ECE321 ELECTRONICS I
FALL 2006

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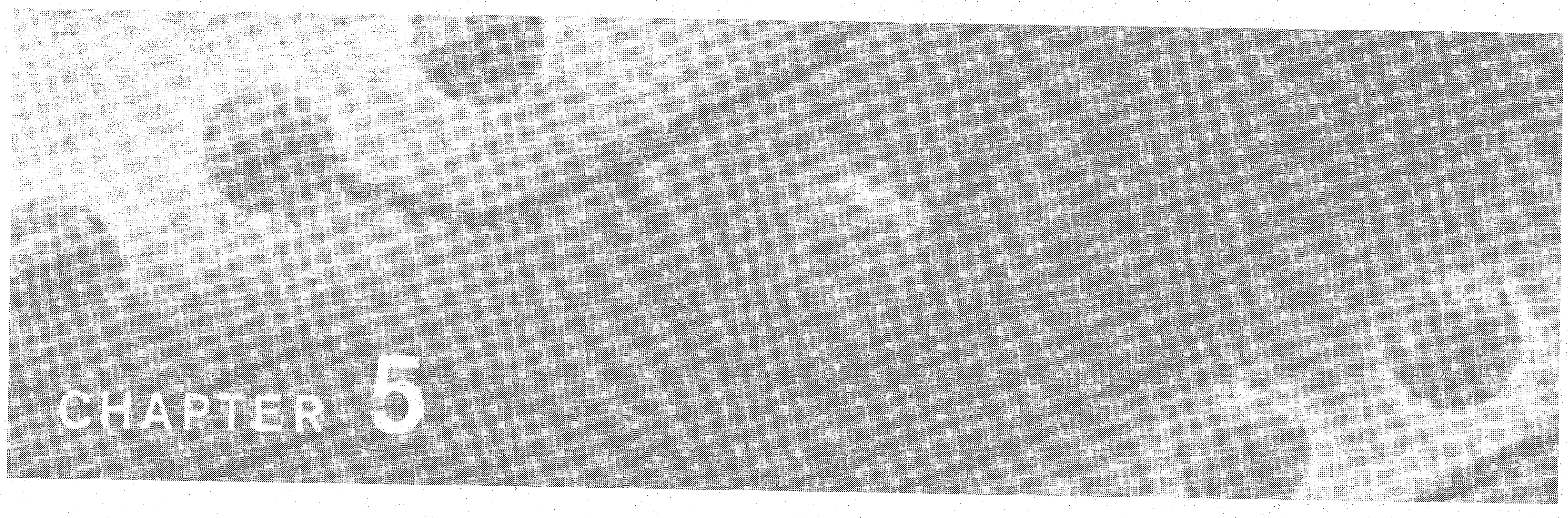
Lecture 13

9th November, 2006



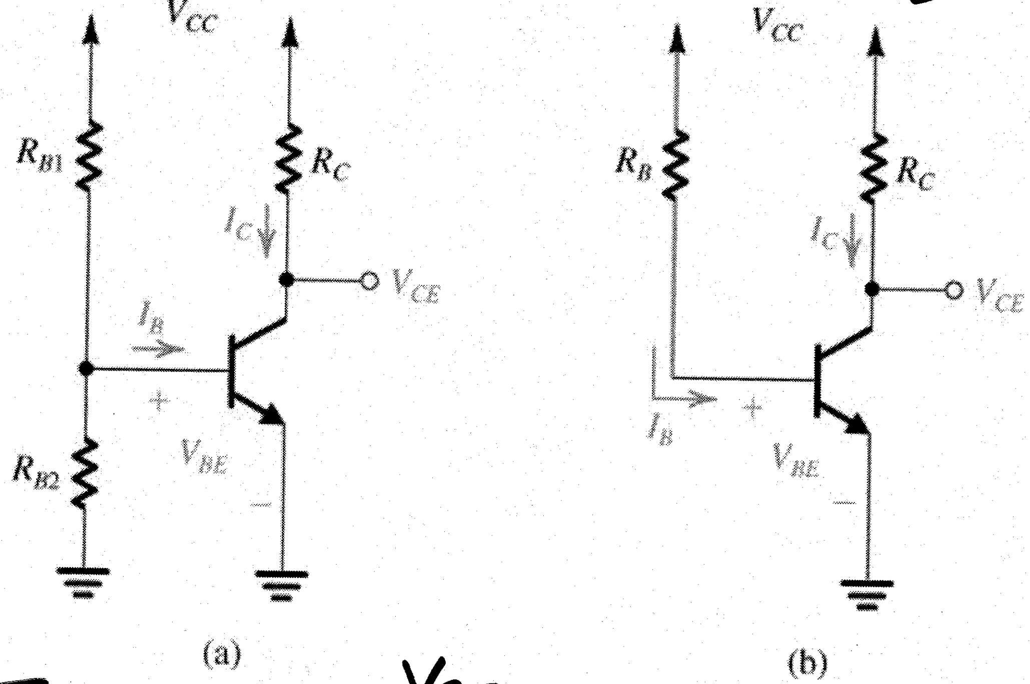
Bipolar Junction Transistors (BJTs)

5.5 Biasing ← DC analysis/design

5.6 Small Signal ← AC analysis/design

Two simple bias schemes

Set V_{BE} | Set I_B



$$I_C = I_S \exp \frac{V_{BE}}{V_T} \qquad I_C = \beta I_B$$

Figure 5.43 Two obvious schemes for biasing the BJT: **(a)** by fixing V_{BE}; **(b)** by fixing I_B. Both result in wide variations in I_C and hence in V_{CE} and therefore are considered to be "bad." Neither scheme is recommended.

Small variations in V_{BE} due to R_1, R_2 tolerances $\longrightarrow$ exp $\longrightarrow$ $\boxed{\text{large } I_C \text{ variations}}$ $\longleftarrow$ β variations

S&S: "Classical" bias
Self-bias
Auto-bias

$\left.\right\}$ R_E negative feedback → defines I_E

Thevenin equiv base circuit

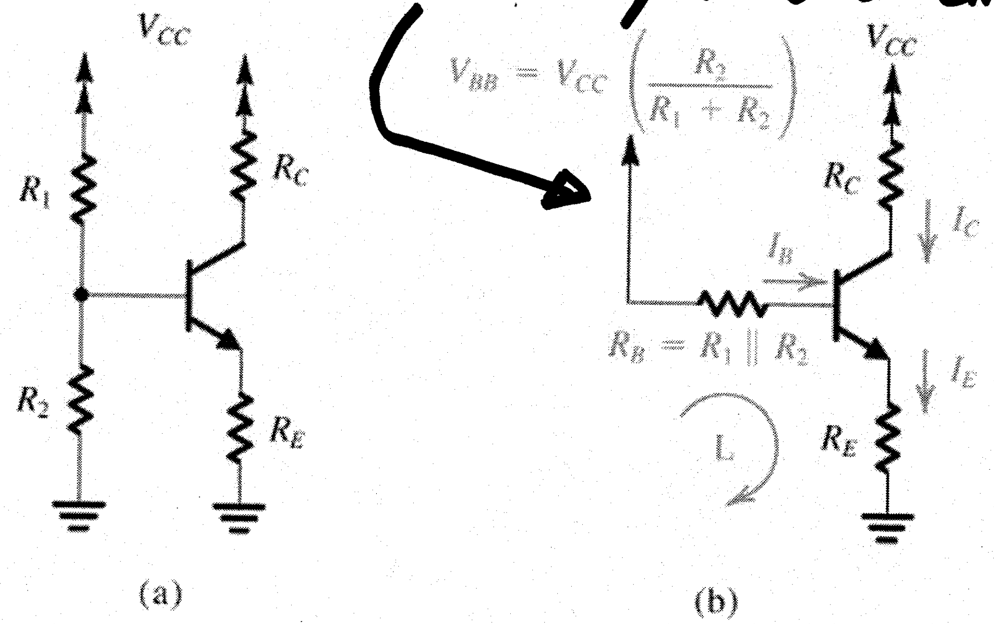


Figure 5.44 Classical biasing for BJTs using a single power supply: **(a)** circuit; **(b)** circuit with the voltage divider supplying the base replaced with its Thévenin equivalent.

$$I_B = \frac{V_{BB} - V_{BE} - I_E R_E}{R_B}$$

$$= \frac{V_{BB} - V_{BE} - (1+\beta) I_B R_E}{R_B}$$

$$= \frac{(V_{BB} - V_{BE})/R_B}{1 + (1+\beta) R_E/R_B} = \frac{V_{BB} - V_{BE}}{R_B + (1+\beta) R_E}$$

But we actually want to control $I_E \approx I_C$, not I_B

$$I_E = (1+\beta) I_B = \frac{(1+\beta)(V_{BB} - V_{BE})}{R_B + (1+\beta) R_E}$$

$$= \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{1+\beta}} \approx \frac{V_{BB}}{R_E} \quad \text{if } V_{BB} \gg V_{BE} \;\&\; R_B \ll (1+\beta) R_E$$

$V_{BB} >> V_{BE}$ minimizes effects of V_{BE} variation with β
and thermal effects

$R_B << (1+\beta)R_E$ minimizes effects of β variations

Typically:

