Electrical Package Design
TKK 2009 Lecture 2

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Nanoelectronics & Packaging Lab

Electrical Package Design

- Lecture topics A: Introduction
  - CMOS; R, L, & C
- Lecture topics B: Transmission lines
  - Z_0 , velocity, lossless lines, lossy lines
- Lecture topics C: Transmission lines
  - Transmission line reflections
- Lecture topics D: Transmission lines
  - Crosstalk
- Lecture topics E: Electromagnetism & Modeling
- Lecture topics F: Electromagnetic Compatibility

Lecture topics A: Introduction CMOS; R, L, & C

1. Interconnect modeling
2. Resistance, inductance, & capacitance
   - R, L, & C
3. Skin effect
4. Ground planes
5. MOS devices and CMOS
6. Delta-I (ΔI) switching noise

1. Interconnect modeling

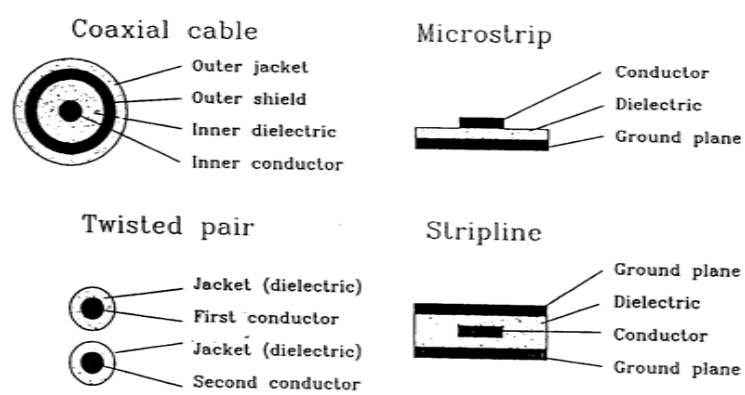
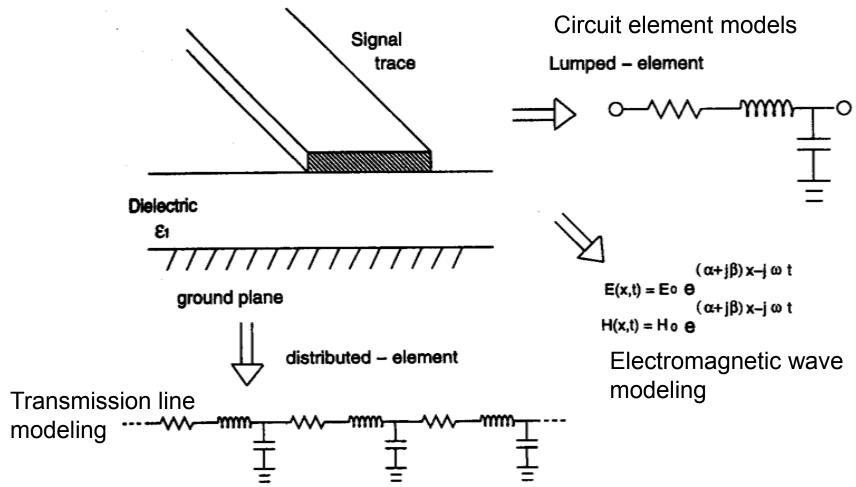


Figure 4.4 Cross sections of popular transmission line geometrics.

4.2.1 Ideal Distortionless, Lossless Transmission Line

Three modeling approaches:



(a) $\omega L >> R$, (b) $R >> \omega L$, (c) $R \approx 0$

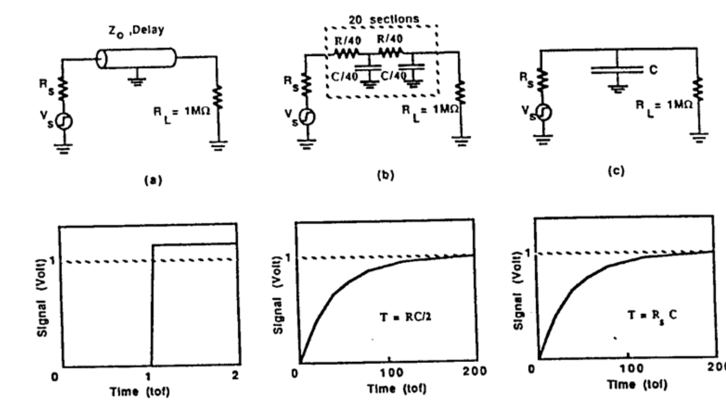
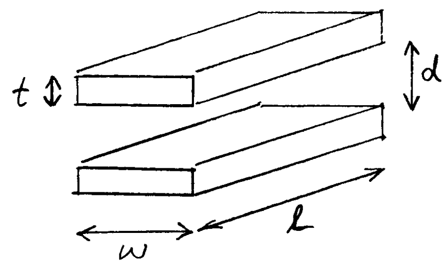


Figure 5-7 Simplified SPICE circuits & signal load responses for limiting propagation regimes. (a)Time-of-flight ($R_S<Z_O$) (b)lossy-line diffusion (c)R_S-limited; time scales are in units of time-of-flights

2. R, G, L, and C

- Conductor:
- $R = \rho L/A = \rho L/wt$
  - $R/L = \rho/wt$



- Dielectric:
- $G = \sigma A/d = \sigma Lw/d$
  - $G/L = \sigma w/d$
- $C = \varepsilon A/d = \varepsilon Lw/d$
  - $C/L = \varepsilon w/d$

- Note: L=length in this slide; used for inductance later

Capacitance/unit length: Generalized geometries

(a) "Visual" approximations

(b) Method of images

(c) Graphical approximations

(d) Experimental field plotting

(e) Numerical techniques (mesh/iteration)

(f) Analytical: $V=\Sigma V_i$ for charges Q_i

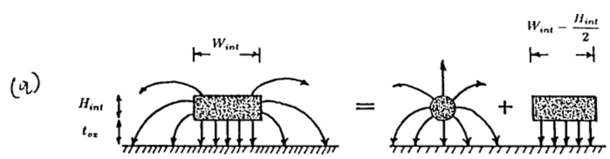


FIGURE 4.2 Modeling of the contribution of fringing fields to interconnection capacitance. Total wire capacitance can be thought of having two components: a parallel plate capacitance determined by the perpendicular field lines between the wire and the ground plane and a fringing field component, which can be approximated by the capacitance of a cylindrical wire with a diameter equal to interconnection thickness [4.3]. Reprinted by permission of Addison-Wesley Publishing Co.

reduced by $H_{int}/2$ to account for some second-order effects [4.4]. This yields the interconnection capacitance per unit length C_{int} as

$$C_{int} = \epsilon_{ox}\left\{\frac{W_{int}}{t_{ox}} - \frac{H_{int}}{2t_{ox}} + \frac{2\pi}{\ln\left[1+\frac{2t_{ox}}{H_{int}}\left(1+\sqrt{1+\frac{H_{int}}{t_{ox}}}\right)\right]}\right\}. \quad (4.5)$$

(g) Method of Moments: Example

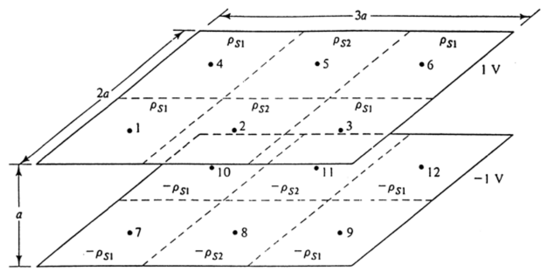


Figure 5.10. For finding the capacitance of a parallel-plate capacitor by the method of moments.

For point 2

$$\frac{\rho_{S2}a}{\pi\varepsilon_0}\ln(1+\sqrt{2}) + \frac{\rho_{S1}a^2}{4\pi\varepsilon_0}\left(\frac{2}{a}+\frac{2}{\sqrt{2}a}-\frac{2}{\sqrt{2}a}-\frac{2}{\sqrt{3}a}\right) + \frac{\rho_{S2}a^2}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{a}-\frac{1}{\sqrt{2}a}\right) = 1 \qquad (5.31b)$$

or

$$2.9101\rho_{S1} + 0.4226\rho_{S2} = \frac{4\pi\varepsilon_0}{a} \qquad (5.32a)$$

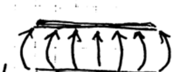
$$0.8453\rho_{S1} + 2.8184\rho_{S2} = \frac{4\pi\varepsilon_0}{a} \qquad (5.32b)$$

Method of Moments Example

PARALLEL PLATE CAPACITOR

$C = \epsilon \frac{A}{d}$

neglects fringing fields



Field curvature ⟹ non-uniform surface charge distribution ρ_s

eg. $3a \times 2a \times a$ capacitor. Charges $\rho_{S1}a^2$, $\rho_{S2}a^2$ by symmetry. Assume charges at center of □'s.

Potential at point 1 due to ρ_{S1} at point 4

⟹ $\frac{(\rho_{S1} \times a^2)}{4\pi\epsilon\, a}$ ⟸ q, ⟸ distance

Potential at point 1 due to distributed charge $\rho_{S1}a^2$ on square #1 $= \frac{(\rho_{S1} \times a^2)}{\pi\epsilon a} \ln(1+\sqrt{2})$

∴ For fixed plate potentials 1v, -1v

Square #1: $V_1 = \frac{\rho_{s1} a}{\pi \epsilon} \ln(1+\sqrt{2}) + \frac{\rho_{s1} a}{4\pi\epsilon}\left(\frac{1}{2} + 1 + \frac{1}{\sqrt{5}} - 1 - \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}} - \right.$

$\left. + \frac{\rho_{s2} a}{4\pi\epsilon}\left(1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right) = 1v$

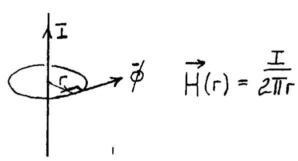
Square #2: $V_2 = \frac{\rho_{s2} a}{\pi\epsilon} \ln(1+\sqrt{2}) + \frac{\rho_{s1} a}{4\pi\epsilon}\left(2 + \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{3}}\right)$

$+ \frac{\rho_{s2} a}{4\pi\epsilon}\left(1 - 1 - \frac{1}{\sqrt{2}}\right) = 1v$

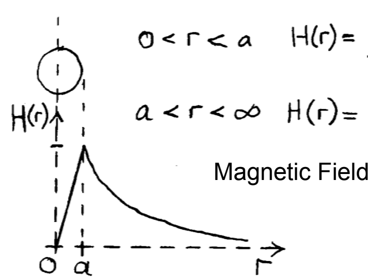
Solve: $\rho_{s1} = 3.8378\,\epsilon/a \quad \rho_{s2} = 3.3075\,\epsilon/a$

∴ Total $Q = 21.9662\,\epsilon a$ & $C = Q/a = 10.983\,\epsilon_0$ (cf $6\epsilon_0$

Inductance of a Straight Wire



$\vec{H}(r) = \frac{I}{2\pi r}$



$0 < r < a \quad H(r) = \frac{\pi r^2}{\pi a^2} \cdot I \frac{1}{2\pi r}\hat{\phi} = \frac{Ir}{2\pi a^2}\hat{\phi}$

$a < r < \infty \quad H(r) = \frac{I}{2\pi r}\hat{\phi}$

Internal/External Inductances

r_w

↑ Internal inductance l_i

↑ External inductance $l_e \gg l_i$ usually

$l = l_i + l_e \approx l_e$

Also $l_e g = \mu \sigma$

$l_e c = \mu \epsilon$

g, c leakage conduct/capac

σ, ε, μ for external medium

$\oint_c \underline{H} \cdot d\underline{l} = I_{enclosed}$ → Circular contour at r

$H(r) \cdot 2\pi r = I$

$H(r) = \frac{I}{2\pi r}$

Internal Inductance

low freq — uniform current.

