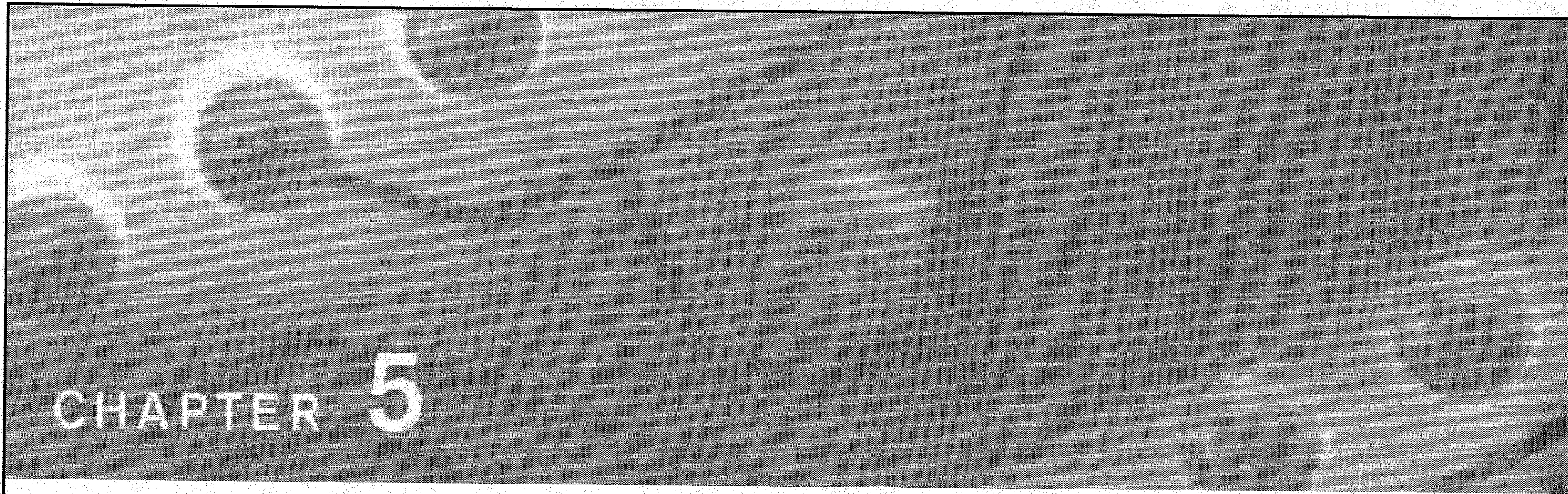
ECE321 ELECTRONICS I
FALL 2006

PROFESSOR JAMES E. MORRIS

Lecture 15

16th November, 2006



Bipolar Junction Transistors (BJTs)

5.8 High Frequency Effects

C_π (fwd bias BE) & C_μ (rev bias CB)

5.9 CE Frequency Response

Low freq effects: coupling & bypass C

Base-Emitter junction

Active (or saturated) mode $\Longrightarrow$ B-E is forward biased

$\therefore$ Stored charge Q_n (electrons for NPN) $= \frac{W^2}{2D_n} i_C = \tau_F i_C$

where $\tau_F = \frac{W^2}{2D_n}$ = "forward bias base-transit time"

The small signal capacitance due to changes in stored charge:

Small signal <u>diffusion</u> capacitance $C_{de} = \frac{dQ_n}{dv_{BE}} = \tau_F \frac{di_C}{dv_{BE}}$

$$= \tau_F g_m = \tau_F \frac{I_C}{V_T}$$

ALSO: B-E <u>junction</u>/transition capacitance $C_{je} = \frac{C_{jeo}}{\left(1 - \frac{V_{BE}}{V_{OE}}\right)^m}$

$m \sim 0.5$, $V_{OE} \sim 0.9\text{V}$

BUT — formula doesn't work well for forward bias, so usually use a ROUGH approximation $C_{je} \sim 2 C_{jeo}$

Collector-Base junction

Active mode — reverse biased

Depletion capacitance

$$C_\mu = \frac{C_{\mu o}}{\left(1 + \frac{V_{CB}}{V_{oc}}\right)^m}$$

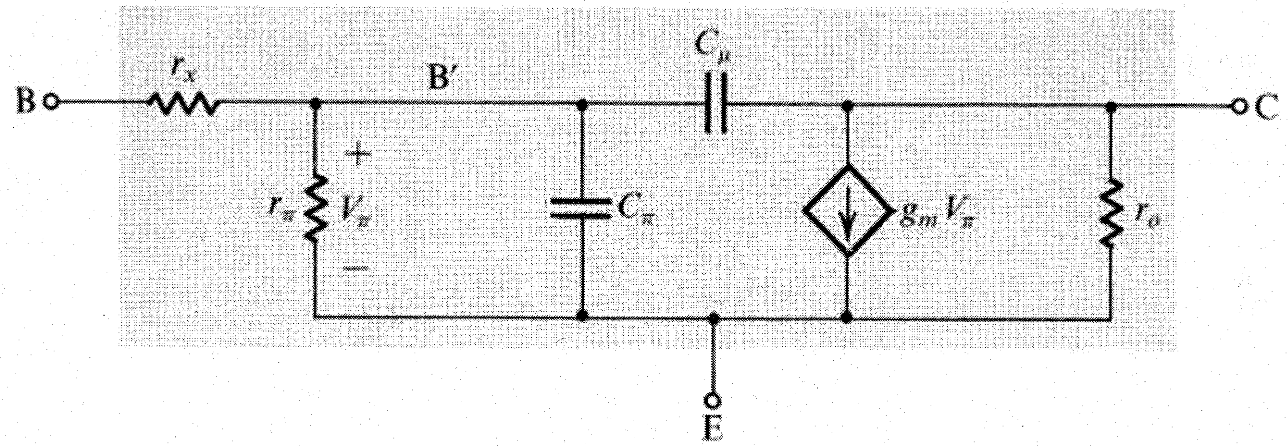
$m \sim 0.2 \rightarrow 0.5$

$V_{oc} \sim 0.75\,V$

HIGH FREQUENCY HYBRID-π MODEL

$C_\pi \longrightarrow$ (B-E junction) $C_{de} + C_{je}$ Typically 10's of pF

$C_\mu \longrightarrow$ (C-B junction) Typically fractions of a pF



$r_x \longrightarrow$ bulk "spreading resistance" of the base silicon

$r_x = r_{bb'} \sim 10\text{'s of ohms} \ll r_\pi$

Figure 5.67 The high-frequency hybrid-π model.

Note r_x defines an internal B′ node. Actually "distributed" parameters — use "lumped" approximation.