(a) make $V_{BB} \sim \frac{1}{3} V_{CC}$

(b) make $\frac{V_{CC}}{R_1+R_2} \sim (0.1 \text{ to } 1) \times I_E$

(c) include $V_{BE} \sim 0.7\text{v}$ in formula, rather than completely neglected

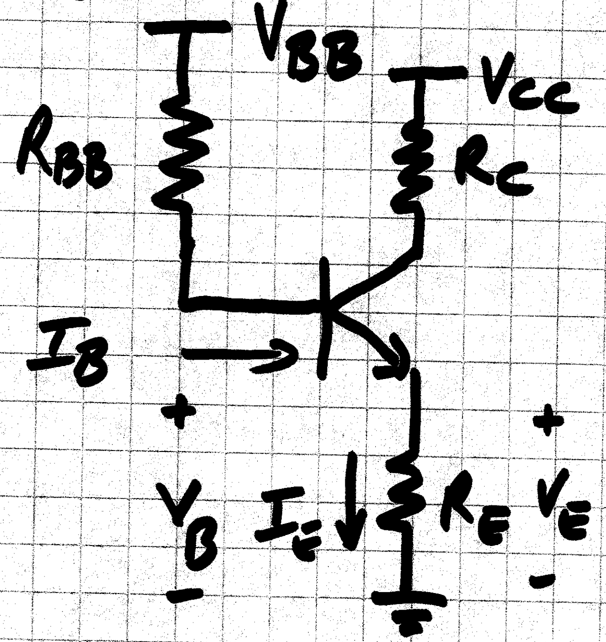
(b) $\rightarrow$ Say $I_E \sim 10\text{mA}$, $\beta = 100 \rightarrow I_B \sim 0.1\text{mA}$

If "standing current" $V_{CC}/(R_1+R_2) = 10\text{mA}$

Base current "drawn off" $= I_B \sim 0.2\text{mA}$ to 0.05mA
as $\beta = 50$ to 200

i.e. minimizes variation in $I_B R_B$ with β

Negative feedback effect with R_E



Assume ideal design for nominal β.

If β increases (due to temperature or different transistor)

I_E tends to increase

$\therefore V_E$ increases

and for $V_{BE} \sim$ constant, V_B increases

$\therefore I_B = \dfrac{V_{BB} - V_B}{R_B}$ decreases, decreasing I_E back <u>towards</u> design value

Exercise 5.32
(refers to Example 5.13)

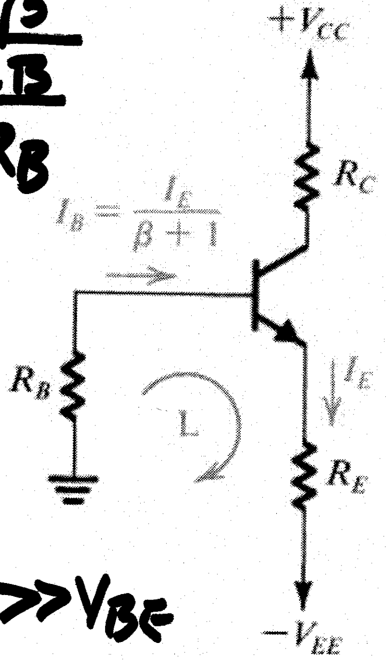
2 Supply voltage bias (+, −, and ground)

$$I_E = (1+\beta) I_B = \frac{1+\beta}{R_B}\left(0 - V_{BE} - I_E R_E - (-V_{EE})\right)$$

$$= \frac{(V_{EE} - V_{BE})\frac{1+\beta}{R_B}}{1 + (1+\beta) R_E / R_B}$$

$$= \frac{V_{EE} - V_{BE}}{R_E + \frac{R_B}{1+\beta}}$$

$$\Rightarrow \frac{V_{EE}}{R_E} \text{ for } V_{EE} \gg V_{BE}, \quad R_E \gg \frac{R_B}{1+\beta}$$



Introduces the idea of constant current source emitter bias.

Figure 5.45 Biasing the BJT using two power supplies. Resistor R_B is needed only if the signal is to be capacitively coupled to the base. Otherwise, the base can be connected directly to ground, or to a grounded signal source, resulting in almost total β-independence of the bias current.

Exercise D5.33

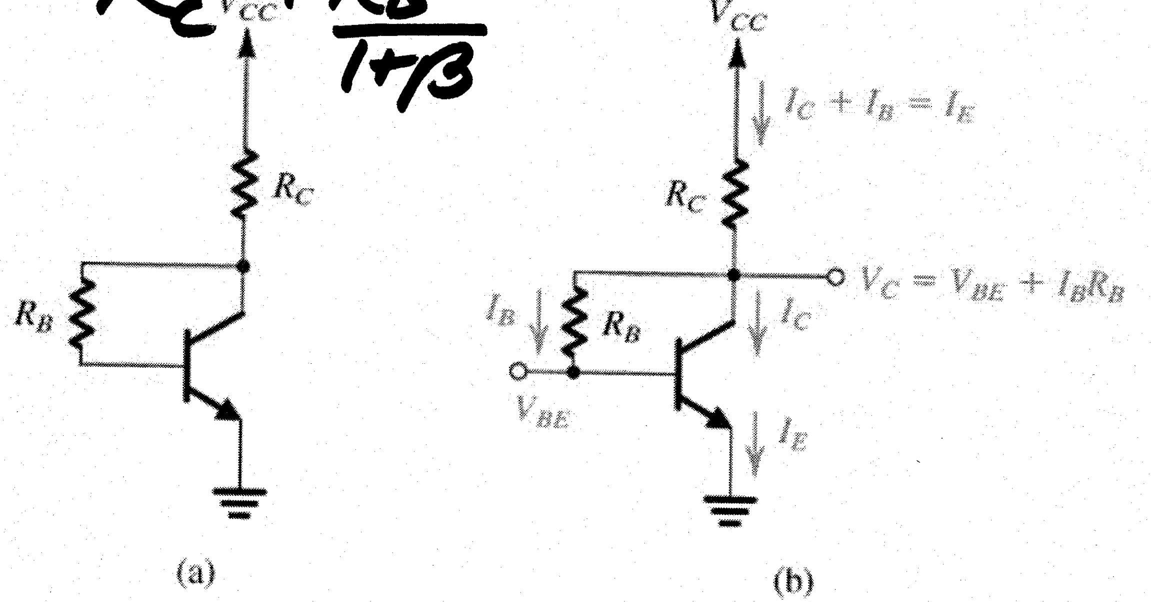
Collector-Base Feedback Bias

$$V_{CC} = I_E R_C + \frac{I_E}{1+\beta} R_B + V_{BE}$$

$$\therefore I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{1+\beta}} \longrightarrow \approx \frac{V_{CC}}{R_C} \text{ for}$$

$V_{CC} \gg V_{BE}$

$R_C \gg \frac{R_B}{1+\beta}$



Ex D5.34 $V_{CC} = 10v$ $\beta = 100$ $I_E = 1mA$ $\pm 2v$ swing at V_C

$I_C \approx I_E = 1mA$ i.e. $V_{CE} = 2.3v$

Figure 5.46 **(a)** A common-emitter transistor amplifier biased by a feedback resistor R_B. **(b)** Analysis of the circuit in (a).

for $V_{CESAT} \sim .3v$

$\therefore R_C = (10 - 2.3v)/1mA = 7.7K$

$R_B = \frac{(2.3 - 0.7)v}{1mA/100} = 160K\Omega$

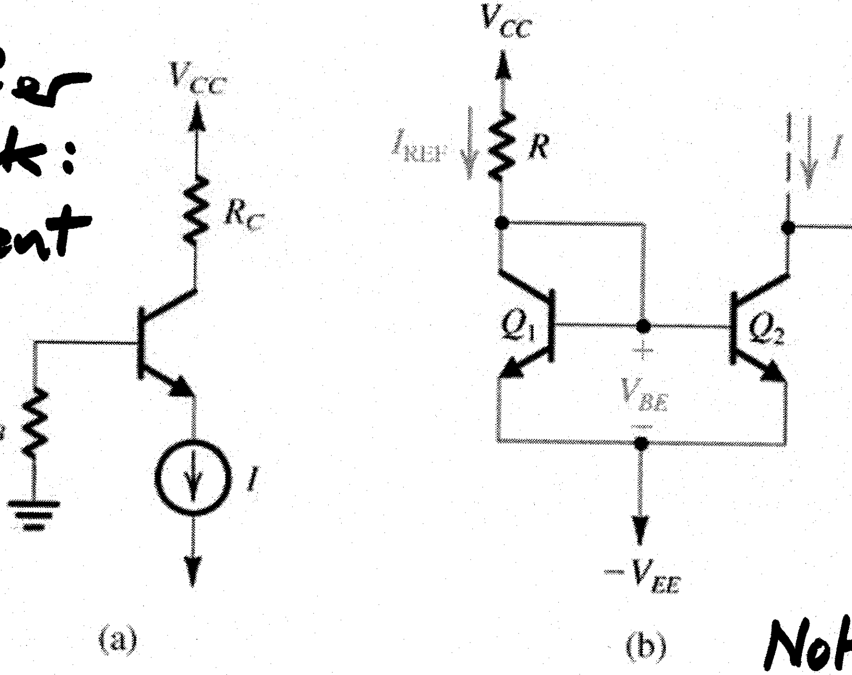
OR $R_C \rightarrow (10 - 2.3) / \frac{1mA}{.99} \sim 7.6K$

$R_B \rightarrow \frac{2.3 - 1.6}{1mA/101} \sim 162K\Omega$

Constant Current Source Bias

$$I_{REF} = \frac{V_{CC} - V_{BE} - (-V_{EE})}{R} \approx \frac{V_{CC} + V_{EE}}{R}$$

Refer back: Constant current emitter bias from 2-supply bias circuit.



