0v	5v	5v	ON	OFF	OFF	0v
		0v			ON	0v
	0v	5v		ON	OFF	0v
		0v			ON	0v

$$Y = A \cdot B \cdot C$$

v_y = 5v requires all diodes OFF; no current flows & needs all inputs = 5v

Example 3.2

(a) Assume D_1 off D_2 on



∴ -3.3v
∴ D_1 ON

Conflict!
Try D_1 on D_2 on
(works)

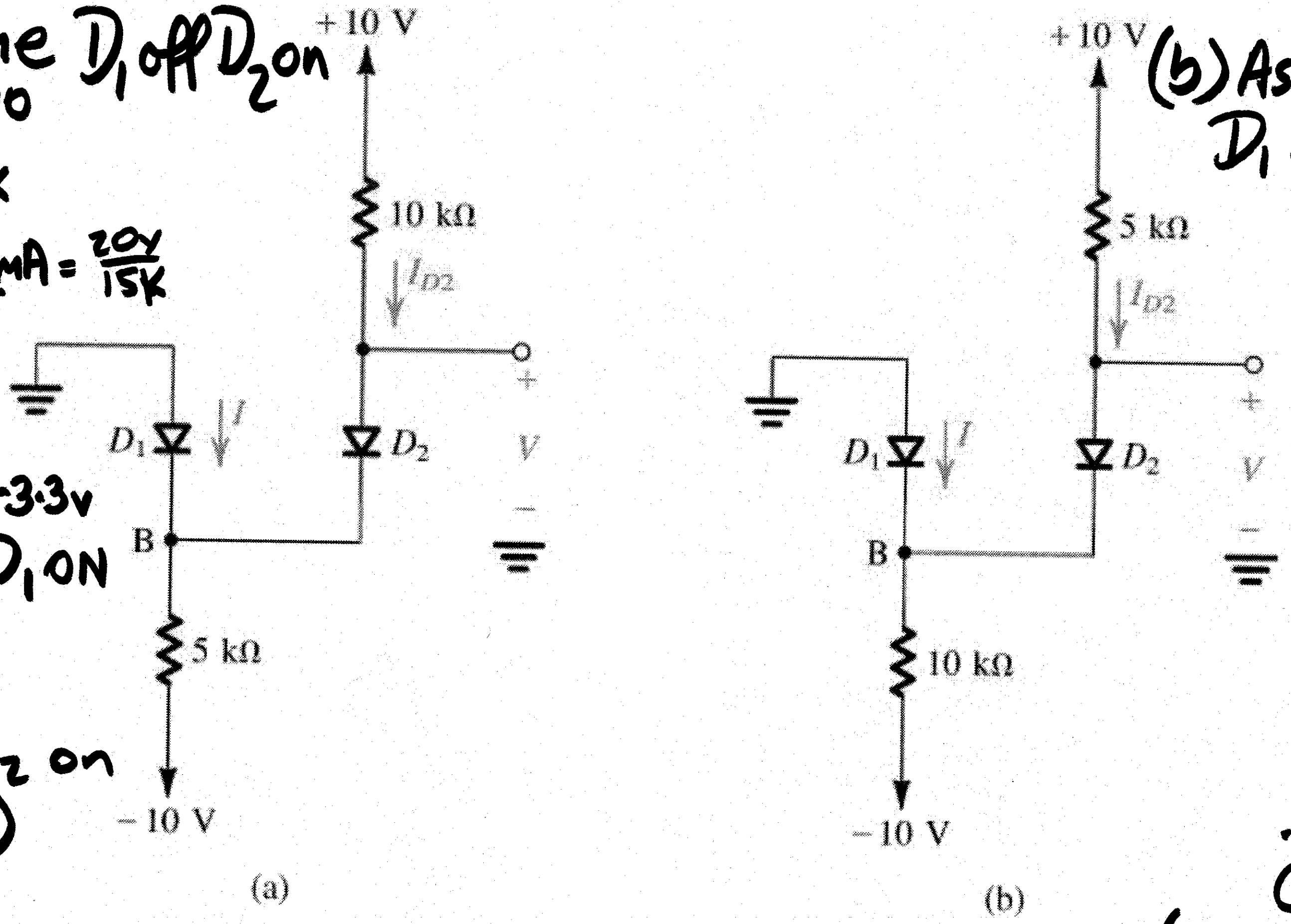
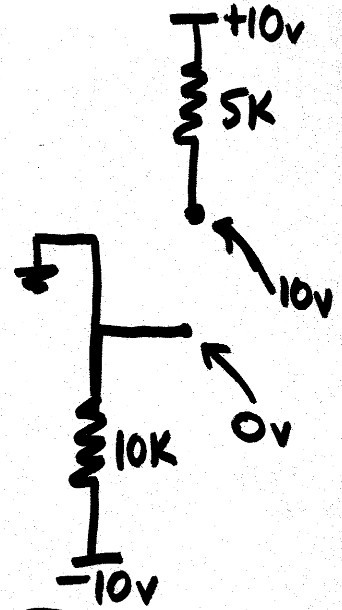


Figure 3.6 Circuits for Example 3.2.

(b) Assume
D_1 on D_2 off



∴ D_2 on,
Conflict!
(D_1 off D_2 on works)

Main point here is:
(1) Assume "state," eg. D_1 on, D_2 off
(2) Analyze circuit, (voltages & currents)
for $D_{ON} \rightarrow$ s.c. & $D_{OFF} \rightarrow$ o.c. eg D_1 s.c. D_2 o.c.
(3) Check result against assumption; if conflict, revise assumption

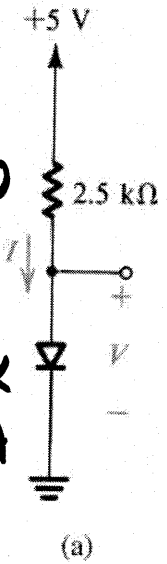
Ex. 3.4.

Fwd bias

$\therefore V = 0$

&

$I = \frac{5V}{2.5K} = 2mA$



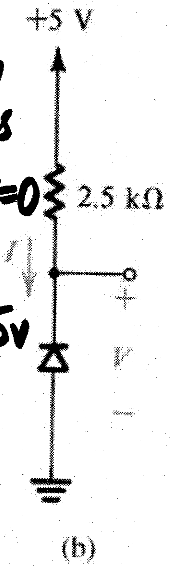
(a)

Rev bias

$\therefore I = 0$

&

$V = 5V$

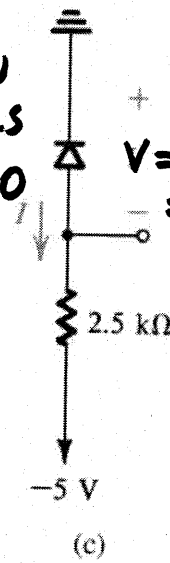


(b)

Rev bias

$\therefore I = 0$

$V = 0_V - (-5V) = +5V$

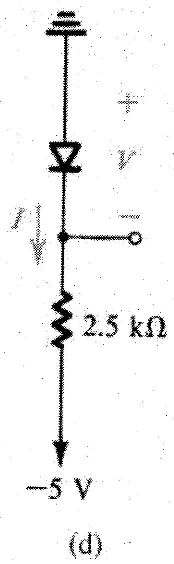


(c)

Fwd bias

$\therefore V = 0$

& $I = \frac{0-(-5V)}{2.5K} = 2mA$



(d)

Assume D_1 on, D_2 & D_3 off

$\therefore V = 1V$

$\therefore D_2$ & D_3 on

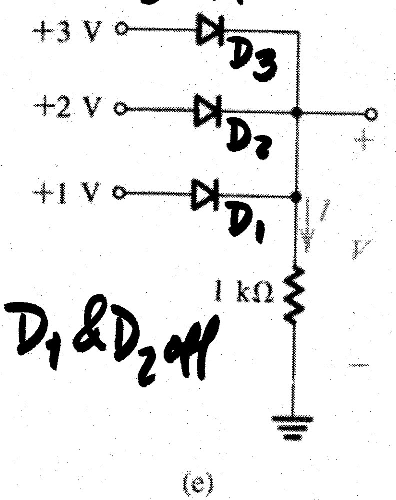
Try again:

Assume D_3 on, D_1 & D_2 off

$\therefore V = 3V$

$\therefore D_2, D_3$ off

OK.