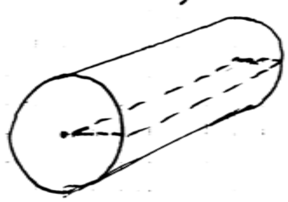
$$I_{encl,i} = \pi r^2 \left(\frac{I}{\pi r_w^2}\right) = I\left(\frac{r}{r_w}\right)^2$$

$$\therefore\ H(r)\cdot 2\pi r = I\left(\frac{r}{r_w}\right)^2 \Rightarrow H(r) = \frac{I}{2\pi r_w^2}\cdot r$$

&

$$\psi_i = \int_S \underset{\sim}{B}(r)\cdot \underset{\sim}{ds} \Rightarrow \psi_i \times \ell = \mu_0 \int_0^{r_w} \frac{I}{2\pi r_w^2}\, r \cdot \ell\, dr$$

↑per unit length

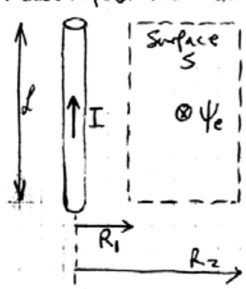


$$\psi_i = \frac{\mu_0 I}{4\pi}$$

$$\therefore\ l_i = \frac{\mu_0}{4\pi}\ \text{H/m}$$

External Inductance: Basic formula

Need to find total external flux



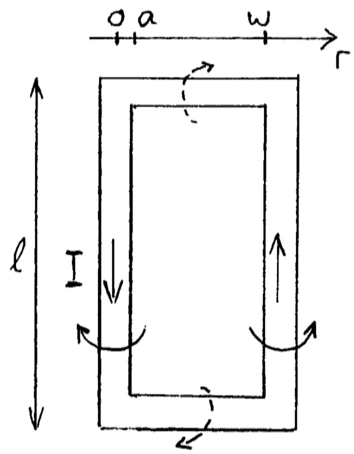
$$\psi_e = \int_S \underset{\sim}{B}(r)\cdot \underset{\sim}{ds} = \int_{R_1}^{R_2} \mu_0 \frac{I}{2\pi r}\, dr = \frac{\mu_0 I}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

Note: To find $l_e = \frac{\psi_e}{I} = \frac{\mu_0}{2\pi}\ln\left(\frac{R_2}{R_1}\right)$

over all space $\left(\begin{matrix} R_2 \to \infty \\ R_1 = r_w \end{matrix}\right)$

requires assumption of specific geometries

Compare:-



$$L = \frac{\Phi}{I} = \frac{1}{I}\int_S \vec{B}\cdot d\vec{S}$$

$$= \frac{\mu}{I}\int \vec{H}\cdot d\vec{S}$$

$$\approx \frac{2\mu l}{I}\left[\int_0^a \frac{Ir}{2\pi a^2}dr + \int_a^w \frac{I}{2\pi r}dr\right]$$

$$L/l = 2\left[\frac{\mu}{4\pi}\left(1 + 2\ln\frac{w}{a}\right)\right]$$

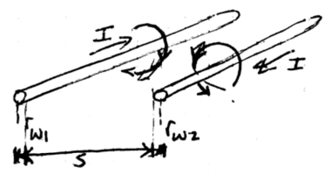
Hence "partial" inductance

$$L = \frac{\mu_0}{4\pi} + \frac{\mu_0}{2\pi}\ln\frac{w}{a}$$

$$= L_i + L_e \approx L_e$$

External Inductance: 2-wire line

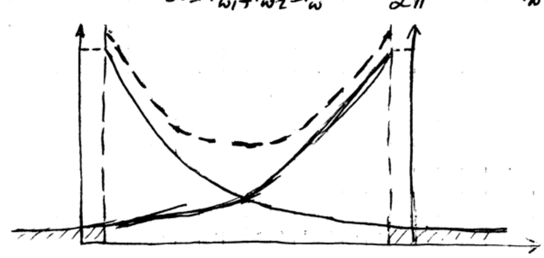
(a) 2-wire line



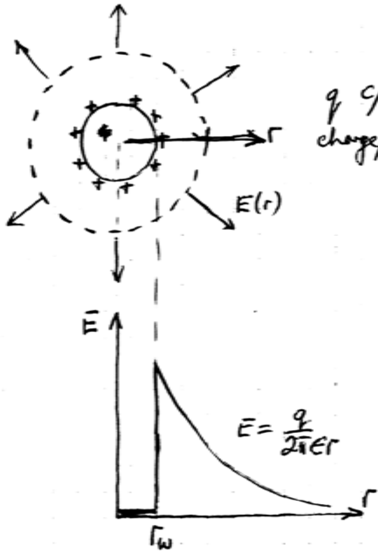
Total flux between wires

$$l_e = \frac{\psi_e}{I} = \frac{\mu_0 I}{2\pi}\left[\ln\left(\frac{s + r_{w1}}{r_{w1}}\right) + \ln\left(\frac{s + r_{w2}}{r_{w2}}\right)\right]$$

$$\underset{s \gg r_{w1} = r_{w2} = r_w}{\Longrightarrow} \frac{\mu_0}{2\pi}\ln\frac{s^2}{r_w^2} = \frac{\mu_0}{\pi}\ln\frac{s}{r_w}$$



Straight Wire: Radial Electric Field



q C/m charge/unit length

$$\oint_S \epsilon \underset{\sim}{E} \cdot \underset{\sim}{ds} = Q_{encl.}$$

$$\epsilon E (2\pi r \times \mathcal{L}) = q \times \mathcal{L}$$

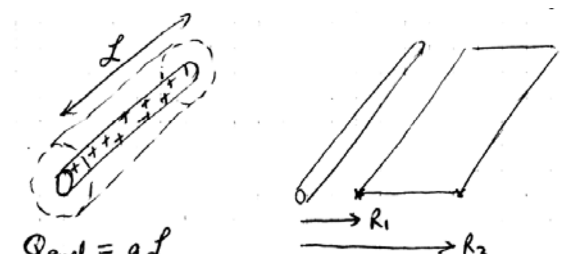
$$\therefore E(r) = \frac{q}{2\pi\epsilon r}$$

$$\& \ V = -\int_C \underset{\sim}{E} \cdot \underset{\sim}{d\ell}$$

$$= -\int_{R_2}^{R_1} \frac{q}{2\pi\epsilon r}\, dr$$

$$= \frac{q}{2\pi\epsilon} \ln\left(\frac{R_2}{R_1}\right)$$

Magnetic and Electric Fields: $(2\pi)^{-1}\ln(R_2/R_1) = \varepsilon V/q$ (electric) $= \psi_e/I\mu_0$



$Q_{encl} = q\mathcal{L}$

Area $= 2\pi r \times \mathcal{L}$

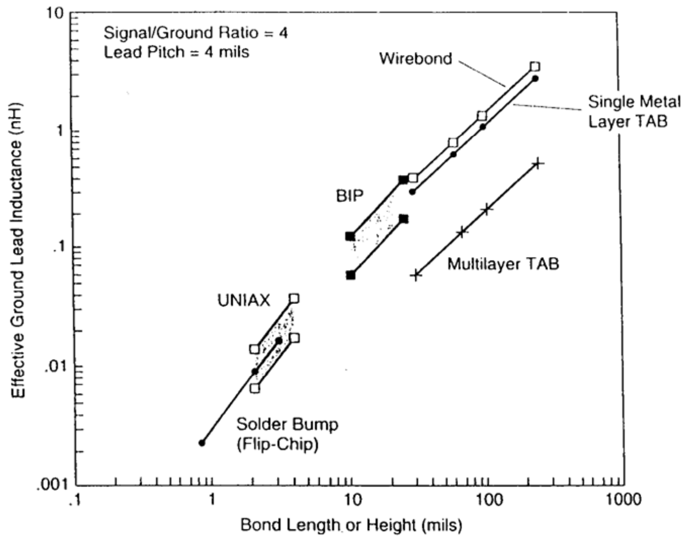
& note $\frac{V\epsilon}{q} = \frac{\psi_e}{\mu_0 I}$

$$\ell_e = \frac{\psi_e}{I} = \mu_0 \epsilon \frac{V}{q} = \mu_0 \epsilon C^{-1}$$

↑ Capac/unit length

i.e. $L_e.C$ = constant = $\mu_0\varepsilon$

Lead inductances



3. Skin Effect

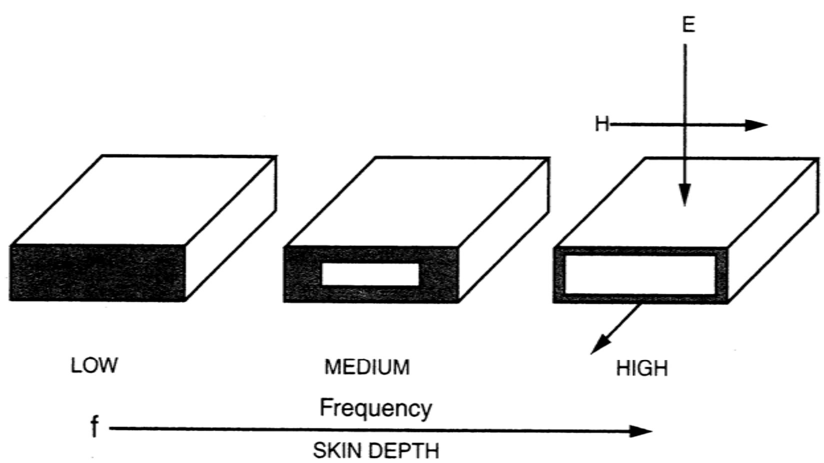


FIGURE 13.19 Effect of frequency on conductor cross section.