& $V_{BE1} = V_{BE2}$

$\therefore I_{E1} = I_{E2}$

$I = I_{REF}$

Current Mirror

Note $V_{CE1} = V_{BE} > V_{CE})_{SAT}$

$\therefore$ Forward Active

Ex 5.35(a) $V_{CC} = 10v$ $I = 1mA$ $R_B = 100K$ $R_C = 7.5K$

Find V_B, V_E, V_C for $\beta = 100$

Figure 5.47 **(a)** A BJT biased using a constant-current source I. **(b)** Circuit for implementing the current source I.

$I = 1mA = I_E$ $\therefore I_C = 0.99mA$ & $V_C = 10 - (.99)7.5 = 2.575v$

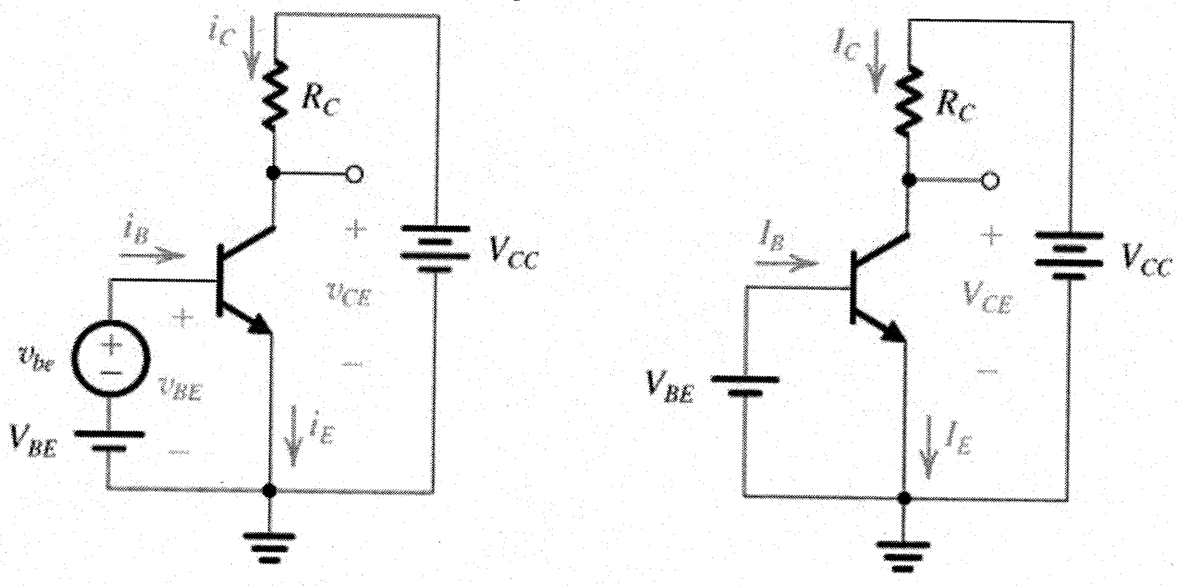
$I_B = 1mA/101$ $\therefore V_B = -100K \times 1mA/101 = -0.99v$ & $V_E = -1.69v$

(b) Find R for $I = 1mA$ ($V_{EE} = 10v$) $\therefore R = (20v - .7v)/1mA = 19.3K\Omega$.

Small Signal (ac) Operation/Models

Complete circuit & signals
e.g. $v_{BE} = V_{BE} + v_{be}$

DC bias only



(a) (b)

$$i_C = I_C + i_c = I_S \exp(V_{BE} + v_{be})/V_T = I_C \exp\frac{v_{be}}{V_T} \approx I_C\left(1 + \frac{v_{be}}{V_T}\right)$$

if $v_{be} \ll V_T$

Figure 5.48 **(a)** Conceptual circuit to illustrate the operation of the transistor as an amplifier. **(b)** The circuit of (a) with the signal source v_{be} eliminated for dc (bias) analysis.

"Small signal approximation"

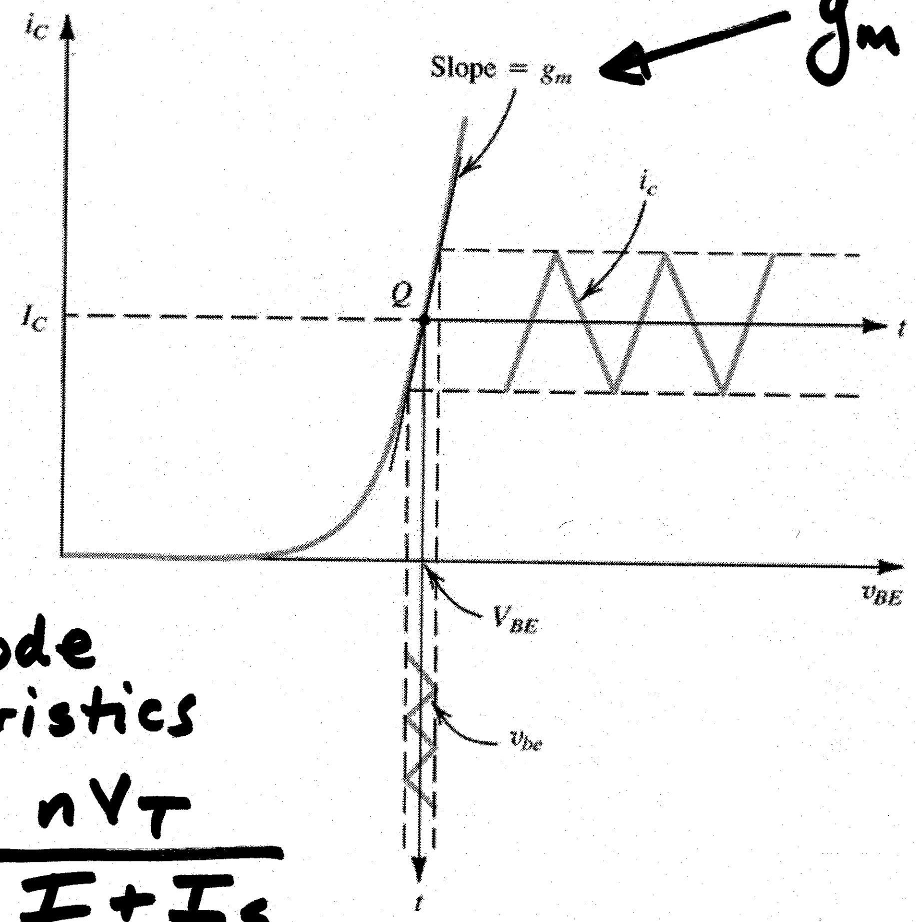
So $i_c \approx I_C \frac{v_{be}}{V_T} \Longrightarrow g_m v_{be}$ where $g_m = I_C/V_T$

"Transconductance"

i_C vs v_{BE} characteristics

Slope

$g_m = \frac{I_C}{V_T}$



Remember diode characteristics

$r_D = \frac{1}{\text{slope}} = \frac{nV_T}{I + I_S}$

$\approx nV_T/I$

Figure 5.49 Linear operation of the transistor under the small-signal condition: A small signal v_{be} with a triangular waveform is superimposed on the dc voltage V_{BE}. It gives rise to a collector signal current i_c, also of triangular waveform, superimposed on the dc current I_C. Here, $i_c = g_m v_{be}$, where g_m is the slope of the i_C–v_{BE} curve at the bias point Q.

Small Signal Input Resistance

$$i_B = \frac{i_C}{\beta} \approx \frac{I_C}{\beta} + \frac{I_C}{\beta}\cdot\frac{v_{be}}{V_T}$$

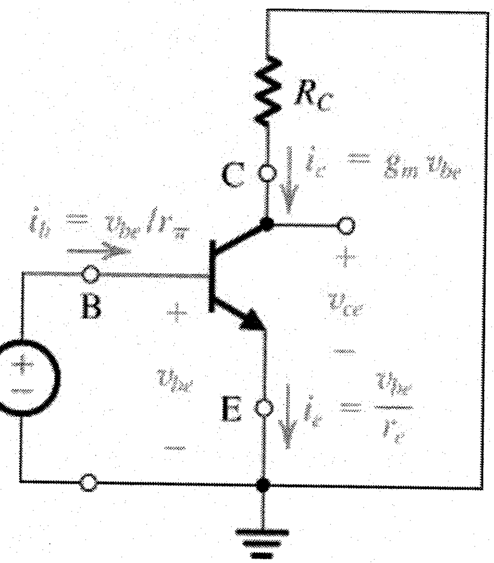
(small signal approximation)

$$\& \quad i_b = \frac{I_C}{\beta}\,\frac{v_{be}}{V_T} = \frac{g_m}{\beta}\,v_{be}$$

Define $r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$

$= \beta\,\frac{V_T}{I_C}$

$= \frac{V_T}{I_B}$



$$\boxed{v_{be} = i_b r_\pi = i_e r_e \quad \therefore r_\pi = \frac{i_e}{i_b} r_e = (1+\beta) r_e}$$

Figure 5.50 The amplifier circuit of Fig. 5.48(a) with the dc sources (V_{BE} and V_{CC}) eliminated (short circuited). Thus only the signal components are present. Note that this is a representation of the signal operation of the BJT and not an actual amplifier circuit.