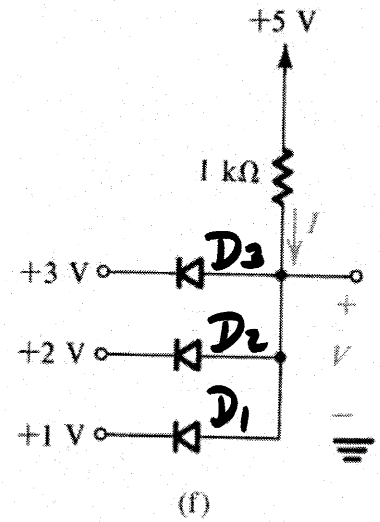
(e)

Assume D_1 on, D_2 & D_3 off

$\therefore V = +1V$

$\therefore D_2$ & D_3 off

OK.

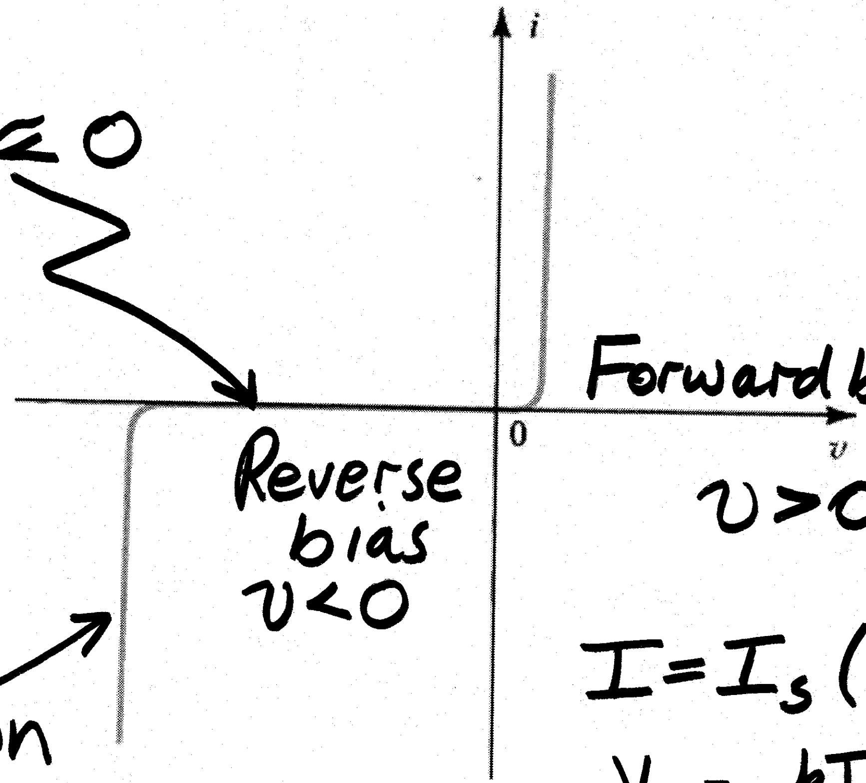


(f)

Figure E3.4

Ex. 3.5

Actual Characteristics



$I \approx 0$

Forward bias

$v > 0$

Reverse bias

$v < 0$

Breakdown

$$I = I_s \left(\exp \frac{v}{nV_T} - 1 \right)$$

$V_T = kT/q$ $\approx 25\,mV$ at $25°C$

$k = 1.38 \times 10^{-23}\,J/K$ Boltzmann's constant

$q = 1.6 \times 10^{-19}\,C$ electronic charge

T absolute temperature

V_T is the "thermal" voltage

$n = 1 \rightarrow 2$

Typically $n=1$ in ICs, $n=2$ discretes.

Figure 3.7 The i–v characteristic of a silicon junction diode.

I_s: Reverse saturation current

OR Scale current ($\propto$ area)

$$I = I_s \left(\exp \frac{v}{nV_T} - 1 \right)$$

$$\approx I_s \exp \frac{v}{nV_T} \qquad \text{for } v >> nV_T,\; I >> I_s$$

$$\text{so} \quad v = nV_T \ln (I/I_s)$$

Now for $I_1(V_1)$ and $I_2(V_2)$

$$V_2 = nV_T \ln(I_2/I_s)$$
$$V_1 = nV_T \ln(I_1/I_s)$$

$$V_2 - V_1 = nV_T \ln(I_2/I_1)$$
$$\approx nV_T \cdot 2.3 \log_{10}(I_2/I_1)$$

I Increases decade as V increases 60–120 mV (depends on V_T)

For "Fully" conducting diode at $V \sim 0.6 \rightarrow 0.8$ volts

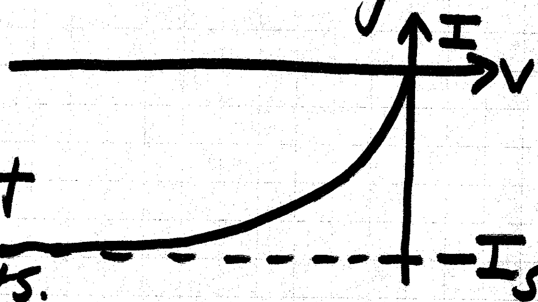
→ negligible current for $V \lesssim 0.5$v

i.e. "cut-in" voltage — turn-on voltage

For $v < 0$ $\quad I = -I_s \left(1 - \exp \frac{-|v|}{nV_T} \right)$

Theoretical reverse "saturation" current

In fact $I_{rev} >> I_s$ due to 2nd order effects.



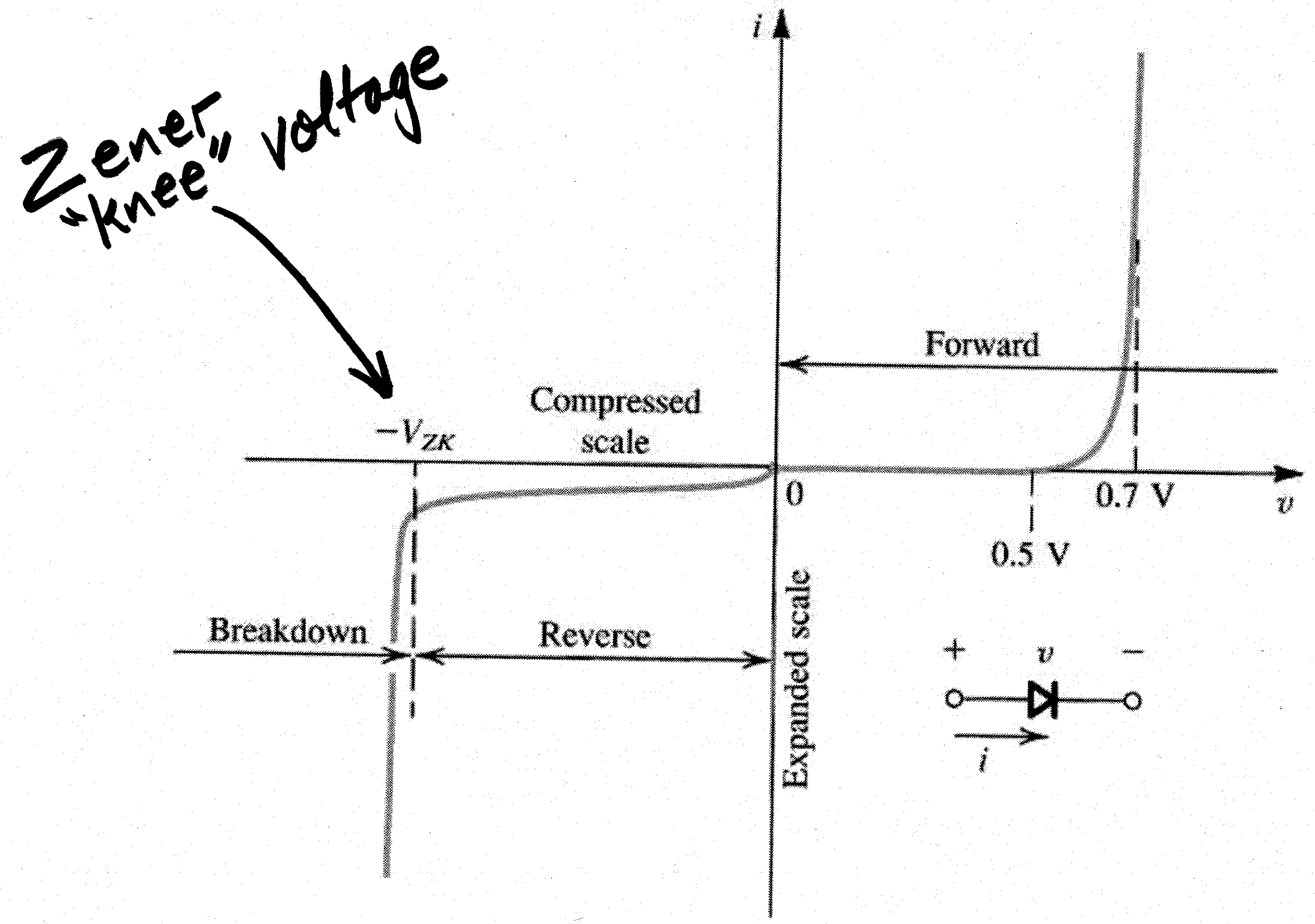
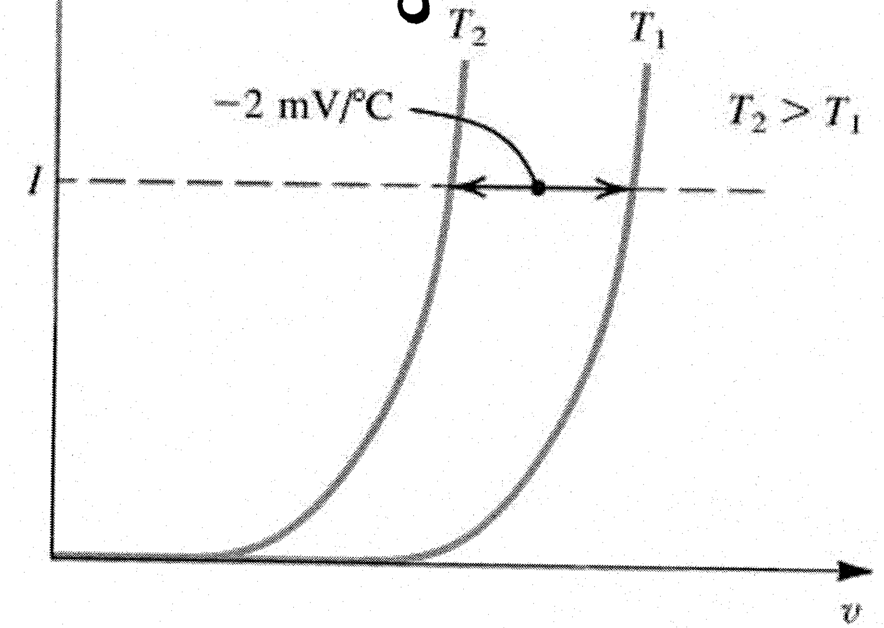


