ECE321 Electronics I
Fall 2006

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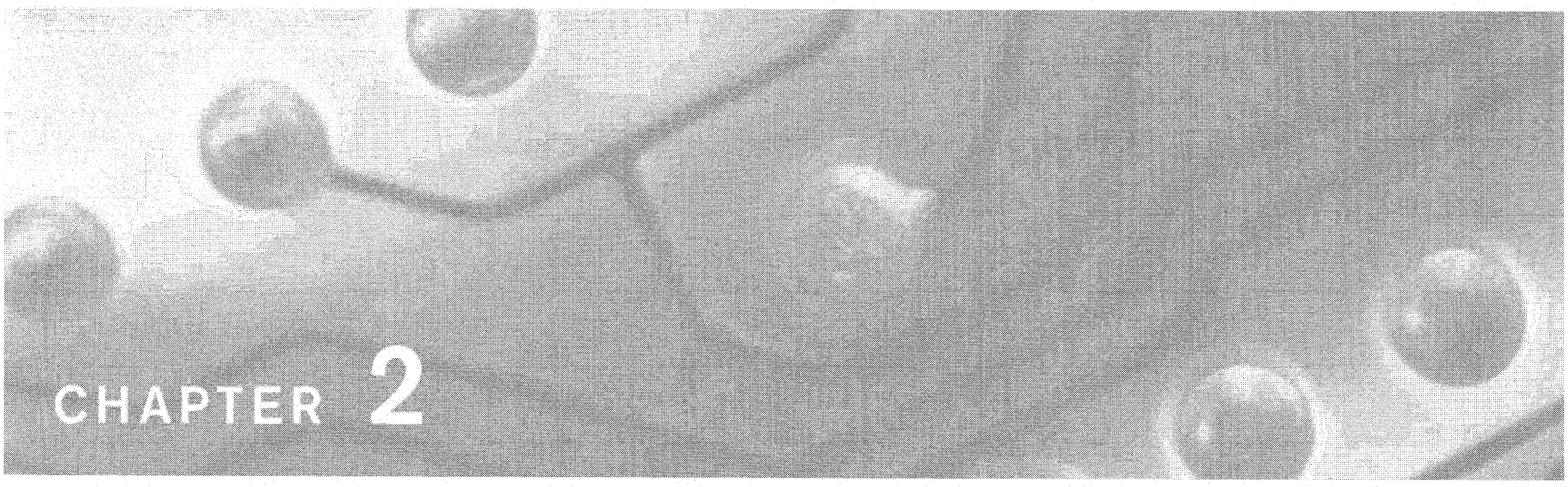
Lecture 2

28th September, 2006



Operational Amplifiers

2.1 Ideal op-amps ← Revision

2.2 Inverting Amplifier

2.3 Non-Inverting Amplifierr

Revision
except
2.2.2/2.3.3

Ideal Op-amp

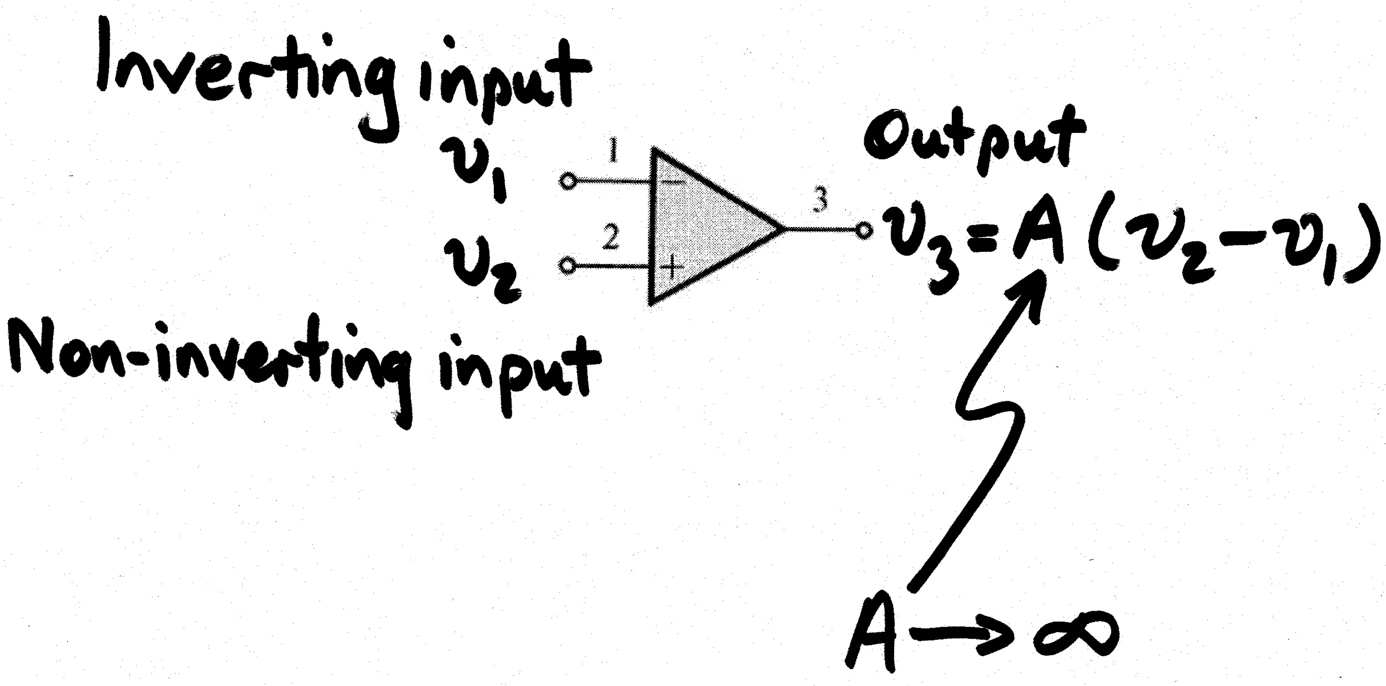
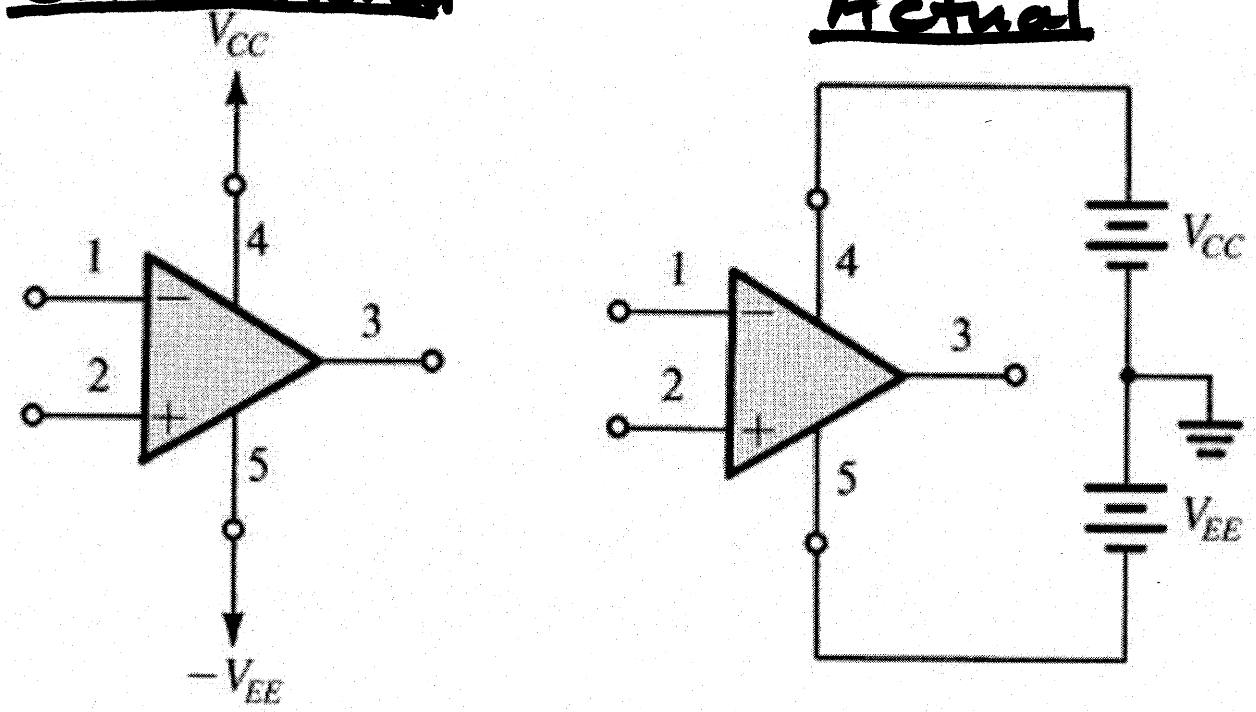