Define r_e (emitter resistance)

$= v_{be}/i_e$

$= v_{be}/(i_c/\alpha)$

$= v_{be}/(I_C v_{be}/\alpha V_T)$

$= v_{be}/(I_E v_{be}/V_T)$

$= V_T/I_E$

$= \alpha V_T/I_C = \alpha/g_m$

$\approx 1/g_m$

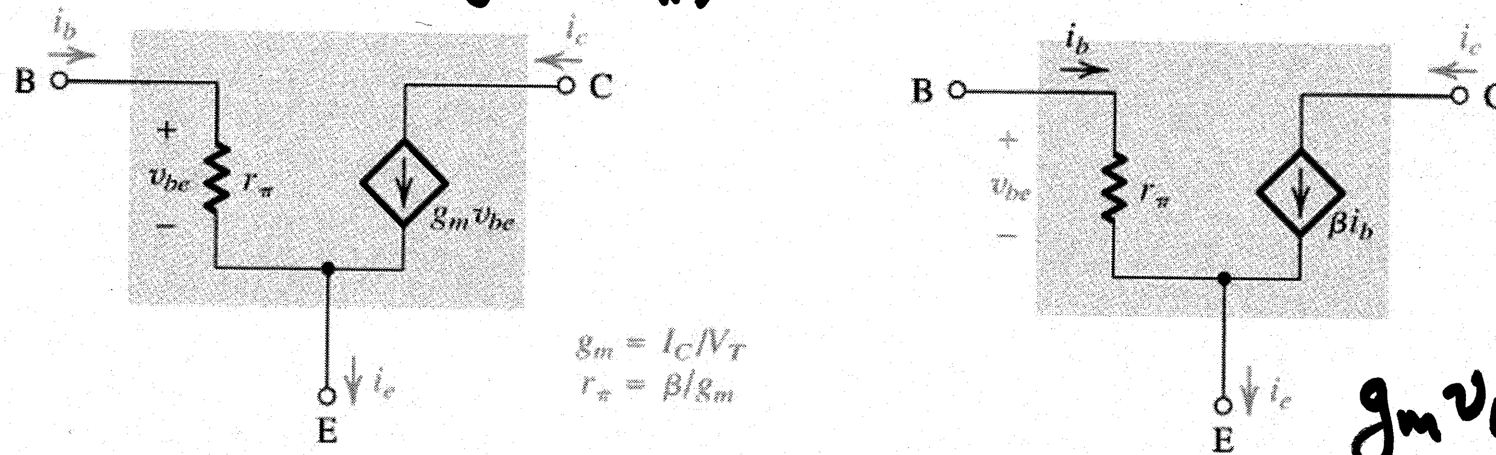
Exercise 5.37

Exercise 5.38

Hybrid Π Small Signal Model

$$i_e = i_b + i_c$$
$$= \frac{v_{be}}{r_\pi} + g_m v_{be}$$
$$= v_{be}\left(g_m + 1/r_\pi\right)$$

$$= \frac{v_{be}}{r_\pi}\left(1 + g_m r_\pi\right)$$
$$= \frac{v_{be}}{r_\pi}\left(1+\beta\right) = v_{be} \Big/ \left(\frac{r_\pi}{1+\beta}\right)$$
$$= v_{be}/r_e$$



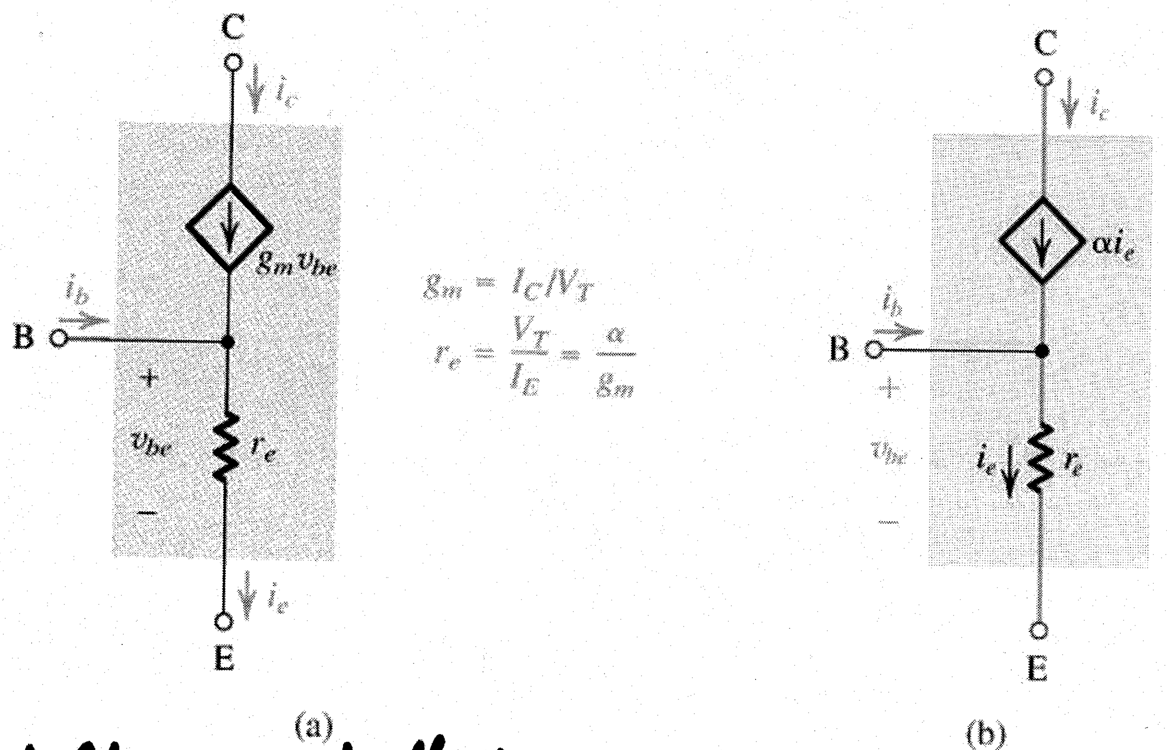
(a) (b)

$$g_m v_{be} = g_m i_b r_\pi$$
$$= \beta i_b$$

Same circuits
Use whichever suits
(Given values, personal preference)

Figure 5.51 Two slightly different versions of the simplified hybrid-π model for the small-signal operation of the BJT. The equivalent circuit in **(a)** represents the BJT as a voltage-controlled current source (a transconductance amplifier), and that in **(b)** represents the BJT as a current-controlled current source (a current amplifier).

T-model : Uses r_e instead of r_π

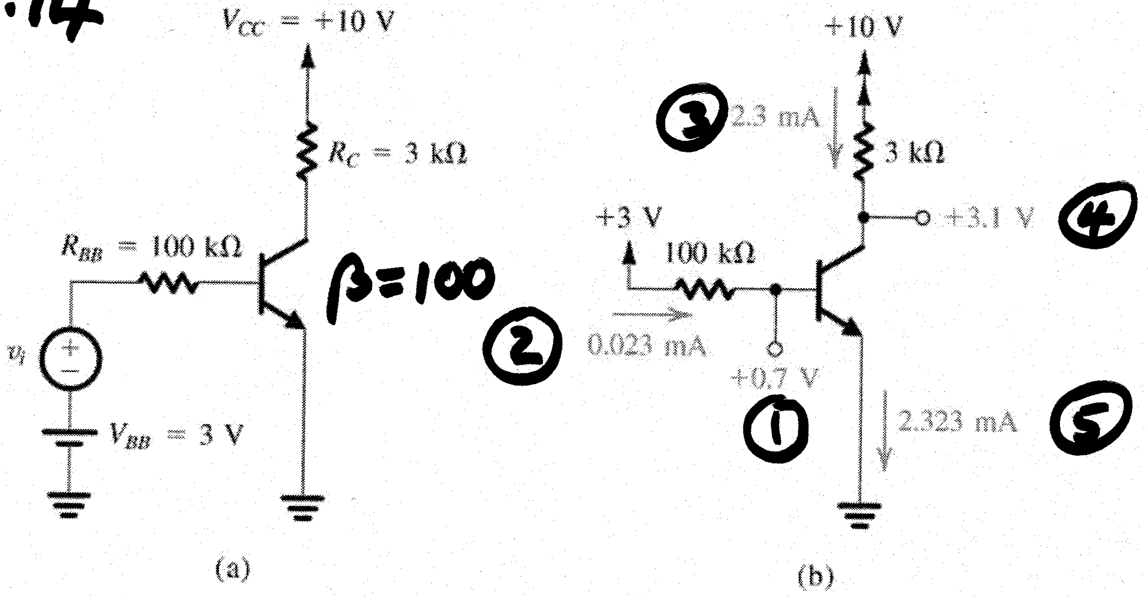


Voltage controlled

Current controlled

Figure 5.52 Two slightly different versions of what is known as the *T model* of the BJT. The circuit in **(a)** is a voltage-controlled current source representation and that in **(b)** is a current-controlled current source representation. These models explicitly show the emitter resistance r_e rather than the base resistance r_π featured in the hybrid-π model.