Figure 3.8 The diode i–v relationship with some scales expanded and others compressed in order to reveal details.

Thermal Effects

Voltage TC



OR I_S doubles ~ every 5°C increase in T (for fixed V)

Figure 3.9 Illustrating the temperature dependence of the diode forward characteristic. At a constant current, the voltage drop decreases by approximately 2 mV for every 1°C increase in temperature.

Ex. 3.6

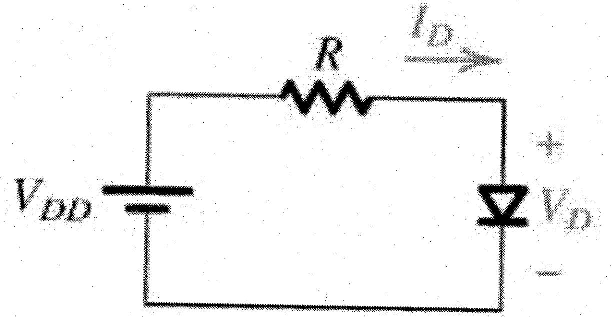
Ex. 3.7

Ex. 3.8

Ex. 3.9

Modeling Diode

1. Exponential model



$$I_D \approx I_S \exp V_D/nV_T$$

$$I_D = (V_{DD} - V_D)/R \longrightarrow V_D = V_{DD} - I_D R$$

$$\therefore I_D = I_S \exp (V_{DD} - I_D R)/nV_T$$

Transcendental Equation in I_D

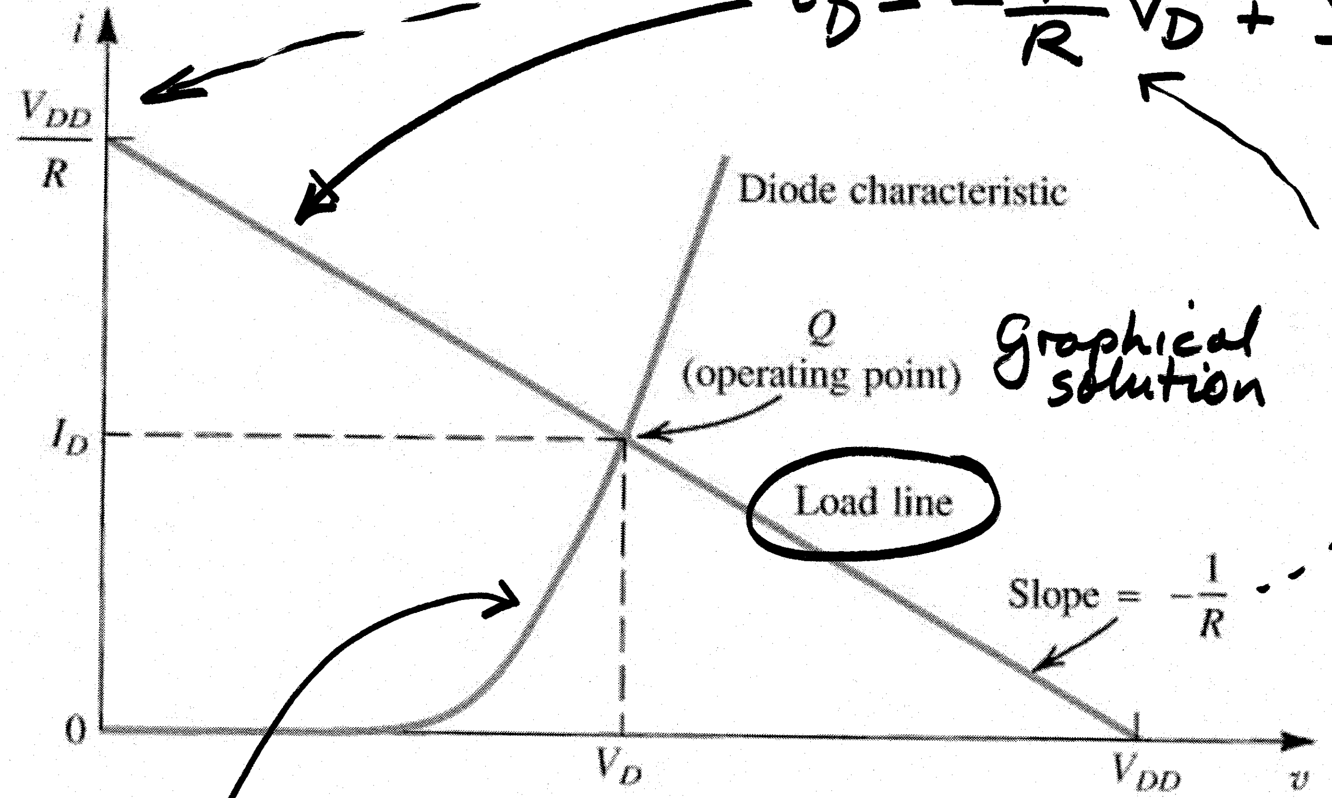
Figure 3.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.

No closed solution

Hence other techniques

(a) Graphical solution

$i_D = -\frac{1}{R} V_D + \frac{V_{DD}}{R}$



$i_D \approx I_s \exp V_D/nV_T$

Figure 3.11 Graphical analysis of the circuit in Fig. 3.10 using the exponential diode model.