Figure 2.1 Circuit symbol for the op amp.

Implied Power Supply Connections

Conventional **Actual**



(a) (b)

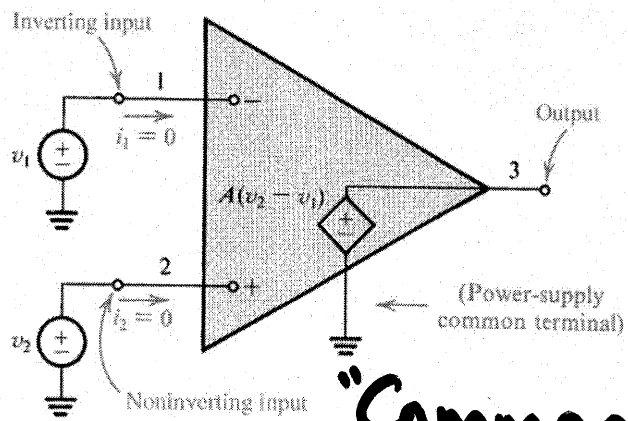
"Conventional" format common in transistor ckts, etc but power supplies usually omitted entirely in op-amp circuit diagrams.

Figure 2.2 The op amp shown connected to dc power supplies.

$R_{in} = \infty$

So i_1 & $i_2 = 0$

$R_{OUT} = 0$



"Common" connection

Ideal Op-amp equivalent circuit

Figure 2.3 Equivalent circuit of the ideal op amp.

Actually $A_v \longrightarrow A_d$ (differential gain) $= v_{od}/v_{Id}$

& A_{cm} (common-mode gain) $= \dfrac{v_o)_{cm}}{v_I)_{cm}}$

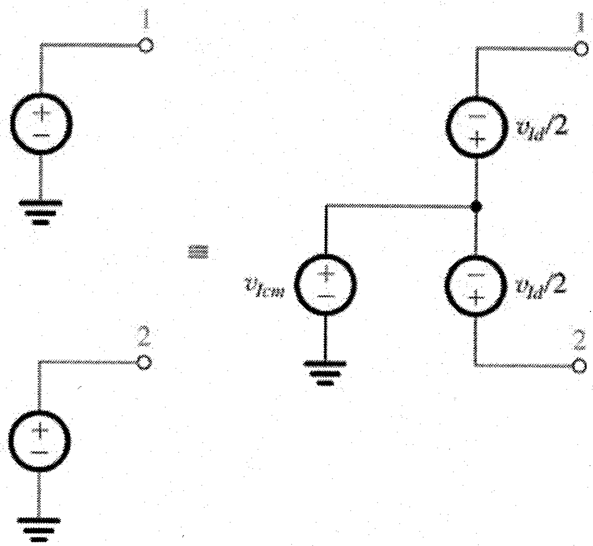
Average values

$v_{id} = v_2 - v_1$

$v_{ICM} = \dfrac{v_2 + v_1}{2}$

$v_1(\text{inv}) = v_{ICM} - v_{Id}/2$

$v_2(\text{non-inv}) = v_{ICM} + v_{Id}/2$



$A_{CM} \rightarrow 0$ in the ideal opamp

Figure 2.4 Representation of the signal sources v_1 and v_2 in terms of their differential and common-mode components.

Ex.2.2. Ideal op-amp, except OL gain $A = 10^3$.
Opamp used in a feedback amplifier; 2 voltages measured — find the third. (Identify CM & diff components of v_I)

Note: "Ideal" ⟶ $A_{CM} = 0$

$v_3 = A(v_2 - v_1)$ $\therefore v_1 = v_2 - v_3/A$ $\therefore v_2 = v_1 + v_3/A$

	v_1	v_2	v_3	
(a)	?	0v	2v	$\therefore v_1 = -2\,mV$ & $v_{Id} = v_2 - v_1 = 2\,mV$; $v_{Icm} = (v_1 + v_2)/2 = -1\,mV$
(b)	?	+5v	−10v	$\therefore v_1 = +5.01\,v$; $v_{Id} = -10\,mV$; $v_{Icm} = 5.005\,v$
(c)	1.002v	0.998v	?	$\therefore v_3 = -4\,v$; $v_{Id} = -4\,mV$; $v_{Icm} = 1\,v$
(d)	−3.6v	?	−3.6v	$\therefore v_2 = -3.6036\,v$; $v_{Id} = -3.6\,mV$; $v_{ICM} = -3.6018\,v$

Ex. 2.3. Find $v_3(v_1, v_2)$. Find OL (open-loop) gain A for $G_m = 10\,mA/V$, $R = 10\,k\Omega$, $\mu = 100$.