Example 5.14



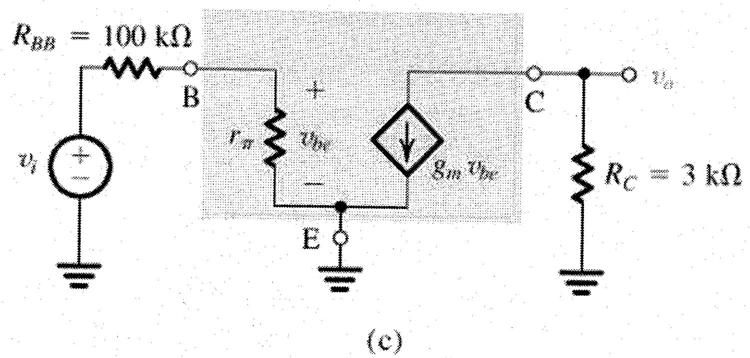


Figure 5.53 Example 5.14: **(a)** circuit; **(b)** dc analysis; **(c)** small-signal model.

Hence

$$g_m = \frac{I_C}{V_T} = \frac{2.3\,mA}{25\,mV} = 92\,mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100\,V}{92\,mA} = 1.09\,K\Omega$$

$$v_{be} = \frac{1.09}{100 + 1.09} v_i$$

so $v_o = -3K \times 92\,mA \cdot \frac{1.09}{101.09} v_i$

$\approx -3.04\,v_i$

Example 5.15 (Waveforms for Example 5.14)

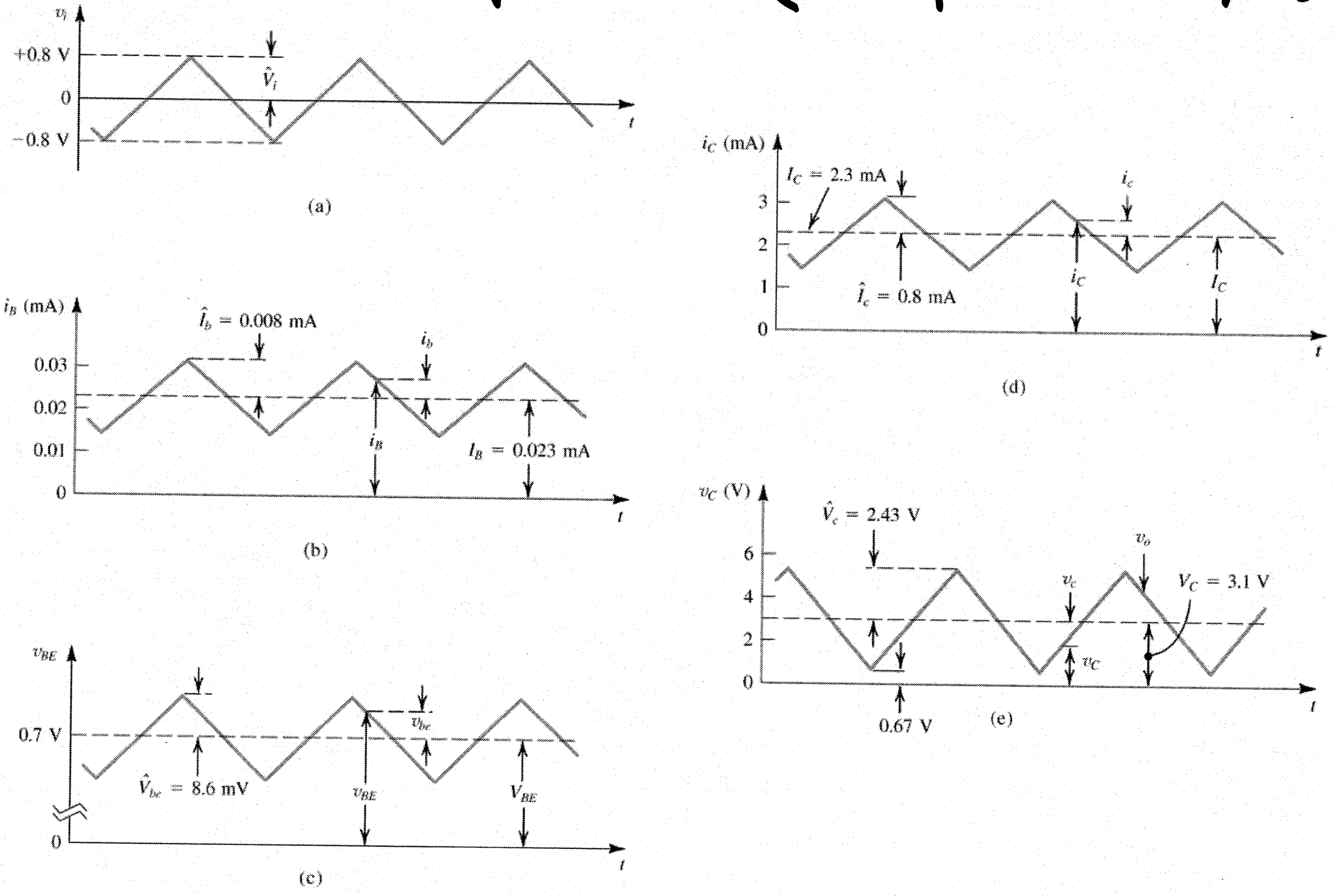


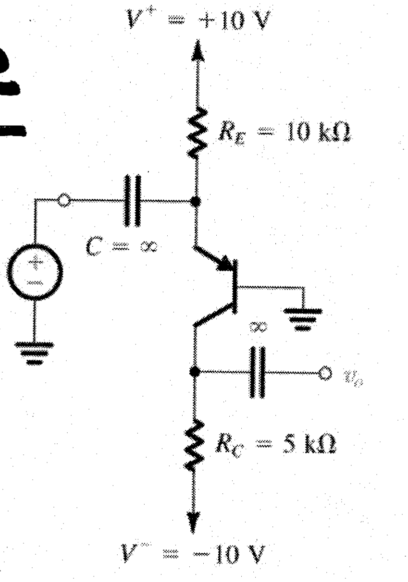
Figure 5.54 Signal waveforms in the circuit of Fig. 5.53.

PNP Common-Base

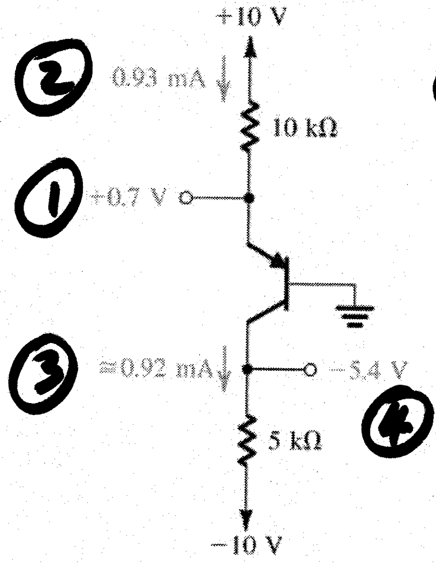
Input to E
Output from C.

Example 5.16

Coupling capacitors
$X_{DC} \rightarrow \infty$
$X_{AC} \rightarrow 0$



(a)

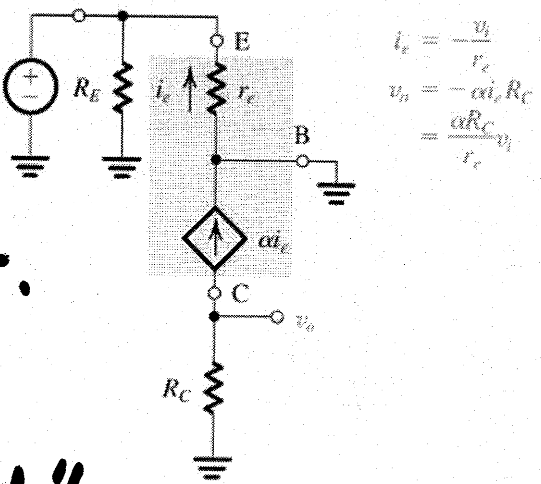


(b)

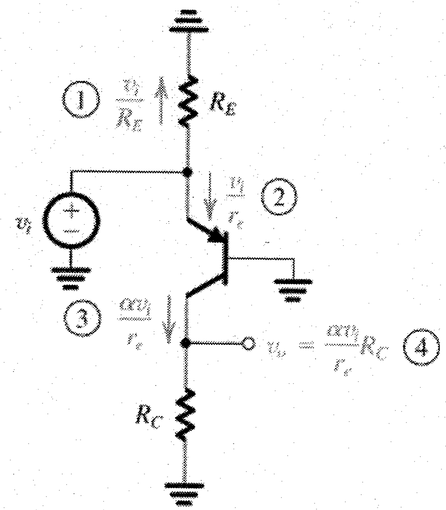
Small signal models :-

1. X_c's $\rightarrow 0$ short circuit.

2. Concept of "AC ground"



(c)



(d)

Small signal analysis without/bypassing the small signal model.