(b) Iterative Solution

Example 3.4 : $V_{DD} = 5\text{v}$ $R = 1K\Omega$ Diode 1mA at 0.7v

& $\Delta V = 0.1\text{v}$ per decade current

$V_2 - V_1 = 2.3\, nV_T \log_{10} I_2/I_1$

$\therefore$ $2.3 nV_T = 0.1\text{v}$

So $V_2 = V_1 + 0.1 \log I_2/I_1$

Iteration 1: Assume $V_1 = 0.7\text{v}$

$$\therefore I_1 = \frac{V_{DD} - V_D}{R} = \frac{5 - 0.7}{1K} = 4.3\text{mA}$$

Iteration 2:

$\therefore$ Re-calc $V_2 = 0.7 + 0.1 \log(4.3\text{mA}/1\text{mA}) = 0.7633\text{v}$

$$\& \; I_2 = \frac{5 - 0.763}{1K}\text{v} = 4.237\text{mA}$$

Iteration 3:

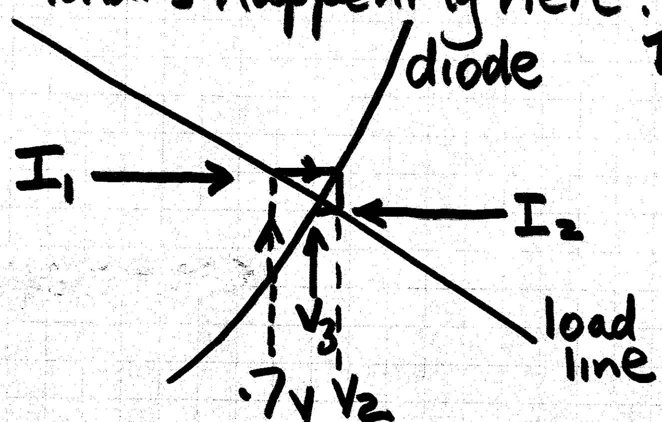
Re-calc $V_3 = 0.7 + 0.1 \log(4.237)$ OR $0.763 + 0.1 \log\left(\frac{4.237}{4.3}\right)$

$= 0.7627\text{v} \approx 0.763\text{v}$

$$I_3 = \frac{5 - .7627}{1K} \approx 4.237\text{mA}$$

$\therefore$ Converged.

What's happening here?



Assume V

Then Calculate I from load line

" " V " diode

" " I " load line

etc.

Caution:

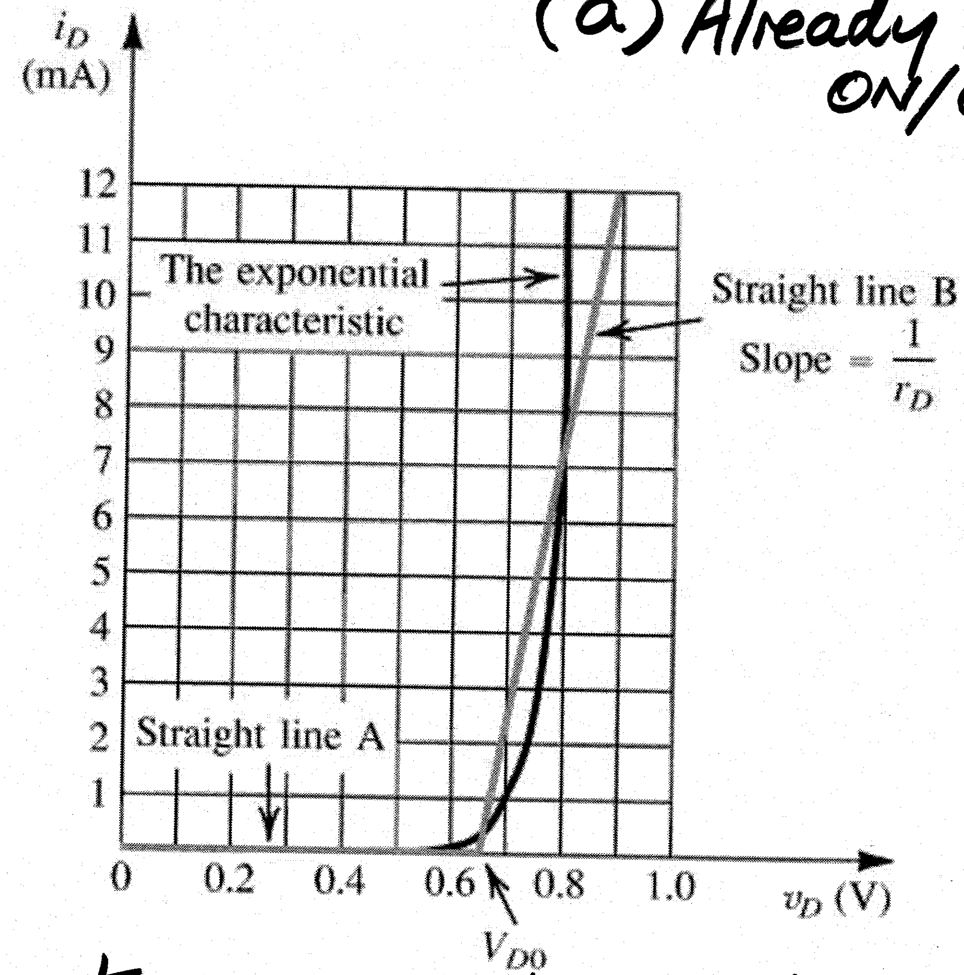
Spiral direction $R > r_D$

Depends on I, V from load line or diode

Opposite for $R < r_D$.

Linear Piecewise Models

(a) Already saw the "ideal" ON/OFF switch model

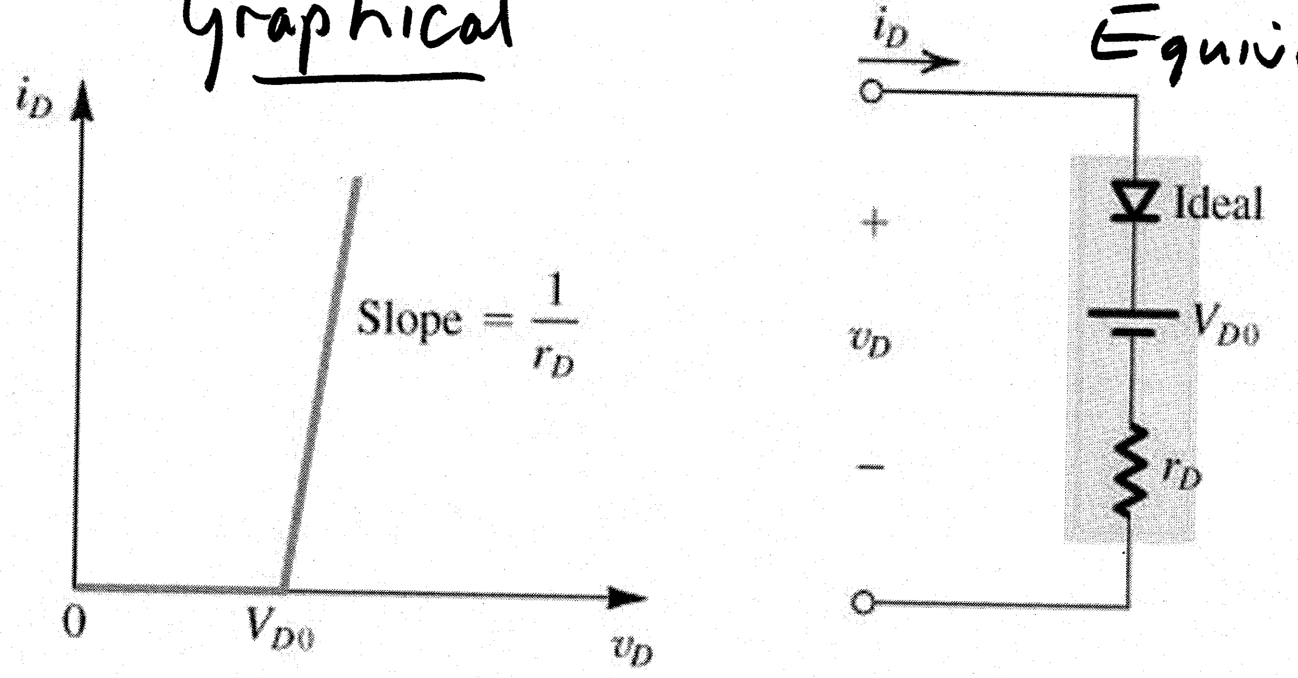


(b) Add turn-on voltage V_{DO} (or "cut-in") and forward slope $1/r_D$

Figure 3.12 Approximating the diode forward characteristic with two straight lines: the piecewise-linear model.