C_π and C_μ lead to high frequency effects that can be represented by a frequency-dependent β ← SHORT-CIRCUIT current gain
Also known as h_{fe}

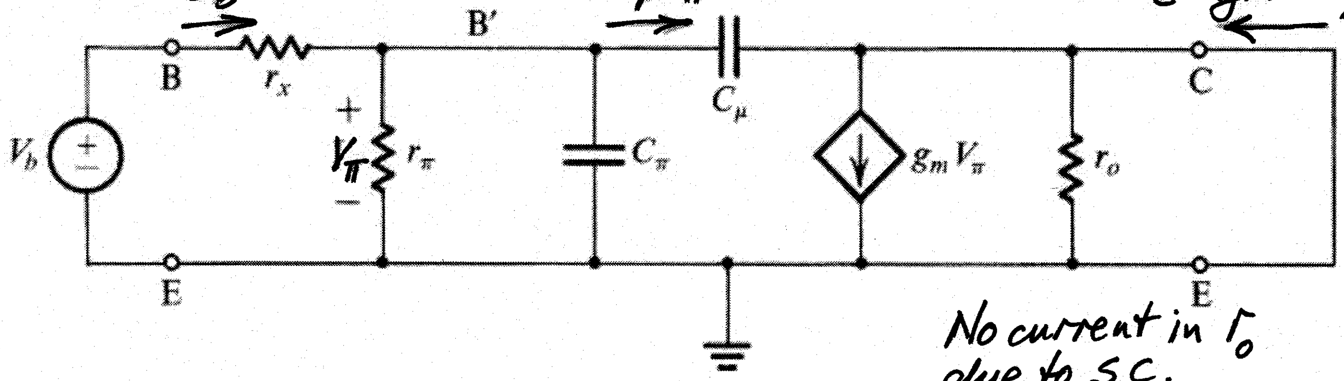
$$I_c = g_m V_\pi - sC_\mu V_\pi$$

$$V_\pi = I_b\,(r_\pi \,\|\, C_\mu \,\|\, C_\pi)$$

$$= \frac{I_b}{1/r_\pi + sC_\mu + sC_\pi}$$

Note: CE S.C.

I_b → $sC_\mu V_\pi$ → $I_c = (g_m - sC_\mu)V_\pi$ ←



Note short circuit.(sc) Differs from later treatments.

No current in r_o due to S.C.

$$h_{fe} = \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{1/r_\pi + sC_\mu + sC_\pi} \approx \frac{g_m}{1/r_\pi + s(C_\mu + C_\pi)} = \frac{g_m r_\pi}{1 + s\,r_\pi(C_\mu + C_\pi)}$$

$$= \frac{\beta_0}{1 + j(\omega/\omega_\beta)}$$

where $\omega_\beta = \dfrac{1}{r_\pi(C_\mu + C_\pi)}$

Figure 5.68 Circuit for deriving an expression for $h_{fe}(s)$; I_c/I_b.

Note approximation that C_μ current $\ll g_m V_\pi$. So C_μ current is included in the denominator, not in numerator.

Frequency dependence of h_{fe} & summary:

$$g_m = \frac{I_C}{V_T}$$

$$r_o \approx \frac{|V_A|}{I_C}$$

$$r_\pi = \frac{\beta}{g_m}$$

$$C_\pi = C_{de} + C_{je}$$

$$C_\mu = \frac{C_{jco}}{\left(1 + \frac{V_{CB}}{V_{CO}}\right)^m}$$

$$C_{de} = \tau_F g_m \qquad C_{je} \sim 2C_{jeo}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

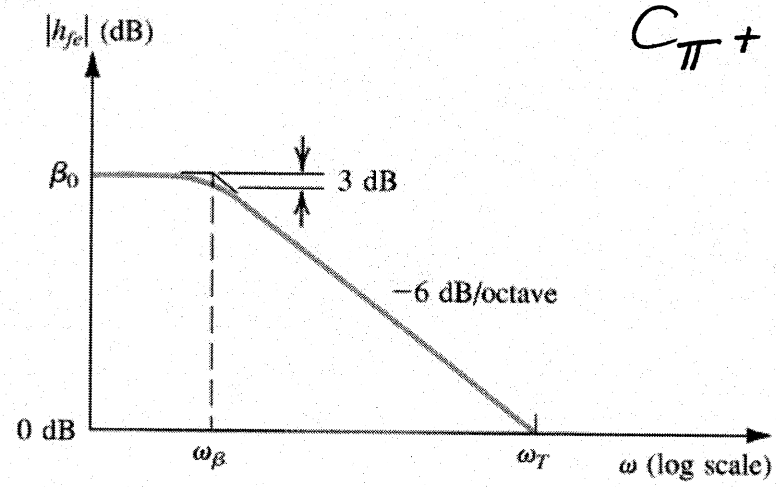


Figure 5.69 Bode plot for uh_{fe}u.

Unity gain bandwidth: $|h_{fe}| = \frac{\beta_o}{\left[1 + (\omega/\omega_\beta)^2\right]^{1/2}} \longrightarrow 1$ when $\omega = \omega_T$

i.e. $\beta_o^2 = 1 + (\omega_T/\omega_\beta)^2$

$$\omega_T \approx \beta_o \omega_\beta$$

Unity gain bandwidth (BW)

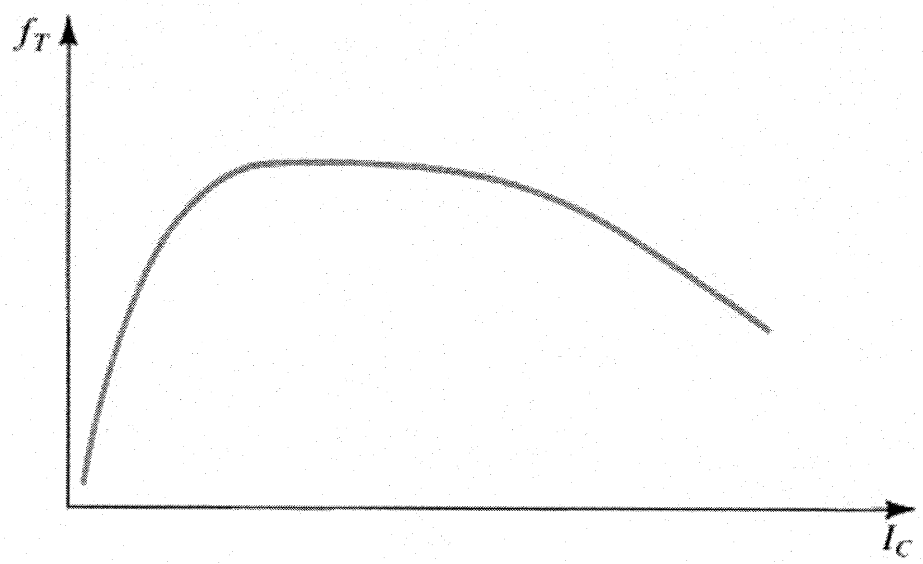


Figure 5.70 Variation of f_T with I_C.