Figure 5.55 Example 5.16: **(a)** circuit; **(b)** dc analysis; **(c)** small-signal model; **(d)** small-signal analysis performed directly on the circuit.

Note: Capacitors still S.C. &
$V^+, V^- \rightarrow$ AC ground

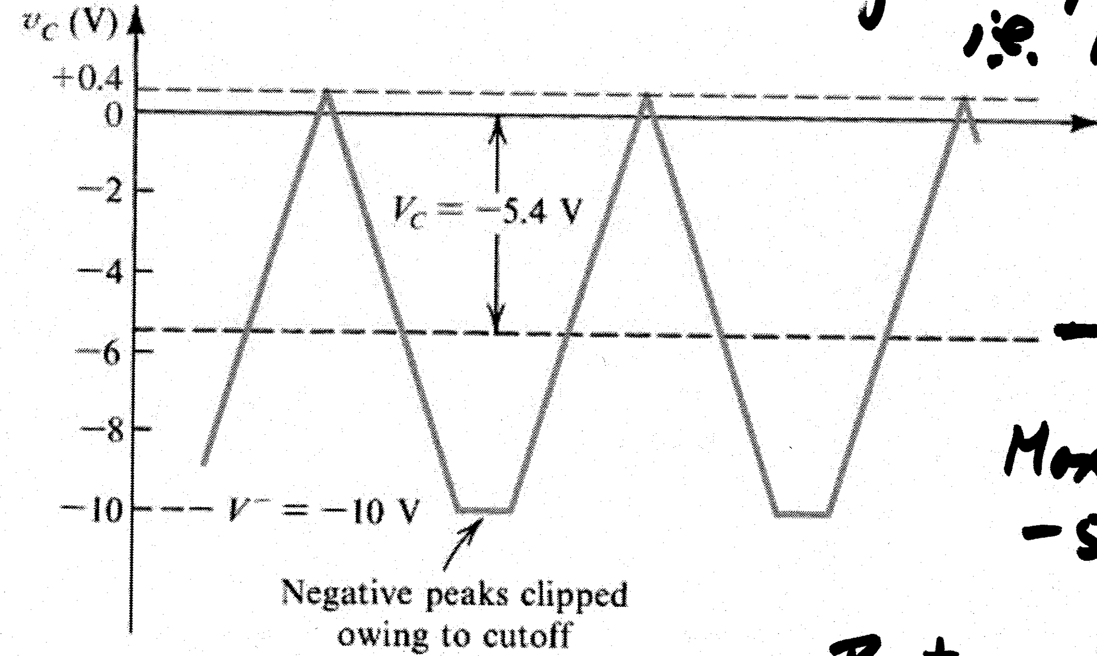
(a) Fixed DC ∴ AC signal = 0
(b) Power supply outputs :
Actually large C to ground.

Small signal model

Use to estimate large signal swing limitations etc!

BUT — clearly exceeding the limits of the small signal approximation i.e. ignores device non-linearities

Example 5.16



$-5.4\text{v} = V_C$

Max +ve swing

$-5.4 \rightarrow +0.4$

i.e. 5.8v

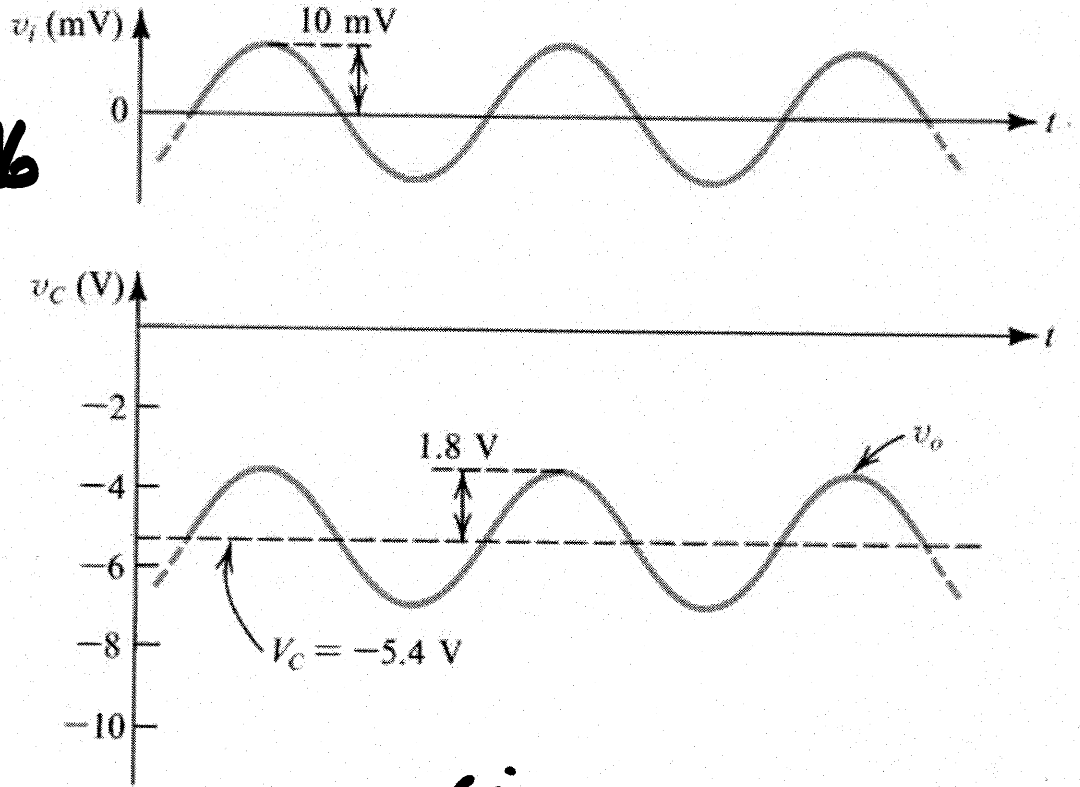
But $-5.4\text{v} - 5.8\text{v} = -11.2\text{v}$

$< V^- = -10\text{v}$

$\therefore$ Exceeds negative swing limit!

Figure 5.56 Distortion in output signal due to transistor cutoff. Note that it is assumed that no distortion due to the transistor nonlinear characteristics is occurring.

Example 5.16
Continued



CB → As $\begin{Bmatrix} v_i \\ v_e \\ v_{be} \end{Bmatrix}$ incr , $\begin{Bmatrix} i_c \\ i_e \end{Bmatrix}$ incr $\therefore \begin{Bmatrix} v_o \\ v_c \end{Bmatrix}$ incr

Figure 5.57 Input and output waveforms for the circuit of Fig. 5.55. Observe that this amplifier is noninverting, a property of the common-base configuration.

∴ output in phase with input

Add Early Effect to the Hybrid-π Model.

$\longrightarrow r_o$

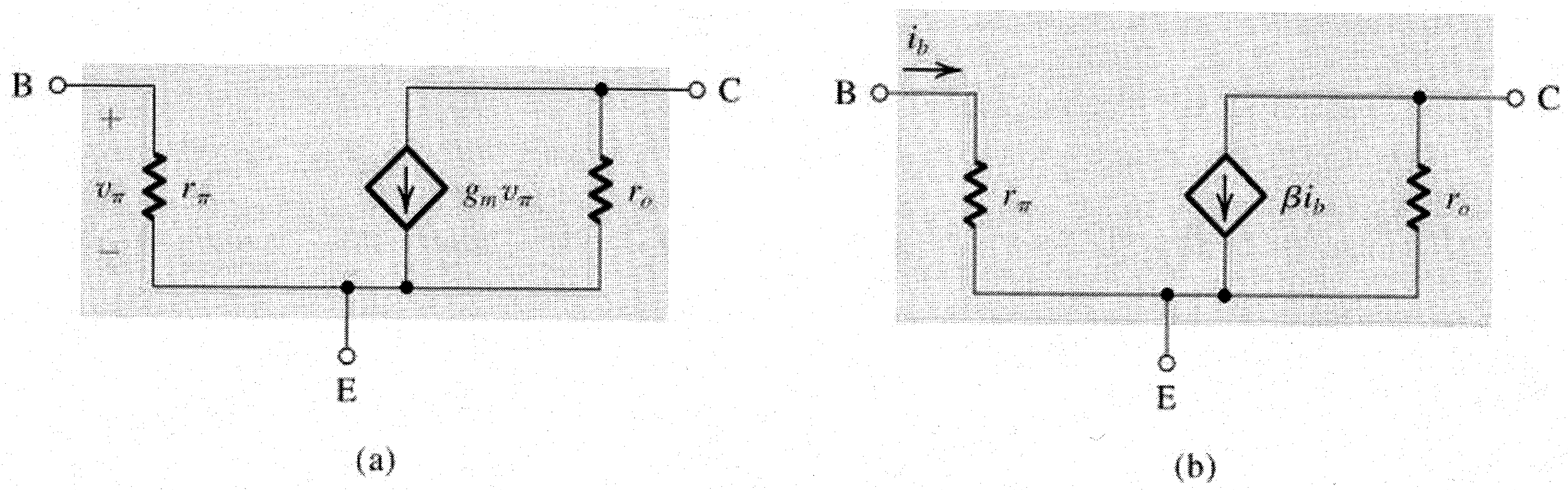
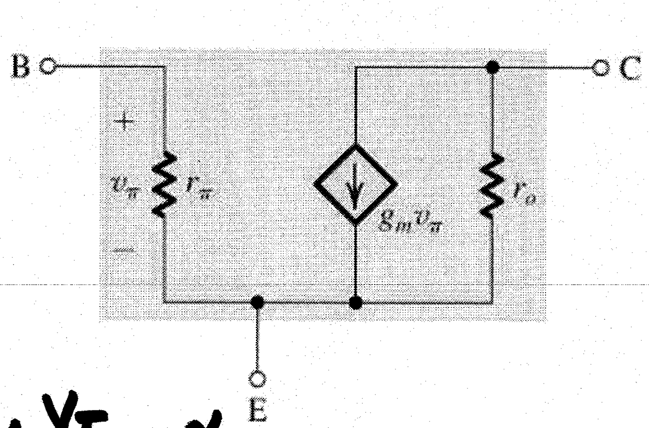
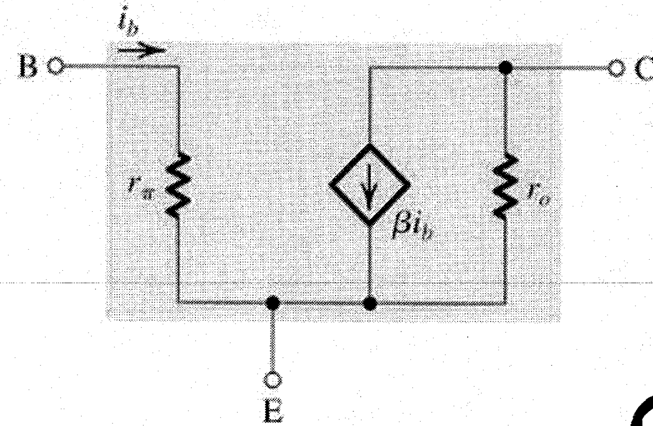