Graphical

Equivalent circuit

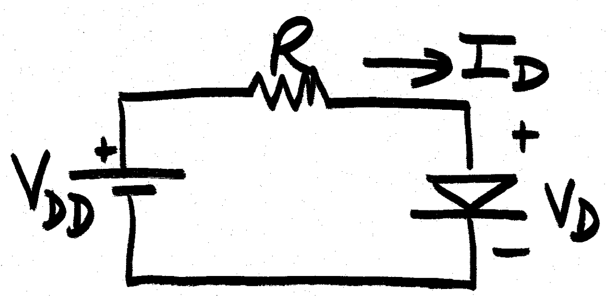


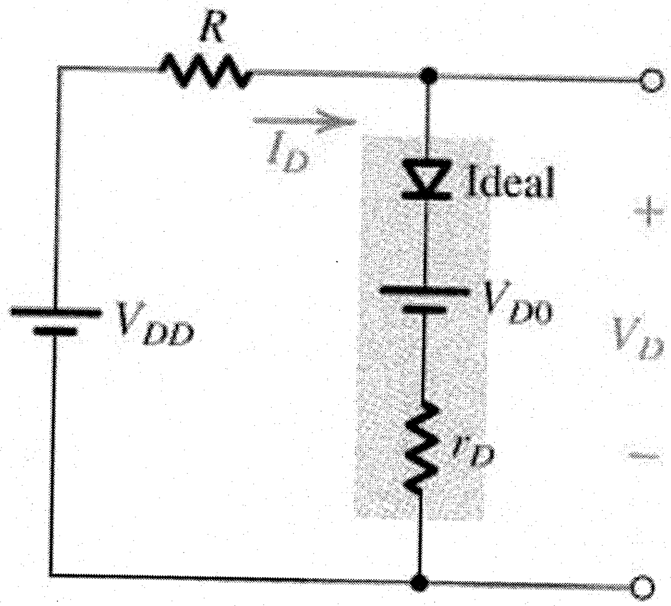
(a) (b)

$$i_D = \frac{v_D - V_{D0}}{r_D} \quad \text{for } v_D > V_{D0}$$

$$= 0 \quad \text{for } v_D < V_{D0}$$

Figure 3.13 Piecewise-linear model of the diode forward characteristic and its equivalent circuit representation.





Replace diode by equivalent circuit
i.e. by model

Figure 3.14 The circuit of Fig. 3.10 with the diode replaced with its piecewise-linear model of Fig. 3.13.

(c) Simplification:
Constant Voltage $V_D (\approx 0.7v)$

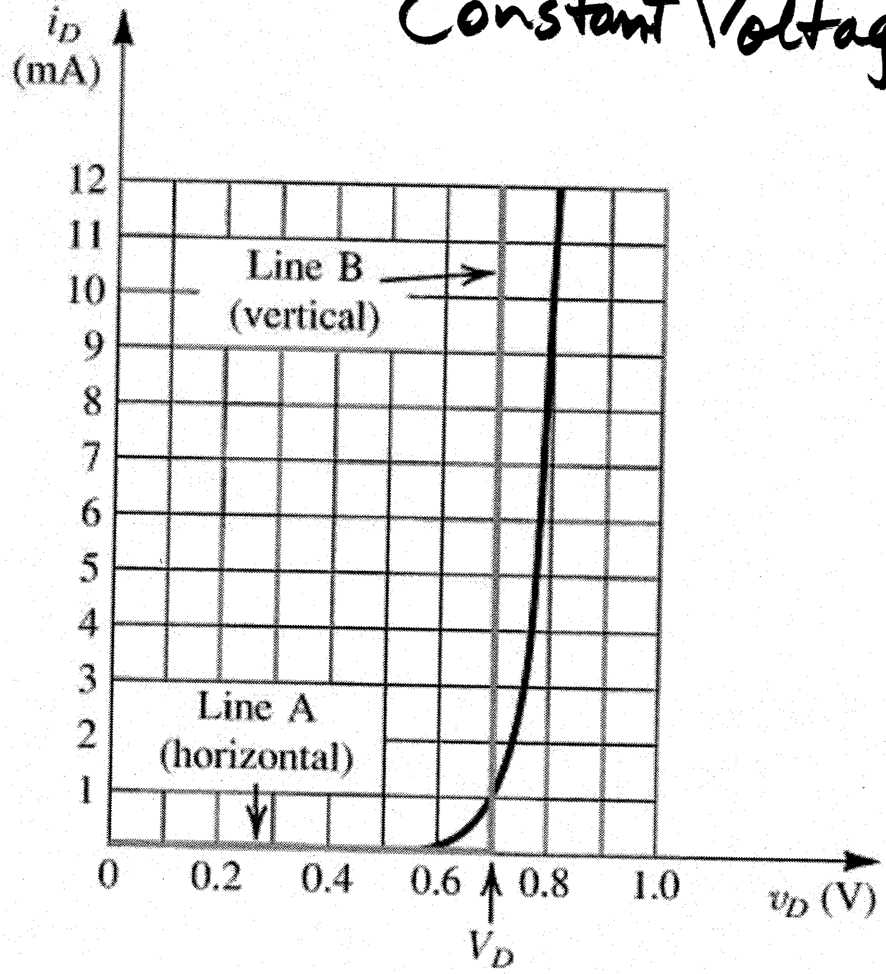
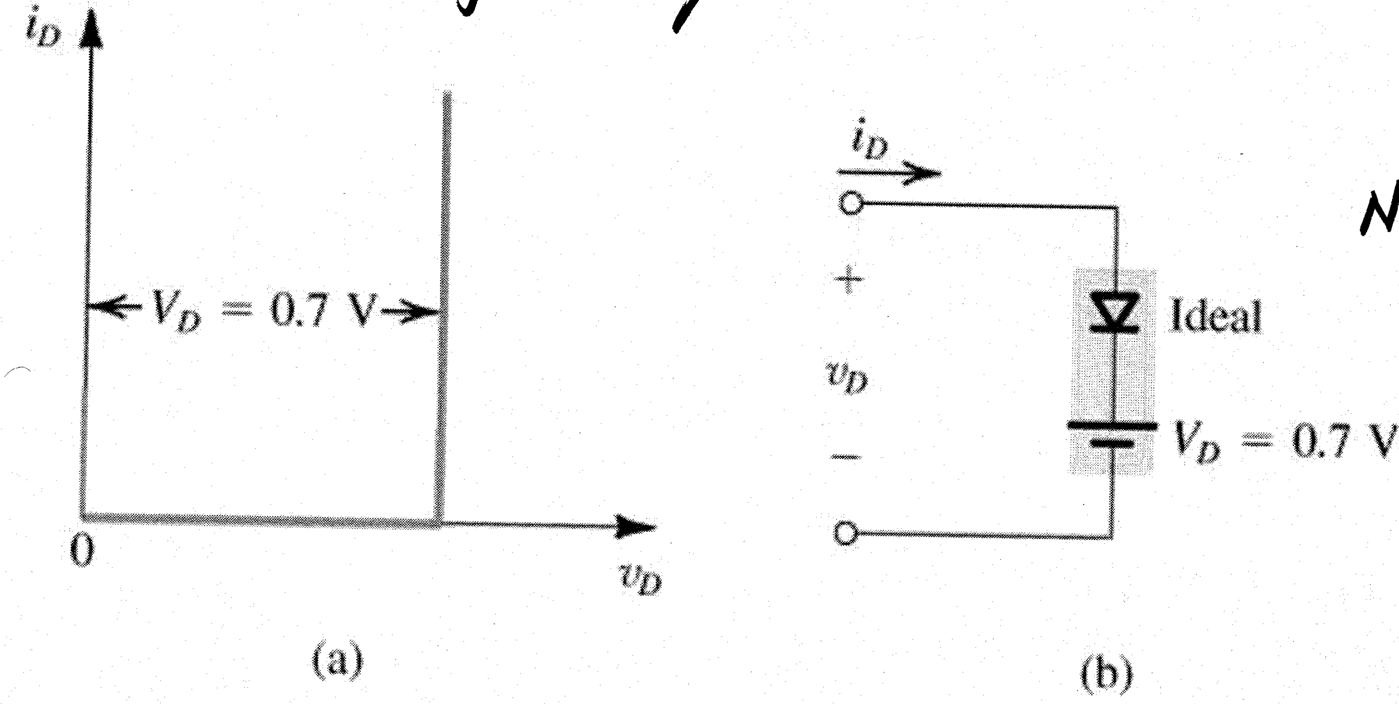


Figure 3.15 Development of the constant-voltage-drop model of the diode forward characteristics. A vertical straight line (B) is used to approximate the fast-rising exponential. Observe that this simple model predicts V_D to within ± 0.1 V over the current range of 0.1 mA to 10 mA.