$$v_3 = \mu v_d = \mu (G_m v_2 - G_m v_1) R$$
$$= \mu G_m R (v_2 - v_1).$$

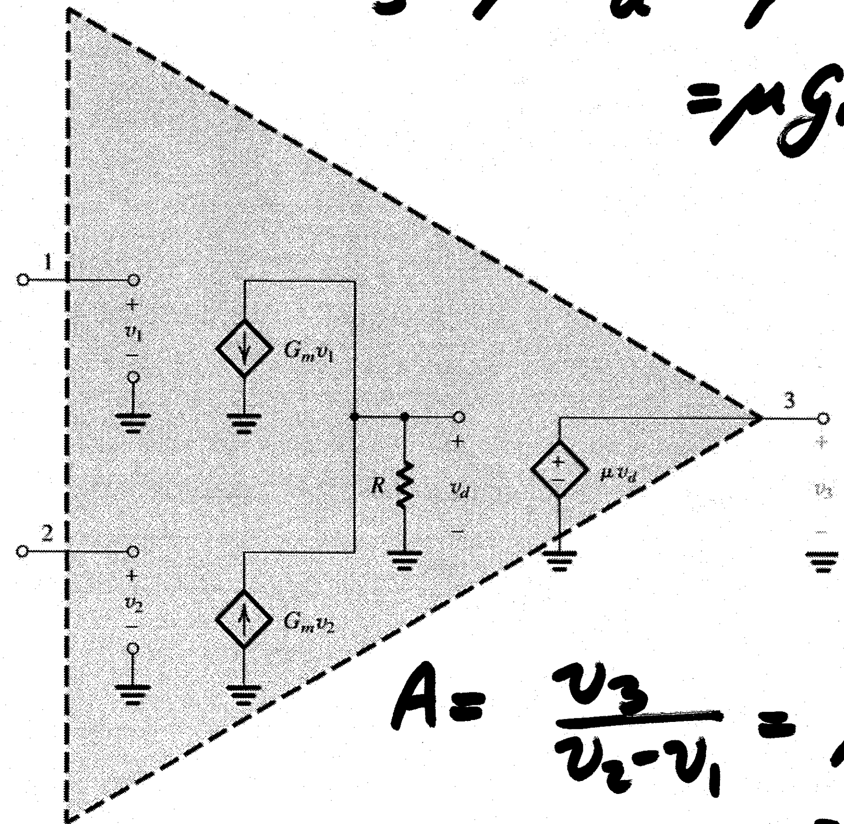


Figure E2.3

$$A = \frac{v_3}{v_2 - v_1} = \mu G_m R$$
$$= 100 \times 10^{-2} \times 10^4$$
$$= 10^4\ V/V$$
$$A|_{dB} = 20 \log_{10} 10^4 = 80\,dB$$

VIRTUAL SHORT CIRCUIT

$$v_3 = A(v_2 - v_1)$$

$$\therefore v_2 - v_1 = v_3/A \xrightarrow[\text{\& if } v_3 \text{ remains finite}]{A \to \infty} 0$$

i.e. $v_1 = v_2$

General analysis principle:

with negative feedback (which keeps v_3 finite) & $A \approx \infty$

then $v_1 \approx v_2$

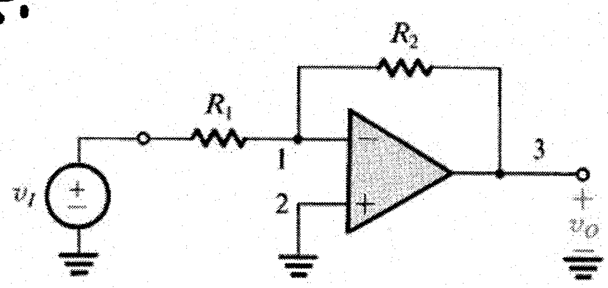


Figure 2.5 The inverting closed-loop configuration.

So v_1 sees a "virtual short circuit"

or here a "virtual ground"

i.e. $v_2 = 0 \therefore v_1 = 0$

even with no physical connection

Inverting Amplifier

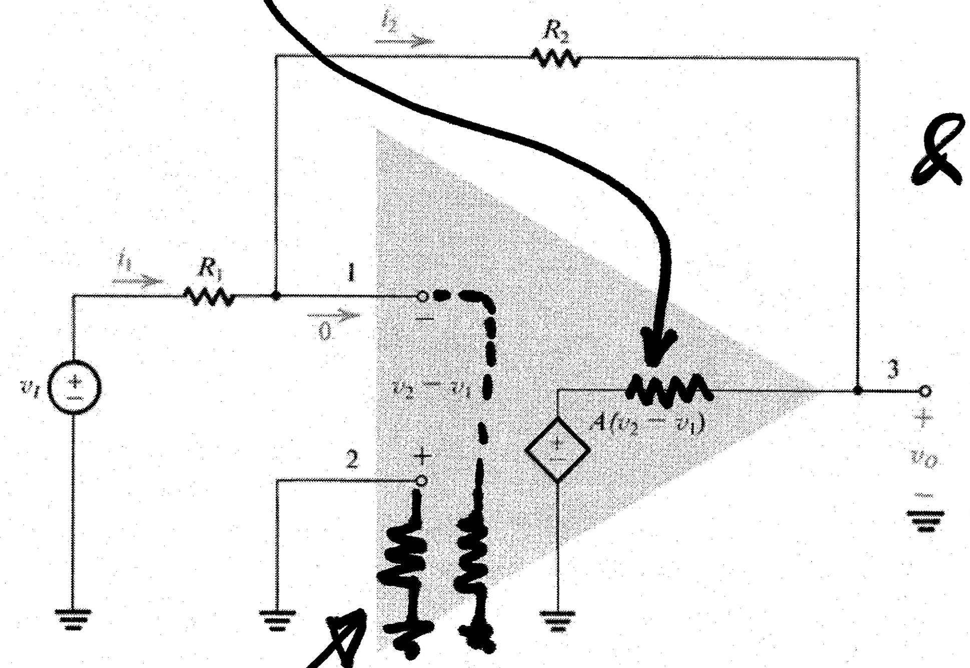
Ideal, so $R_{out} = 0\Omega$

Negative feedback & OL gain $A = \infty$

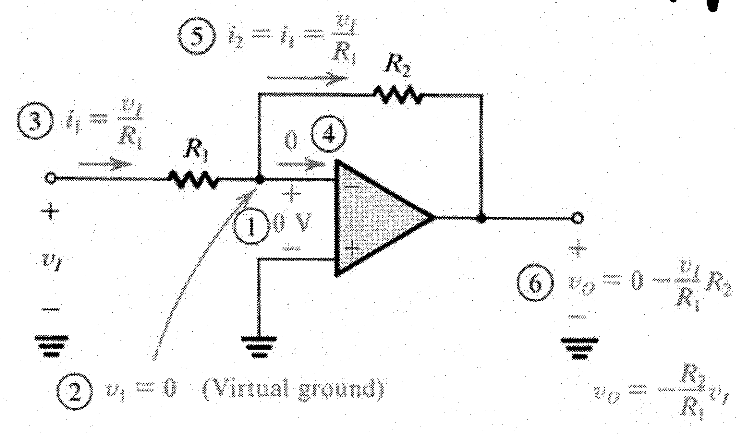
$\therefore v_1 = v_2 = 0$

$\therefore i_1 = v_I / R_1$

& $R_{in1} = \infty$, $\therefore i_2 = i_1 = v_I/R_1$



(a)



(b)

Ideal, so R_{in1} & $R_{in2} = \infty$

$(R_{out} = 0)$ $v_o = v_1 - i_2 R_2$

$= 0 - i_2 R_2$

$= -\frac{R_2}{R_1} v_I$

Figure 2.6 Analysis of the inverting configuration. The circled numbers indicate the order of the analysis steps.

