ECE321 ELECTRONICS I
FALL 2006

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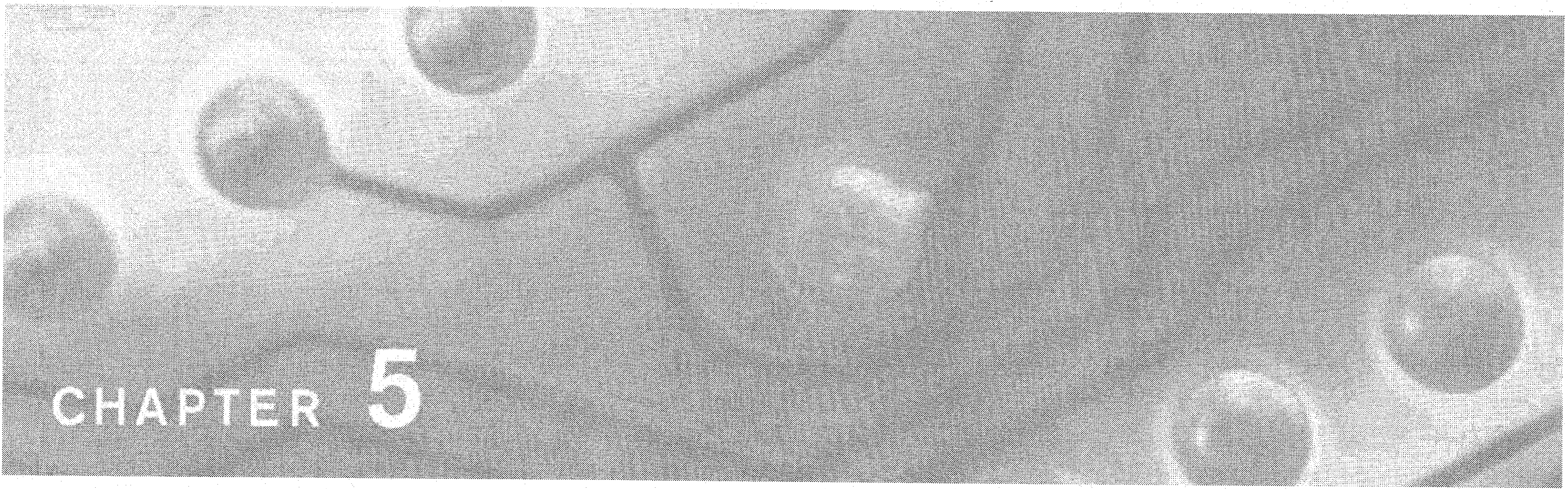
Lecture 11

31st October, 2006



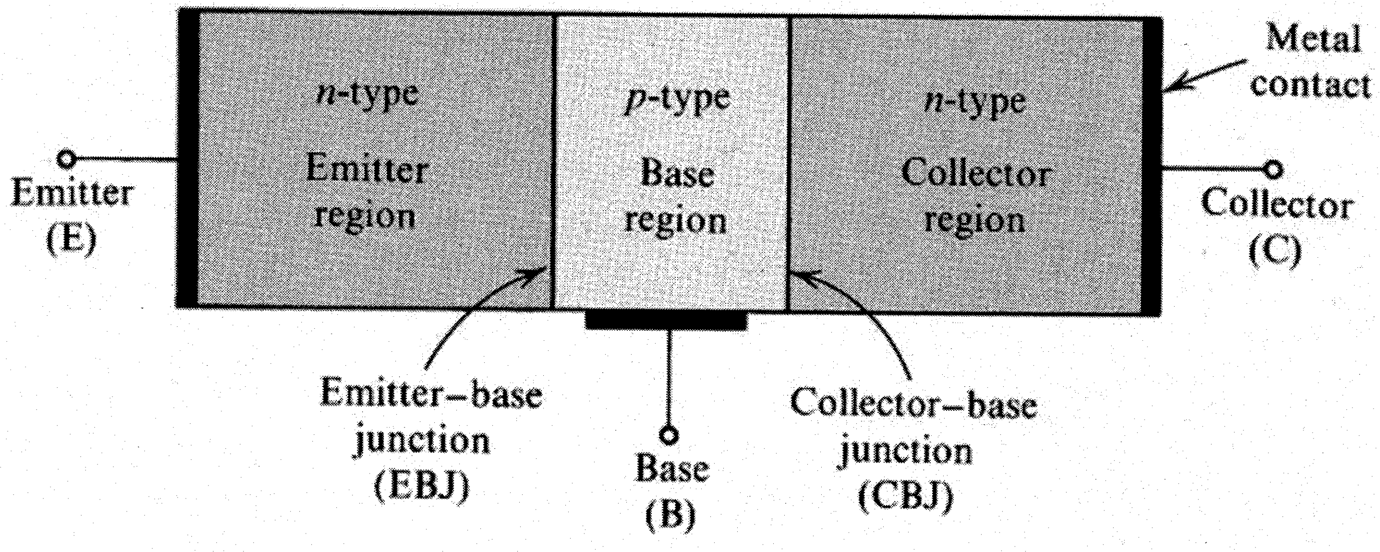
CHAPTER 5

Bipolar Junction Transistors (BJTs)

5.1 Device Structure & Physics

5.2 I – V Characteristics ← Convert 5.1 info to circuit applications

NPN BJT



Emitter Base Collector

Figure 5.1 A simplified structure of the *npn* transistor.

PNP BJT

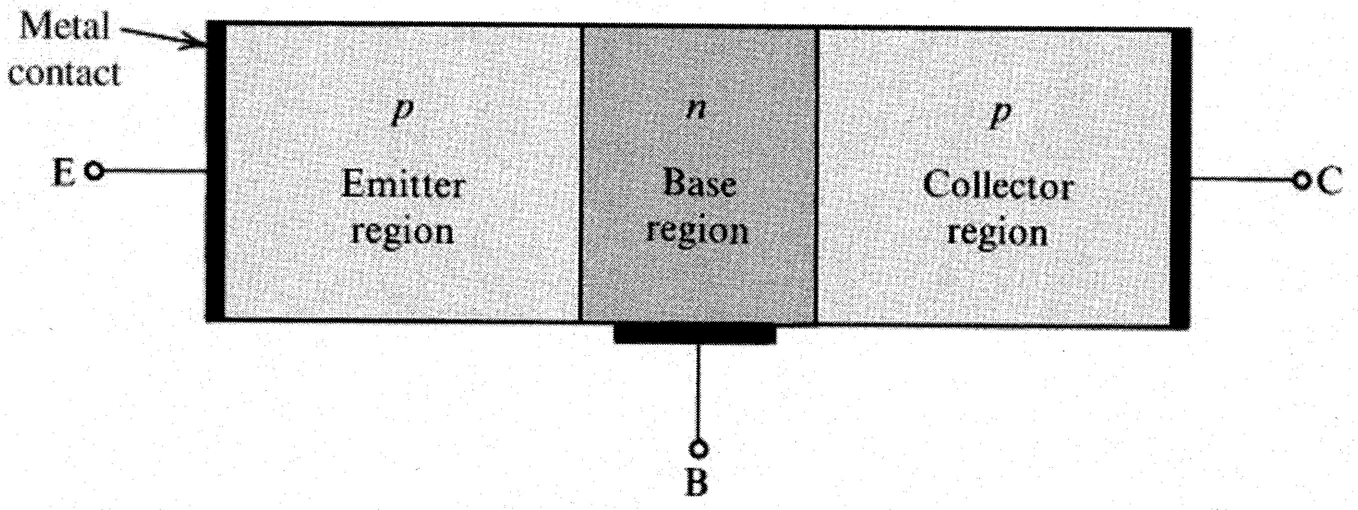


Figure 5.2 A simplified structure of the *pnp* transistor.

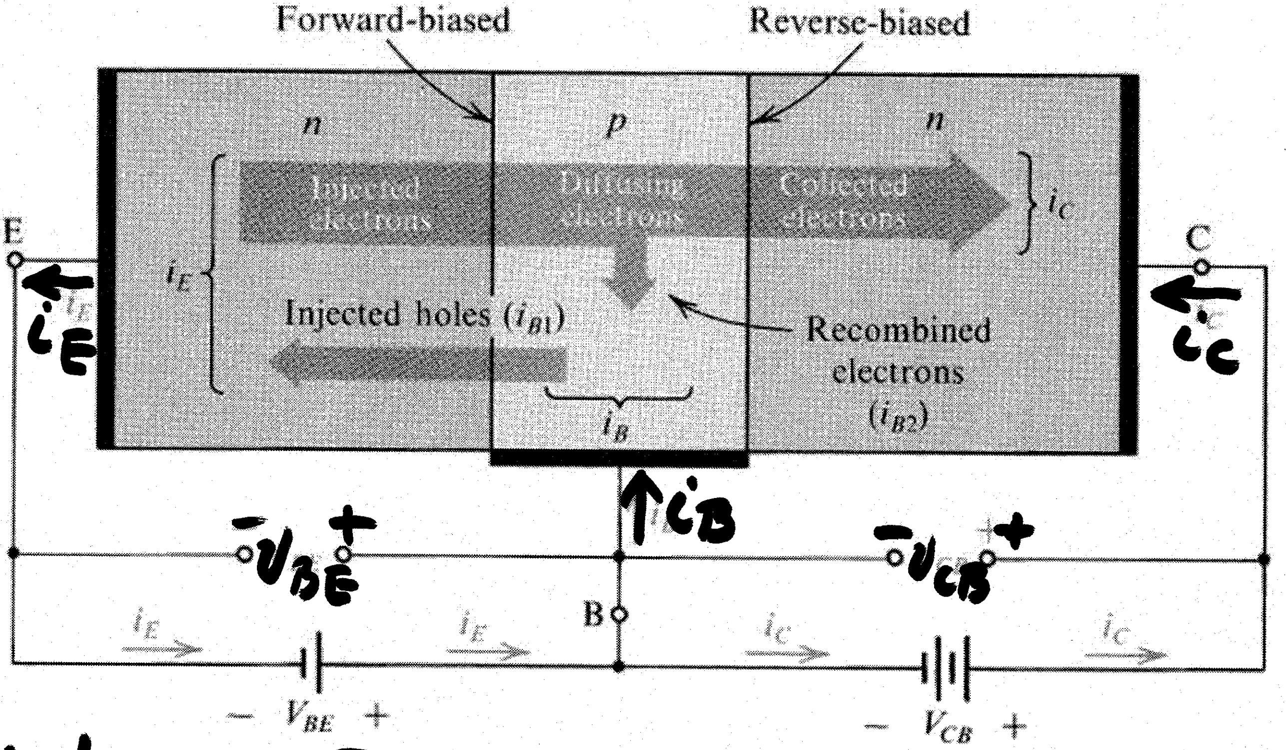
Table 5.1 BJT Operation Modes

Mode	Emitter-Base Junction Bias	Collector-Base Junction Bias
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Reverse Active	Reverse	Forward
Saturation	Forward	Forward