8. SKIN EFFECT

Consider wire to consist of concentric cylinders

Inductance of outer rings < Inductance of center rings

At high frequency, current path seeks lower inductance → outer "skin"

SKIN DEPTH $\delta = \left(\frac{2\rho}{\omega\mu}\right)^{1/2}$

As current area decreases, resistance increases ($R = \rho\, \ell/A$)

$R \propto 1/\delta \propto freq^{1/2}$

and $L_{SKIN} \propto freq^{-1/2}$

Skin Reactance $\omega L_{SKIN} \propto freq^{1/2}$

($L_{internal} \ll L_{external}$)

$R = \rho\ell/\pi r^2 \rightarrow R_{skin} = \rho\ell/2\pi r\delta$ as $f \rightarrow \infty$, and

$\omega L_{int} = \omega(\mu_0/8\pi)\,\ell \rightarrow R_{skin}$ as $f \rightarrow \infty$

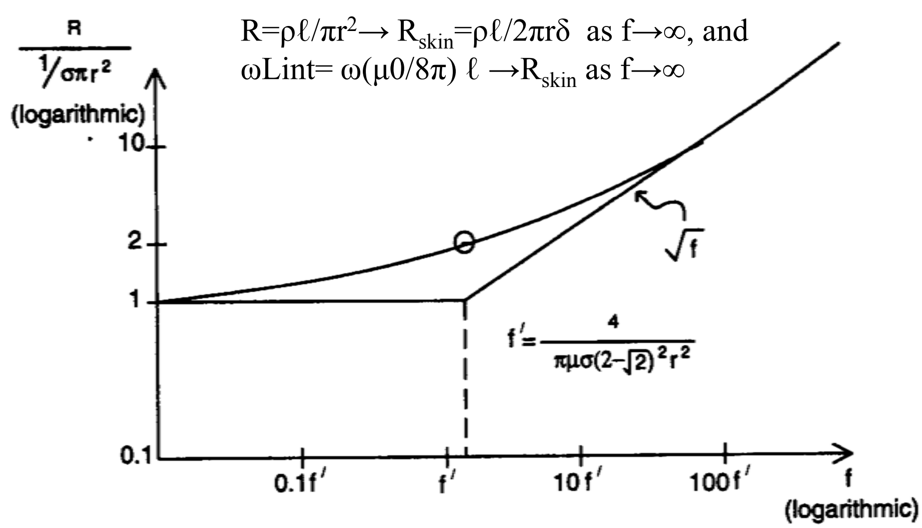


Figure 2.25 *R*(*f*) for a wire of circular cross section, radius *r*.

8. Skin Effect.

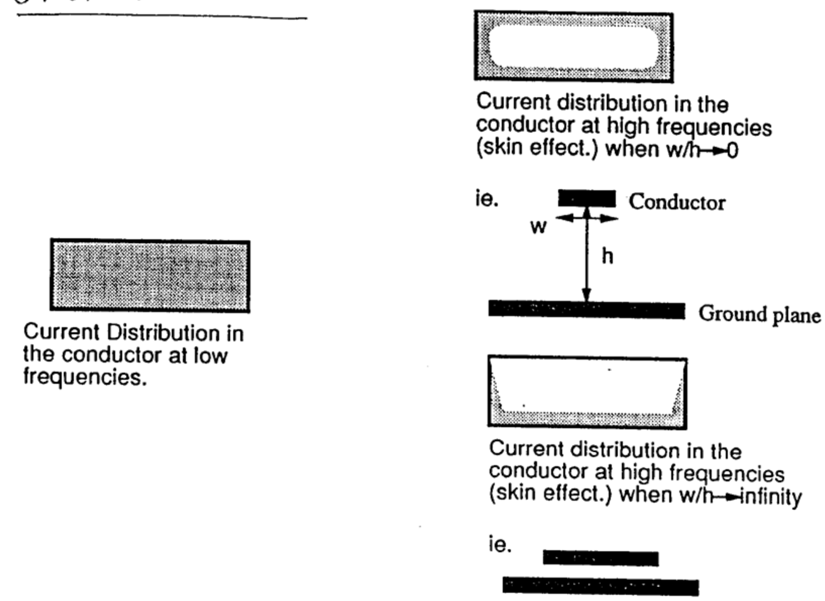


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Figure 7.5 The skin effect.

4. Ground planes

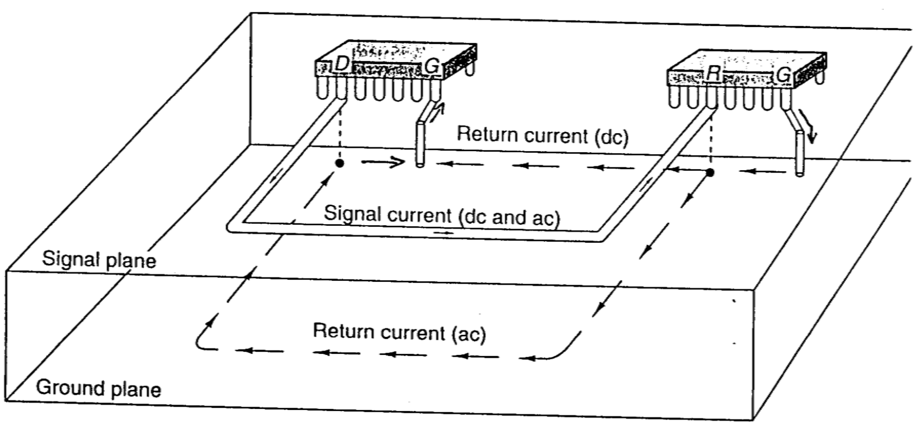


Figure 8.16 Paths for dc and ac ground returns on a multilayer board.

Loss of coupled return → inductance, EMI

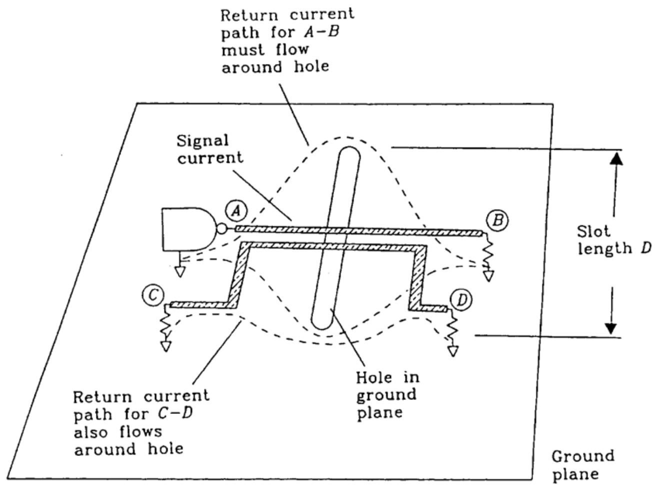
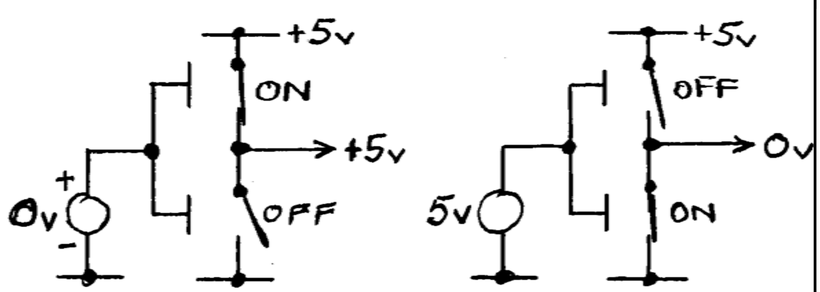


Figure 5.8 Crosstalk in a slotted ground plane.

5. MOS devices and CMOS



Derivation of MOSFET characteristic equations

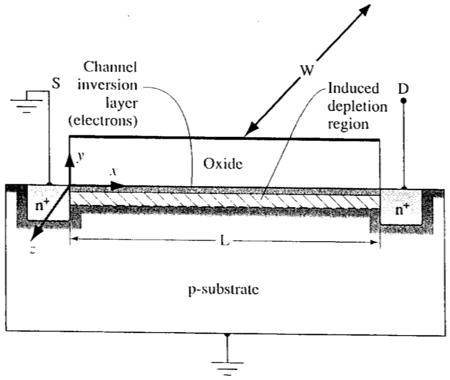


Figure 10.42 Geometry of a MOSFET for I_D versus V_{DS} derivation

$$I(x) = \int_y \int_z J(x)\, dy\, dz$$
$$= W \cdot \mu \int_y \sigma(x) \frac{dV(x)}{dx} \cdot dy$$
$$I(x)\, dx = \mu W q(x)\, dV(x)$$
and $$q(x) = \frac{\epsilon_{ox}}{t_{ox}}[V_g - V(x)]$$
$$= C_{ox}[(V_g - V_T) - V(x)]$$
$$I_{DS}\int_0^L dx = \mu W C_{ox} \int_0^{V_{DS}} [(V_g - V_T) - V(x)]\, dV_x$$
$$I_{DS} \cdot L = \mu W C_{ox}[(V_g - V_T)V_{DS} - V_{DS}^2/2]$$
$$I_{DS} = \frac{\mu}{2}\frac{W}{L} C_{ox}[2(V_g - V_T) - V_{DS}]V_{DS}$$

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \mu \frac{W}{L} C_{ox}[(V_g - V_T) - V_{DS}]$$
$\rightarrow 0$ when $V_{DS} = V_g - V_T$

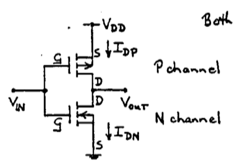
Substitute: $$I_{DS}\big]_{SAT} = \frac{\mu}{2}\frac{W}{L} C_{ox}(V_g - V_T)^2$$
$$= \frac{\mu}{2}\frac{W}{L} C_{ox} V_{DS}^2$$

Now set $(\mu W/2L)C_{ox} = k \rightarrow$

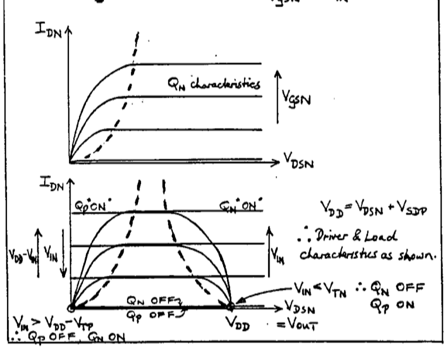
CMOS dynamic characteristics (part 1)

CMOS INVERTER

Both enhancement MOSFETs.