Constant Voltage Equivalent Circuit



Note: $r_D \rightarrow 0$

(d) Ideal $\longrightarrow$ set $V_{DO} = 0$ here

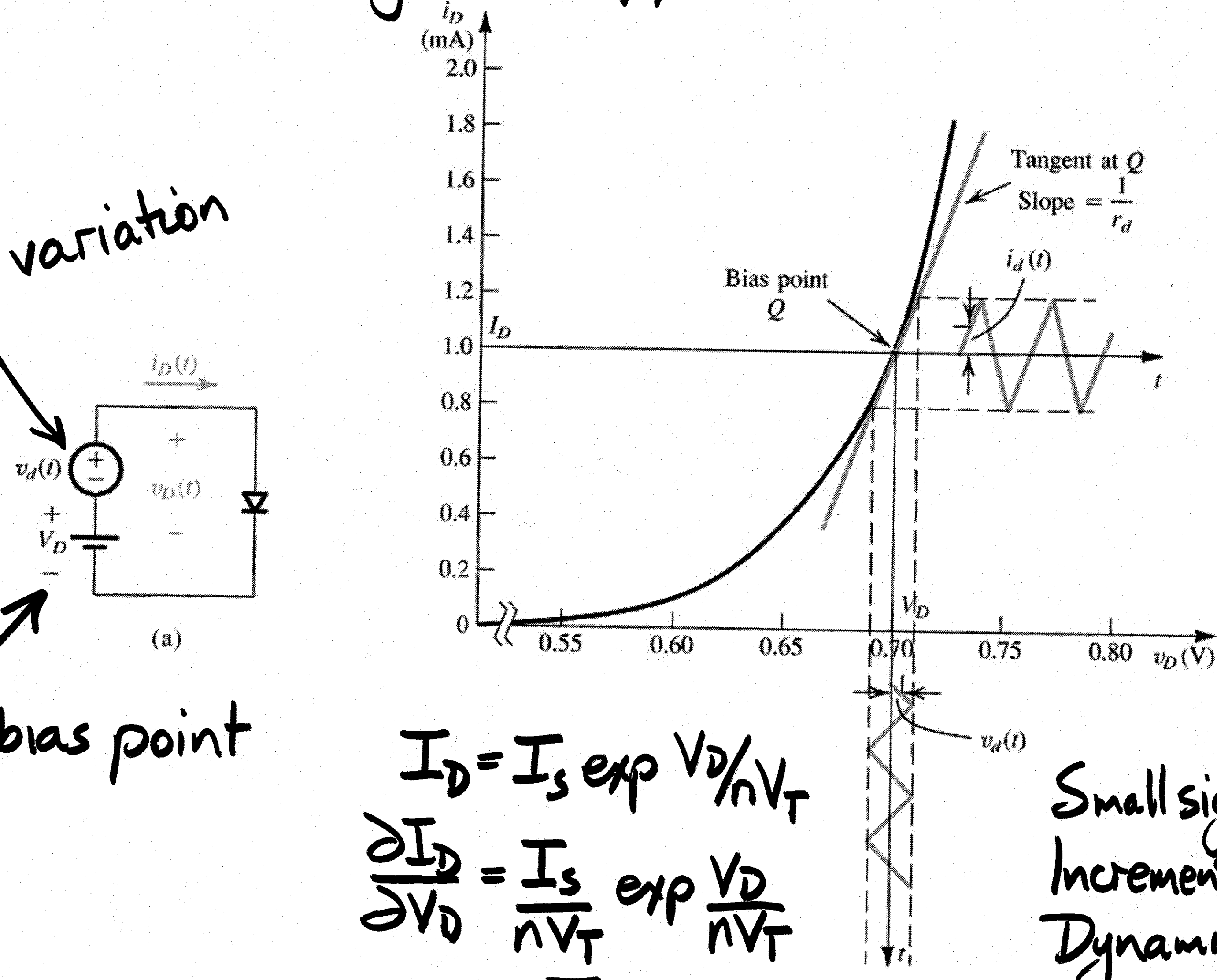
Figure 3.16 The constant-voltage-drop model of the diode forward characteristics and its equivalent-circuit representation.

Ex. D3.12

3. Small Signal Approximation

AC variation

DC bias point



$$I_D = I_s \exp V_D / nV_T$$

$$\frac{\partial I_D}{\partial V_D} = \frac{I_s}{nV_T} \exp \frac{V_D}{nV_T}$$

$$= \frac{I_D}{nV_T}$$

Small signal resistance

Incremental resistance

Dynamic resistance

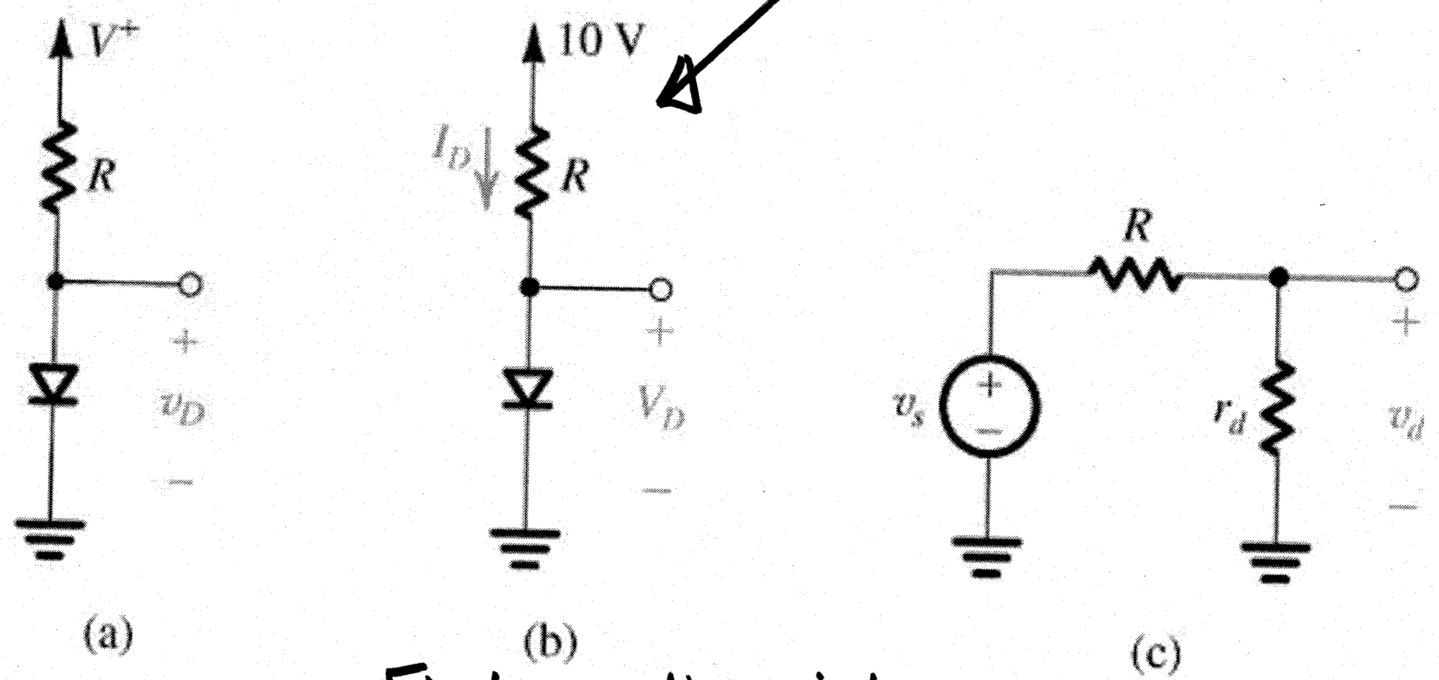
AC resistance

Figure 3.17 Development of the diode small-signal model. Note that the numerical values shown are for a diode with $n = 2$.

$$\therefore \; r_D = \frac{\partial V_D}{\partial I_D} = \frac{nV_T}{I_D}$$

Example 3.6

DC



Find operating point Q (V_D, I_D)

Find r_D at Q

$$r_D = \frac{nV_T}{I_D}$$

$$v_d = \frac{r_d}{R + r_d} v_s$$

Figure 3.18 **(a)** Circuit for Example 3.6. **(b)** Circuit for calculating the dc operating point. **(c)** Small-signal equivalent circuit.