Linear amplification — Active

Digital/switching — Cutoff & Saturation

NPN Active Mode : BE Fwd Bias
CB Rev Bias



BE diode ~0.7v

Electrons are minority carriers in base

Emitter doping >> Base doping

$N_{DE} \gg N_{AB}$

$\therefore I_E \approx I_{nE}$

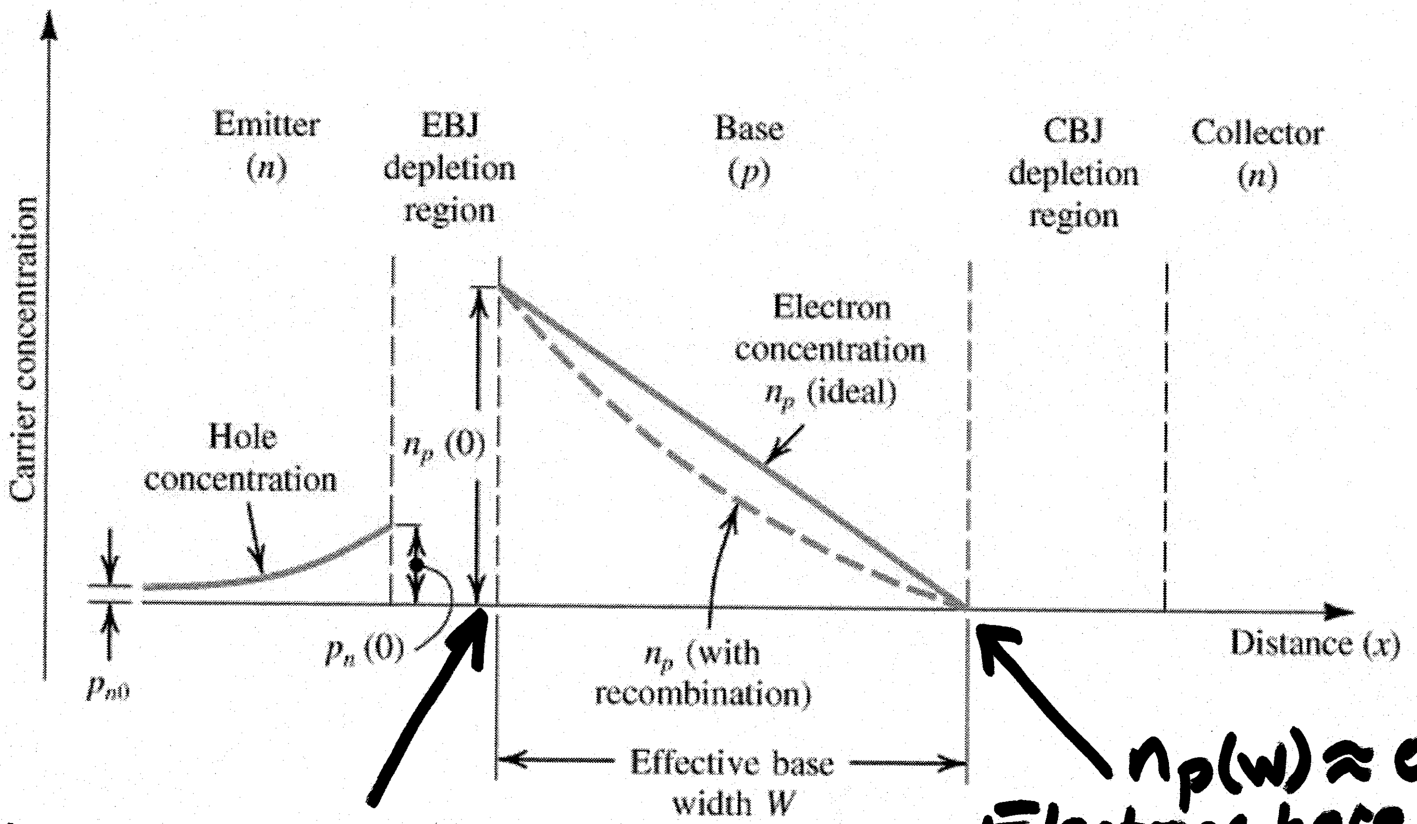
Figure 5.3 Current flow in an *npn* transistor biased to operate in the active mode. (Reverse current components due to drift of thermally generated minority carriers are not shown.)

Key point: Base very thin, so electrons from emitter reach C-B junction before many lost to recombinati

But electrons accelerated across C-B junction into collector

E-B junction as a diode (forward bias)

Then consider recombination in base



$n_p(0) = n_{p0} \exp v_{BE}/V_T$

$n_p(W) \approx 0$

Electrons here are accelerated across the C-B junction

Figure 5.4 Profiles of minority-carrier concentrations in the base and in the emitter of an *npn* transistor operating in the active mode: $v_{BE} > 0$ and $v_{CB} \geq 0$.

$$\therefore I_n)_{base} = A_E\, q\, D_n \frac{dn_p(x)}{dx} = A_E\, q\, D_n \left(-\frac{n_p(0)}{W}\right)$$

$i_c = I_n = I_s \exp(v_{BE}/V_T)$ where $I_s = A_E\, q\, D_n\, n_i^2/N_A W$

and note $i_C = A_E q D_n \frac{n_i^2}{N_A W} \exp \frac{v_{BE}}{V_T}$

is (ideally) independent of V_{CB}

Base Current:

$$i_B = i_{B1} + i_{B2}$$

where i_{B1} = hole current I_p base → emitter

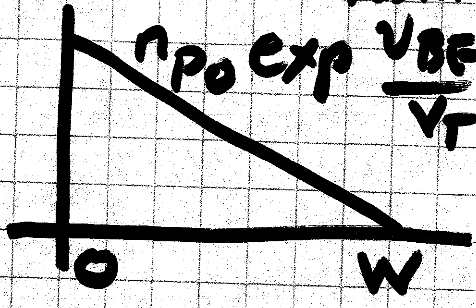
$$= A_E q \frac{D_p n_i^2}{L_p N_D} \exp \frac{v_{BE}}{V_T}$$

and i_{B2} → hole current to replace base holes lost to recombination with electrons from emitter

Minority carrier lifetime in base = τ_b

i.e. Q_n electron charge in base recombines with holes in τ_b

$$\therefore i_{B2} = \frac{Q_n}{\tau_b} = A_E q \cdot \frac{1}{2} n_p(0) W / \tau_b$$



$$= \frac{A_E q W n_i^2}{2 N_A \tau_b} \exp \frac{v_{BE}}{V_T}$$

$$\therefore\ i_B = i_{B1} + i_{B2} = I_s\left(\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}\right)\exp\frac{v_{BE}}{V_T}$$

where $I_s = A_E q D_n n_i^2 / W N_A$

Rewrite $i_B = \left(\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}\right) i_C = i_C/\beta$

$\therefore$ CE current gain $\beta = \left(\qquad\qquad\qquad \right)^{-1}$

β high $\longrightarrow W \ll L_p,\ N_A \ll N_D$

Common emitter (CE): i_B input, i_C output

$$i_E = i_C + i_B = i_C\left(1 + \frac{1}{\beta}\right) = \frac{1+\beta}{\beta} i_C$$

OR $\quad i_C = \frac{\beta}{1+\beta} i_E = \alpha i_E \quad$ where $\alpha \lesssim 1$

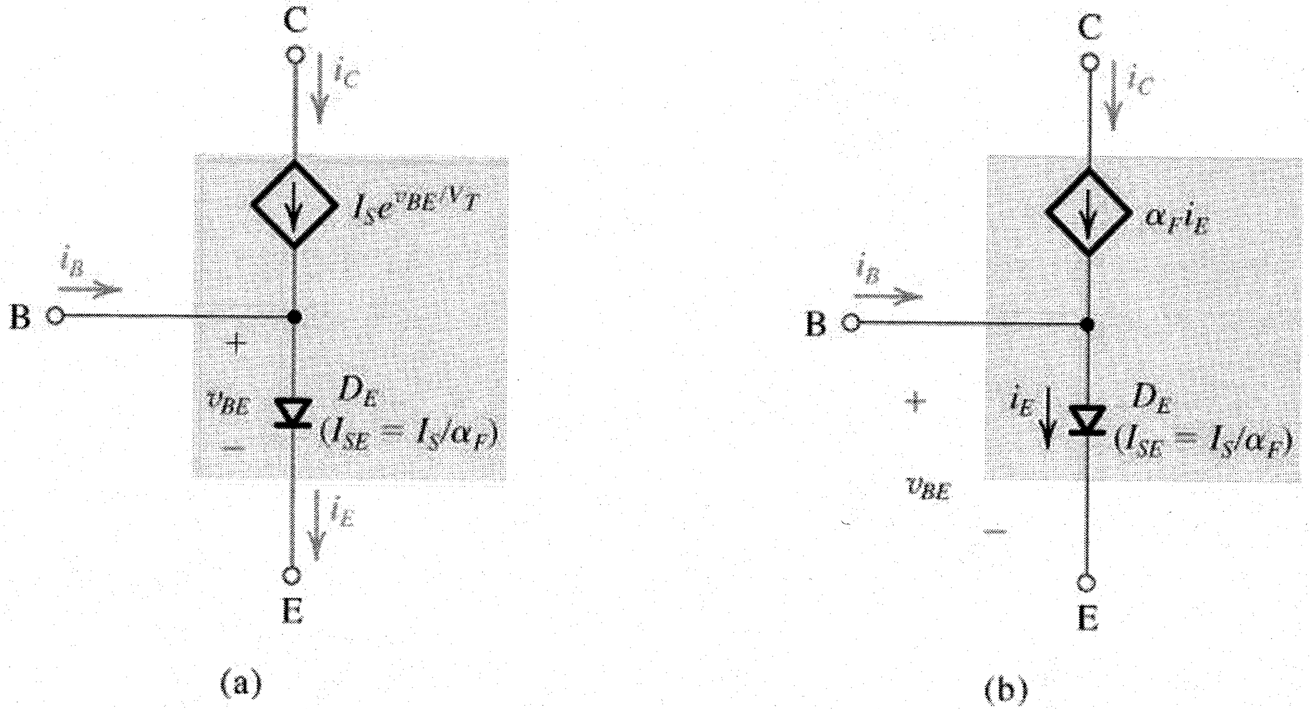
Common base (CB): i_E input, i_C output

$\alpha = \frac{\beta}{1+\beta}$ & $\beta = \frac{\alpha}{1-\alpha}$

$\alpha \to \alpha_F$ (Forward active) also α_R (Reverse active)

$\beta \to \beta_F$ $\qquad\qquad\qquad\qquad\quad$ β_R

Large Signal BJT Models: Forward Active Mode



i_C voltage controlled (i.e. by v_{BE})

i_C current controlled (i.e. by i_E)

Figure 5.5 Large-signal equivalent-circuit models of the *npn* BJT operating in the forward active mode.

Exercises

- 5.1
  - NPN v_{BE}=0.7V at i_C=1mA. Find v_{BE} at i_C=0.1mA, 10mA
- 5.2
  - Find α range for β=50 to 150.
- 5.3
  - NPN I_B=14.46μA for I_E=1.460mA, and V_{BE}=0.7V. Find α, β, and I_S
- 5.4
  - I_C=10mA. Find β and I_B for α = 0.99, 0.98.

iccuser:

BJT Structure

Collector junction surrounds emitter

$\rightarrow \alpha_F \sim 1$

Not symmettrical $\therefore \alpha_F, \beta_F \neq \alpha_R, \beta_R \ll \alpha_F, \beta_F$

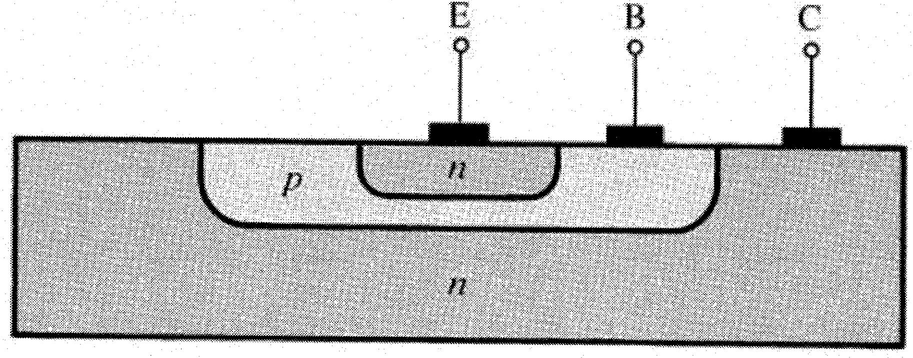


Figure 5.6 Cross-section of an *npn* BJT.

Reverse Active Model:

CB forward bias
EB reverse bias

As for Active Mode, except $\alpha_F \rightarrow \alpha_R$

$BE \leftrightarrow CB$

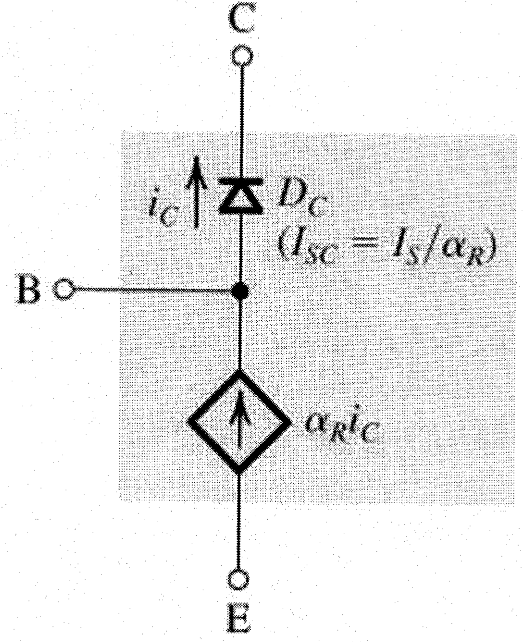


Figure 5.7 Model for the *npn* transistor when operated in the reverse active mode (i.e., with the CBJ forward biased and the EBJ reverse biased).

$$\alpha_F I_{SE} = I_S = \alpha_R I_{SC}$$

$$\alpha_R << \alpha_F$$

$$\therefore I_{SC} >> I_{SE}$$

OR: for same currents I, $V_{CB})_{\text{fwd bias}} \ll V_{EB})_{\text{fwd bias}}$

Ebers-Moll Model

Combines
Forward Active
& Reverse Active
models

Covers all possible
modes.