Figure 5.58 The hybrid-π small-signal model, in its two versions, with the resistance r_o included.

Exercise 5.40





$$g_m = \frac{I_C}{V_T}$$

$$r_e = \frac{V_T}{I_E} = \alpha \frac{V_T}{I_C} = \frac{\alpha}{g_m}$$

$$r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} = \frac{\beta}{g_m}$$

$$= (1+\beta) r_e$$

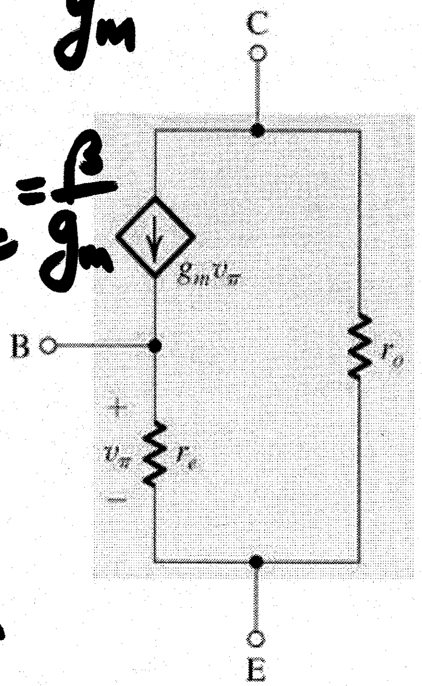
&

$$\frac{1}{r_e} = \frac{1}{r_\pi} + g_m$$

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\alpha = \frac{\beta}{1+\beta}$$

$$1+\beta = \frac{1}{1-\alpha}$$



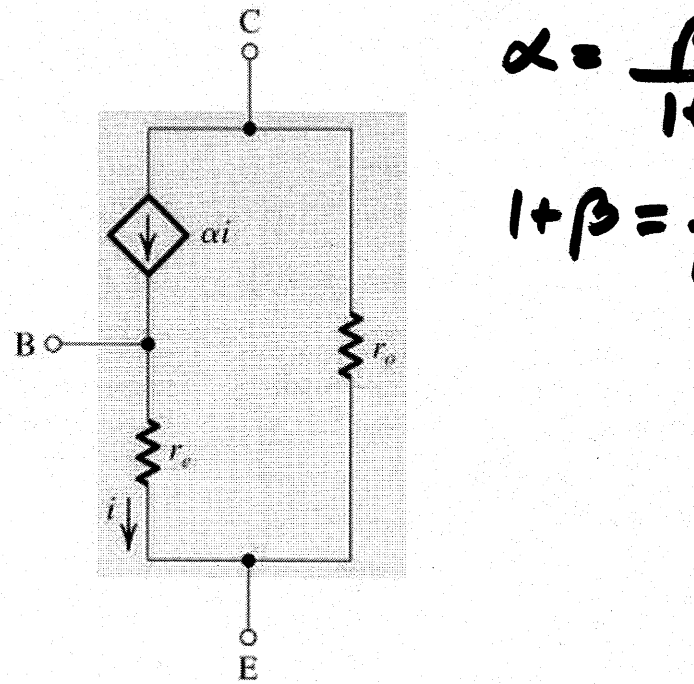


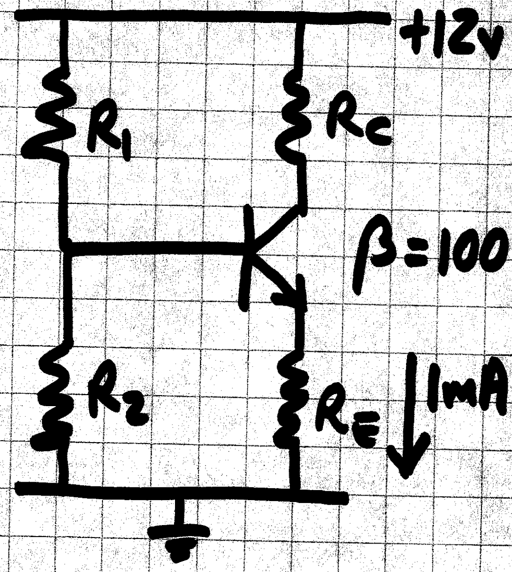
Table 5.4

Small Signal Model Summary

$$\boxed{\frac{I_E}{V_T} = \frac{I_B}{V_T} + \frac{I_C}{V_T}}$$

$$r_o \approx \frac{|V_A|}{I_C}$$

Ex 5.32 Refers to Example 5.13



Design sets $V_B = V_{cc}/3 = 4v$, so

(neglecting V_{BE}) $V_{CE} \sim 1/3 V_{cc} \longrightarrow 1/3 V_{cc} - V_{BE}$

$V_{cc} - V_c \sim 1/3 V_{cc}$

and $V_{CE} \sim 1/3 V_{cc}$ for signal swing

Example 5.13 $\longrightarrow$ Design 1

$$\frac{V_{cc}}{R_1+R_2} = 0.1 I_E = 0.1mA$$

$V_E = 4 - .7 = 3.3v$

$\therefore R_E = \frac{3.3v}{1mA} = 3.3K$

$\therefore R_1 + R_2 = \frac{12v}{.1mA} = 120K$

& $\frac{R_2}{R_1+R_2} \approx \frac{4}{12}$ for $I_B \sim 0$

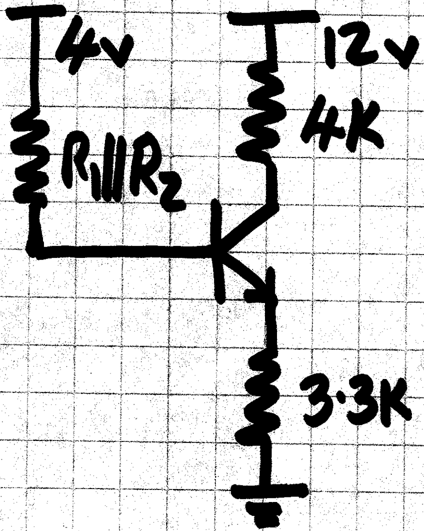
$R_1 = 80K \quad R_2 = 40K$

Check on I_E : $\frac{4 - 0.7}{3.3K + \frac{(80 \| 40)K}{101}} = 0.93mA$ Too Low!

So try Design 2 $\frac{V_{cc}}{R_1+R_2} = 1 \times I_E = 1mA \quad R_1 = 8K \quad R_2 = 4K$

Check I_E : $\frac{4 - 0.7}{3.3K + \frac{(8 \| 4)K}{101}} = 0.99mA$ OK!

Ex 5.32 Check effect of β variation $100 \pm 50 = 50 \rightarrow 150$ on I_E as % I_{nom}. for both designs.