Ex. D3.16

Exponential → Linear piecewise

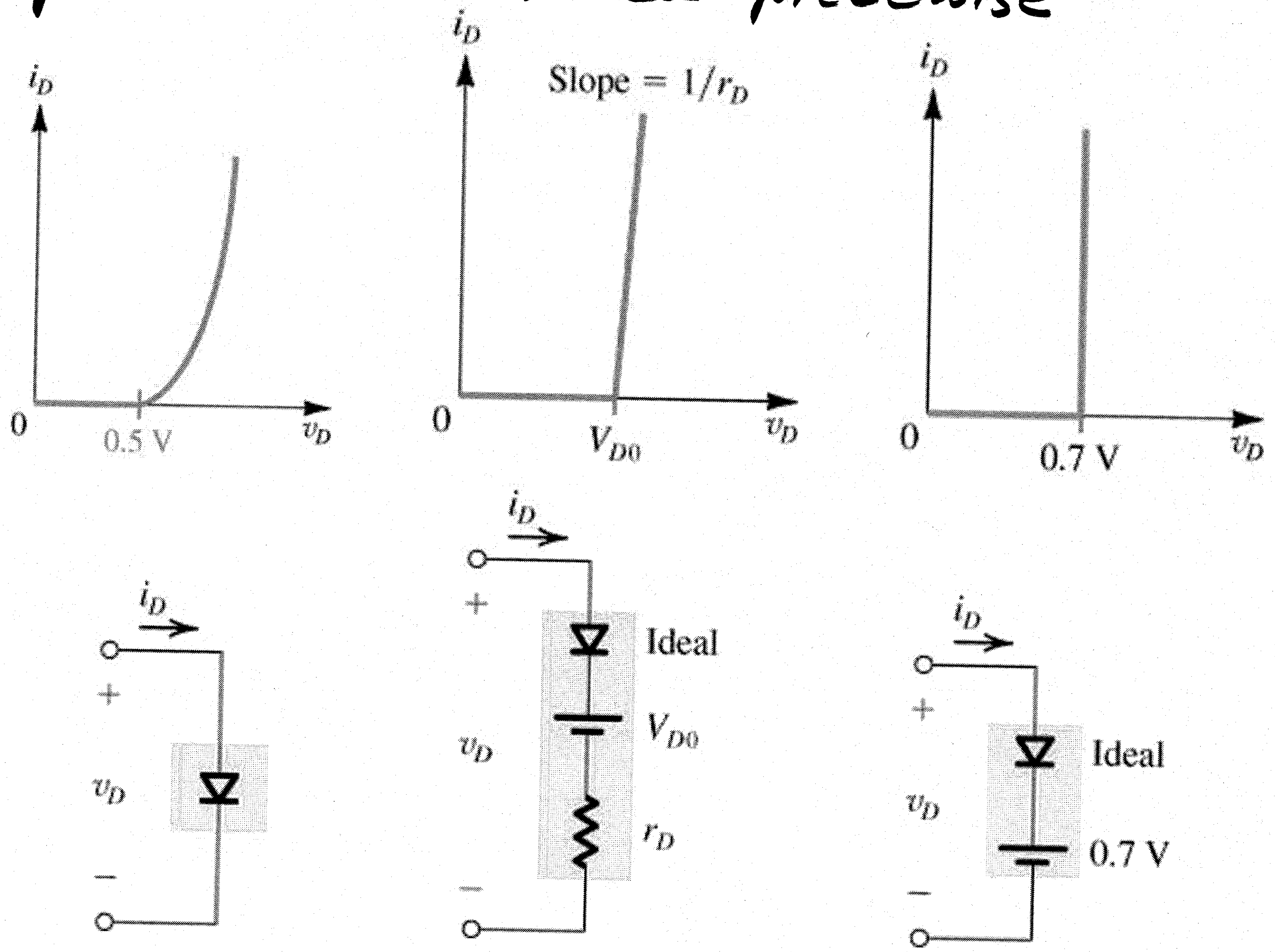
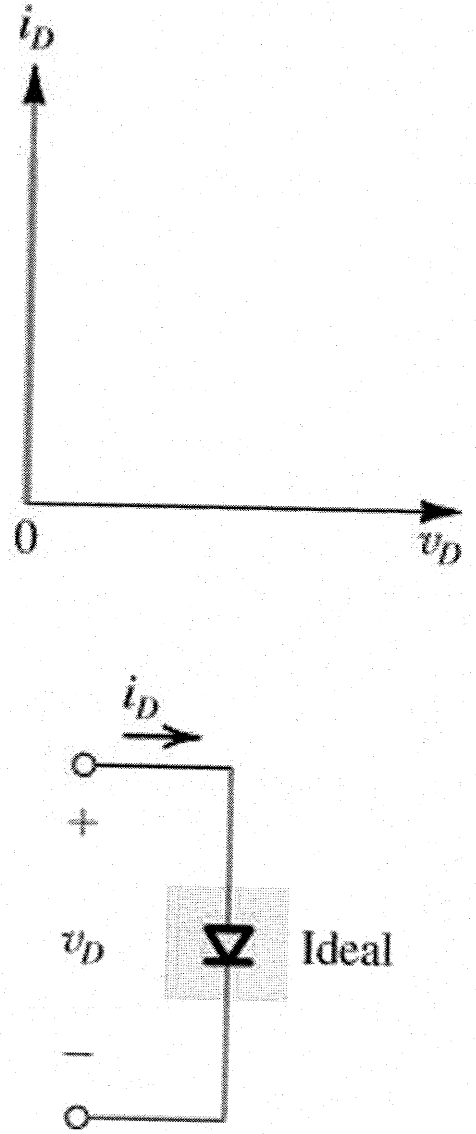


Table 3.1 Modeling the Diode Forward Characteristic

MODELING SUMMARY

(Ideal)



Small - signal

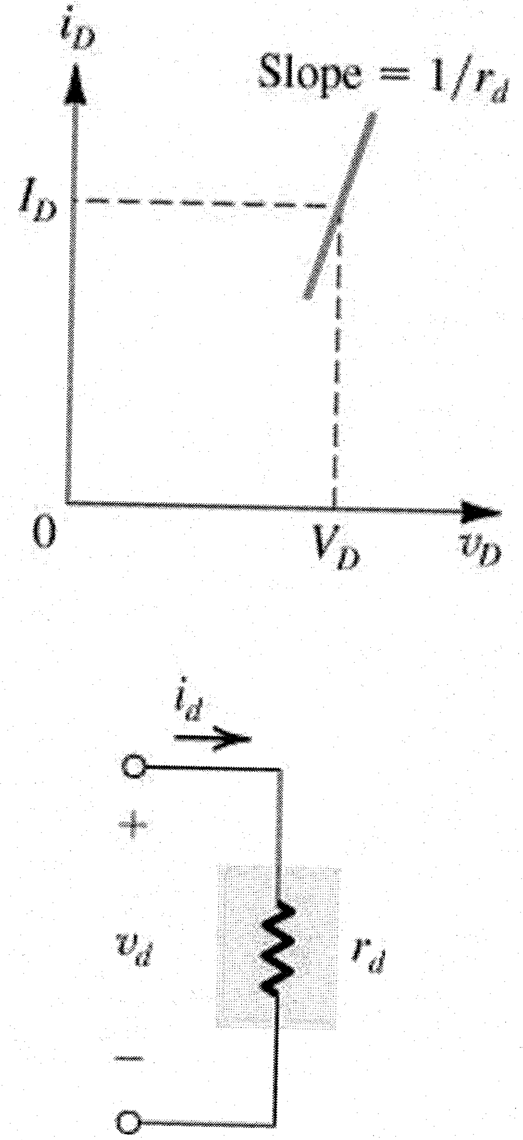


Table 3.1 ***(Continued)***

MODELING SUMMARY

SUMMARY: New Concepts

Non-linear device → Characteristic
Exponential I-V formula
Fwd/Rev/Breakdown regions
Thermal effects
Small-signal resistance