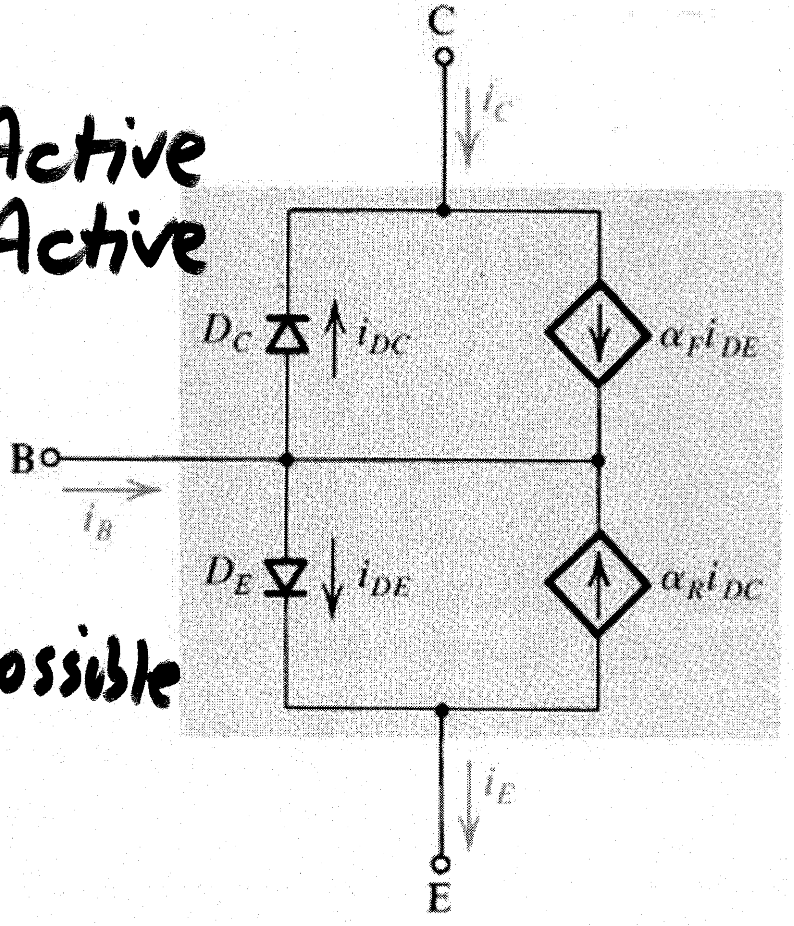


Figure 5.8 The Ebers-Moll (EM) model of the *npn* transistor.

See text p388 — from the Ebers-Moll circuit model:

$$\left.\begin{aligned} i_E &= i_{DE} - \alpha_R i_{DC} \\ i_C &= -i_{DC} + \alpha_F i_{DE} \\ i_B &= (1-\alpha_F) i_{DE} + (1-\alpha_R) i_{DC} \end{aligned}\right\} \quad \begin{aligned} &\text{where} \\ &i_{DE} = I_{SE}\left(\exp\frac{v_{BE}}{V_T} - 1\right) \\ &i_{DC} = I_{SC}\left(\exp\frac{v_{BC}}{V_T} - 1\right) \end{aligned}$$

Substitute for i_{DE}, i_{DC} $\downarrow$ & $I_{SE} = I_S/\alpha_F$
$I_{SC} = I_S/\alpha_R$

Gives:

$$i_E = \left(\frac{I_S}{\alpha_F}\right)\left(\exp\frac{v_{BE}}{V_T} - 1\right) - I_S\left(\exp\frac{v_{BC}}{V_T} - 1\right)$$

$$i_C = I_S\left(\exp\frac{v_{BE}}{V_T} - 1\right) - \left(\frac{I_S}{\alpha_R}\right)\left(\exp\frac{v_{BC}}{V_T} - 1\right)$$

& $$i_B = \left(\frac{I_S}{\beta_F}\right)\left(\exp\frac{v_{BE}}{V_T} - 1\right) + \left(\frac{I_S}{\beta_R}\right)\left(\exp\frac{v_{BC}}{V_T} - 1\right)$$

In active mode, for $v_{BE} \sim 0.6$ to 0.8v & $v_{BC} < 0$

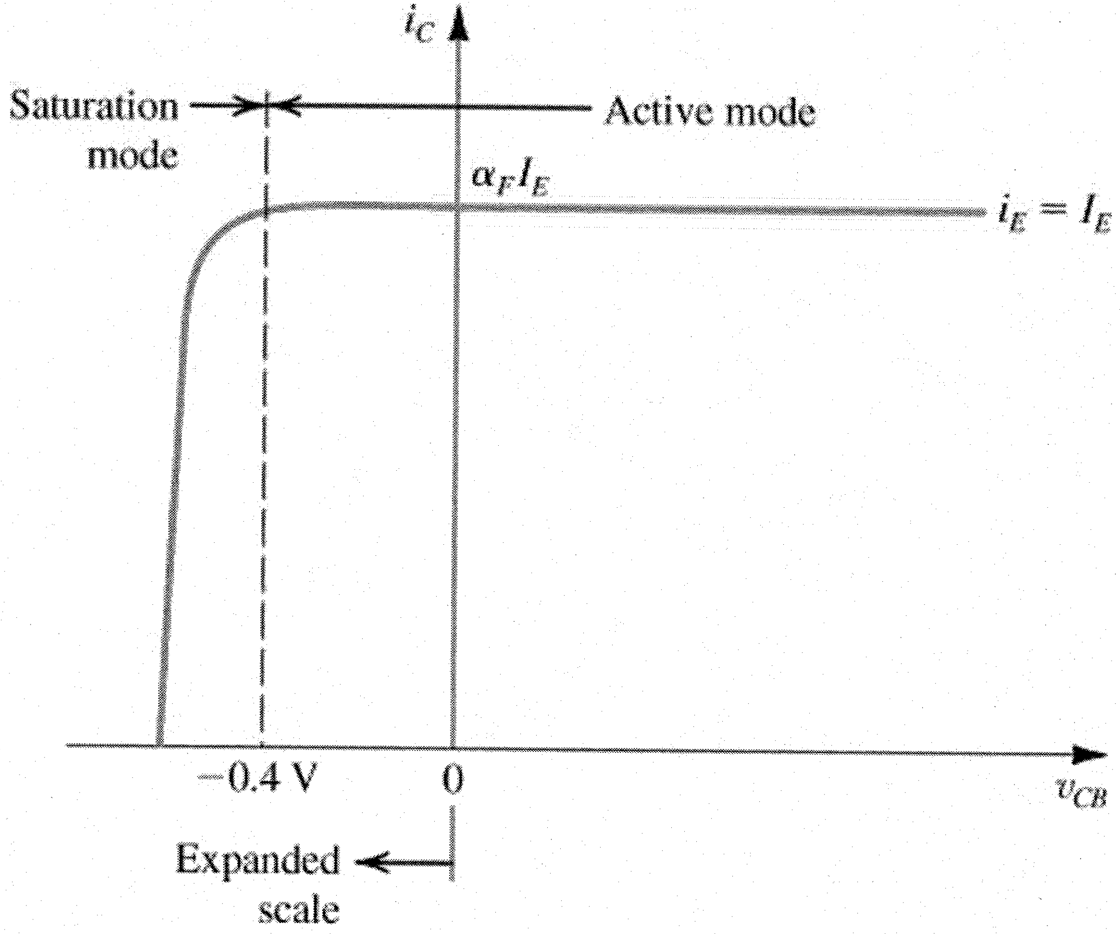
$\exp\frac{v_{BC}}{V_T} \sim 0$, so

$$\begin{aligned} i_E &\approx (I_S/\alpha_F)\exp\frac{v_{BE}}{V_T} + I_S(1 - 1/\alpha_F) \\ i_C &\approx I_S \exp v_{BE}/V_T + I_S(1/\alpha_R - 1) \\ i_B &\approx (I_S/\beta_F)\exp v_{BE}/V_T - I_S(1/\beta_F + 1/\beta_R) \end{aligned}$$

Exercise

- 5.6
  - BJT α_F=0.99, α_R=0.02, I_S=10^{-15}A:
  - Calculate second term on RHS of equations 5.31, 5.32, 5.33, and verify they can be neglected.
  - Calculate i_E, i_C, and i_B for v_{BE}=0.7V.

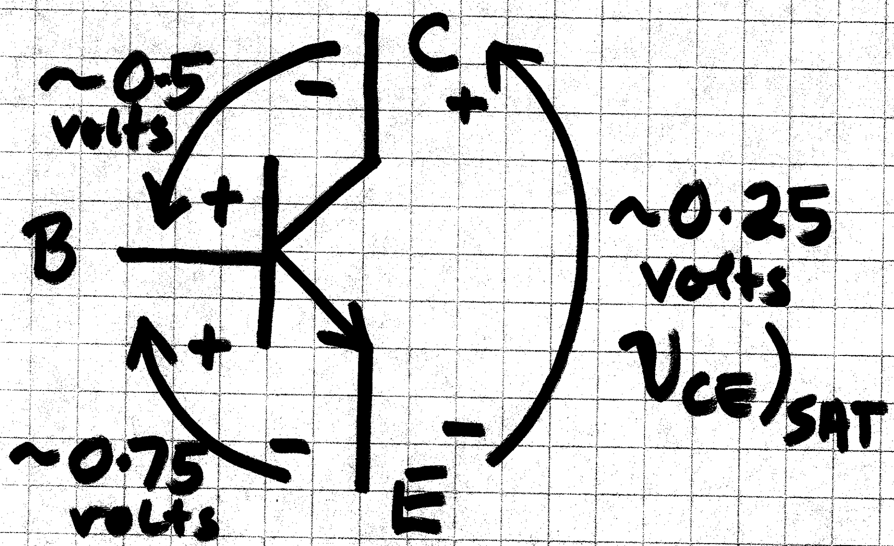
Start considering saturation mode:



C-B junction not effectively forward biassed until $v_{CB} \lesssim -0.5$v

Figure 5.9 The i_C–v_{CB} characteristic of an *npn* transistor fed with a constant emitter current I_E. The transistor enters the saturation mode of operation for $v_{CB} < -0.4$ V, and the collector current diminishes.

Hence forward active mode until saturation at $v_{CB} \lesssim -0.4$v



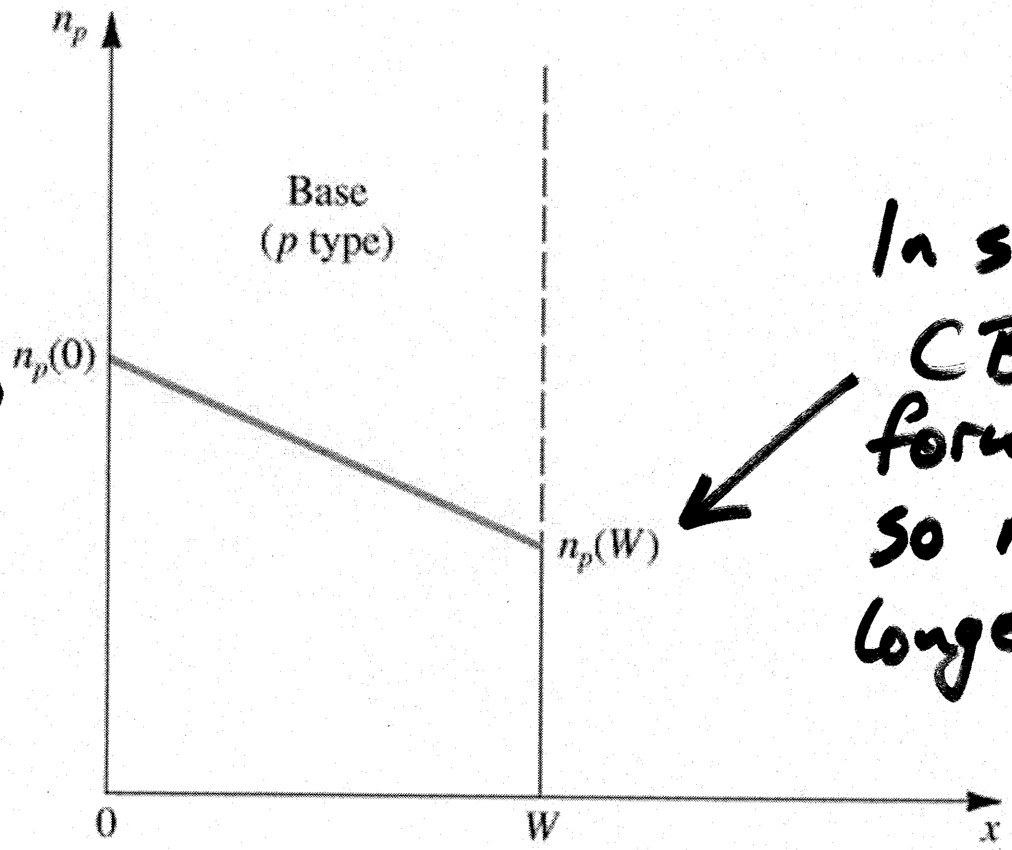
Typical saturation

$$i_C = I_S \exp\frac{v_{BE}}{V_T} - \frac{I_S}{\alpha_R}\exp\frac{v_{BC}}{V_T}$$

Forward bias EB   Forward bias CB

$\longrightarrow 0$ as $v_{BC} \longrightarrow \sim -0.5$ volts

EB forward biassed as for active



In saturation CB now forward biassed, so $n_p(w)$ no longer zero

Figure 5.10 Concentration profile of the minority carriers (electrons) in the base of an *npn* transistor operating in the saturation mode.