Trade high A for accuracy & designability. One op-amp eg 741 used in many applications

$\therefore$ CL gain $v_o/v_I = -R_2/R_1$

Another inverting op-amp application (Example 2.2)

① Neg. f.b. ∴ $v^- \simeq 0$

② ∴ $i_1 = v_I/R_1$

③ $R_{in} = \infty$

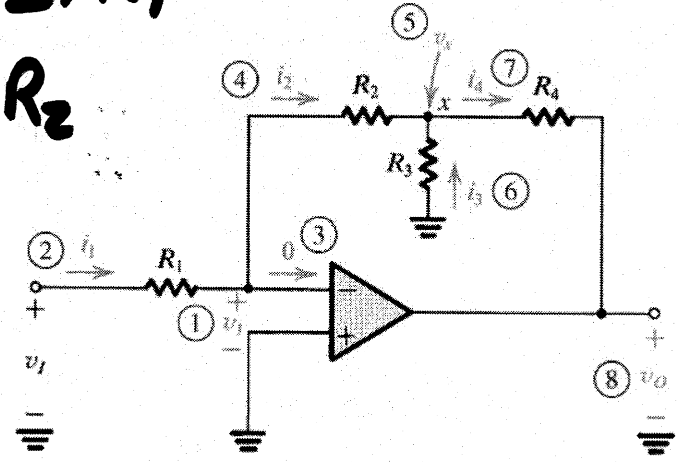
④ ∴ $i_2 = i_1 = v_I/R_1$

⑤ ∴ $v_x = -\left(\frac{v_I}{R_1}\right)R_2$

⑥ ∴ $i_3 = \frac{v_I}{R_1}\frac{R_2}{R_3}$

⑦ $i_4 = i_2 + i_3$

$= \frac{v_I}{R_1}\left(1+\frac{R_2}{R_3}\right)$



⑧ ∴ $v_O = v_x - i_4 R_4 = -\frac{v_I}{R_1}\left(R_2 + R_4 + R_4\frac{R_2}{R_3}\right)$

Figure 2.8 Circuit for Example 2.2. The circled numbers indicate the sequence of the steps in the analysis.

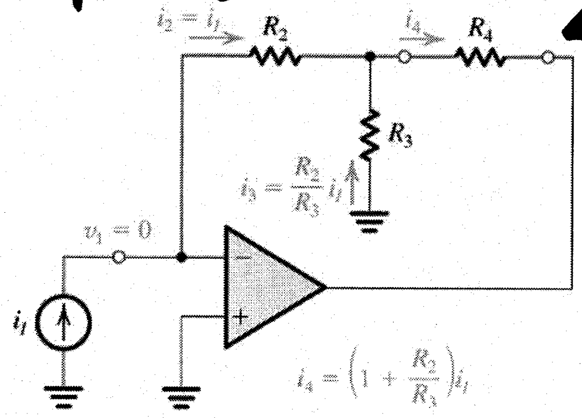
& $v_O/v_I = -(R_2/R_1)(1 + R_4/R_3 + R_4/R_2)$

Note in previous circuit:

$$i_4 = i_2 + i_3$$
$$= i_1 + \frac{R_2}{R_3} i_1$$

$\therefore$ As a current amplifier

$$A_i = \frac{i_4}{i_1} = 1 + \frac{R_2}{R_3}$$



R_4 "floating" load

Figure 2.9 A current amplifier based on the circuit of Fig. 2.8. The amplifier delivers its output current to R_4. It has a current gain of $(1 + R_2/R_3)$, a zero input resistance, and an infinite output resistance. The load (R_4), however, must be floating (i.e., neither of its two terminals can be connected to ground).

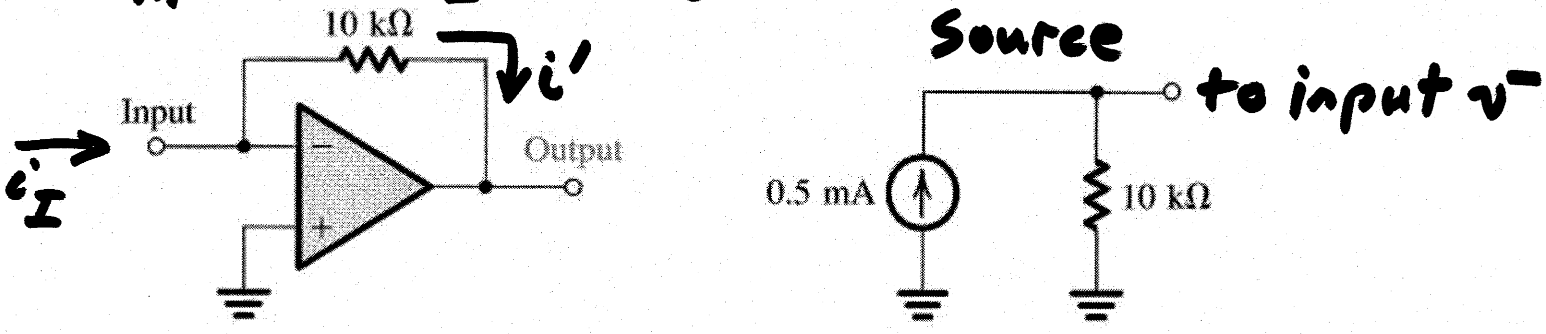
Transresistance Amplifier (voltage out / current in)

Ex. 2.5.

(a) Find $R_{in} = v^-/i_I = 0$ (since $v^- = 0$)

Find $v_o = v^- - i' \, 10K = 0 - 10K \cdot i_I = -10^4 i_I$

Find $R_m = v_o/i_I = -10^4$



(a) (b)

(b) $v^- = 0 \quad \therefore i_{10K} = 0 \quad \therefore i_I = 0.5mA$

$\therefore v_o = -10K \times 0.5mA = -5v$

Figure E2.5

Ex 2.6. Neg. f.b. $\therefore v_1 = 0 \quad \therefore i_1 = 1v/1k\Omega = 1mA$

$R_{in} = \infty \quad \therefore i_2 = 1mA \quad \therefore v_o = -10K \times 1mA = -10v$