High Frequency Hybrid-π including r_o

See back (Fig 5.69) for relationship summary.

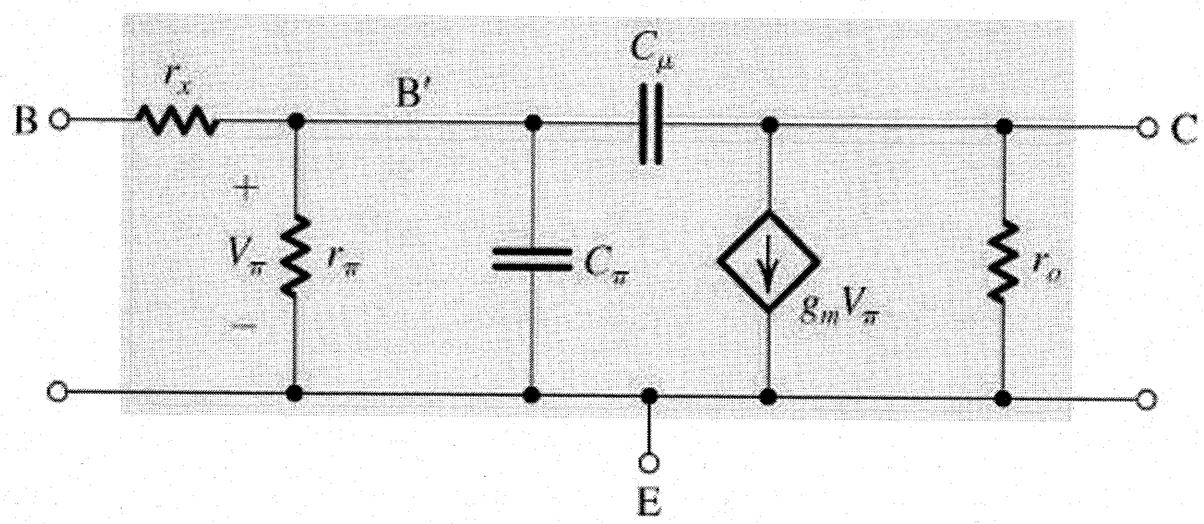


Table 5.7

Exercise 5.48
Exercise 5.49
Exercise 5.50

C_{C1}, C_{C2} coupling capacitors — $X_C \sim 0$ at "mid-band" frequencies

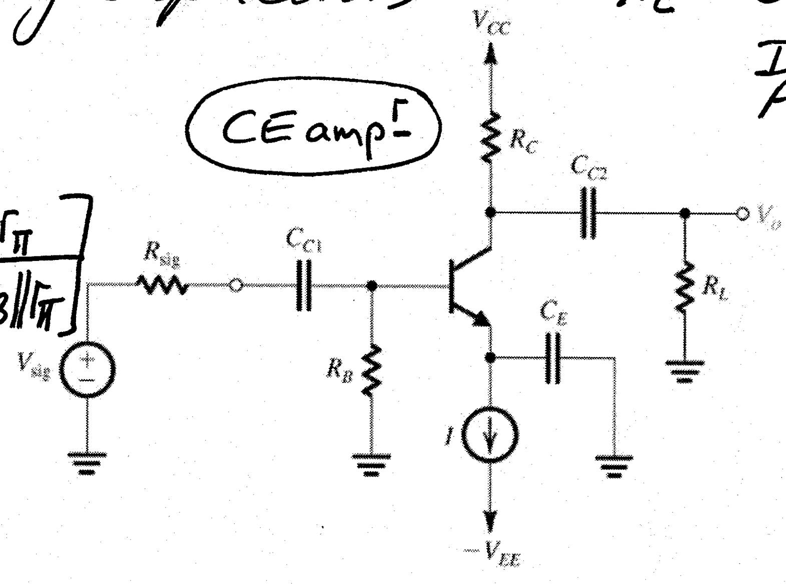
DC block
AC short

(CE amp!)

Mid-band gain:

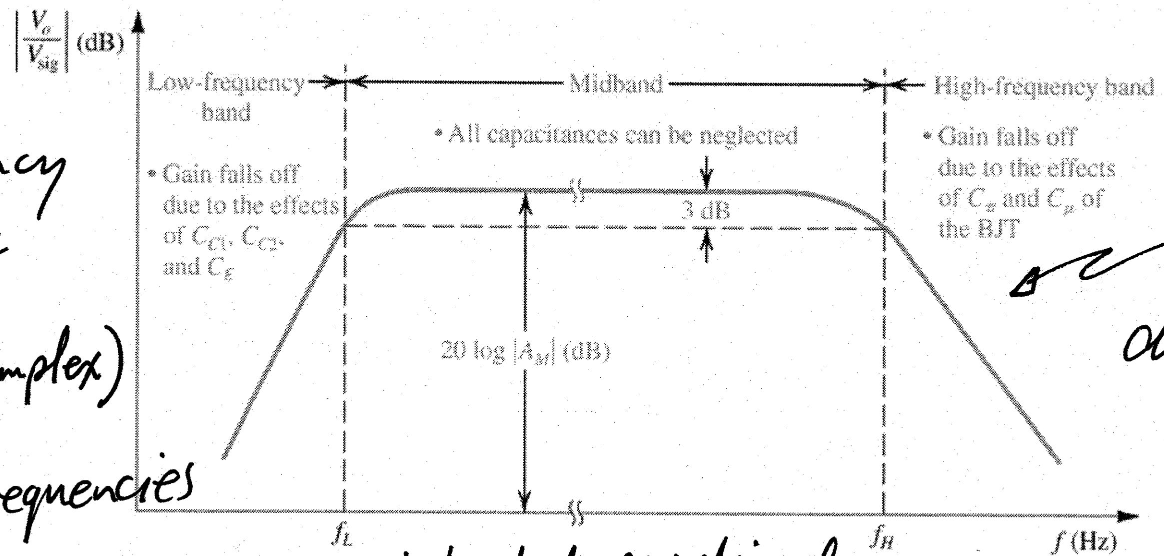
$$A_{MB} = -g_m\left[r_o \| R_C \| R_L\right]\left[\frac{R_B \| r_\pi}{R_{sig} + R_B \| r_\pi}\right]$$

But at lower frequencies, $X_C \neq 0$



(a)

Low frequency roll-off due to C_{C1}, C_{C2} – and C_E (more complex)



High frequency roll-off due to C_π, C_μ.

$C_\mu \ll C_\pi$, but C_μ dominant due to "Miller Effect"

Usually corner frequencies staggered.