PNP BJT

All similar, but
e⁻'s ⟷ holes
polarities, etc
reversed

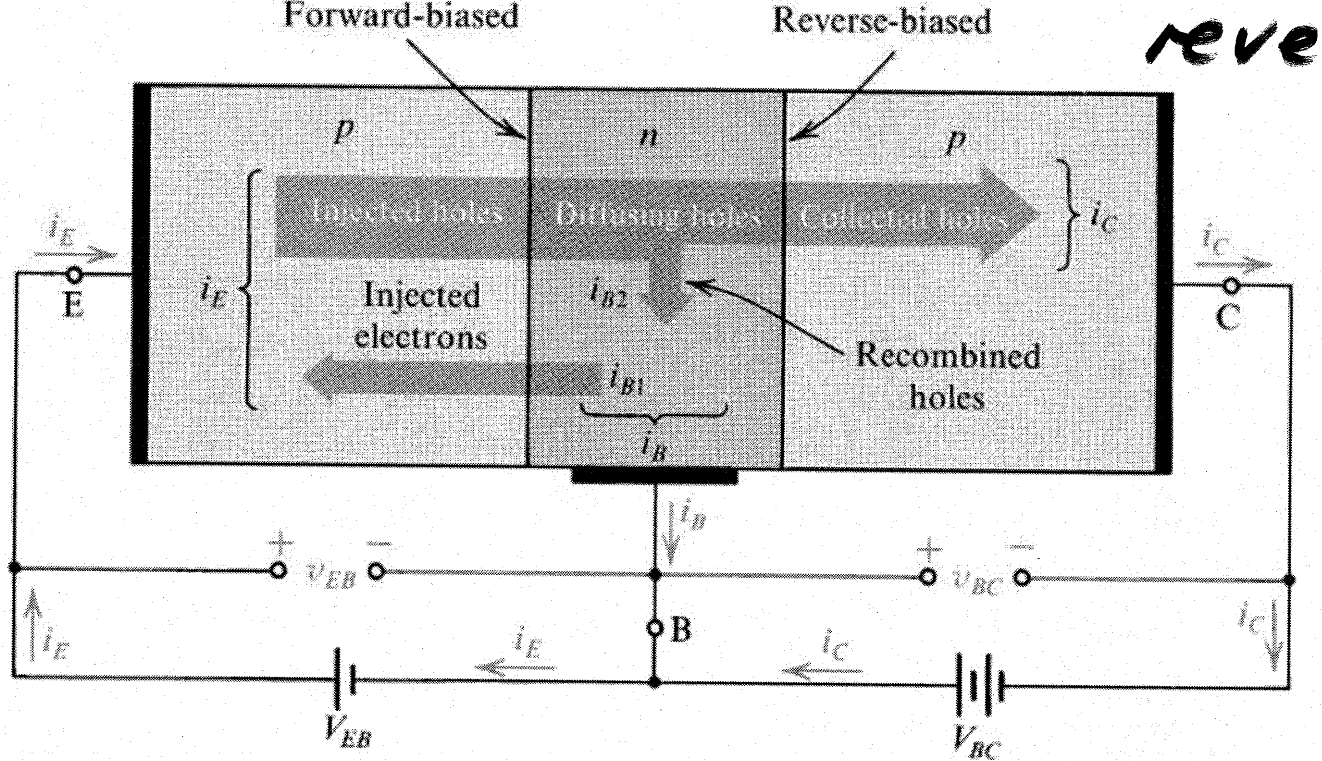


Figure 5.11 Current flow in a *pnp* transistor biased to operate in the active mode.

PNP Large Signal Model

Forward Active Mode

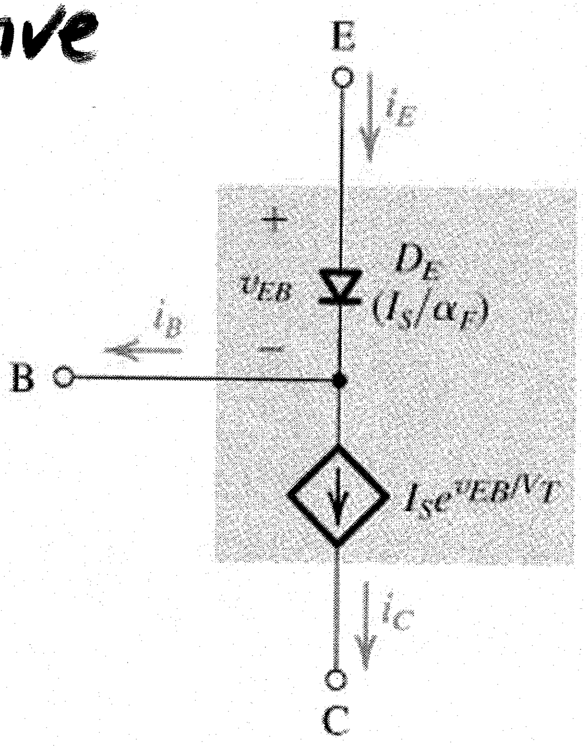
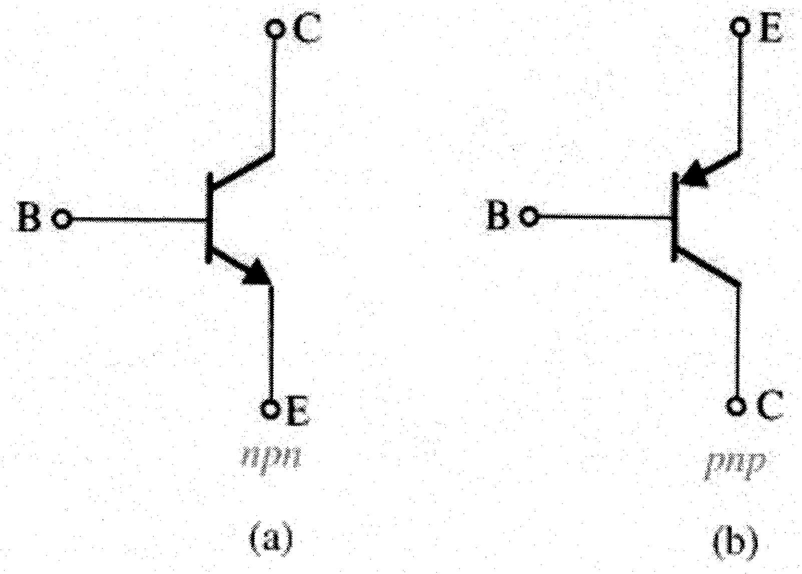


Figure 5.12 Large-signal model for the *pnp* transistor operating in the active mode.

NPN PNP



Emitter arrow

Figure 5.13 Circuit symbols for BJTs.

Forward Active Bias Systems

See also Table 5.2:

$$i_C = I_S \exp\frac{v_{BE}}{V_T}, \quad i_B = i_C/\beta, \quad i_E = i_C/\alpha$$

So $i_C = \alpha i_E = \beta i_B$, $i_E = (1+\beta) i_B$, $i_B = (1-\alpha) i_E$

$$= \frac{i_E}{1+\beta}$$

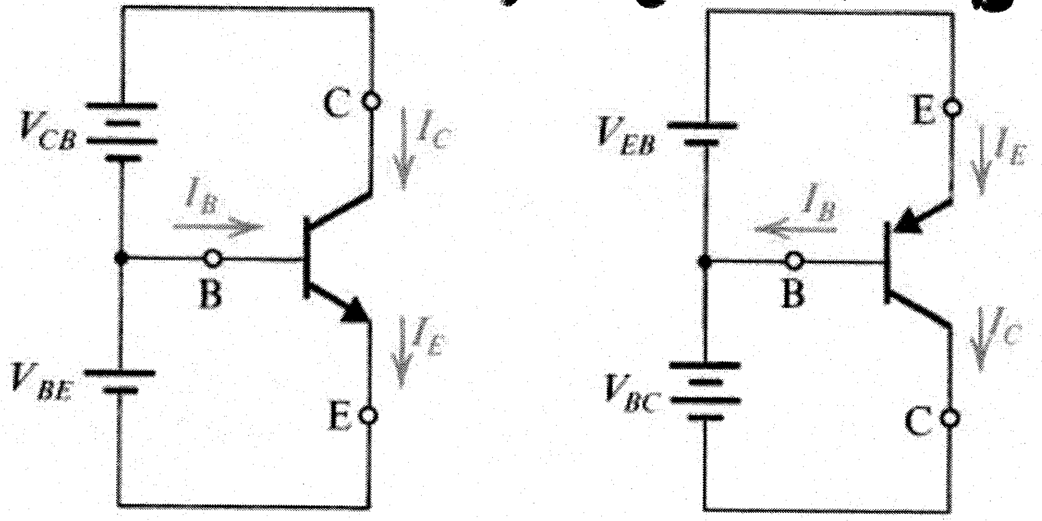


Figure 5.14 Voltage polarities and current flow in transistors biased in the active mode.

Note also: $n = 1$

Reverse leakage: $I_C = \beta I_B + (1+\beta) I_{CBO}$

where $I_{CBO} = I_S +$ leakage currents

Temperature dependent, doubles ~ every 5 °C

MSE

Exercises

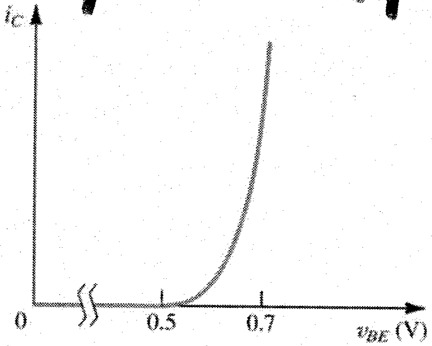
- 5.10
- 5.11

$i_C - v_{BE}$ characteristic

Note $i_E - v_{BE}$ is forward bias diode

$i_C = I_S \exp v_{BE}/V_T$

$i_E = \frac{I_S}{\alpha} \exp v_{BE}/V_T$



$i_B = \frac{I_S}{\beta} \exp v_{BE}/V_T$

All follow same basic shape with current scale factor

Figure 5.16 The i_C–v_{BE} characteristic for an *npn* transistor.