$\therefore i_L = -\dfrac{10v}{1K} = -10mA$

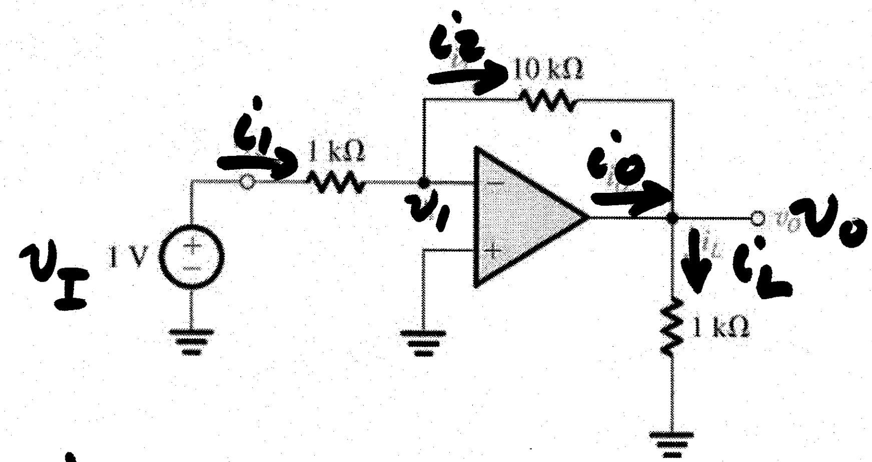


Figure E2.6

$i_o = i_L - i_2 = (-10mA) - (1mA) = -11mA$

$v_o/v_I = -10v/1v = -10 \longrightarrow 20\log 10 = 20dB$

$i_L/i_I = -10mA/1mA = -10 \longrightarrow 20\log 10 = 20dB$

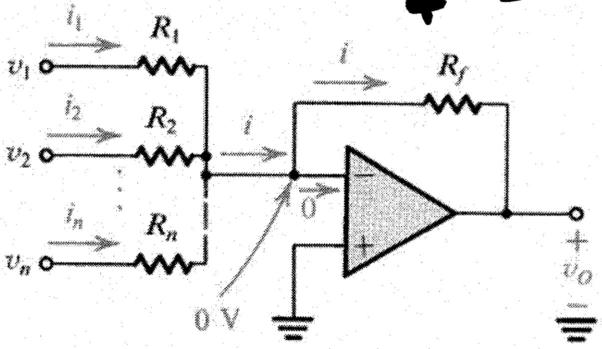
$P_o/P_I = (-10)(-10) = 100 \longrightarrow 10\log 100 = 20dB$

Weighted Summing Circuit (D to A)

$v^- = 0 \quad \therefore \; i_i = v_i / R_i \qquad i = 1, \ldots, n$

$\therefore \; i = \sum_n i_i = \sum_n v_i / R_i$

& $v_o = -i R_f = -R_f \sum_n v_i / R_i$



$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots + \frac{R_f}{R_n}v_n\right)$$

Ex D2.7 $v_o = -(v_1 + 5v_2)$. Find R_1, R_2, R_f so $i \leq 1mA$ for $v_o \leq 10v$.

Say $v_o = 10v$ (max), then $i(max) = 1mA \Rightarrow R_f = 10K$

$\therefore \; R_1 = 10K, \; R_2 = 2K$

Figure 2.10 A weighted summer.

But: DESIGN requirement $i \leq 1mA \longrightarrow R_1 = R_f \geq 10K, \; R_2 = R_1/5$

Also, preferred values $\longrightarrow R_1 = R_f = 4.7K, \; R_2 = 1.0K$?

Appendix G, 5% range $\longrightarrow$ 11K, 2.2K or 12K, 2.4K or 15K, 3K or 18K, 3.6K

If coefficients have different signs
→ cascade 2 inverting amp[r]s
for the positive coeffs.

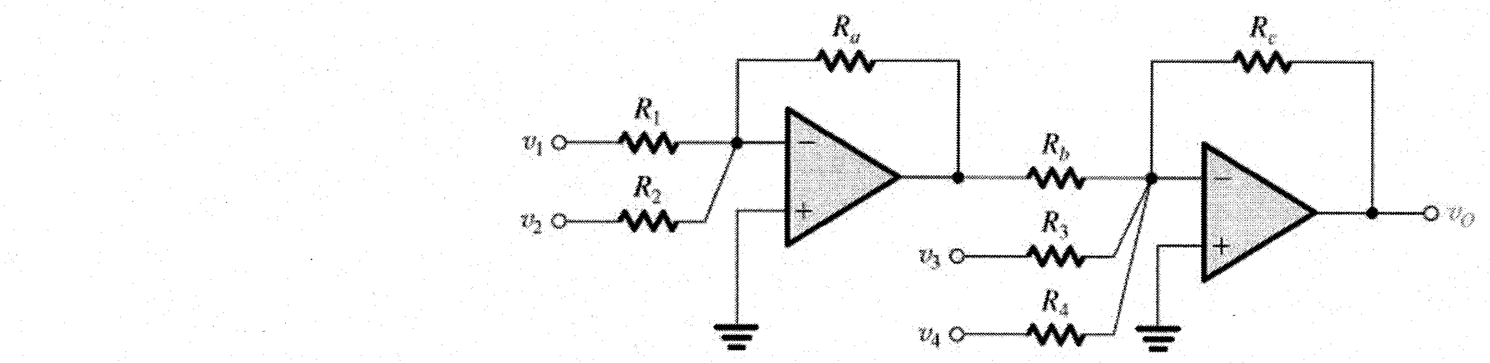


Figure 2.11 A weighted summer capable of implementing summing coefficients of both signs.

Non-Inverting Amplifier

$v^- = v^+ = v_I$

etc

Same approach

$R_{in} = \infty$

$\therefore i_{in_1} = i_{in_2} = 0$

etc

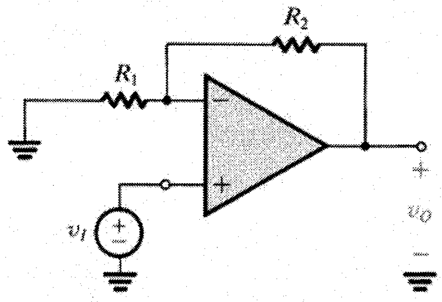


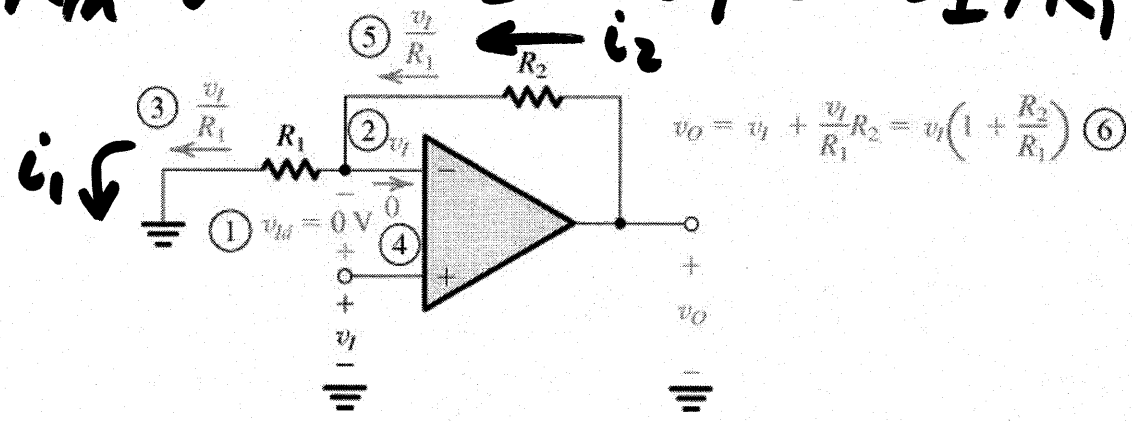
Figure 2.12 The noninverting configuration.