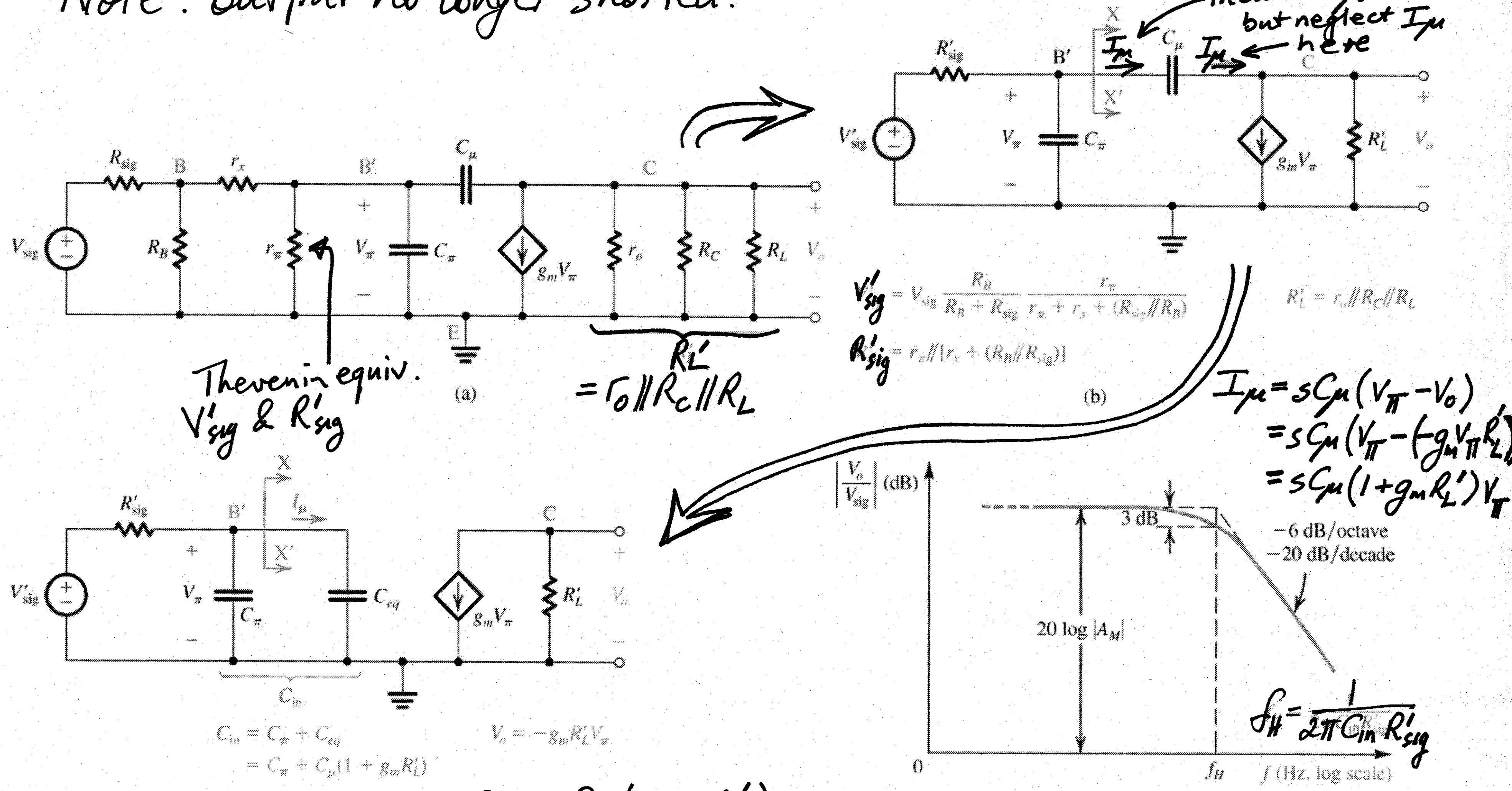
Intended operational range

(b)

Figure 5.71 **(a)** Capacitively coupled common-emitter amplifier. **(b)** Sketch of the magnitude of the gain of the CE amplifier versus frequency. The graph delineates the three frequency bands relevant to frequency-response determination.

Bandwidth $= f_H - f_L \approx f_H$

Amplifier equivalent circuit: High frequencies — X_{C1}, X_{C2}, X_{CE} all s.c.
Note: output no longer shorted.



$\therefore C_{eq} = C_\mu(1 + g_m R'_L)$

so $I_\mu = sC_{eq} \cdot V_\pi$

Figure 5.72 Determining the high-frequency response of the CE amplifier: **(a)** equivalent circuit; **(b)** the circuit of (a) simplified at both the input side and the output side; **(c)** equivalent circuit with C_μ replaced at the input side with the equivalent capacitance C_{eq}; **(d)** sketch of the frequency-response plot, which is that of a low-pass STC circuit.

This treatment misses the second corner frequency due to the I_μ flowing to the output circuit.