B-E Diode temperature dependence

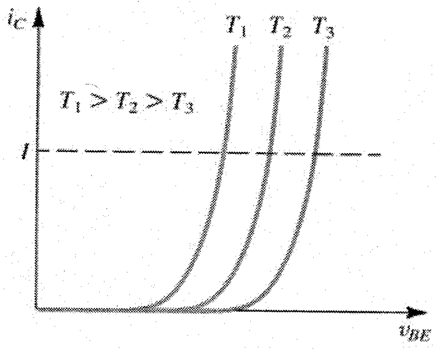
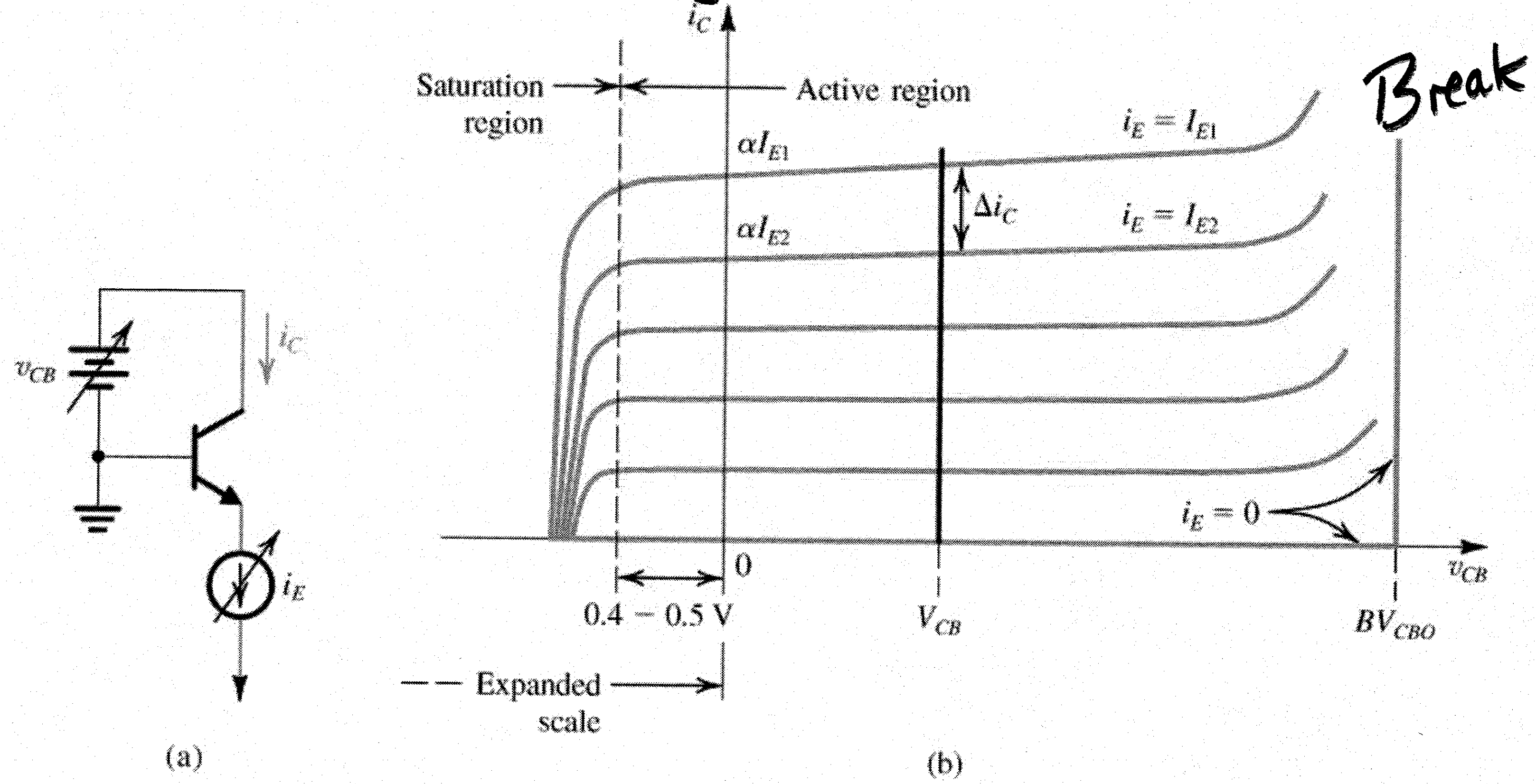


Figure 5.17 Effect of temperature on the i_C–v_{BE} characteristic. At a constant emitter current (broken line), v_{BE} changes by –2 mV/°C.

Common Base Characteristics

$|i_c \text{ vs } v_{CB}|_{\text{const } I_E}$



Breakdown

(a)

(b)

V_B constant → CB

Note i_c not constant for constant i_E

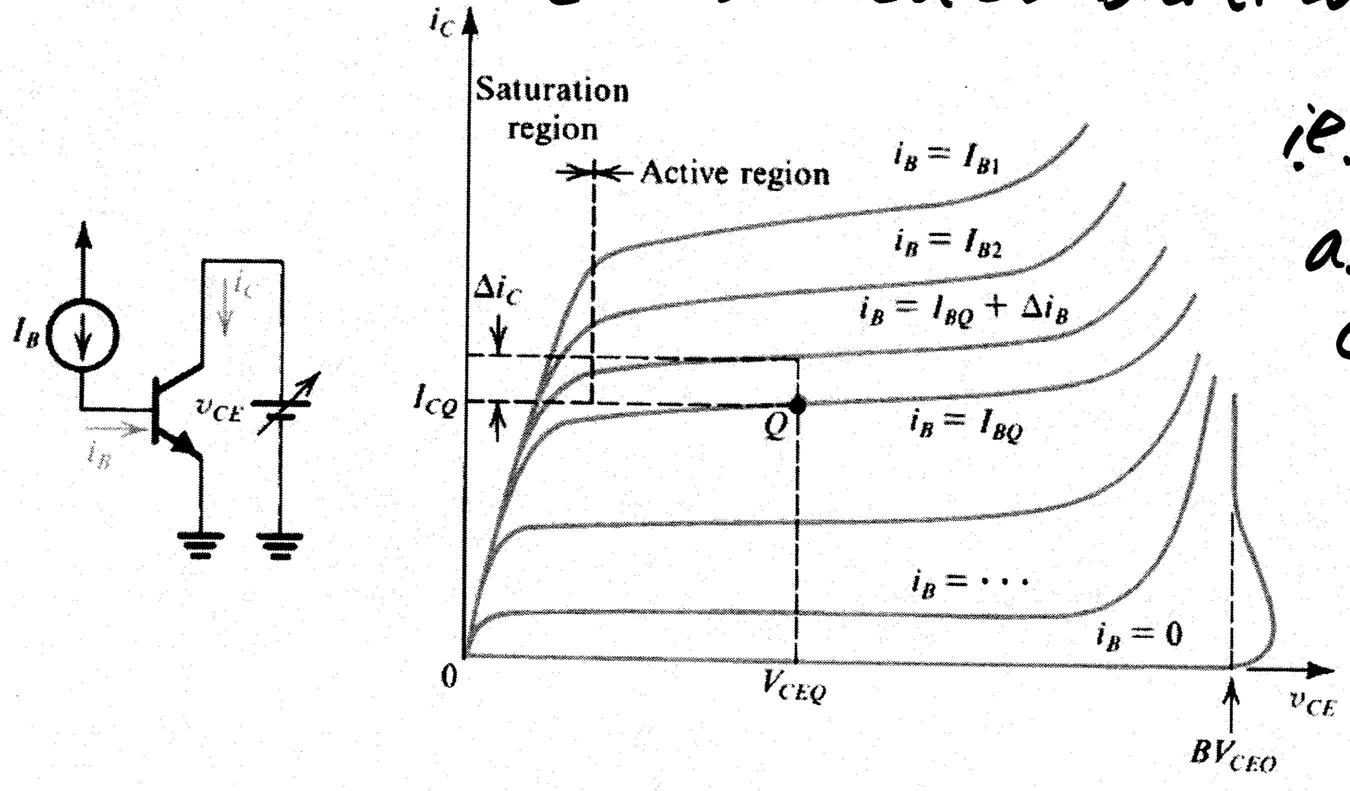
Figure 5.18 The i_C–v_{CB} characteristics of an *npn* transistor.

Small signal $\alpha = \dfrac{\Delta i_c}{\Delta i_E} \approx$ large signal α

Common Emitter Characteristics

Note slope of characteristics

Until now, i_c assumed constant with v_{BE} or i_B



(a) (b)

i.e. $\beta = \beta_F$ assumed constant

Figure 5.21 Common-emitter characteristics. Note that the horizontal scale is expanded around the origin to show the saturation region in some detail.

Early Voltage

Increase V_{CE} increases $V_{CB} = V_{CE} - V_{BE}$

Increased V_{CB} → increases C-B junction depletion width → decreases effective base width W → increases α, β, I_S

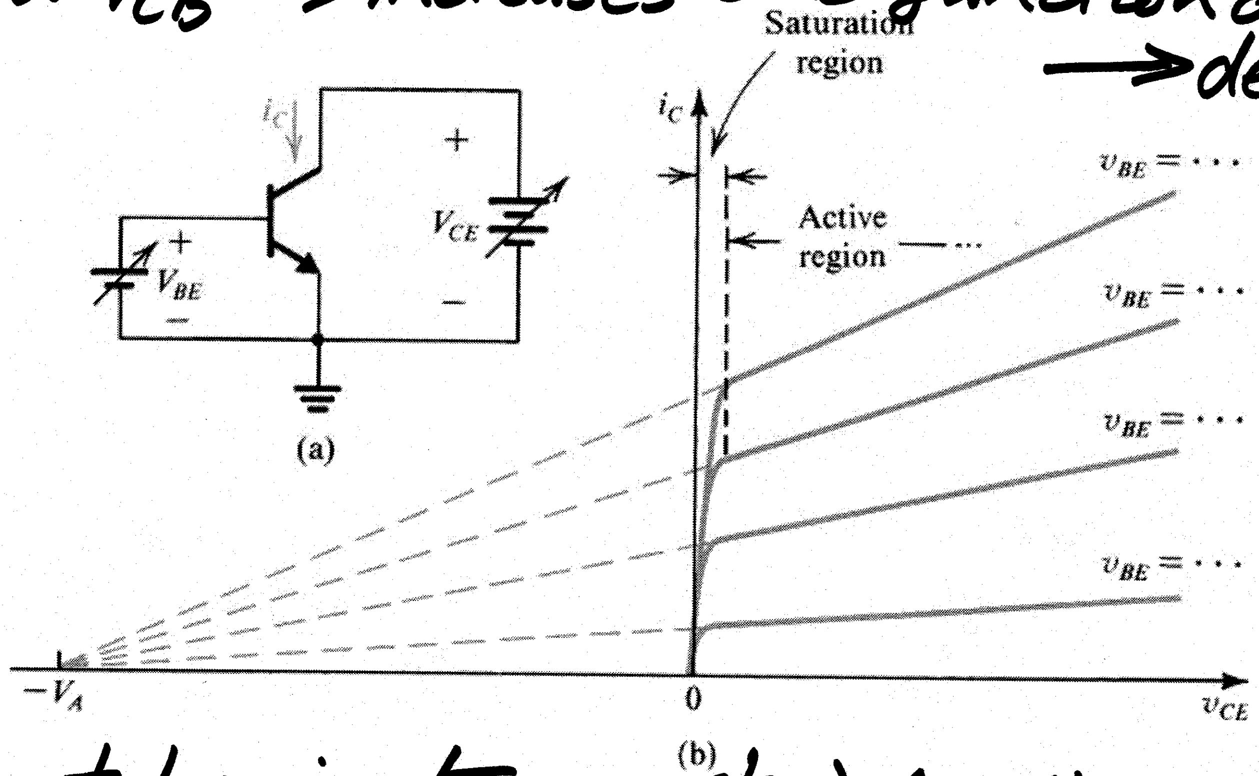


Figure 5.19 **(a)** Conceptual circuit for measuring the i_C–v_{CE} characteristics of the BJT. **(b)** The i_C–v_{CE} characteristics of a practical BJT.

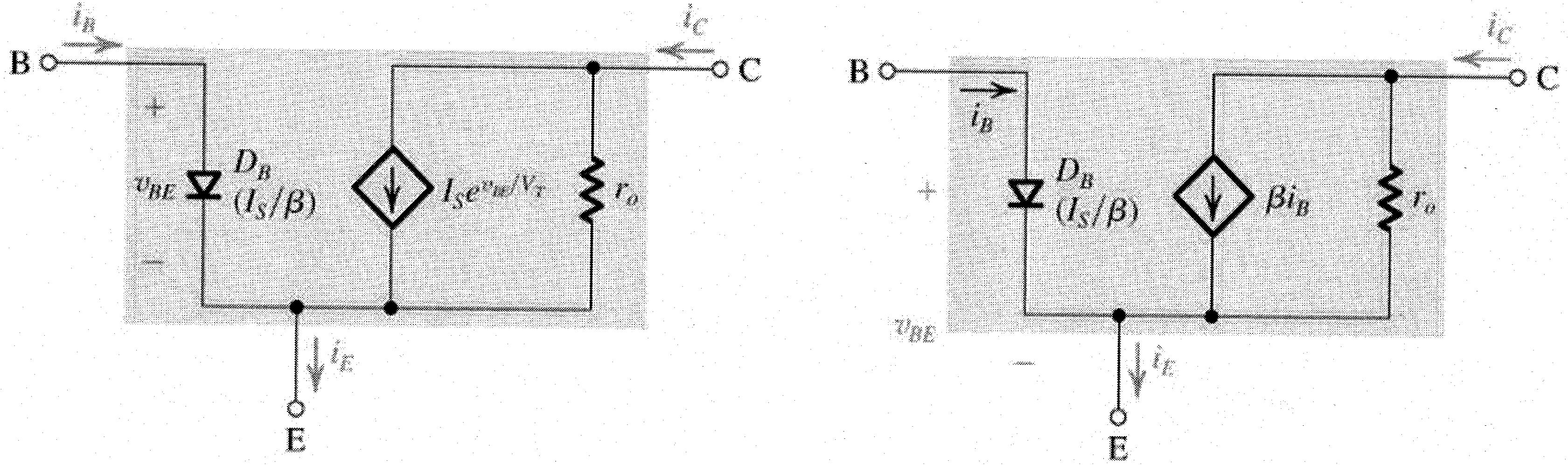
Represent by $i_C = \left(I_S \exp \frac{v_{BE}}{V_T}\right) \cdot \left(1 + v_{CE}/V_A\right)$

$$r_o = \left\{ \partial i_C / \partial v_{CE} \Big|_{v_{BE}} \right\}^{-1} = V_A / \left(I_S \exp v_{BE}/V_T\right)$$

$$= V_A / \left[I_C / \left(1 + \frac{v_{CE}}{V_A}\right) \right] = \frac{V_A + V_{CE}}{I_C}$$

Equivalent Circuit Model

Modify forward active model to include r_o



(a)

(b)

$$v_{BE} \text{ controls } i_C \overset{\text{equivalent}}{\longleftrightarrow} i_B \text{ controls } i_C$$

Figure 5.20 Large-signal equivalent-circuit models of an *npn* BJT operating in the active mode in the common-emitter configuration.