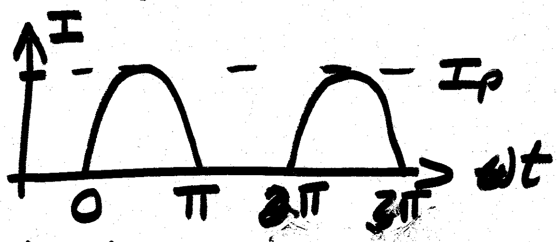
Modeling → Exponential formula
Linear piecewise → Ideal
(Constant V) → Turn-on
→ Fwd slope
(Turn-on/battery + resistance)

Solutions → Linear (equivalent circuit) models
Graphical
Iterative
Incremental/small signal

Applications → AC voltage rectification
Diode logic

Ex 3·5 AC voltmeter — Full scale when $\underline{\text{average}}$ $I = 1mA$

$I = v_I/(R+50)$ for $v_I > 0$
$I = 0$ for $v_I < 0$



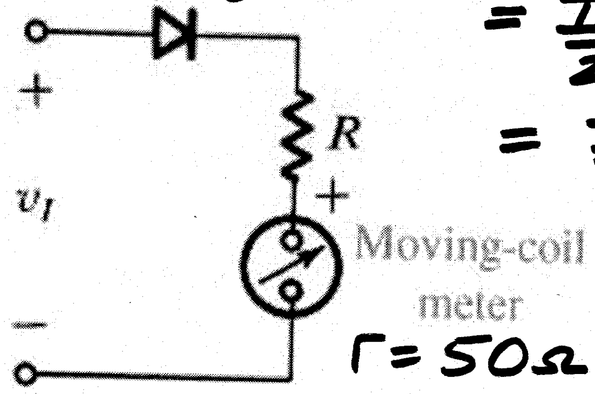
$$\text{Av. } I = \frac{1}{2\pi}\int_0^{\pi} I_p \sin\omega t \, d(\omega t)$$

$$= \frac{I_p}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{I_p}{2\pi}\left(-(-1)-(-1)\right)$$

$$\Rightarrow \frac{I_p}{\pi}$$

(Hint provided)



$r = 50\Omega$

Need $\dfrac{10v}{\pi(R+50\Omega)} = 10^{-3}A$ i.e. $R = \dfrac{10}{\pi}K\Omega - 50\Omega$

$\approx 3.133 K\Omega$

Figure E3.5 10v for 20v p-p fullscale (given)

Ex 3.6 Si diode with $n=1.5$. Find ΔV for I change $0.1mA \to 10mA$

$$V_2 - V_1 = nV_T \ln(I_2/I_1)$$

$$\Delta V = 1.5 \times 25mV \times \ln(100) \approx 37.5mV \times 2.3 \times 2$$

$$= 2.3 \times 75mV = 172.5mV$$

Ex. 3.7 Si diode with $n=1$, $v=0.7v$ at $i=1mA$. Find ΔV for $i=0.1mA$ & $i=10mA$

$$\Delta V = 1 \times 2.3 \times 25mV \times \log_{10} 10 \quad \leftarrow 10mA$$

$$= 57.5mV$$

$\left(0.1mA \to \log_{10} 10^{-1} \right.$
$\left. \to -57.5mV \right)$

$\therefore$ $0.1mA \to 0.7 - 0.06v \approx 0.64v$

$10mA \to 0.7 + 0.06v \approx 0.76v$

OR $\quad 1mA = I_s \exp \dfrac{0.7v}{25mV} \quad \therefore I_s = 10^{-3} \exp{-28} = 6.9 \times 10^{-16} A$

Then $V = 25mV \ln(I'/6.9 \times 10^{-16}) \longrightarrow$ same result.

$\uparrow 10^{-2}$ & 10^{-4}

Ex.3.8. $I_s = 10^{-14}A$ at $25°C$, I_s incr $15\%/°C$. Find I_s at $125°C$

$\Delta T = 125°C - 25°C = 100°C$, $\quad I_s = 10^{-14}A \times 1.15^{\Delta T}$

$$= 10^{-14} \times 1.15^{100} A$$

$$= 1.17 \times 10^{-5} A$$

Ex 3.9 High current diode, approx constant reverse leakage

$V = 1\text{v}$ at $20^\circ C$, find V at $40^\circ C$ & $0^\circ C$

$V = 1\text{v}$

$\therefore I_s)_{20^\circ C} = 1\mu A$

Assume I_s doubles every $10^\circ C$

$\therefore I_s)_{40^\circ C} = 4\mu A$

& $I_s)_{0^\circ C} = 0.25\mu A$

$\therefore V)_{40^\circ C} = 4\text{v}$

$V)_{0^\circ C} = 0.25\text{v}$

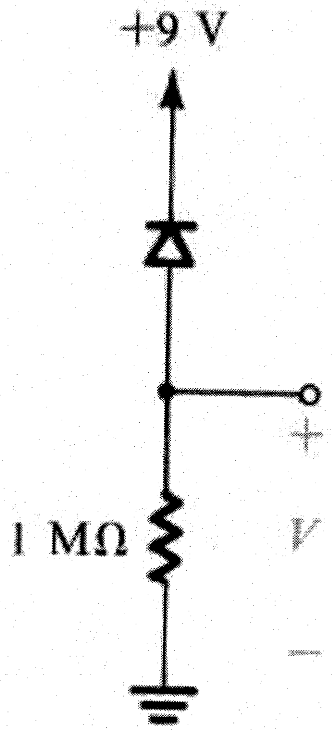


Figure E3.9

Ex D.3.12. Design for $V_o = 2.4v$. Diodes 1mA at 0.7v

$\Delta V = 0.1\ V/\text{decade current}$

$\therefore \Delta V = 0.1v$
$= 2.3 n V_T \log 10$
$= 2.3 n V_T$

Need $2.4v/3$
$= 0.8\ v/\text{diode}$

0.1 volt/decade current

$\therefore 0.8v \rightarrow 10mA$

$$\therefore R = \frac{10 - 2.4v}{10mA} = \frac{7.6v}{10^{-2}A} = 760\Omega$$

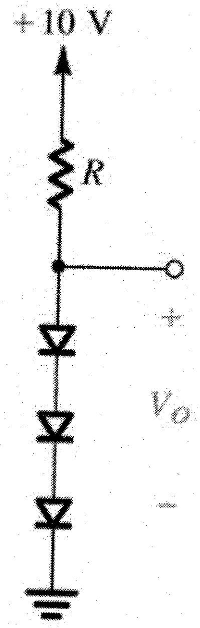
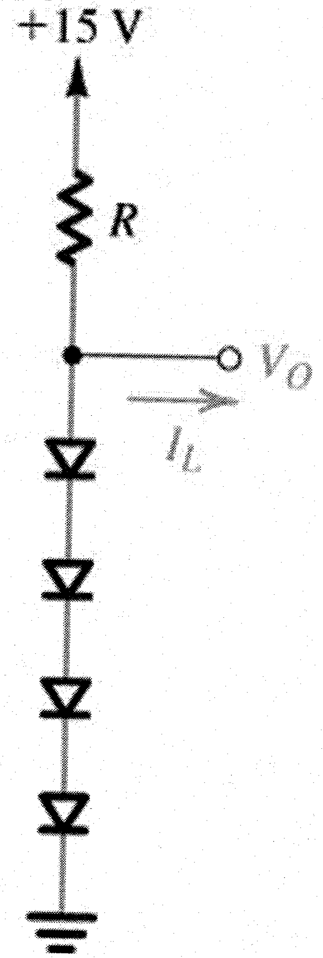


Figure E3.12

Ex. D3.16. $V_0 = 3v$ when $I_L = 0$.
V_0 changes $40mV/mA$ of I_L.

Find R and diode junction area relative to .7v, 1mA diode. $n=1$

$V_D = 0.75$ for
$V_0 = 3v$ at $I_L = 0$, &
$I_D R = 12v$



$\Delta V = 40mV/mA$,
i.e. $10mV/mA$ per diode
i.e. $r_d = 10\Omega$

$$\therefore \frac{nV_T}{I_D} = \frac{25mV}{I_D} = 10\Omega$$

$\therefore I_D = 2.5mA \longrightarrow R = \frac{12v}{2.5mA} = 4.8K\Omega$

Need $I_D = 2.5mA$ at $0.75v$

Figure E3.16

$$I_2 = I_1 \exp \frac{V_2 - V_1}{nV_T} = 1mA \exp \frac{0.75 - 0.70}{.025} = (\exp 2)mA$$

$$\therefore \text{Area} = \frac{2.5mA}{\exp 2 \ mA} = \frac{2.5}{7.4} = 0.34$$