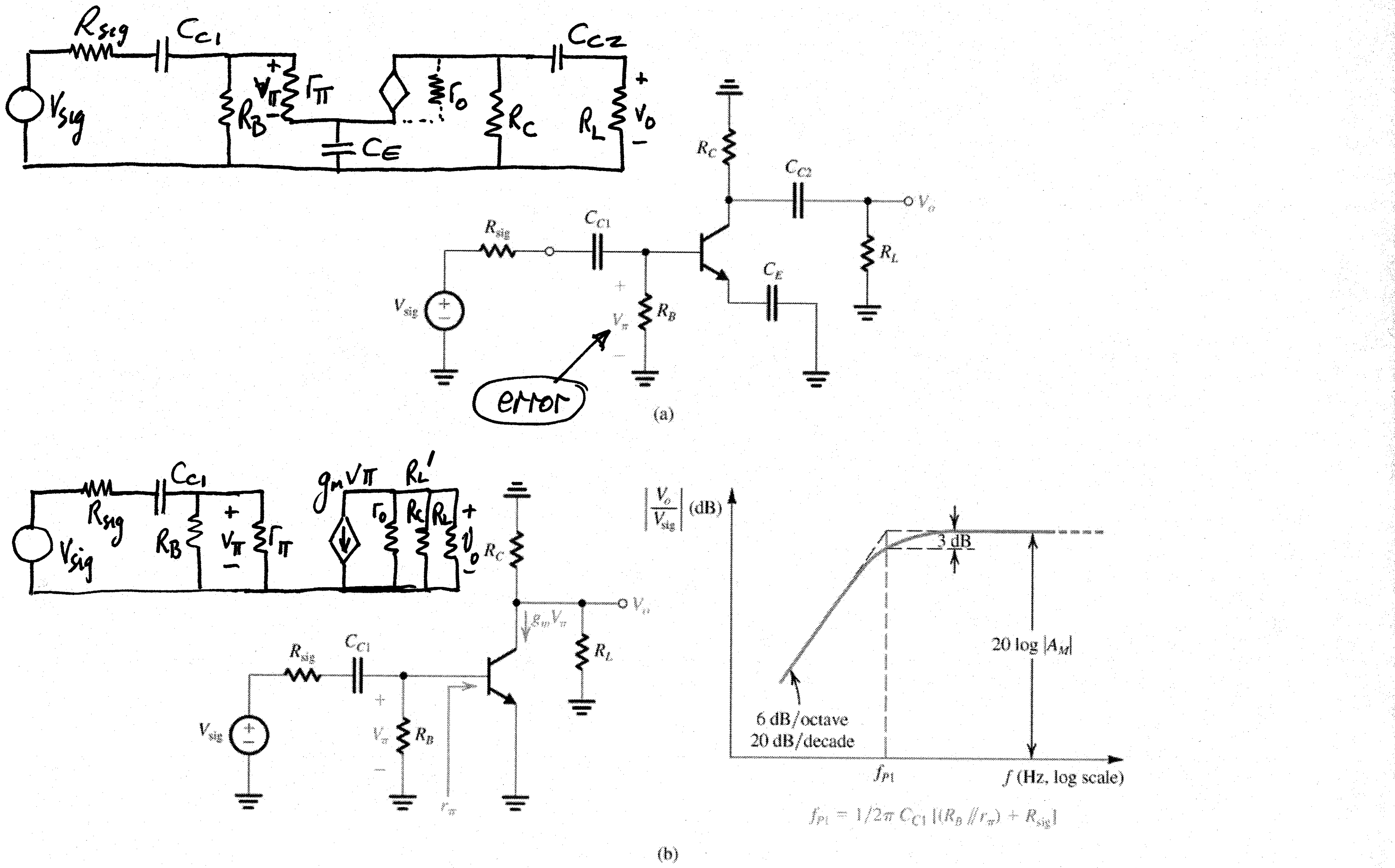


Figure 5.73 Analysis of the low-frequency response of the CE amplifier: **(a)** amplifier circuit with dc sources removed; **(b)** the effect of C_{C1} is determined with C_E and C_{C2} assumed to be acting as perfect short circuits;

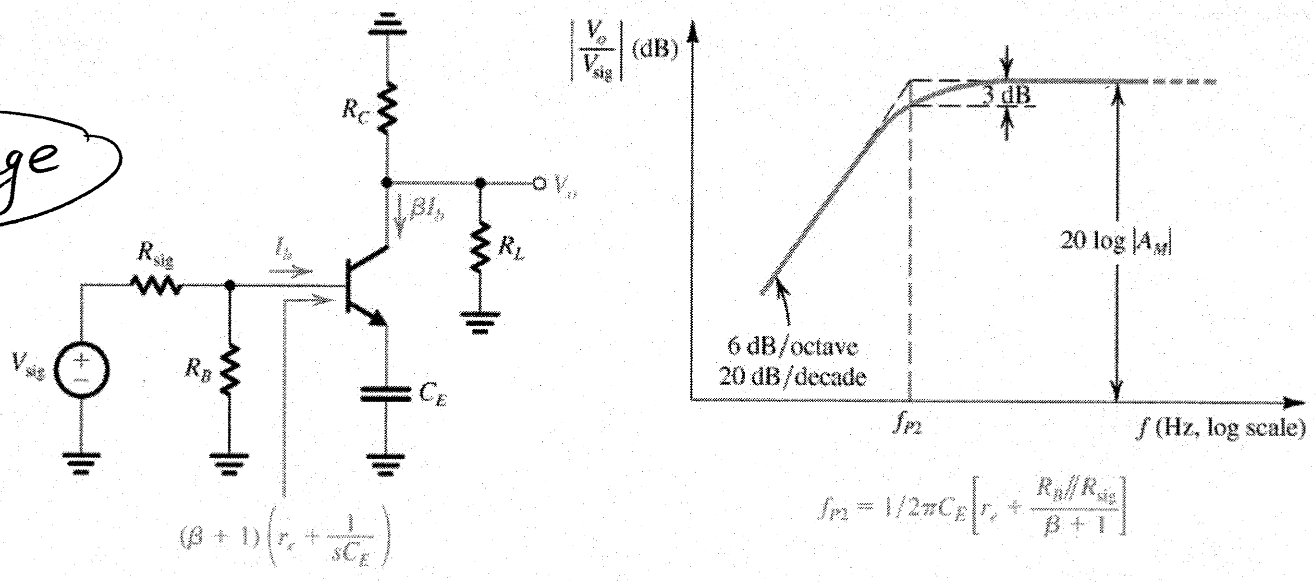
$$\frac{V_o}{V_{sig}} = -g_m R_L' V_\pi = -g_m R_L' \frac{r_\pi \| R_B}{(r_\pi \| R_B) + R_{sig} + 1/sC_{C1}} = A_{MB} \frac{sR_s' C_{C1}}{1 + sR_s' C_{C1}}$$

$$= -g_m R_L' \frac{r_\pi / R_B}{r_\pi \| R_B + R_{sig}} \; \frac{1}{1 + 1/sC_{C1}(r_\pi \| R_B + R_{sig})}$$