① $v^- = v^+$ for neg fb & $A \to \infty$

② $\therefore v^- = v_I$

③ $\therefore i_1 = v_I / R_1$

④ $R_{in} = \infty \therefore i_2 = i_1 = v_I / R_1$ ⑤



⑥ $\therefore v_O = v^- + i_2 R_2 = v_I + \frac{v_I}{R_1} R_2$

$= v_I (1 + R_2/R_1)$

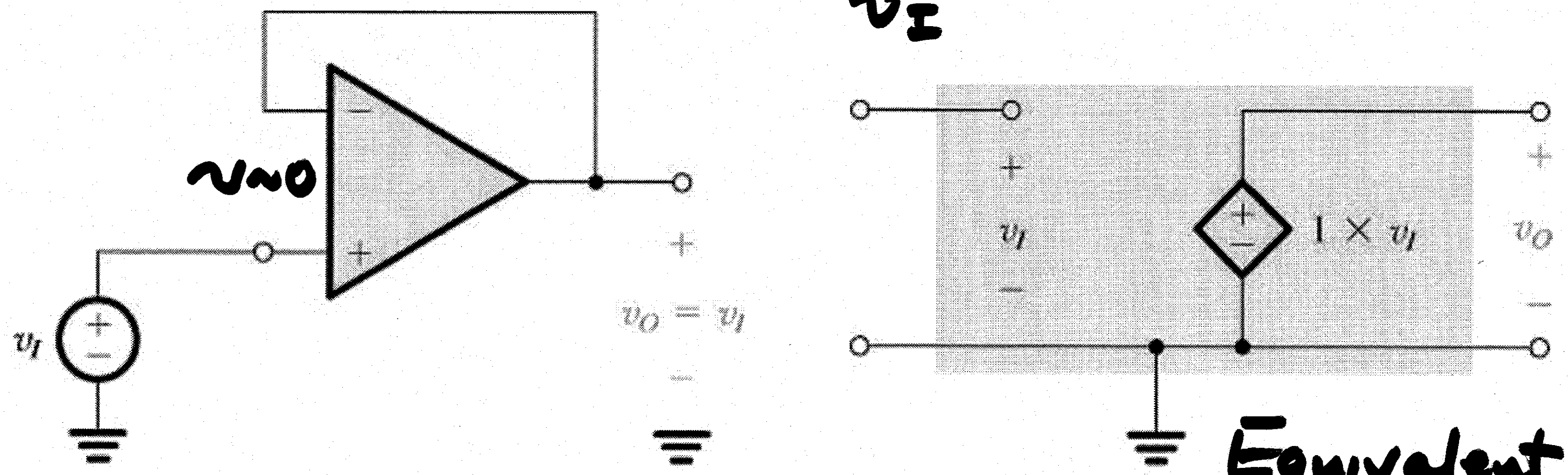
Figure 2.13 Analysis of the noninverting circuit. The sequence of the steps in the analysis is indicated by the circled numbers.

$\therefore A_{CL} = v_O / v_I = 1 + R_2/R_1$

VOLTAGE FOLLOWER

$$v_O = v^- = v^+ = v_I$$

$$\therefore A_v = \frac{v_O}{v_I} = +1$$



(a)

(b)

Equivalent circuit

$R_{in} = \infty$

$R_{out} = 0$

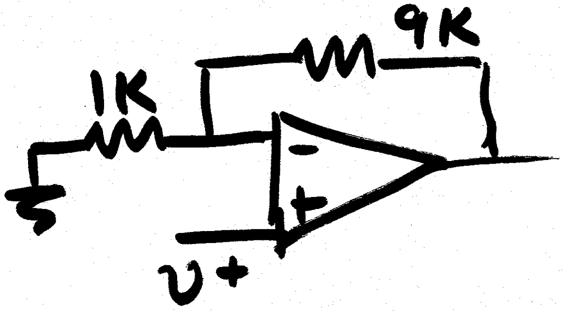
So what's the point of $A = 1$?

Buffer $\longrightarrow$ $R_{in} = \infty$

$R_{out} = 0$

Figure 2.14 **(a)** The unity-gain buffer or follower amplifier. **(b)** Its equivalent circuit model.

Ex 2.9 Superposition



$$v_o = \left(1+\frac{9K}{1K}\right)v^+ = 10v^+$$

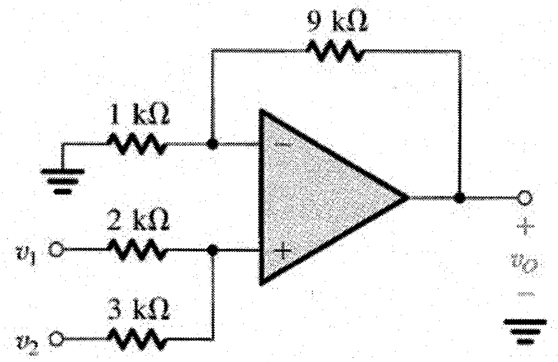
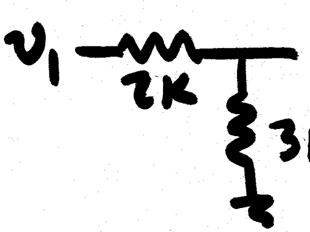
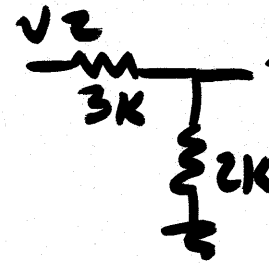


Figure E2.9

Find v^+



$v_1^+ = \frac{3}{5}v_1$



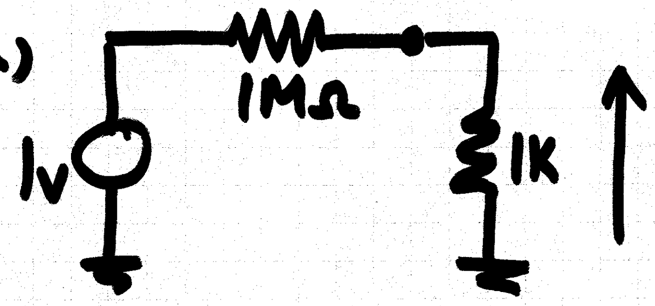
$v_2^+ = \frac{2}{5}v_2$

$$\therefore v_o = 10\left(\frac{3}{5}v_1 + \frac{2}{5}v_2\right) = 6v_1 + 4v_2$$