$V_{DSP} = V_{OUT} - V_{DD}$
$V_{GSP} = V_{IN} - V_{DD}$
$V_{DSN} = V_{OUT}$
$V_{GSN} = V_{IN}$



CMOS INVERTER (cont)

As V_{IN} increases from 0 volts :-

(i) For $V_{IN} < V_{TN}$, Q_N OFF $\therefore I = 0$

$\therefore V_{GSP} = V_{IN} - V_{DD} \sim -V_{DD}$, Q_P not saturated.

$V_{OUT} = V_{DD}$ — see diagram.

(ii) For $V_{IN} > V_{TN}$, Q_N begins to turn ON

Q_N saturated, Q_P not saturated — see diagram (for $V_{IN} < V_{DSN} + V_{TN}$)

$\therefore I_{DN} = I_{DP}$

gives $k_N (V_{GSN} - V_{TN})^2 = k_P[2(V_{GSP} - V_{TP})V_{DSP} - V_{DSP}^2]$

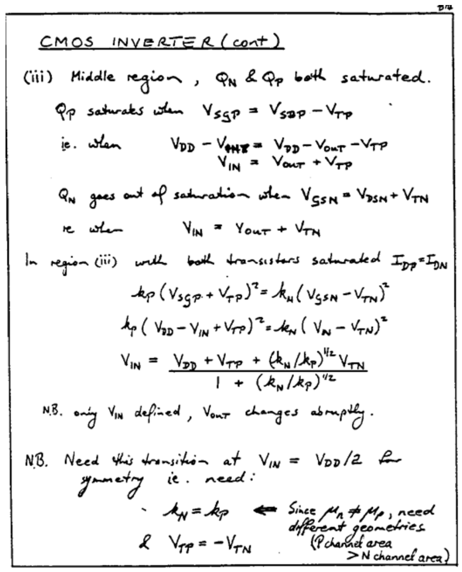
$\Rightarrow k_P[2(V_{SGP} + V_{TP})V_{SDP} - V_{SDP}^2]$

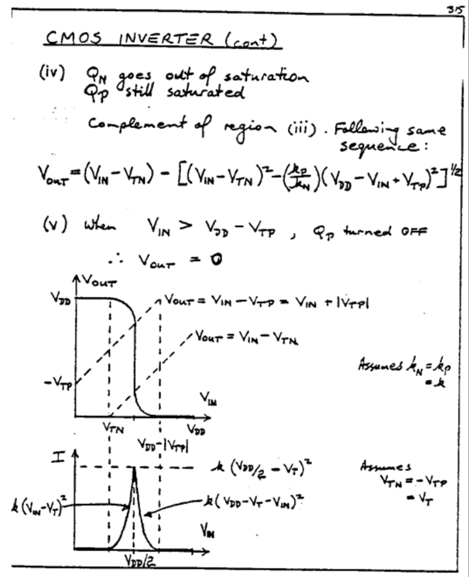
$k_N (V_{IN} - V_{TN})^2 = k_P[2(V_{DD} - V_{IN} + V_{TP})(V_{DD} - V_{OUT}) - (V_{DD} - V_{OUT})^2]$

i.e. $V_{OUT} = V_{DD} + (V_{DD} - V_{IN} + V_{TP}) - [(V_{DD} - V_{IN} + V_{TP})^2 - \frac{k_N}{k_P}(V_{IN} - V_{TN})^2]^{1/2}$

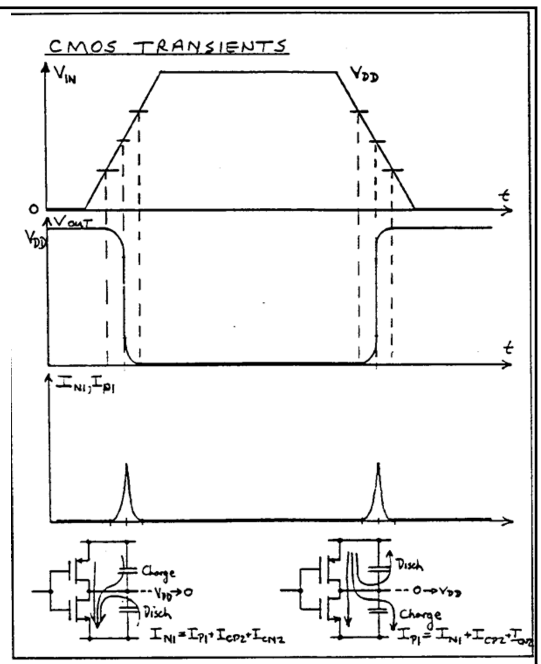
N.B. $V_{TN} > 0$, $V_{TP} < 0$ } for enhancement

CMOS dynamic characteristics (part 2)





Switching current (1)



Switching current (2)

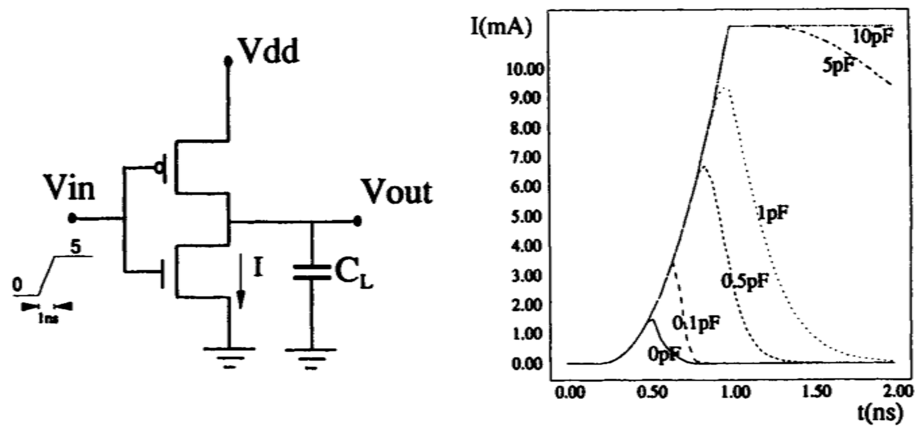
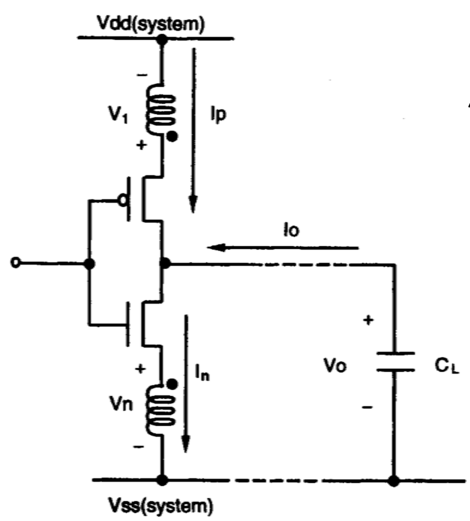


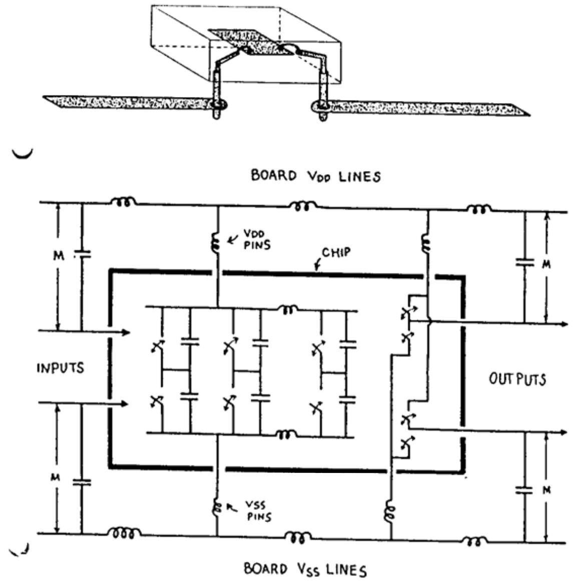
Figure 2.75 N-channel MOSFET current for various sizes of C_L.

6. Delta-I (ΔI) switching noise



ΔV = L di/dt

Figure 2.83 Driver circuit for the calculation of the mutual effect between L_{vss} and L_{vdd} on V_n.



GURE 7.10 Power-distribution path in an integrated circuit. The IC die, nding wire, distribution layer on the chip carrier, and the package pins and wer lines on the board are shown. The electrical model of this structure is ıstrated at the bottom. Here M represents the mutual inductance between the nal and power/ground planes.

Example to show typical values

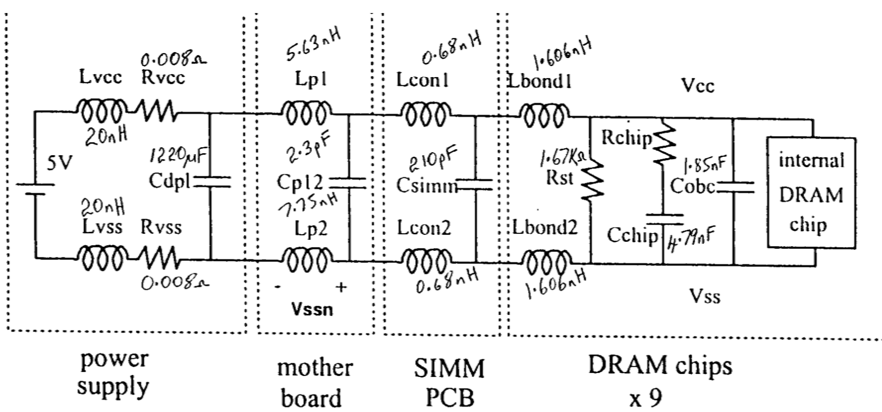


Fig. 15. Simplified equivalent circuit of the test setup shown in Fig. 14.