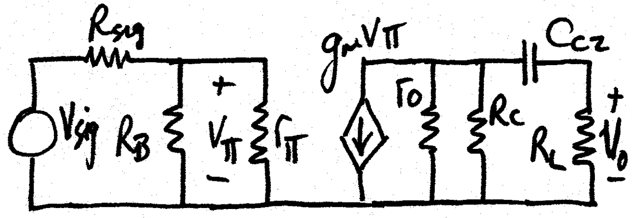
where $R_s' = R_{sig} + (r_\pi \| R_B)$

& $A_{MB} = -g_m R_L' \cdot \frac{r_\pi \| R_B}{R_s'}$

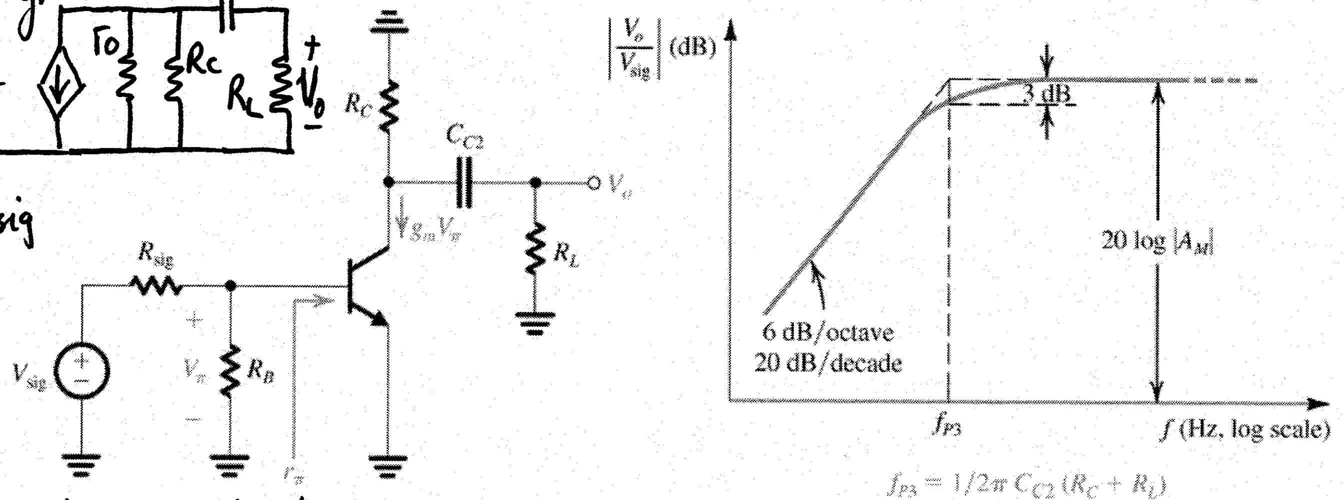
Next page

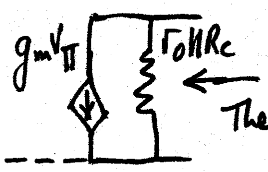


(c)



$$V_\pi = \frac{r_\pi \| R_B}{R_B \| r_\pi + R_{sig}} V_{sig}$$

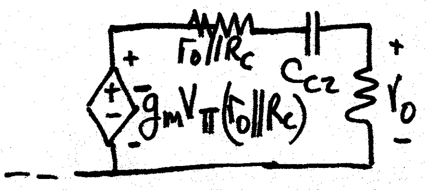




Thevenin equivalent

(d)

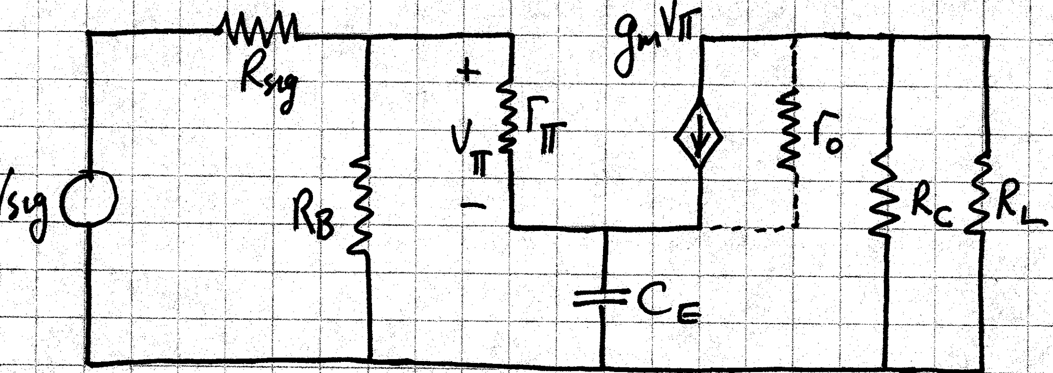
Figure 5.73 *(Continued)* **(c)** the effect of C_E is determined with C_{C1} and C_{C2} assumed to be acting as perfect short circuits; **(d)** the effect of C_{C2} is determined with C_{C1} and C_E assumed to be acting as perfect short circuits;



$$V_o = \frac{-R_L}{R_L + (r_o \| R_c) + 1/sC_{C2}}\, g_m V_\pi (r_o \| R_c) = -g_m \frac{R_L (r_o \| R_c)}{R_L + (r_o \| R_c)} \frac{1}{1 + 1/sC_{C2}[R_L + r_o \| R_c]}$$

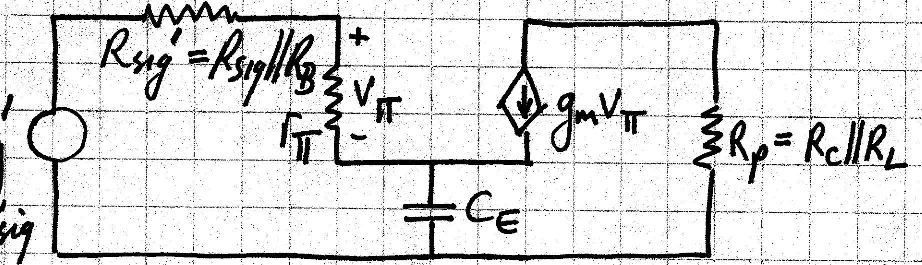
Hence $V_o/V_{sig} = A_{MB} \dfrac{s R_M C_{C2}}{1 + s R_M C_{C2}}$ where $R_M = R_L + r_o \| R_c$

Emitter Case



T-equivalent probably easier here.
Or even βi_b instead of V_π. But show any equivalent circuit will work.

However, neglect r_o or algebra gets much messier!



$$V_{sig}' = \frac{R_B}{R_B + R_{sig}} V_{sig}$$

$$V_o = -g_m V_\pi R_p$$

$$V_{sig}' = R_{sig}'\left(\frac{V_\pi}{r_\pi}\right) + \frac{1}{sC_E}\left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) + V_\pi$$

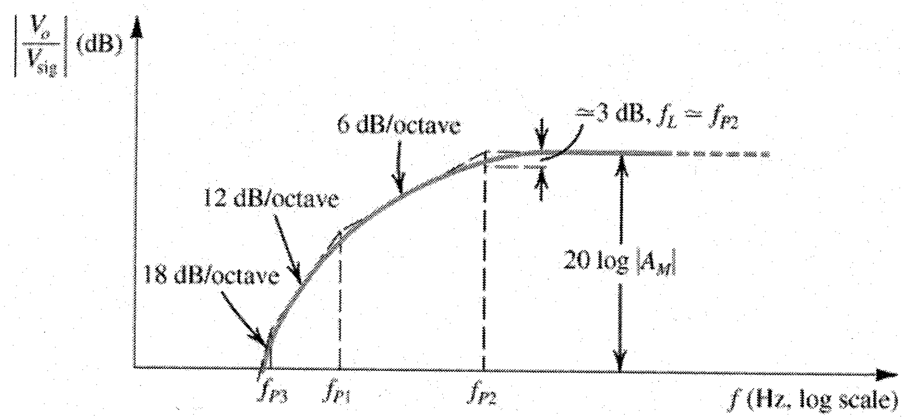
$$\therefore \frac{V_o}{V_{sig}'} = \frac{-g_m R_p}{\frac{R_{sig}'}{r_\pi} + \frac{1}{sC_E}\left(\frac{1}{r_\pi} + g_m\right) + 1} = -g_m R_p \frac{r_\pi}{r_\pi + R_{sig}' + \frac{1}{sC_E}\left(\frac{1}{r_\pi} + g_m\right)}$$

$$= -g_m R_p \frac{r_\pi}{r_\pi + R_{sig}'} \cdot \frac{1}{1 + \frac{1}{sC_E} \cdot \frac{1 + g_m r_\pi}{r_\pi + R_{sig}'}} \Rightarrow A_{MB} \frac{sR'C_E}{1 + sR'C_E}$$

where $R' = \frac{r_\pi + R_{sig}'}{1 + g_m r_\pi} = \frac{r_\pi + R_{sig}'}{1 + \beta}$

$= r_e + \frac{R_B \| R_{sig}}{1 + \beta}$

But note assumed other two X_C's $= 0$ for each case.
Must be wrong for two!!