Design 1 : $R_B = 40K \| 80K = \frac{40 \times 80}{120} K = \frac{80K}{3}$

Design 2 : $R_B = 4K \| 8K = \frac{8K}{3}$

$I_E = ?$ formula or first principles

$$I_B = \frac{4 - .7 - 3.3K \times (1+\beta) I_B}{R_B} = \frac{3.3/R_B}{1 + \frac{3.3K(1+\beta)}{R_B}}$$

$$\therefore I_E = (1+\beta) I_B = \frac{3.3\,(1+\beta)}{R_B + (1+\beta)\,3.3K}$$

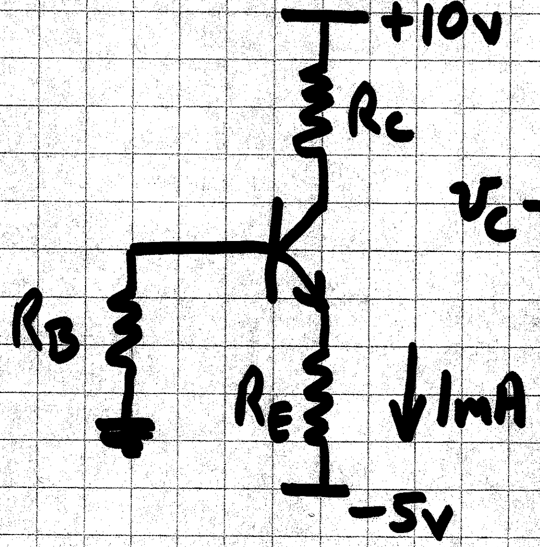
$$= 3.3 \Big/ \left(3.3K + \frac{R_B}{1+\beta}\right)$$

$\therefore$ I_E for $R_B = \frac{80K}{3}$, $\beta = 50 \longrightarrow 0.86\text{mA}$
$100 \longrightarrow 0.93\text{mA}$
" " " ", $\beta = 150 \longrightarrow 0.95\text{mA}$

Text answers for R_E changed to 3K.

" " " $= \frac{8K}{3}$, $\beta = 50 \longrightarrow 0.984$ mA
$100 \longrightarrow 0.990$ mA
" " " ", $\beta = 150 \longrightarrow 0.995$ mA

Ex D.5.33 Design for max <u>Common-Base</u> gain & max collector signal swing $\pm 2v$.



$V_C \rightarrow \pm 2v$ swing

CB $\therefore$ base common

& can use $R_B = 0$

$\therefore V_E \approx -0.7v$

& $R_E = \dfrac{-0.7v - (-5v)}{1mA}$

$= 4.3K.$

& for $I_C \approx I_E = 1mA$

& V_C min value $\;-0.7v + V_{CE})_{SAT}$ ← onset

$\approx -0.4v$

$2v$ swing $\longrightarrow V_C = -0.4 + 2\,v = 1.6v$

$\therefore R_C = \dfrac{10 - 1.6v}{1mA} = 8.4K.$

Ex 5.37 Typical values: Find g_m, r_e, r_π at bias $I_C = 1mA$
($\beta = 100$)

$$g_m = \frac{I_C}{V_T} = \frac{1mA}{25mV} = 40mA/V$$

$$r_e = \alpha/g_m \text{ or } V_T/I_E = \frac{0.99}{40mA/V} = 0.99(25\Omega) \approx 25\Omega$$

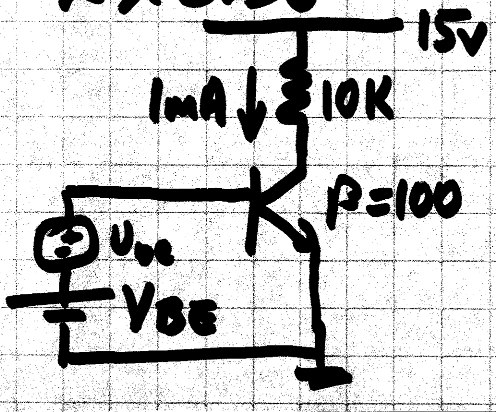
$$r_\pi = \beta/g_m \text{ or } V_T/I_B = \frac{100}{40mA/V} = 2.5K\Omega$$

Small Signal Voltage Gain

$$A_v = \frac{v_c}{v_{be}} = -g_m R_C$$
$$= -\frac{I_C}{V_T} R_C$$

$$v_C = V_{CC} - i_C R_C$$
$$= V_{CC} - (I_C + i_c) R_C$$
$$= V_C - i_c R_C$$
$$v_c = -i_c R_C = -(g_m v_{be}) R_C$$

Ex 5.38



$$\frac{v_c}{v_{be}} = -\frac{1mA}{25mV} 10K = -400$$

$$v_{be} = .005 \sin\omega t \longrightarrow v_C(t) = (15 - 1mA \times 10K)v - 400 \times .005 \sin\omega t$$
$$= 5v - 2v \sin\omega t$$

$$\& \; i_B(t) = \frac{I_C}{\beta} + \frac{v_{be}}{r_\pi} = \frac{1mA}{100} + \frac{.005 \sin\omega t}{100/(1mA/25mV)} = 10 + 2 \sin\omega t \; \mu A$$

Ex 5.40 Transistor Early Voltage $V_A = 100v$

(a) Find V_B, V_E, V_C (b) Find g_m, r_π, r_o (c) Z grounded, i.e. CE

Input X: $v_{sig}, R_{sig} = 2K$

Output Y: $R_L = 8K$. Find $\frac{v_y}{v_{sig}}$

(d) Error if neglect r_o?

(a) $I_B = \frac{I_E}{1+\beta} = \frac{1mA}{101}$ ∴ $V_B = 0 - \frac{1mA}{101} 10K \approx -0.1v$

∴ $V_E = -0.1v - 0.7v = -0.8v$

$V_C = 10 - 8K \times \frac{I_E}{\alpha} \approx 10 - 8 \times 1$

$= 2v$

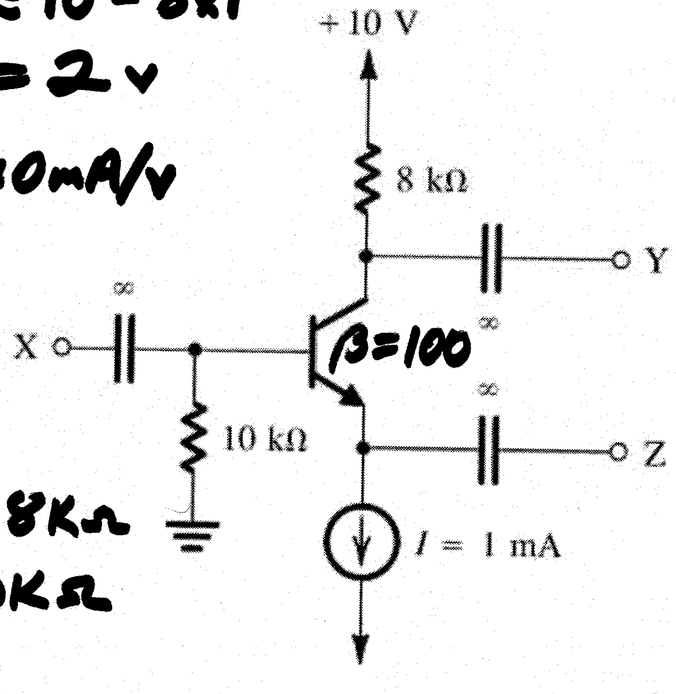
(b) $g_m = \frac{I_c}{V_T} \approx \frac{1mA}{25mV} = 40mA/v$

$r_\pi = \beta/g_m = \frac{100}{40 \times 10^{-3}}$

$= 2.5K$

$r_o = \frac{102.8v}{1mA} = 102.8K\Omega$

$\approx 100K\Omega$



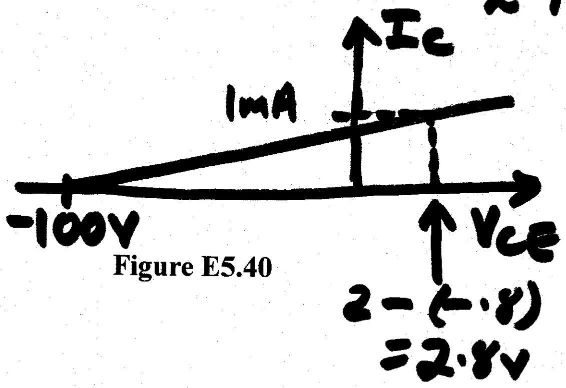
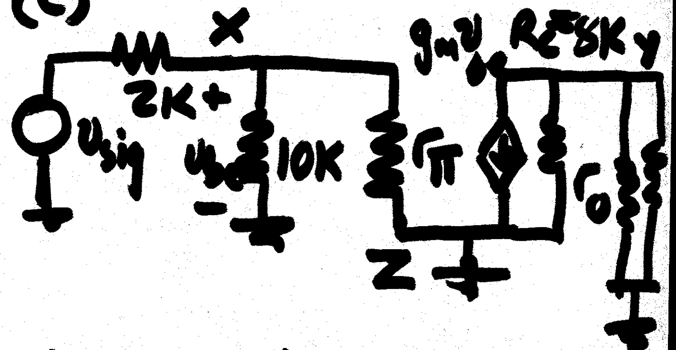


Figure E5.40

(c)



$v_{be} = \frac{10K || r_\pi}{2K + 10K || r_\pi} v_{sig}$

$\left(10K || 2.5K = \frac{2.5 \times 10}{12.5} K = 2K\right)$

∴ $v_{be} = v_{sig}/2$

& $v_y = -g_m v_{be} (r_o || R_c || R_L)$ ← $100K || 4K$

$v_y/v_{sig} = -\frac{40 \times 10^{-3}}{2} \frac{400}{104} 10^3 = 76.9$

(d) Neglect r_o ⟶ $\frac{40 \times 10^{-3}}{2} 4 \times 10^3 = 80$, ~3.9% error