B/C/D. Transmission line effects

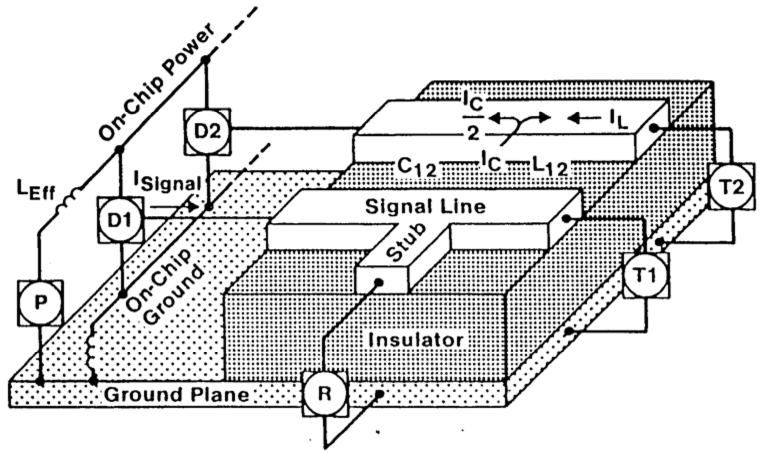


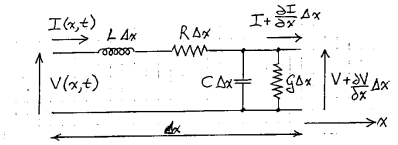
Figure 1-12. Causes of Reflected, Coupled, and Switching (ΔI) Noises.

Lecture topics B: Transmission lines

(Z_0 , velocity, lossless, lossy)

1. Transmission line theory
   - Characteristic impedance Z_0
   - Attenuation, dispersion, velocity
2. Z_0 calculations
3. Lossless line
   - Distortionless transmission
   - Shape factor
4. Z_0 for practical geometries
5. Effects of loss
   - Distortion

1. Transmission line theory



$$V - \left(V + \frac{\partial V}{\partial x}\Delta x\right) = R\Delta x\, I + L\Delta x \,\frac{\partial I}{\partial t}$$

ie. $$-\frac{\partial V}{\partial x} = RI + L\frac{\partial I}{\partial t}$$

Assume $V(x,t) = V(x)\,e^{j\omega t}$, $I(x,t) = I(x)\,e^{j\omega t}$

then $$-\frac{dV(x)}{dx} = (R + j\omega L)\,I(x)$$

And similarly
$$I - \left(I + \frac{\partial I}{\partial x}\Delta x\right) = G\Delta x\, V + C\Delta x \frac{\partial V}{\partial t}$$

gives $$-\frac{dI(x)}{dx} = (G + j\omega C)\,V(x)$$

These give
$$\frac{d^2V(x)}{dx^2} = (R + j\omega L)(G + j\omega C)\,V(x)$$
& $$\frac{d^2I(x)}{dx^2} = (G + j\omega C)(R + j\omega L)\,I(x)$$

2nd order differential equation has solution:
$$V(x) = A\,e^{-\lambda x} + B\,e^{+\lambda x}$$
where $\lambda = [(R + j\omega L)(G + j\omega C)]^{1/2}$

ie. $$V(x,t) = V(x)\,e^{j\omega t} = A\,e^{j\omega t - \lambda x} + B\,e^{j\omega t + \lambda x}$$

Forward wave (damped λ) → ; ← Reverse wave (damped λ)

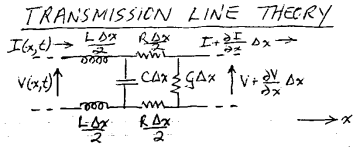
Similarly $$I(x) = -(R + j\omega L)^{-1}\frac{dV(x)}{dx}$$
$$= -\frac{\left[-A e^{-\lambda x} + B e^{+\lambda x}\right]\lambda}{(R + j\omega L)}$$
$$= \frac{A\,e^{-\lambda x} - B\,e^{+\lambda x}}{(R + j\omega L)^{1/2}}(G + j\omega C)^{1/2}$$
$$= \frac{A\,e^{-\lambda x} - B\,e^{+\lambda x}}{Z_0}$$

where the Characteristic Impedance is
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Note: units Ω

Summary

TRANSMISSION LINE THEORY



$$V(x,t) = A\,e^{j\omega t - \lambda x} + B\,e^{j\omega t + \lambda x}$$

Forward wave → ; ← Reverse wave

$$I(x,t) = \frac{A}{Z_0}e^{j\omega t - \lambda x} - \frac{B}{Z_0}e^{j\omega t + \lambda x}$$

where $$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \text{(units Ω)}$$
Characteristic Impedance

& $$\lambda = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$
Attenuation (α), Dispersion (β)

Wave velocity $v = \omega/\beta$

Attenuation, dispersion, velocity:-
$$\lambda = [(R + j\omega L)(G + j\omega C)]^{1/2}$$
Write $\lambda = \alpha + j\beta$

Real part α is attenuation
Imag. part β is dispersion

$$\lambda^2 = \alpha^2 - \beta^2 + 2j\beta\alpha = (RG - \omega^2 LC) + j\omega(RC + GL)$$
$$\therefore \alpha^2 - \beta^2 = RG - \omega^2 LC$$
$$2\alpha\beta = \omega(RC + GL)$$

Substitute for $\beta = \frac{1}{2}\frac{\omega}{\alpha}(RC + GL)$
$$\alpha^2 - \frac{\omega^2}{4\alpha^2}(RC + GL)^2 = (RG - \omega^2 LC)$$
$$(\alpha^2)^2 - (\alpha^2)(RG - \omega^2 LC) - \frac{\omega^2}{4}(RC + GL)^2 = 0$$
$$\alpha^2 = \frac{1}{2}\left\{(RG - \omega^2 LC) \pm \left[(RG - \omega^2 LC)^2 + \omega^2(RC + GL)^2\right]^{1/2}\right\}$$
& $$\beta^2 = \alpha^2 - (RG - \omega^2 LC)$$
$$= \frac{1}{2}\left\{-(RG - \omega^2 LC) \pm \left[(RG - \omega^2 LC)^2 + \omega^2(RC + GL)^2\right]^{1/2}\right\}$$

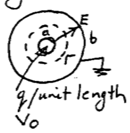
By comparison with standard wave equations:
$$V(x,t) = A\,e^{j\omega t - \lambda x} = A\,e^{j\omega t}\,e^{-(\alpha + j\beta)x}$$
$$= A\,e^{-\alpha x}\,e^{j\omega\left(t - \frac{\beta}{\omega}x\right)}$$
i.e. velocity $v = \omega/\beta$

2. Z_0 calculations

- Need L, C
- Also gives β, v
- Easiest geometry is coaxial line
  - (radial symmetry)

(a) Dielectric: C/unit length

GAUSS'S LAW $\epsilon \oint \underline{E} \cdot d\underline{s} = \sum q_i$



$\epsilon E \, 2\pi r (\ell) = q(\ell)$

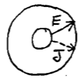
$\therefore E = \frac{q}{2\pi\epsilon} \cdot \frac{1}{r}$

$\therefore V_0 = -\int_b^a E \cdot dr = \frac{q}{2\pi\epsilon} \ln \frac{b}{a}$

Capacitance/unit length $C = q/V_0 = 2\pi\epsilon / \ln\left(\frac{b}{a}\right)$

(b) Leakage conductance: G/unit length

(b) FOR LEAKAGE CONDUCTANCE: G /unit length



Current density $\underline{J} = \sigma \underline{E}$

& using E, V_0 from above $E = \frac{1}{r} \frac{V_0}{\ln(b/a)}$

Conductance/unit length $G = I/V_0 = \frac{J(r)\, 2\pi r \times \ell}{V_0} / \ell$

$= \frac{\sigma}{V_0} 2\pi r \frac{1}{r} \frac{V_0}{\ln(b/a)}$

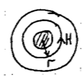
$= 2\pi\sigma / \ln\left(\frac{b}{a}\right)$

Note: for $G \ll \omega C$ need $\sigma \ll \omega\epsilon$

or $\omega \gg \sigma/\epsilon$

(c) Inductance: L/unit length

(c) FOR INDUCTANCE : L /unit length



$B = \mu \underline{H} = \mu \frac{I}{2\pi r}$

$I = \oint \underline{H} \cdot d\underline{\ell}$

$\phi = \int \underline{B} \cdot d\underline{s}$

Magnetic flux $\phi = \int_a^b B dr \ell = \frac{\mu I}{2\pi} \ln(b/a)$

$L = \frac{\phi/\ell}{I} = \frac{\mu}{2\pi} \ln(b/a)$

↑ /unit length.

3. Lossless lines

In practice $G \ll \omega C$ in practical systems

i.e. set $G \approx 0$

$Z_0 \Rightarrow \sqrt{\frac{L}{C} + \left(\frac{R}{j\omega C}\right)}$

$\alpha^2 \Rightarrow \frac{1}{2} \omega^2 LC \left\{ \left[1 + \left(\frac{R}{\omega L}\right)^2\right]^{1/2} - 1 \right\}$

$\beta^2 \Rightarrow \frac{1}{2} \omega^2 LC \left\{ \left[1 + \left(\frac{R}{\omega L}\right)^2\right]^{1/2} + 1 \right\}$

In practice $G \ll \omega C$ ie. set $G \approx C$

Also usually assume $R \ll \omega L$ for simplicity (not always a valid assumption) ie. set $R \approx 0$

Then:

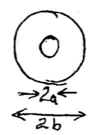
$Z_0 \Rightarrow \sqrt{L/C}$ Real, but non-dissipative

$\alpha \Rightarrow 0$ ie. lossless

$\beta \Rightarrow \omega\sqrt{LC}$

and $v \Rightarrow \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

e.g. Coaxial line



$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$ H/m

$C = 2\pi\epsilon / \ln(b/a)$ F/m

$\therefore Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{\ln(b/a)}{2\pi}$

$= \sqrt{\frac{\mu}{\epsilon}} \times$ Shape factor

$= \sqrt{\frac{\mu_0}{\epsilon_0}} \times \frac{\text{Shape factor}}{\sqrt{\epsilon_r}}$

$= 376.7\,\Omega \left(S.F./\epsilon_r^{1/2} \right)$

$v = 1/\sqrt{\mu\epsilon}$

$\approx \frac{(\mu_0 \epsilon_0)^{-1/2}}{\sqrt{\epsilon_r}}$

$= \frac{c}{\sqrt{\epsilon_r}}$

TABLE 5.2 CONDUCTANCE, CAPACITANCE, AND INDUCTANCE PER UNIT LENGTH FOR SOME STRUCTURES CONSISTING OF INFINITELY LONG CONDUCTORS HAVING THE CROSS SECTIONS SHOWN IN FIG. 5.12