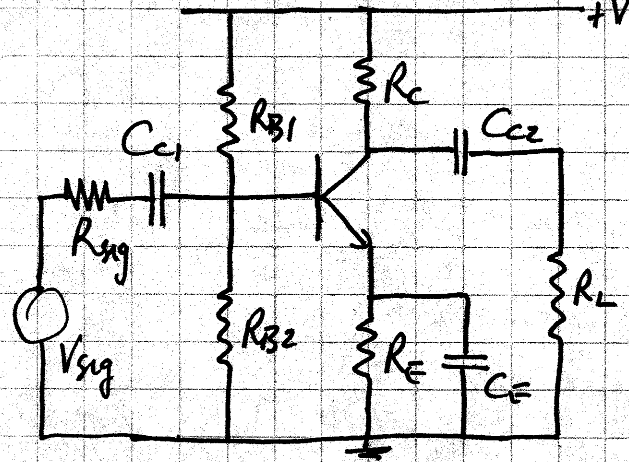
(e)

Note 3 corner frequencies → −60dB/decade at very low frequency.

Figure 5.73 *(Continued)* **(e)** sketch of the low-frequency gain under the assumptions that C_{C1}, C_E, and C_{C2} do not interact and that their break (or pole) frequencies are widely separated.

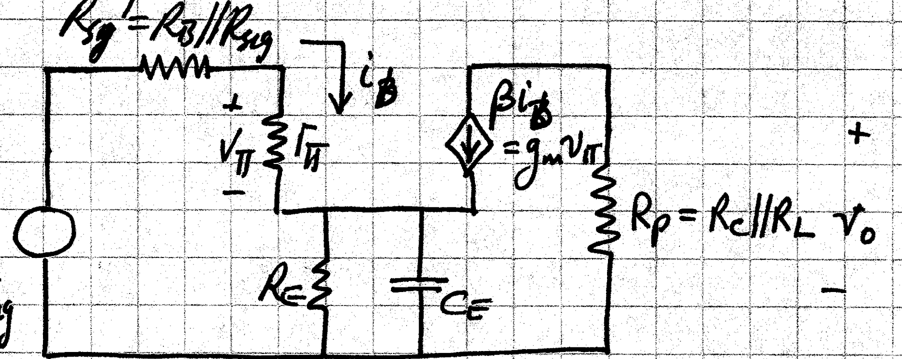
Exercise 5.51

Discrete Emitter Resistor Core



$X_{C1}, X_{C2} \to 0$

$$\Longrightarrow \quad V_{sig}' = \frac{R_B}{R_{sig}+R_B} V_{sig}$$



$$v_\pi = i_b r_\pi \qquad v_E = (1+\beta) i_b (R_E \| C_E)$$

$$v_s = i_b \left(R_s' + r_\pi + (1+\beta)[R_E \| C_E]\right)$$

$$= \frac{v_\pi}{r_\pi}\left(R_s' + r_\pi + (1+\beta)(R_E \| C_E)\right)$$

$$\therefore v_o = -g_m v_\pi R_p = -g_m R_p v_s \frac{r_\pi}{R_s' + r_\pi + (1+\beta) R_E \| C_E} = -g_m R_p v_s \frac{r_\pi}{R_s' + r_\pi} \cdot \frac{1}{1 + \frac{(1+\beta) R_E \| C_E}{R_s' + r_\pi}}$$

$$\therefore \frac{v_o}{v_s} = A_{MB} \frac{1}{1 + \frac{(1+\beta)}{R_s' + r_\pi} \frac{R_E \, 1/sC_E}{R_E + 1/sC_E}} = A_{MB} \frac{1}{1 + \frac{1}{r_x} \frac{R_E}{1 + sR_EC_E}}$$

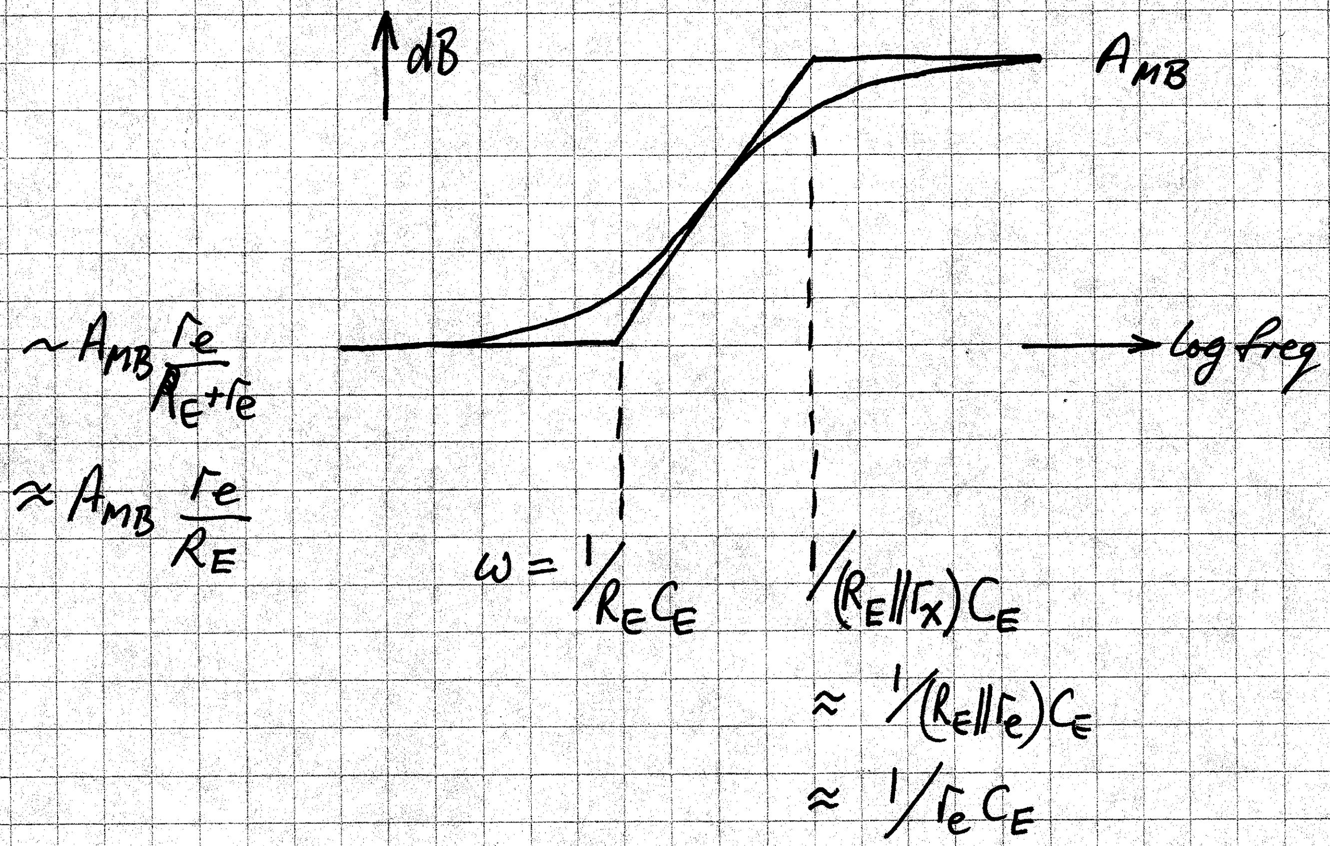
where $r_x = \frac{R_s' + r_\pi}{1+\beta} = r_e + \frac{R_s'}{1+\beta} \approx r_e$