Ex 2.14 Transducer $V_{oc} = 1v$, $R_{source} = 1M\Omega$

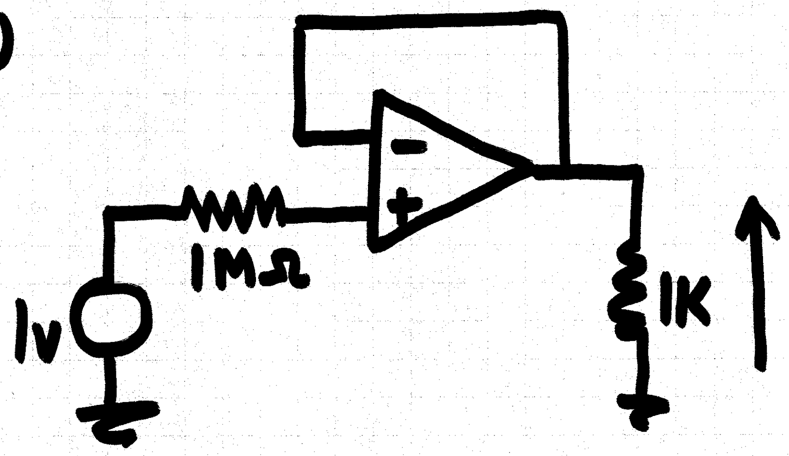
Connect to $1K\Omega$ load (a) directly, & (b) with voltage follower

(a)



$V_L = \frac{1K}{1M + 1K} 1v = \frac{1}{1001} v \approx 1mV$

(b)

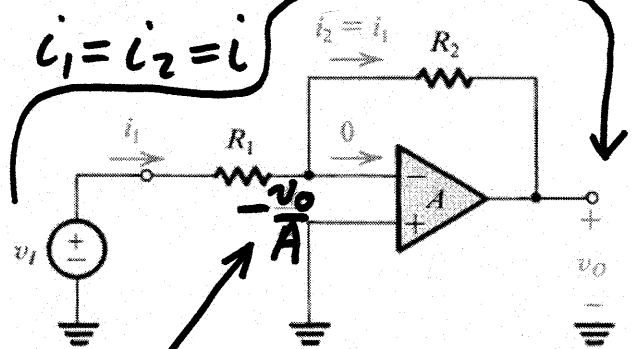


$V_L = 1v$

Inverting Amplifier: OpAmp Gain = A

2

$i_1 = i_2 = i$



3

$$v_I = iR_1 - \frac{v_o}{A}$$

and

$$-\frac{v_o}{A} = v_o + iR_2$$

$$\therefore -\frac{v_o}{A} = v_o + \frac{R_2}{R_1}\left(v_I + \frac{v_o}{A}\right)$$

1

Treatment as before except $v_o = A(v^+ - v^-) \therefore v^- = -v_o/A$

Figure 2.7 Analysis of the inverting configuration taking into account the finite open-loop gain of the op amp.

4 Rearrange $\dfrac{v_o}{v_I} = \dfrac{-R_2}{R_1} \Big/ \left(1 + \dfrac{R_2}{AR_1} + \dfrac{1}{A}\right)$

Continue: <u>Inverting Amplifier</u>

Text's result Eq$^{\underline{n}}$ 2.5

$$\frac{v_o}{v_I} = \frac{-R_2/R_1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}\right)} \xrightarrow[A\to\infty]{} -\frac{R_2}{R_1}$$

Check!

Let's rearrange a bit more

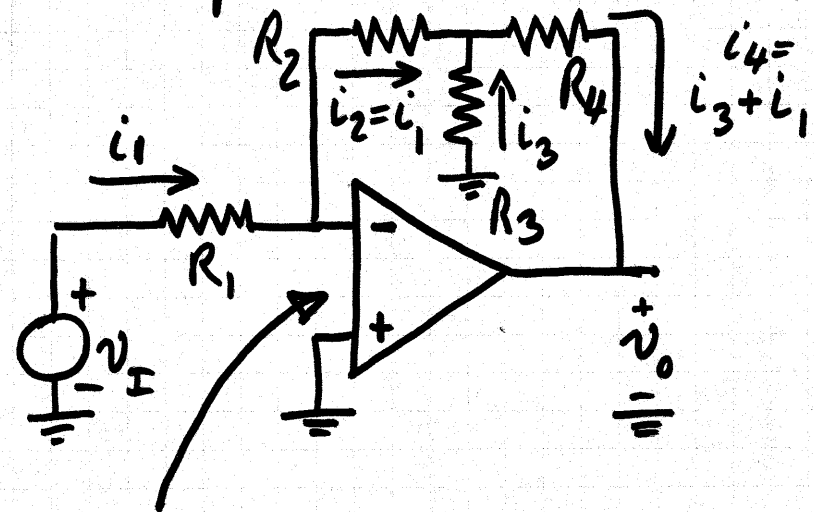
$$\Rightarrow \frac{-(R_2/R_1)A}{A + (R_1+R_2)/R_1}$$

$$= \frac{-A}{1 + A.\frac{R_1}{R_1+R_2}}\left(\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow -\frac{R_2}{R_1+R_2}\cdot\frac{A}{1+A\beta} \quad \text{where } \beta = \frac{R_1}{R_1+R_2}$$

See significance later

Example 2.2. was covered earlier (Fig 2.8) with ideal op-amp



① Now $v^- = -v_o/A$

② $\therefore\; i_1 = \dfrac{v_I + v_o/A}{R_1} = i_2$

③ $\therefore\; v_x = -\dfrac{v_o}{A} - R_2\left(\dfrac{v_I + v_o/A}{R_1}\right)$

④ $i_3 = -\dfrac{v_x}{R_3} = \left[\dfrac{v_o}{A} + \dfrac{R_2}{R_1}\left(v_I + \dfrac{v_o}{A}\right)\right] / R_3$

⑤ $\therefore\; v_o = v_x - (i_2 + i_3)R_4 = -\dfrac{v_o}{A} - \dfrac{R_2}{R_1}\left(v_I + \dfrac{v_o}{A}\right) - \dfrac{R_4}{R_1}\left(v_I + \dfrac{v_o}{A}\right) - \dfrac{R_4}{R_3}\left[\dfrac{v_o}{A} + \dfrac{R_2}{R_1}\left(v_I + \dfrac{v_o}{A}\right)\right]$

⑥ Rearrange →

$$\frac{v_o}{v_I} = \frac{-\left(R_2/R_1 + R_4/R_1 + R_2R_4/R_1R_3\right)}{1 + \left(1 + \frac{R_2}{R_1} + \frac{R_4}{R_1} + \frac{R_2R_4}{R_1R_3} + \frac{R_4}{R_3}\right)/A}$$

$$\xrightarrow[A\to\infty]{} -\left(R_2/R_1 + R_4/R_1 + R_2R_4/R_1R_3\right)$$

check

Non-Inverting Amplifier (Ex 2.12)

Again $v^+ - v^- = v_O/A$ $\therefore$ $v^- = v_I - v_O/A$

Use voltage divider since current i continuous as shown

$\therefore$ $v^- = \frac{R_1}{R_1+R_2} v_O$ (ie. Negative feedback factor $\beta = \frac{R_1}{R_1+R_2}$)