Description	Capacitance per unit length, $\mathscr{C}$	Conductance per unit length, $\mathscr{G}$	Inductance per unit length, $\mathscr{L}$
Parallel-plane conductors, Fig. 5.12(a)	$\varepsilon \frac{w}{d}$	$\sigma \frac{w}{d}$	$\mu \frac{d}{w}$
Coaxial cylindrical conductors, Fig. 5.12(b)	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$
Parallel cylindrical wires, Fig. 5.12(c)	$\frac{\pi\varepsilon}{\cosh^{-1}(d/a)}$	$\frac{\pi\sigma}{\cosh^{-1}(d/a)}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{a}$
Eccentric inner conductor, Fig. 5.12(d)	$\frac{2\pi\varepsilon}{\cosh^{-1}\left(\frac{a^2+b^2-d^2}{2ab}\right)}$	$\frac{2\pi\sigma}{\cosh^{-1}\left(\frac{a^2+b^2-d^2}{2ab}\right)}$	$\frac{\mu}{2\pi}\cosh^{-1}\frac{a^2+b^2-d^2}{2ab}$
Shielded parallel cylindrical wires, Fig. 5.12(e)	$\frac{\pi\varepsilon}{\ln\frac{d(b^2-d^2/4)}{a(b^2+d^2/4)}}$	$\frac{\pi\sigma}{\ln\frac{d(b^2-d^2/4)}{a(b^2+d^2/4)}}$	$\frac{\mu}{\pi}\ln\frac{d(b^2-d^2/4)}{a(b^2+d^2/4)}$

4. Practical Z_0

For $Z_0 = [376.7/\sqrt{\epsilon_r'}] \times s.f.$ Ω

(1)

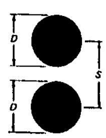


Figure E-2 Cross section of two-wire transmission line

The shape factor of the transmission line, *sf*, depends only on the shapes, sizes, and positions of the conductors. Two-wire transmission line (Figure E-2) has shape factor

$$sf = \frac{1}{\pi}\ln\left[\frac{S}{D} + \left(\frac{S^2}{D^2} - 1\right)^{1/2}\right]$$

when both wires have diameter D. When the wires have diameters $D1$ and $D2$, the shape factor becomes

$$sf = \frac{1}{2\pi}\ln[x + (x^2 - 1)^{1/2}]$$

where

$$x = \frac{4S^2 - D1^2 - D2^2}{2D1D2}$$

(2) Twisting a two-wire transmission line to form a twisted-pair increases the average relative permittivity $\epsilon_{r'}$. For hard insulation with relative permittivity ϵ_r,

$$\epsilon_{r'} \approx 1 + (0.25 + 0.0004\Theta^2)(\epsilon_r - 1),$$

and for soft insulation (Teflon, polyvinyl chloride)

$$\epsilon_{r'} \approx 1 + (0.25 + 0.001\Theta^2)(\epsilon_r - 1),$$

$$\Theta = \arctan(\pi S n) \text{ degrees},$$

and n is the number of twists per meter.

(3) A wire near a ground plane (Figure E-3) has shape factor

$$sf = \frac{1}{2\pi}\ln\left[\frac{2S}{D} + \left(\frac{4S^2}{D^2} + 1\right)^{1/2}\right].$$

(4) A laminar bus (Figure E-4) has shape factor

$$sf \approx \frac{1}{\pi}\ln\left(\frac{4S}{W} + \frac{W}{2S}\right) \quad \text{for } W \le \frac{S}{2},\ T << W,$$

$$sf \approx \frac{2}{\frac{2W}{S} + 2.42 - \frac{0.22S}{W} + \left(1 - \frac{S}{2W}\right)^6} \quad \text{for } W > \frac{S}{2},\ T << W.$$

If an insulator separates the conductors (Figure E-1(c)),

$$\epsilon_{r'} \approx \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\left(4 + 20\frac{S}{W}\right)^{1/2}}.$$

(5) Microstrip (Figure E-5) has shape factor

$$sf \approx \frac{1}{2\pi} \ln\left(\frac{8S}{W} + \frac{W}{4S}\right) \qquad \text{for } W \leq S,\ T << W,$$

$$sf \approx \frac{1}{\dfrac{W}{S} + 2.42 - \dfrac{0.44S}{W} + \left(1 - \dfrac{S}{W}\right)^6} \qquad \text{for } W > S,\ T$$

If an insulator separates the conductors (Figure E-1(c)),

$$\epsilon_{r'} \approx \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\left(4 + 40\dfrac{S}{W}\right)^{1/2}}.$$

(6) Coplanar lines (Figure E-6) have shape factor

$$sf \approx \frac{1}{\pi} \ln\left[2\,\frac{\left(\dfrac{2W}{S} + 1\right)^{1/2} + 1}{\left(\dfrac{2W}{S} + 1\right)^{1/2} - 1}\right] \qquad \text{for } W \leq 2.414S,\ T <<$$

$$sf \approx \frac{\pi}{4 \ln\left[2\left(\dfrac{2W}{S} + 1\right)^{1/2}\right]} \qquad \text{for } W > 2.414S,\ T << W.$$

If the lines are supported by an insulator on one side,

$$1 < \epsilon_{r'} < \frac{\epsilon_r + 1}{2}.$$

(7) Triplate stripline (Figure E-7) has shape factor

$$sf \approx \frac{1}{2\pi} \ln\left[2\,\frac{\exp\left(\dfrac{\pi W}{4S}\right) + 1}{\exp\left(\dfrac{\pi W}{4S}\right) - 1}\right] \qquad \text{for } W \leq 1.117S,\ T << S,$$

$$sf \approx \frac{1}{1.765 + \dfrac{2W}{S}} \qquad \text{for } W > 1.117S,\ T << S.$$

(8) Coaxial cable (Figure E-8) has shape factor

$$sf = \frac{1}{2\pi} \ln\left(\frac{B}{D}\right).$$

(9) Shielded twisted-pair (Figure E-9) has shape factor

$$sf = \frac{1}{\pi} \ln\left(\frac{2S}{D}\,\frac{B^2 - S^2}{B^2 + S^2}\right).$$

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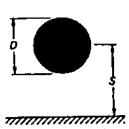

Figure E-3 Cross section of wire near a groundplane

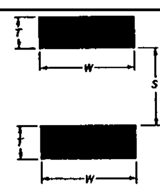

Figure E-4 Cross section of a laminar bus

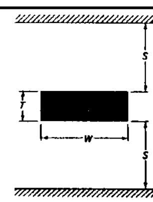

Figure E-7 Cross section of triplate stripline

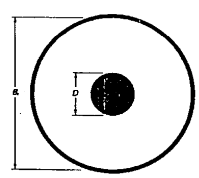

Figure E-8 Cross section of coaxial cable

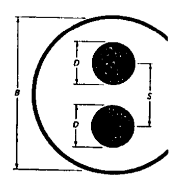

Figure E-9 Cross section of shielded twisted-pair

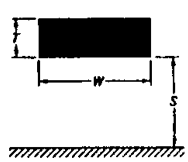

Figure E-5 Cross section of microstrip

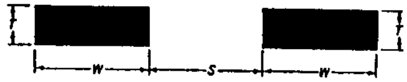

Figure E-6 Cross section of coplanar lines

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5. Lossy lines

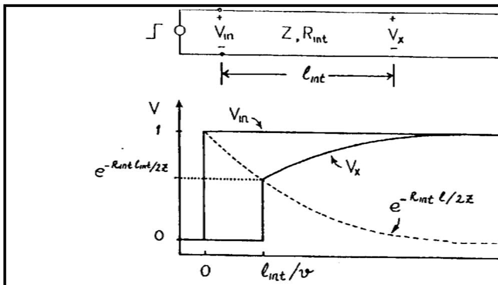

FIGURE 6.10 Waveforms in a lossy transmission line

The response of this line to a unit step input at a distance x from the beginning of the line is [6.1]:

$$V(x,t) = \left\{ e^{-\mathcal{R}x/2Z_0} + \frac{\mathcal{R}x}{2Z_0}\int_{t=x\sqrt{\mathcal{L}\mathcal{C}}}^{t}\left[\frac{e^{-\mathcal{R}t/2\mathcal{L}}}{\sqrt{t^2 - x\sqrt{\mathcal{L}\mathcal{C}}}}\,\frac{I_1\mathcal{R}}{2\mathcal{L}}\sqrt{t^2 - x\sqrt{\mathcal{L}\mathcal{C}}}\right]dt\right\}$$

$$\times u\left(t - x\sqrt{\mathcal{L}\mathcal{C}}\right).$$

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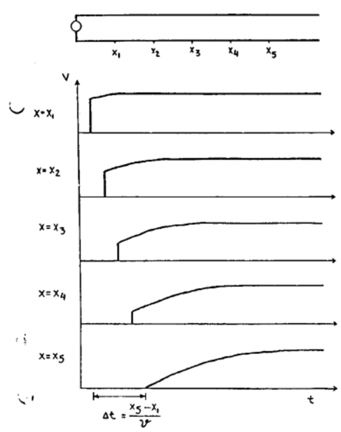
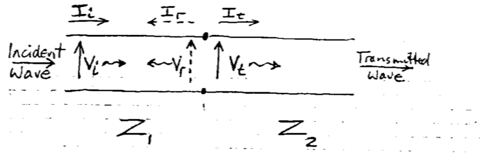


URE 6.11 Waveform propagating along a lossy transmission line. The
it to the transmission line is a unit voltage step, and the response is captured
uccessive points along the line. The size of the step is attenuated as the
eform travels down the line, and at a point far enough from the source, the
onse is like that of a distributed RC line.