$$= A_{MB} \frac{1 + sR_EC_E}{1 + sR_EC_E + \frac{R_E}{r_x}} = \frac{A_{MB}}{1 + \frac{R_E}{r_x}} \frac{1 + sR_EC_E}{1 + s\frac{R_EC_E}{1 + R_E/r_x}}$$

$$= A_{MB} \frac{r_x}{r_x + R_E} \cdot \frac{1 + sR_EC_E}{1 + s\frac{R_E r_x}{R_E + r_x} C_E}$$

$$\xrightarrow[\text{high freq}]{} A_{MB} \frac{\cancel{r_x}}{\cancel{r_x + R_E}} \frac{\cancel{R_E C_E}}{\cancel{\frac{R_E r_x}{r_x + R_E}} \cancel{C_E}} = A_{MB} \qquad \xrightarrow[\text{low freq}]{} A_{MB} \frac{r_e + \frac{R_s'}{1+\beta}}{R_E + r_e + \frac{R_s'}{1+\beta}} \approx A_{MB} \frac{r_e}{R_E + r_e}$$

Bode Plot



Note relevance to lab experiment

Ex 5.48 BJT $I_C = 1mA$, $V_{CB} = -2V$, $\tau_F = 20ps$, $C_{jeo} = 20fF$, $C_{\mu o} = 20fF$
$V_{oe} = 0.9V$, $V_{oc} = 0.5V$, $m_{CBJ} = 0.33$

Find $C_{de} = \tau_F \dfrac{I_c}{V_T} = \dfrac{20\times10^{-12} \times 10^{-3}}{25\times10^{-3}} = 0.8pF$

$C_{je} = 2C_{jeo} = 40fF$

$C_\pi = C_{de} + C_{je} = 0.84pF$

$$C_\mu = \frac{C_{\mu o}}{\left(1 + \frac{|V_{CB}|}{V_{oc}}\right)^{m_{CBJ}}} = \frac{20fF}{\left(1+\frac{2V}{0.5V}\right)^{0.33}} \approx 12fF$$

$$f_T = g_m/2\pi(C_\pi + C_\mu) = \frac{I_c}{V_T}\frac{1}{2\pi(C_\pi+C_\mu)} = \frac{10^{-3}\times10^{12}}{25\times10^{-3}\times2\pi\times0.852} \approx 7.5GHz.$$

Ex 5.49 BJT $I_c = 1mA$, $C_\mu = 2pF$, $|h_{fe}| = 10$ at 50MHz, Find f_T & C_π

$\therefore g_m = \dfrac{1mA}{25mV} = 40mA/V$ $\quad f_T = 500MHz$ $\quad \therefore C_\pi + C_\mu = g_m/2\pi f_T = \dfrac{40\times10^{-3}}{2\pi\times5\times10^8}$
$= 12.7pF$

$\therefore C_\pi = 10.7pF$

Ex 5.50 For the BJT of Ex 5.49: C_π includes approx constant C_{je} (depletion capac) = 2pF
Find f_T at $I_c = 0.1mA$

$C_{de} = \tau_F g_m = \tau_F I_c/V_T$

$C_\pi = C_{de} + C_{je}$ $\therefore C_{de} = (10.7-2)pF = 8.7pF$ at 1mA. So $C_{de}(.1mA) = 0.87pF$

$\therefore C_\pi(.1mA) = 2pF + 0.87pF = 2.87pF$ $\therefore f_T(I_c = 0.1mA) = g_m/2\pi(C_\pi + C_\mu)$
$= 4\times10^{-3}/2\pi(2.87+2)10^{-12} = 130.7MHz$

Ex 5.51 For amplifier of Example 5.18, (← refer to Example 5.18!) find R_L to halve A_{MB}; what is new f_H?

$$A_{MB} = -g_m R_L' \cdot \frac{r_\pi \| R_B}{R_{sig} + r_\pi \| R_B}$$

← halve A_{MB} by halving R_L'

$$\begin{cases} R_L = 5K \\ R_C = 8K \\ r_o = 100K \end{cases}$$

$$R_L' = R_L \| R_C \| r_o = \frac{R_C R_L r_o}{R_C R_L + R_L r_o + r_o R_C} = \frac{8 \times 5 \times 100K}{40 + 500 + 800} \approx 3K\Omega$$

$$\therefore \text{ need } \frac{R_L \times 8 \times 100}{8R_L + 100R_L + 800} = 1.5 \Rightarrow R_L = 1.9K\Omega$$

$$\text{New } f_H = \frac{1}{2\pi C_{in} R_{sig}'}$$

C_{in} dominates since $(1+g_m R_L')C_\mu \gg C_\mu$

$R_{sig}' = 1.65K\Omega$ from Example 5.18

$$C_{in} = C_\pi + C_\mu (1 + g_m R_L')$$

$$= 7pF + 1pF(1 + 40 \times 10^{-3} \cdot 1.5 \times 10^3) = 68pF$$

($7pF$, $1pF$, 40×10^{-3} from example 5.18; 1.5×10^3 new value)

$$\therefore f_H = \frac{1}{2\pi \, 68 \times 10^{-12} \underbrace{1.65 \times 10^3}_{\text{from Example 5.18}}} = 1.42 MHz.$$