Common Emitter Characteristics

Bias point Q

I_{CQ}, I_{BQ}, V_{CEQ}

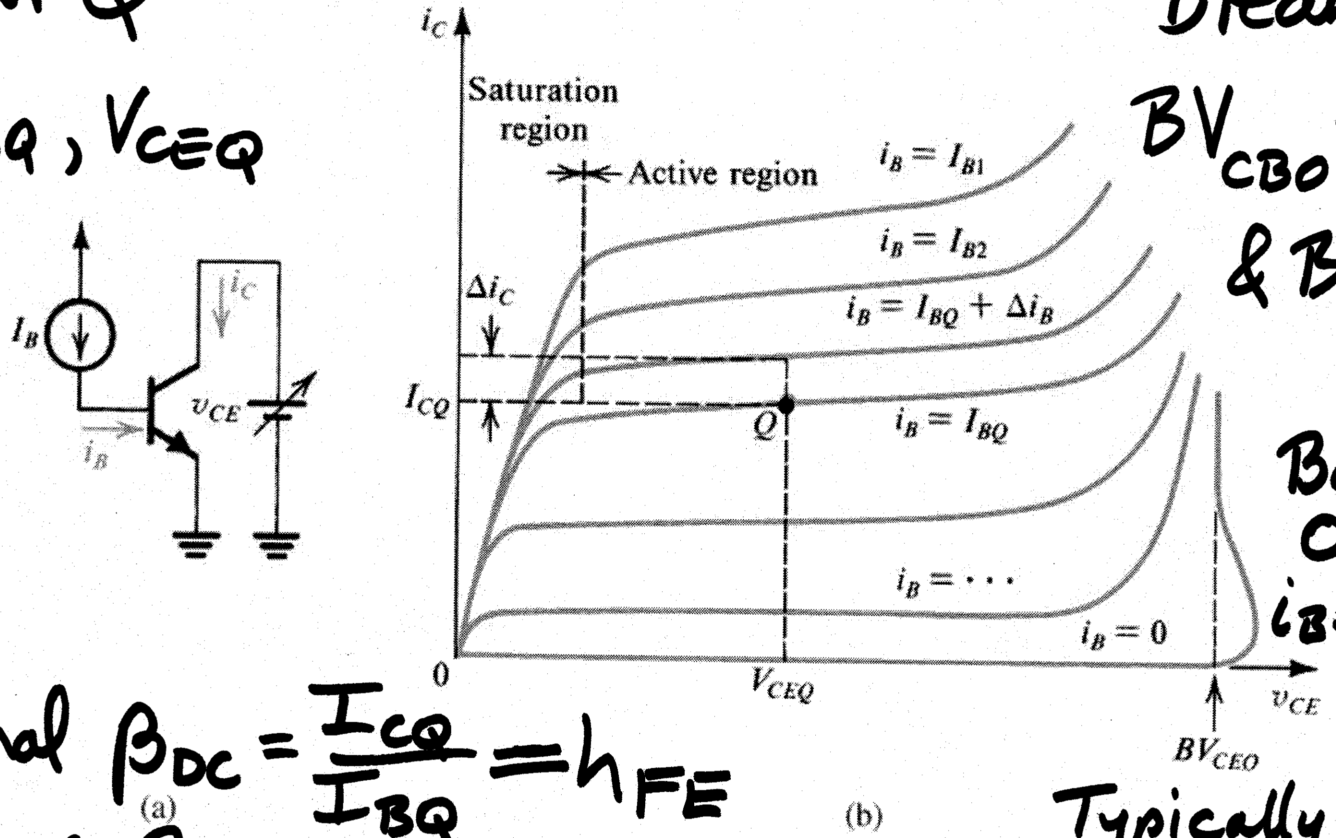
Breakdown

$BV_{CBO} \rightarrow BV_{CEO}$

& $BV_{CEO} \sim \frac{1}{2} BV_{CBO}$

Base O.C. $i_B = 0$

Emitter O.C. $i_E = 0$



Large signal $\beta_{DC} = \frac{I_{CQ}}{I_{BQ}} = h_{FE}$

Small signal $\beta_{ac} = \left.\frac{\Delta I_c}{\Delta I_B}\right|_{V_{CE}=\text{const}} = h_{fe}$

(Incremental/a.c.)

Typically $BV_{CBO} \sim 50v$

& $BV_{EBO} \sim 6$ to $8v$

Figure 5.21 Common-emitter characteristics. Note that the horizontal scale is expanded around the origin to show the saturation region in some detail.

V_{CE} constant, i.e. $v_{ce} = 0$, no signal

Hence "short circuit" current gain

β_{DC} & β_{ac} ~10-20% different

β_{DC} variations

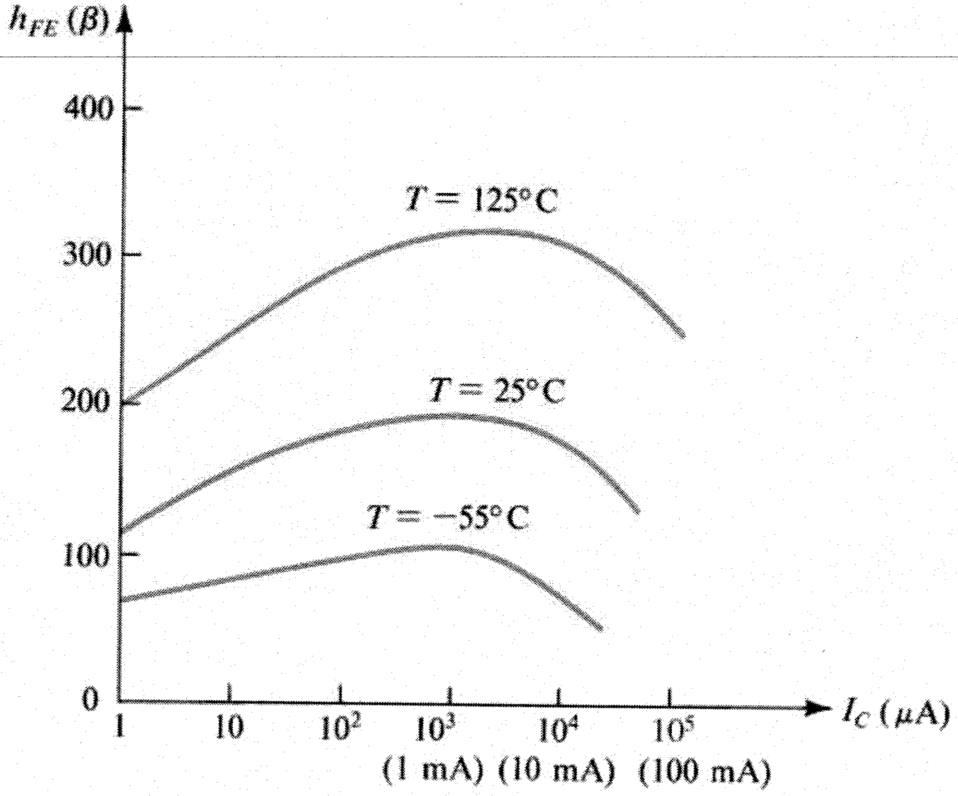
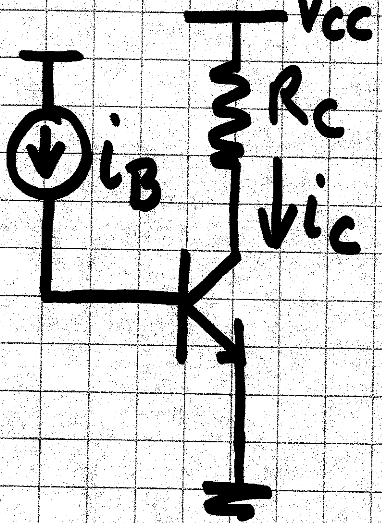
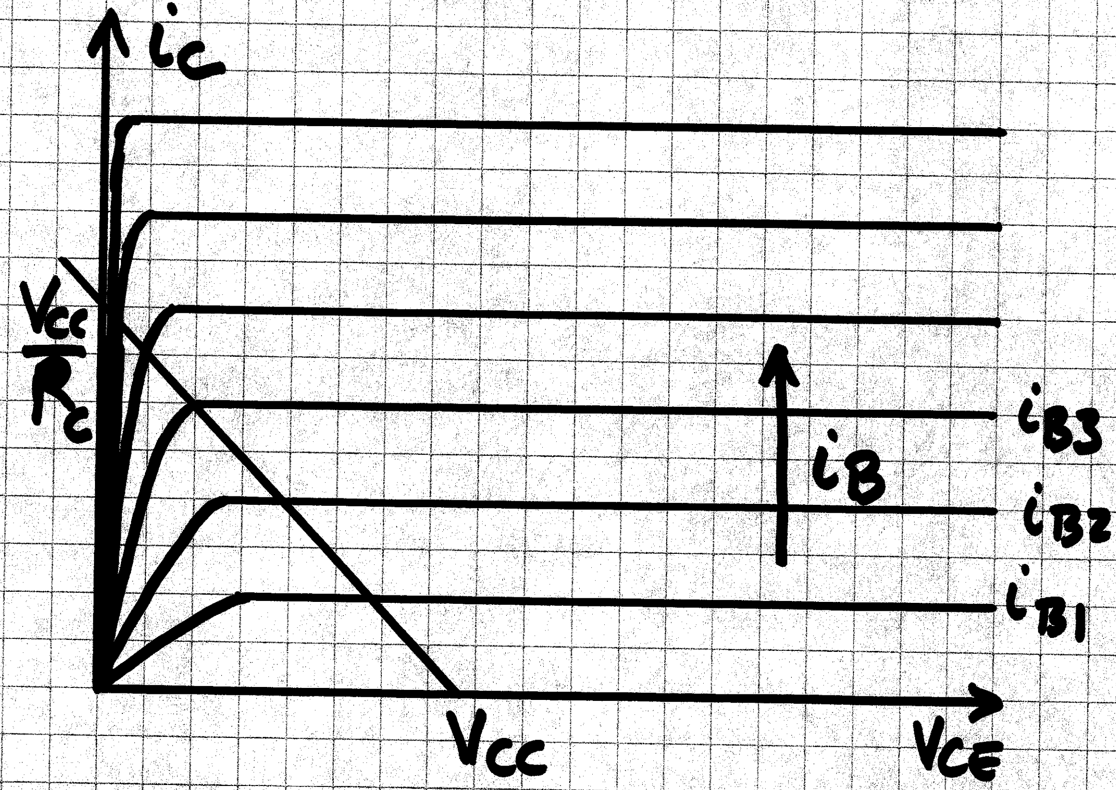


Figure 5.22 Typical dependence of β on I_C and on temperature in a modern integrated-circuit *npn* silicon transistor intended for operation around 1 mA.