Lecture topics C: Transmission lines

Transmission line reflections

1. Reflection coefficients
2. Basic cases:
   - Matched and open circuit
3. Generalized mismatches
   - Source and load
4. Bounce chart/lattice diagram
5. Reflections from discontinuities



1. Reflection Coefficient

At discontinuity $Z_1 \rightarrow Z_2$

and the relationships of the incident wave $V_i = I_i Z_1$ cannot be maintained in the transmitted wave $V_t = I_t Z_2$ without changes in V, I. Solution requires reflected wave $V_r = I_r Z_1$

Boundary conditions $V_i + V_r = V_t$

$I_i - I_r = I_t$

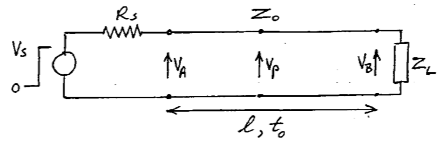
Gives $V_r = \frac{Z_2 - Z_1}{Z_2 + Z_1} V_i = \Gamma_v V_i$ (Voltage) Reflection Coefficient

& $I_r = \Gamma_I I_i = -\Gamma_v I_i$ (Current) Reflection Coefficient

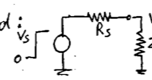
Voltage transmission coefficient $V_t / V_i = 1 + \Gamma_v$

Voltage reflection at load $\Gamma_{VL} = \frac{Z_L - Z_o}{Z_L + Z_o} \rightarrow 0$ for $Z_L = Z_o = R_o$

" " source $\Gamma_{VS} = \frac{R_s - Z_o}{R_s + Z_o} \rightarrow 0$ for $R_s = Z_o$

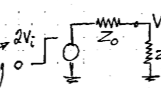


At source end: $V_A = \frac{Z_o}{R_s + Z_o} V_s$



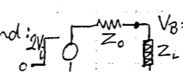
Along line: $V_p = \frac{Z_o}{Z_o + Z_o} 2V_i = V_i$

V_i is propagating signal



Need equivalent $2V_i$ source

At load end: $V_B = \frac{Z_L}{Z_o + Z_L} 2V_i \rightarrow V_i$ if $Z_L = Z_o$



2. 3 significant cases:

$\Gamma_{VL} = (R_L - Z_0) / (R_L + Z_0)$

- Load $R_L = Z_0$ (above) $\Gamma_{VL} = 0$
- Open circuit load:
  - $R_S = Z_0$ ($\Gamma_{VL} = 1$)
  - $R_S = 0$ ($\Gamma_{VS} = -1$)
- Short circuit load $R_L = 0$, $\Gamma_{VL} = -1$

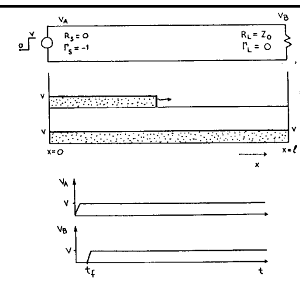


FIGURE 6.17 Termination at the receiving end: $R_S = 0$, $R_L = Z_0$

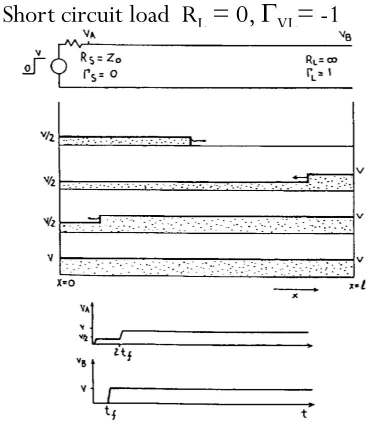


FIGURE 6.18 Termination at the source end: $R_S = Z_0$, $R_L = \infty$

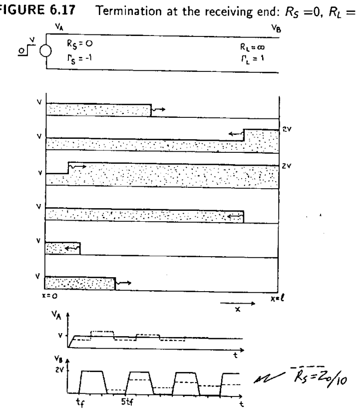


FIGURE 6.16 No termination: $R_S = 0$, $R_L = \infty$

6. WHEN TO USE TRANSMISSION LINE ANALYSIS 243

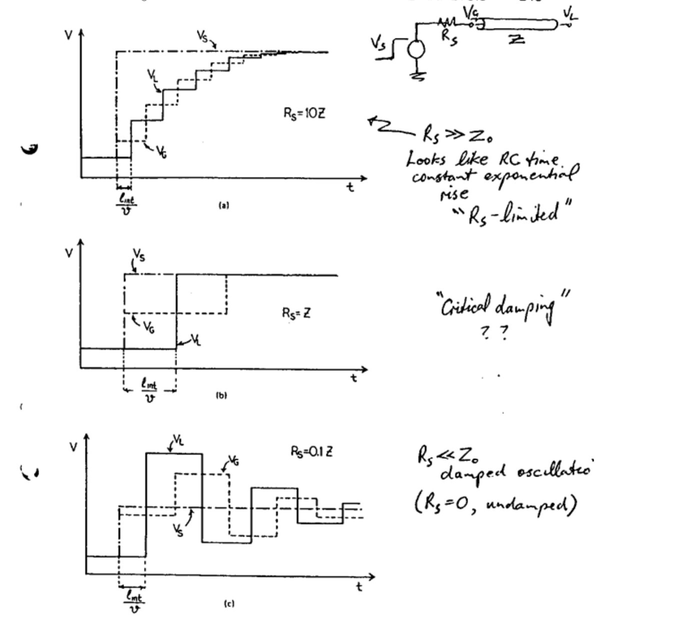


FIGURE 6.8 Waveforms for various R_S and Z_0 combinations in the circuit in Fig. 6.7. As can be seen by the l_{int}/v markers, the time axes of the plots are not drawn at the same scale for clarity. The top plot $R_s = 10Z_0$ has the longest delay.

3. Mismatched load &/or source

- 1 volt pulse, $Z_0 = 78\Omega$
  - Consider $R_L = 78\Omega$ cases first, then vary R_L for
    - (a) $R_S = Z_0$
    - (b) $R_S << Z_0$
    - (c) $R_S >> Z_0$

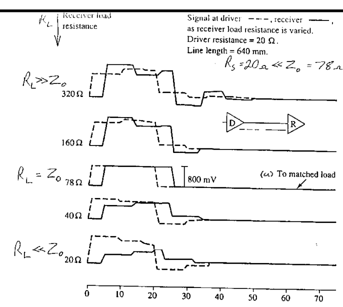


FIGURE 3-9
Multiple reflections on a transmission line. Signal at driver and receiver, driver resis 20 Ω. R_r is varied from low to high values. Characteristic impedance Z_0 of line is 7 waveform is a 20-ns wide trapezoidal pulse with 1 ns rise and fall times and 1 V p into an open circuit.

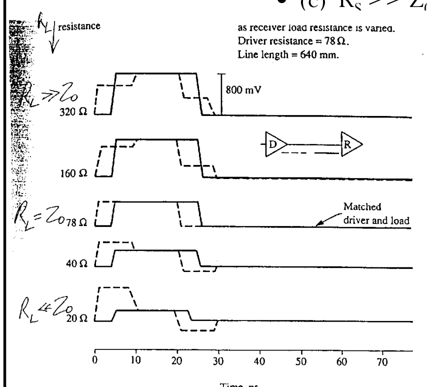


FIGURE 3-10
Same as Figure 3-9, but driver resistance is 78 Ω. $R_S = Z_0$

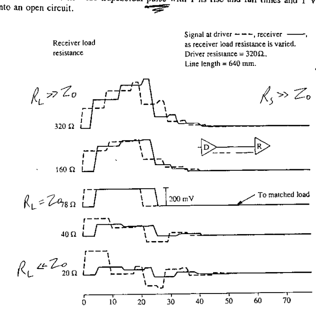


FIGURE 3-11
Same as Figure 3-9, but driver resistance is 320 Ω.

4. Bounce chart (lattice diagram)

- Calculate reflection coefficients
- Distance / time diagram
- Transfer data to waveform plots
- Example: Open circuit line

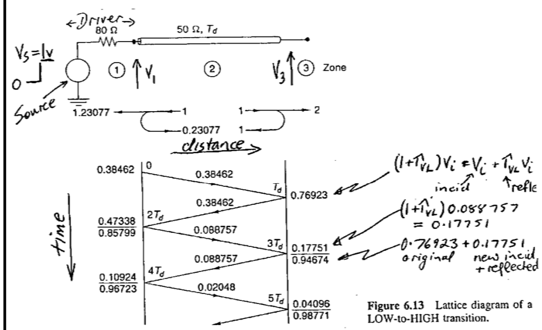


Figure 6.13 Lattice diagram of a LOW-to-HIGH transition.

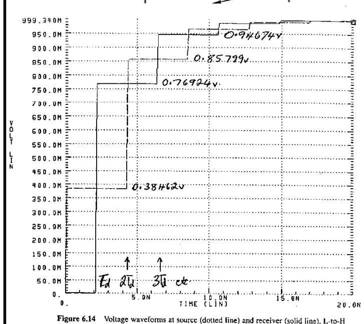


Figure 6.14 Voltage waveforms at source (dotted line) and receiver (solid line). L-to-H transition, $R_L > Z_o$. See Fig. 6.13.

"BOUNCE" CHART / LATTICE DIAGRAM

(1) $R_S = 80\Omega$ $Z_o = 50\Omega$ $R_L = \infty$

∴ $\Gamma_{VS} = \frac{80-50}{80+50} = \frac{3}{13} = 0.23077$ $\Gamma_{VL} = \frac{\infty-50}{\infty+50} = 1$

At $t=0$: $V_1 = \frac{50}{50+80} V_s = \frac{5}{13} v = 0.38462v$

$t=T_d$: Load $V_i = 0.38462v$ $V_r = \Gamma_{VL} V_i = 0.38462v$

& load voltage $V_3 = V_i + V_r = 0.76924v$

$t=2T_d$: Driver Reflected wave $V_r = \Gamma_{VS} \cdot 0.38462v = 0.088757v$

$V_1 = 0.38462v + 0.38462v + 0.088757v$ (original pulse; reflection from load; reflection back to load) $= 0.85799v$

$t=3T_d$: Load Reflected wave $= \Gamma_{VL} \times 0.088757v = 0.088757v$

$V_3 = 0.76924 + .088757 + 0.088757$ (Previous value; $(1+\Gamma_{VL})V_i = 0.17751$ Incid reflection + next refl back) $= 0.94674v$

$t=4T_d$: Driver Reflected wave $= \Gamma_{VS} \cdot 0.088757 = 0.02048v$

$V_1 = 0.85799v + (0.088757 + 0.02048)v = 0.96723v$

5. Discontinuities

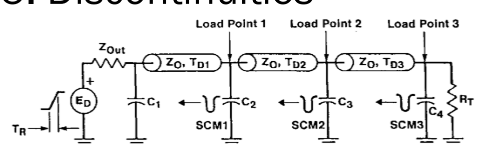
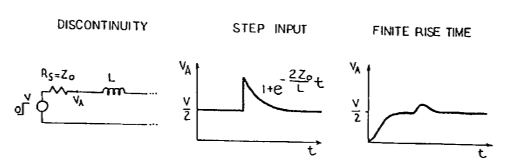


Figure 3-9. Discrete or Distributed Net Configurations. Note reflections fror discontinuities.