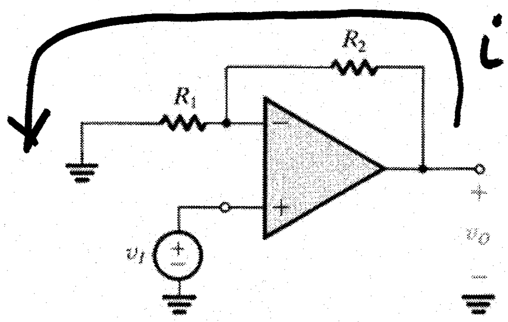


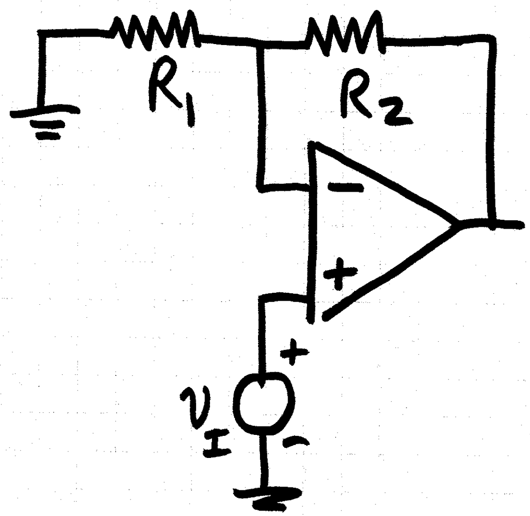
Figure 2.12 The noninverting configuration.

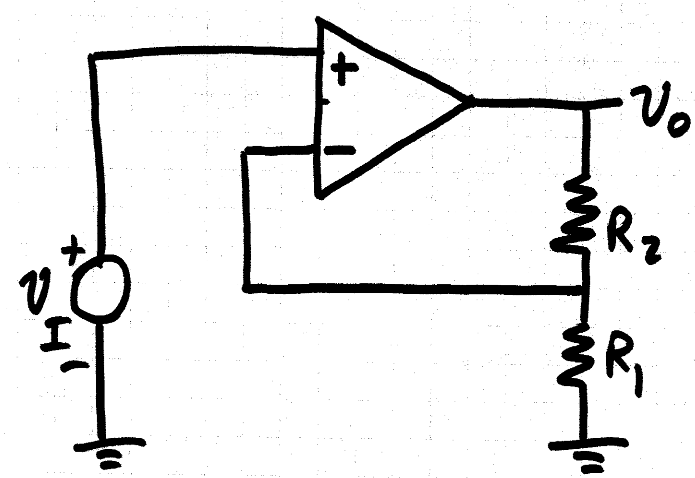
$$\therefore \; v_I - \frac{v_O}{A} = \frac{R_1}{R_1+R_2} v_O \Longrightarrow v_O = v_I \Big/ \left(\frac{1}{A} + \frac{R_1}{R_1+R_2}\right)$$

ie. $$\frac{v_O}{v_I} = \frac{A}{1+A\beta}$$

where $\beta = \frac{R_1}{R_1+R_2}$

Different way of looking at the Non-Inverting Amplifier



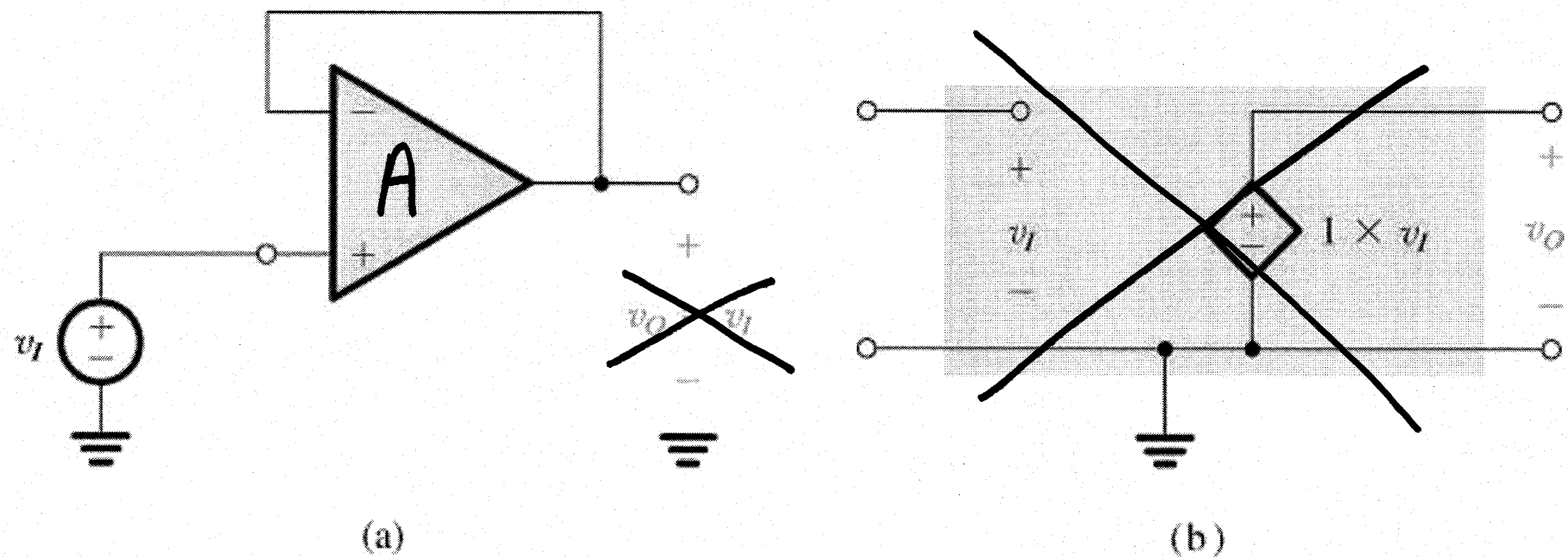


$$v^- = \beta v_o = \frac{R_1}{R_1 + R_2} v_o$$

Problem 2.54

Voltage Follower with finite gain

$A = 1000, 100, 10$



$$v_O = A(v^+ - v^-) = A(v_I - v_O) = \frac{A v_I}{1+A} \xrightarrow[A\to\infty]{} v_I$$

For $A = 1000$ $\quad v_O/v_I = 1000/1001 = 0.999$

$\quad\;\; = 100 \quad\quad\quad\;\; = 100/101 = 0.990$

Figure 2.14 **(a)** The unity-gain buffer or follower amplifier. **(b)** Its equivalent circuit model.

$\quad\;\; = 10 \quad\quad\quad\;\;\; = 10/11 = 0.909$

Assignment #1

Problems at end of chapter

D * 1.44
1.58
1.63
2.8
D 2.33
D 2.46

D → "design" problem
* → harder than others

Selected answers in Appendix H (3 of this week's assignment)

Usually 5 problems/week