Saturation





Increasing I_B → $I_C = \beta I_B$ & $V_{CE} = V_{CC} - I_C R_C$

Works until i_{B3} here, then → saturation

$I_C \rightarrow V_{CC}/R_C$ max as $V_{CE} \rightarrow 0$

In practice $\frac{V_{CC} - V_{CE})_{SAT}}{R_C}$ & $V_{CE})_{SAT}$

Saturation Region, e.g. at X

$\beta_{sat} < \beta_{active}$ $\quad$ $I_{C\,SAT} < \beta I_B$

Overdrive factor

$$= \frac{\beta_F}{(I_C/I_B)_{SAT}}$$

$$= \beta_F \left(\frac{I_B}{I_C}\right)_{SAT}$$

$$\rightarrow \frac{I_B)_{SAT}}{I_C/\beta_F}$$

$$= \beta_F/\beta_{forced}$$

$\longrightarrow 1$ in active region

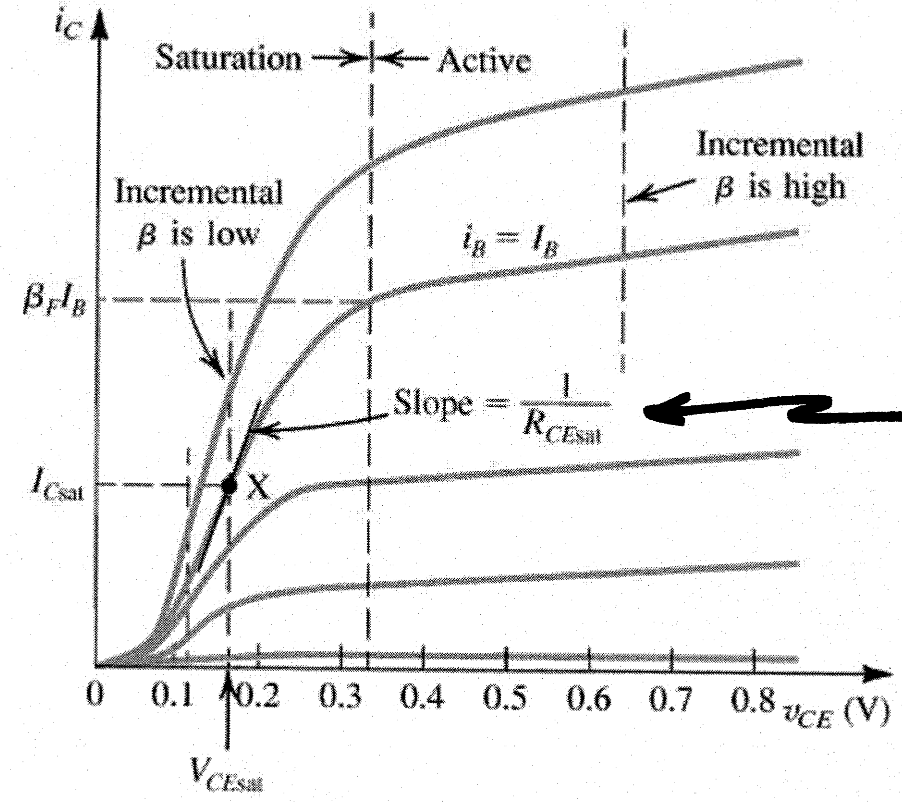
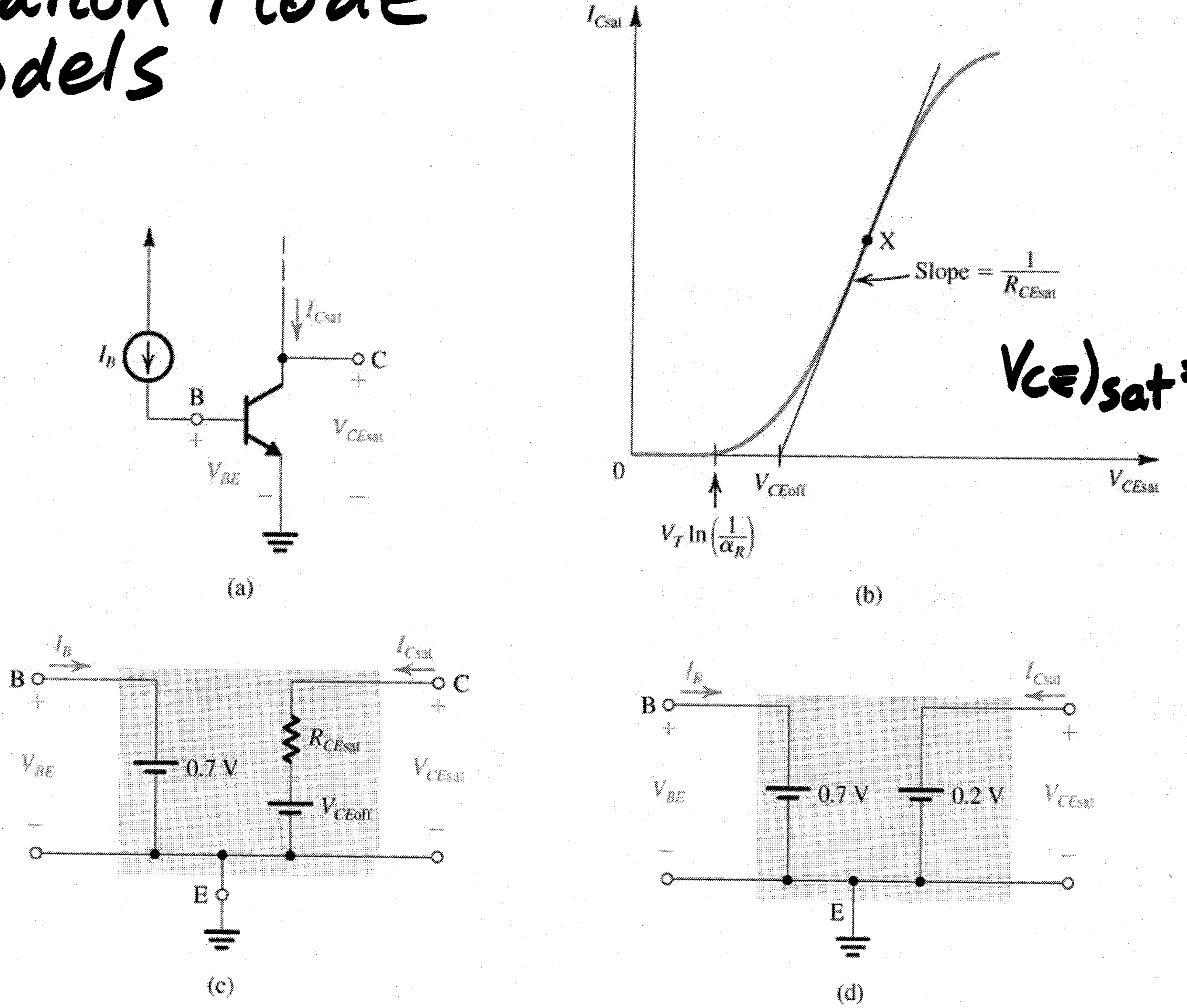


Figure 5.23 An expanded view of the common-emitter characteristics in the saturation region.

See text for discussion of R_{CEsat}

Saturation Mode models



$V_{CE})_{sat} = V_{CE})_{off} + I_{Csat} R_{CEsat}$

Figure 5.24 **(a)** An *npn* transistor operated in saturation mode with a constant base current I_B. **(b)** The i_C–v_{CE} characteristic curve corresponding to $i_B = I_B$. The curve can be approximated by a straight line of slope $1/R_{CEsat}$. **(c)** Equivalent-circuit representation of the saturated transistor. **(d)** A simplified equivalent-circuit model of the saturated transistor.

Ex. 5.18: BV_{CBO}=70V. Find V_O

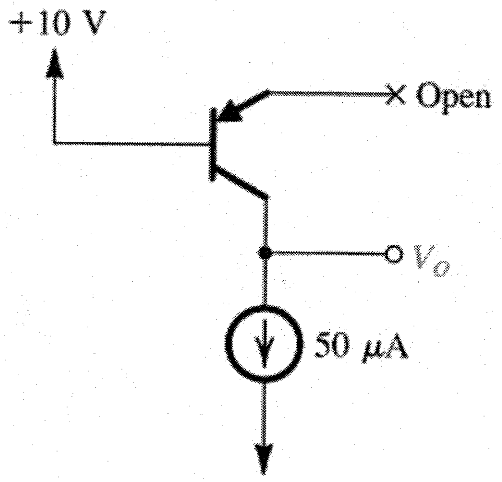
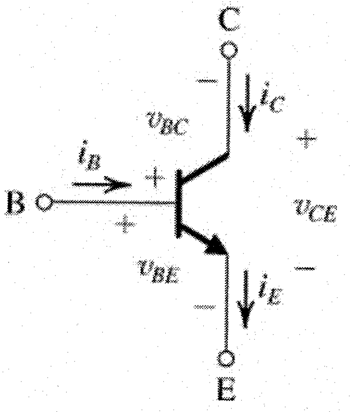


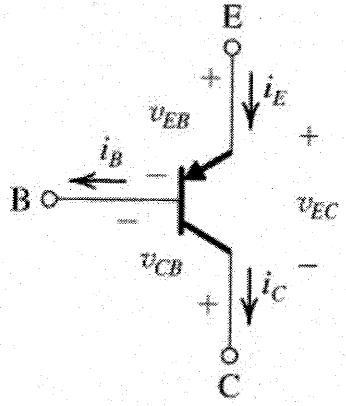
Figure E5.18

Summary of BJT models

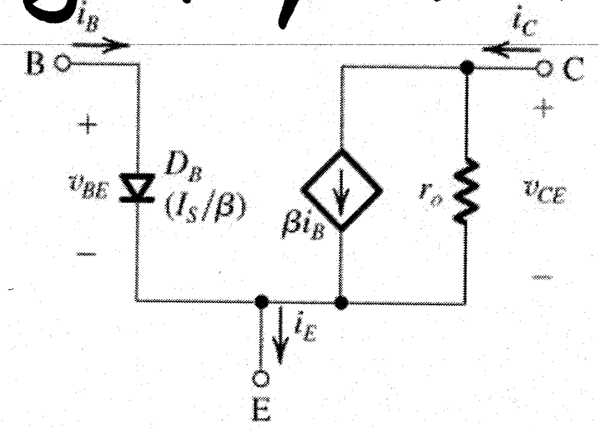
NPN



PNP



Large signal equivalent circuits (including Early Effect)



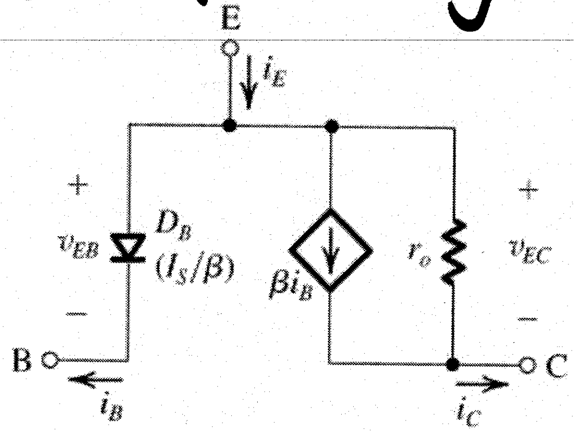


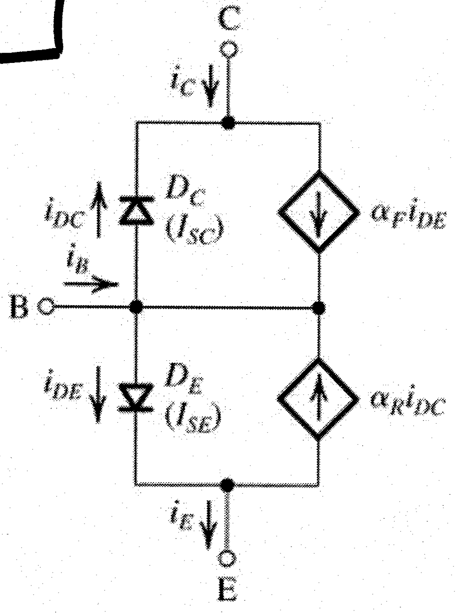
Table 5.3

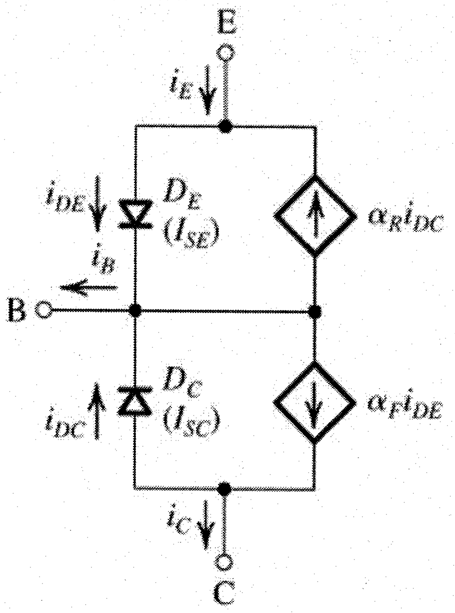
BJT Models continued

NPN

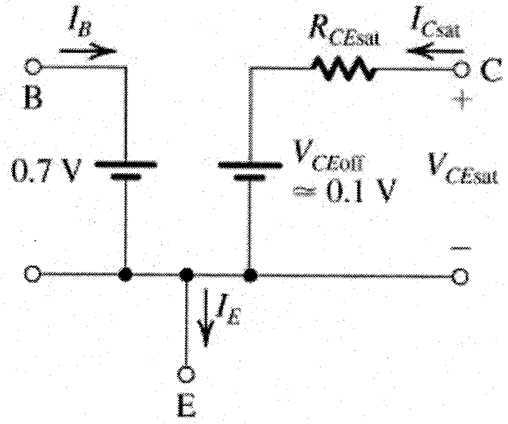
PNP

Ebers-Moll Models





Saturation mode



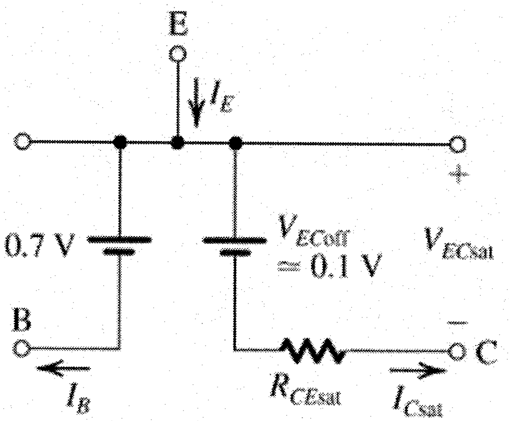


Table 5.3 *(Continued)*

Exercises

- 5.1
  - NPN v_{BE}=0.7V at i_C=1mA. Find v_{BE} at i_C=0.1mA, 10mA
- 5.2
  - Find α range for β=50 to 150.
- 5.3
  - NPN I_B=14.46μA for I_E=1.460mA, and V_{BE}=0.7V. Find α, β, and I_S
- 5.4
  - I_C=10mA. Find β and I_B for α = 0.99, 0.98.

Ex 5.1 NPN with $v_{BE} = 0.7\text{v}$ at $i_c = 1\text{mA}$

Find v_{BE} for $i_c = 0.1\text{mA}, 10\text{mA}$

(i_c controlled by v_{BE} — hense same as earlier diode examples)

Method 1: $i_c = I_s \exp(v_{BE}/V_T)$ Solve for I_s & substitute back.

Method 2: $v_{BE} = V_T \ln(i_c/I_s) \longrightarrow v_{BE}' = 0.7\text{v} + V_T \ln \frac{i_c'}{1\text{mA}}$,

$= .7\text{v} + 2.3 \times 25\text{mV} \log_{10} \frac{i_c'}{1\text{m}}$

$i_c' = 0.1\text{mA}$ $\quad v_{BE}' = .7 - 2.3 \times 25\text{mV} = 0.64\text{v}$

$i_c' = 10\text{mA}$ $\quad v_{BE}' = .7 + 2.3 \times 25\text{mV} = 0.76\text{v}$

Ex 5.2 $\quad \beta = 50$ to 150, find α range

$$\alpha = \frac{50}{51} \text{ to } \frac{150}{151} = 0.982 \text{ to } 0.993$$

Ex 5.3 $i_B = 14.46\mu A$ $i_E = 1.460 mA$ $v_{BE} = 0.7v$

Calculate α, β, I_S

$$\beta = \frac{i_C}{i_B} = \frac{i_E - i_B}{i_B} = \frac{1.446 mA}{14.46\mu A} = 100$$

$$\alpha = \frac{\beta}{1+\beta} = \frac{100}{101} = 0.99 \quad \underline{OR} \quad \frac{i_C}{i_E} = \frac{1.446 mA}{1.460 mA}$$

$$I_S = i_C \exp - \frac{v_{BE}}{V_T} = 1.446 mA \exp - \frac{0.7}{25\times10^{-3}} = 9.998\times10^{-16} A$$

$$\approx 10^{-15} A = 1 fA$$

Ex 5.4 $i_C = 10 mA$

(a) For $\alpha = 0.99$ $i_B = i_C/\beta = i_C \frac{1-\alpha}{\alpha} = \frac{.01}{.99} 10 mA$

$$\approx 0.1 mA$$

$$\beta = \frac{.99}{.01} = 99$$

(b) For $\alpha = 0.98$ $i_B = \frac{.02}{.98} 10 mA \approx 0.2 mA$

$$\beta = \frac{.98}{.02} = 49$$

Exercise

- 5.6
  - BJT $\alpha_F=0.99$, $\alpha_R=0.02$, $I_S=10^{-15}$A:
  - Calculate second term on RHS of equations 5.31, 5.32, 5.33, and verify they can be neglected.
  - Calculate i_E, i_C, and i_B for $v_{BE}=0.7$V.

Ex 5.6 $\alpha_F = 0.99$ $\alpha_R = 0.02$ $I_S = 10^{-15}A$ & $v_{BE} = .7v$

Repeat previous equations for convenience:

$$i_E = (I_S/\alpha_F)\exp\frac{v_{BE}}{V_T} + I_S(1 - 1/\alpha_F)$$

$$i_C = I_S \exp\frac{v_{BE}}{V_T} + I_S(1/\alpha_R - 1)$$

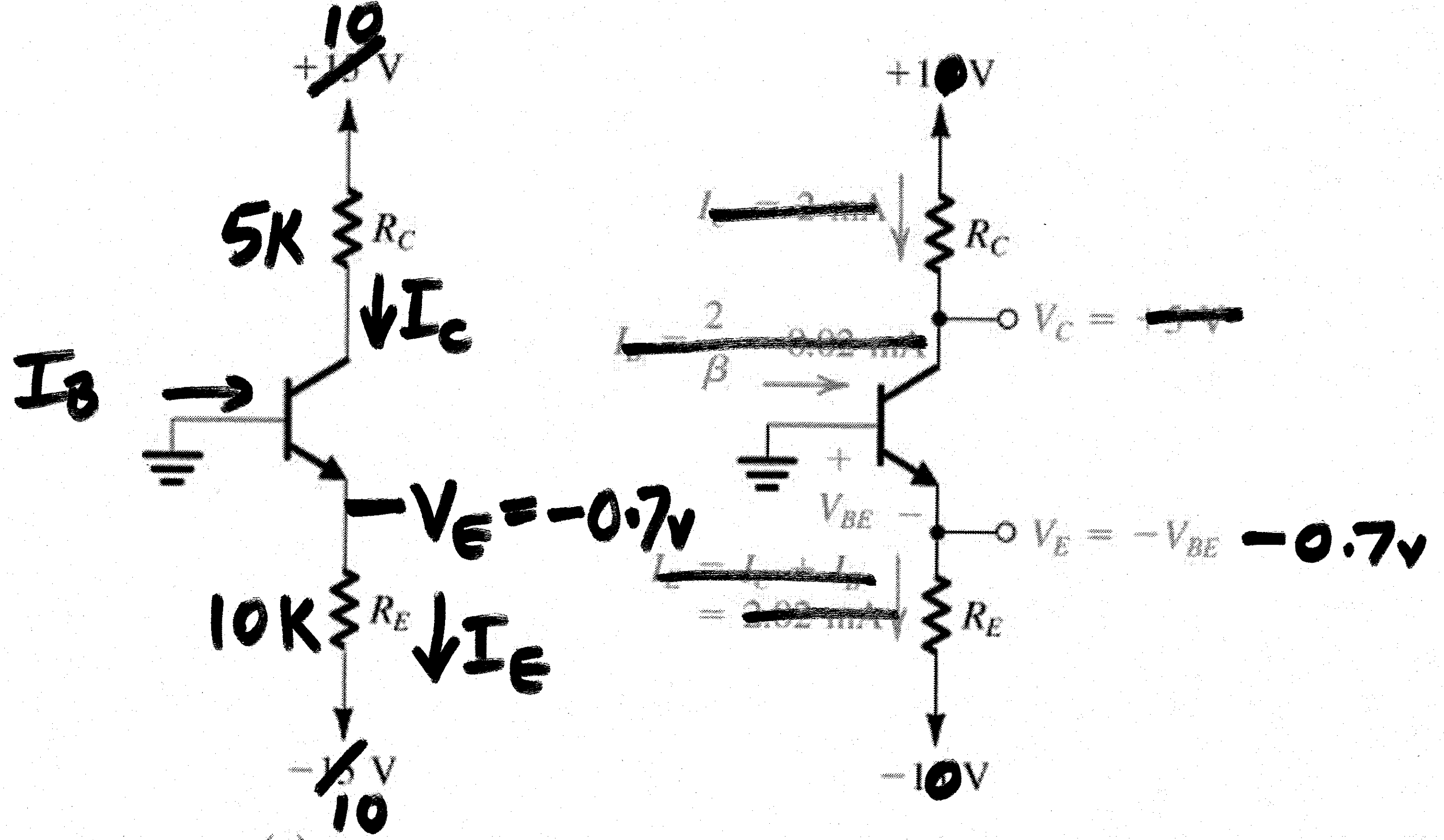
$$i_B = (I_S/\beta_F)\exp\frac{v_{BE}}{V_T} - \underbrace{I_S(1/\beta_F + 1/\beta_R)}_{\text{Show these terms negligible}}$$

$$\beta_F = \frac{.99}{.01} = 99 \qquad \exp\frac{.7v}{.025v} = 1.446 \times 10^{12}$$

$$\beta_R = \frac{.02}{.98} \approx .02 \qquad \therefore I_S \exp\frac{.7}{.025} = 1.446mA$$

$$\therefore \quad \begin{aligned} i_E &\approx 1.46mA - 10^{-17}A \\ i_C &\approx 1.446mA + 4.9 \times 10^{-14}A \\ i_B &\approx 14.5\mu A - \underbrace{3 \times 10^{-17}A}_{\text{negligible}} \end{aligned}$$

Ex 5.10 $\beta = 50$. I_E, I_B, I_C, V_C?



(a) (b)

Figure 5.15 Circuit for Example 5.1.

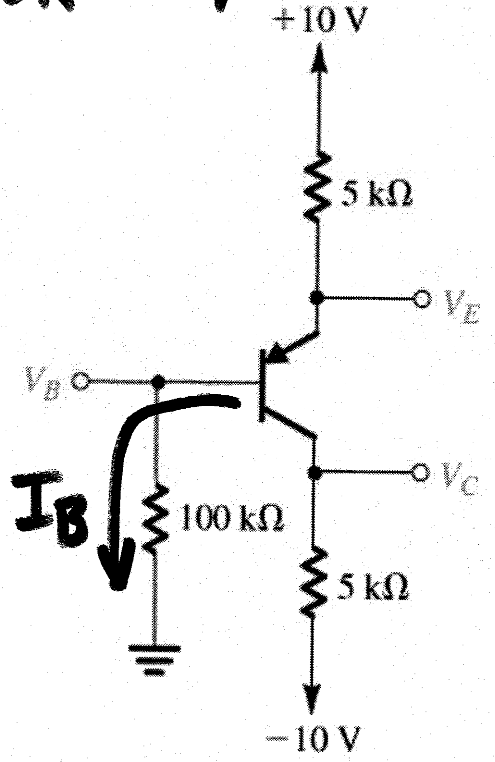
$I_E = \frac{-.7-(-10)v}{10K} = 0.93mA$, $I_B = \frac{I_E}{1+\beta} = \frac{.93mA}{51} = 18.24\mu A$

$I_C = \frac{\beta}{1+\beta} I_E = \frac{50}{51} 0.93mA = .912\,mA$ (Check $I_C + I_B = I_E$)

$V_C = 10v - 5K \times 0.912mA = 5.44$ volts

Ex 5.11 Find α, β, V_C . $V_B = 1v$ $V_E = 1.7v$

$$V_B = 1v \therefore I_B = \frac{1v}{100K} = 10\mu A$$



$$= 1.7v \therefore I_E = \frac{10-1.7v}{5K}$$

$$= 8.3v/5K$$

$$= 1.66mA$$

$$\therefore I_C = I_E - I_B = 1.66 - 0.01\,mA = 1.65mA$$

$$\therefore V_C = -10v + 5K \times 1.65mA = -10v + 8.25v = -1.75v$$

Figure E5.11

$$\alpha = I_C / I_E = 1.65/1.66 = 1 - \frac{.01}{1.66} = 1 - \frac{.06}{10} = .994$$

$$\beta = I_C / I_B = 1.65/0.01 = 165$$

Ex 5.18 Find $V_o = V_C$ if $BV_{BCO} = 70v$

Note: Emitter open circuit

$I_E = 0 \quad \therefore I_B, I_C = 0$ if BJT in active region

Also, not saturation

$\therefore$ C-B breakdown for finite I_C & $\therefore I_B$

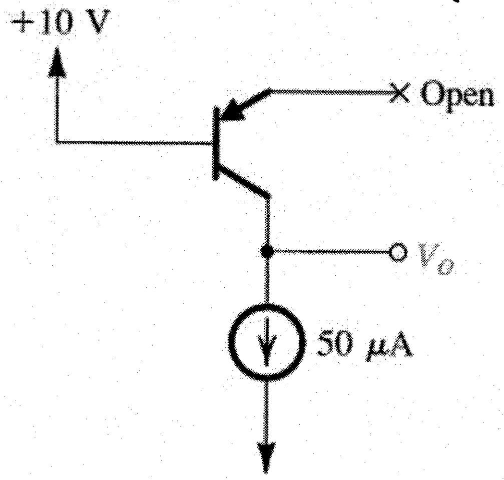


Figure E5.18

$\therefore V_B - V_C = BV_{BCO} = 70v$

& $V_C = +10v - 70v$

$= -60v$