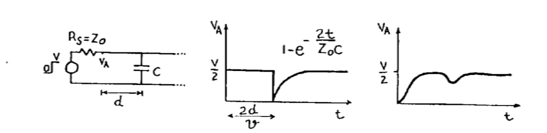
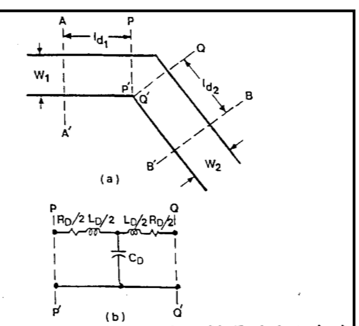


FIGURE 6.6 Inductive and capacitive discontinuities. The transmission line is driven by a signal generator with $R_S = Z_0$, and the reflected waveforms are observed at the source end. The line is either infinitely long or is terminated with $R_L = Z_0$ at the far end. Two types of reflected waveforms are plotted for each discontinuity: one assuming a step input (left) and another assuming a finite-rise-time input (right).



A planar bend - top view; (b) Equivalent circuit

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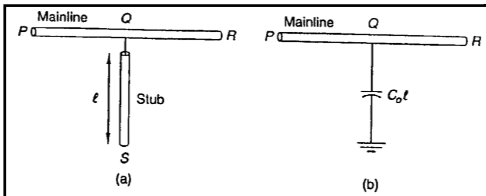
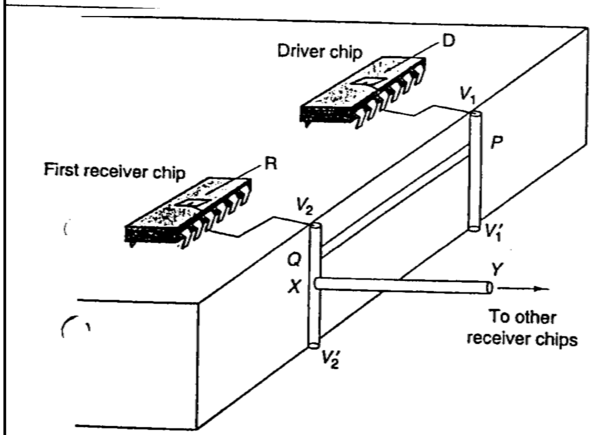


Figure 6.3 A short stub (a) reflects as a capacitor (b).



Best to drive multiple loads from branch at matched source — avoids multiple reflection times from stubs.

Figure 6.4 DV_1, RV_2 are metallizations on the board surface. PQ, XY are board internal traces. V_1V_1', V_2V_2' are through vias. The path DV_1PQXY is the mainline, while RV_2Q, PV_1' and XV_2' are stubs.

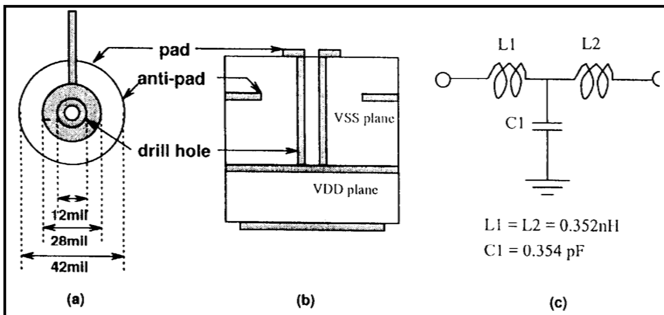
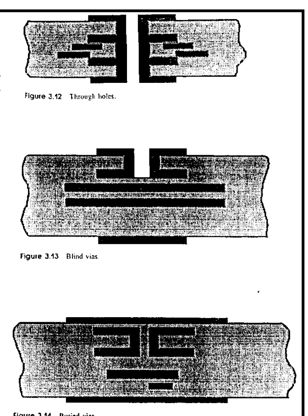


Fig. 10. A via hole (a) top view (b) cross-sectional view (c) equivalent circuit model



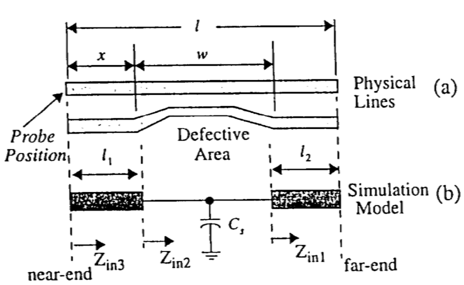


Figure 7. Simulation Model for an Interconnect with Near-Short Defect.

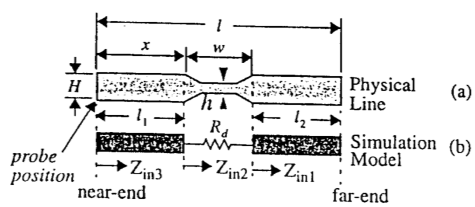
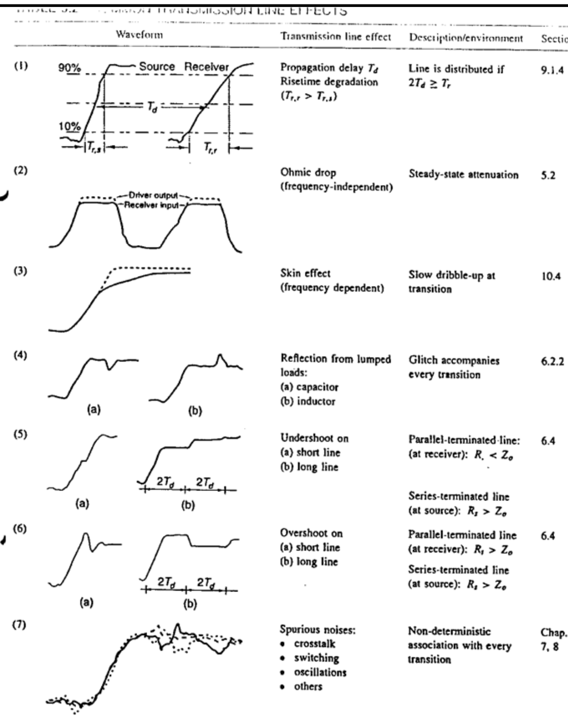


Figure 6. Simulation Model for an Interconnect with Near-Open Defect.

Summary



TRANSMISSION LINE EFFECTS

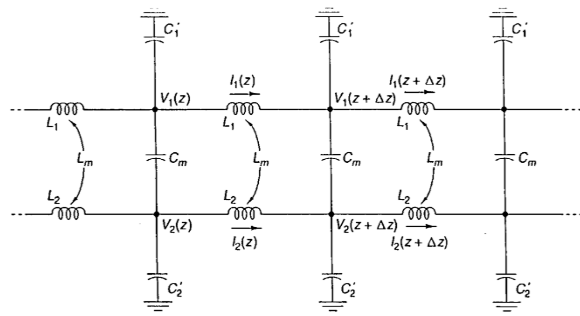
	Waveform	Transmission line effect	Description/environment	Section
(1)		Propagation delay T_d Risetime degradation ($T_{r,r} > T_{r,s}$)	Line is distributed if $2T_d \geq T_r$	9.1.4
(2)		Ohmic drop (frequency-independent)	Steady-state attenuation	5.2
(3)		Skin effect (frequency dependent)	Slow dribble-up at transition	10.4
(4)		Reflection from lumped loads: (a) capacitor (b) inductor	Glitch accompanies every transition	6.2.2
(5)		Undershoot on (a) short line (b) long line	Parallel-terminated line: (at receiver): $R_t < Z_o$ Series-terminated line (at source): $R_s > Z_o$	6.4
(6)		Overshoot on (a) short line (b) long line	Parallel-terminated line (at receiver): $R_t > Z_o$ Series-terminated line (at source): $R_s > Z_o$	6.4
(7)		Spurious noises: • crosstalk • switching • oscillations • others	Non-deterministic association with every transition	Chap. 7, 8

Lecture topics D: Transmission lines Crosstalk

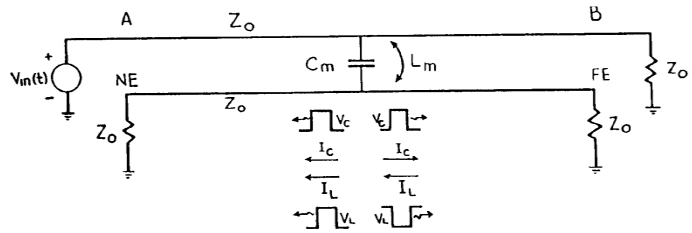
1. Inductive and capacitive coupling
2. Forward and backward noise
3. Far end and near end noise
4. Incremental model and formulae
5. Crosstalk examples
6. Capacitive crosstalk systems
7. No ground plane

1. Inductive and capacitive coupling

Incremental line model



Crosstalk polarities



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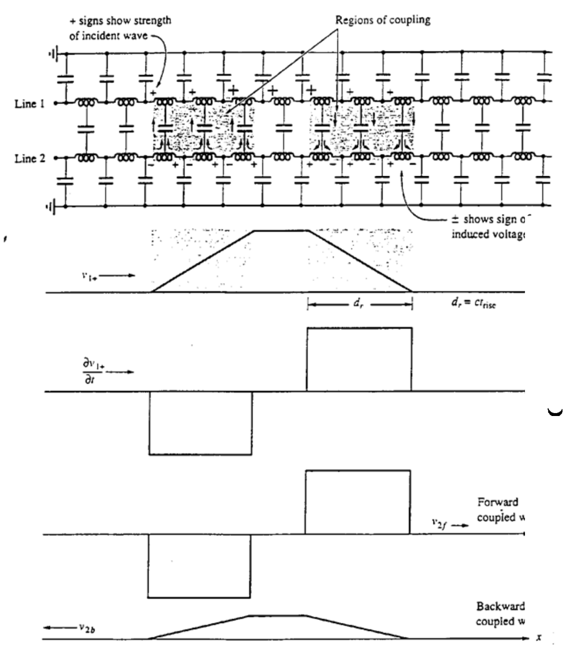
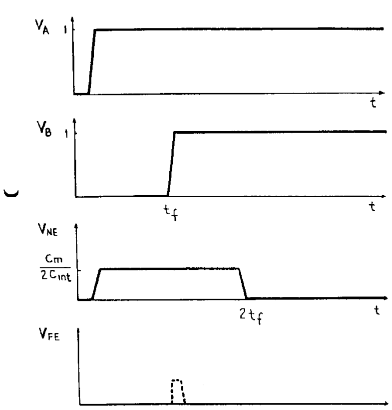
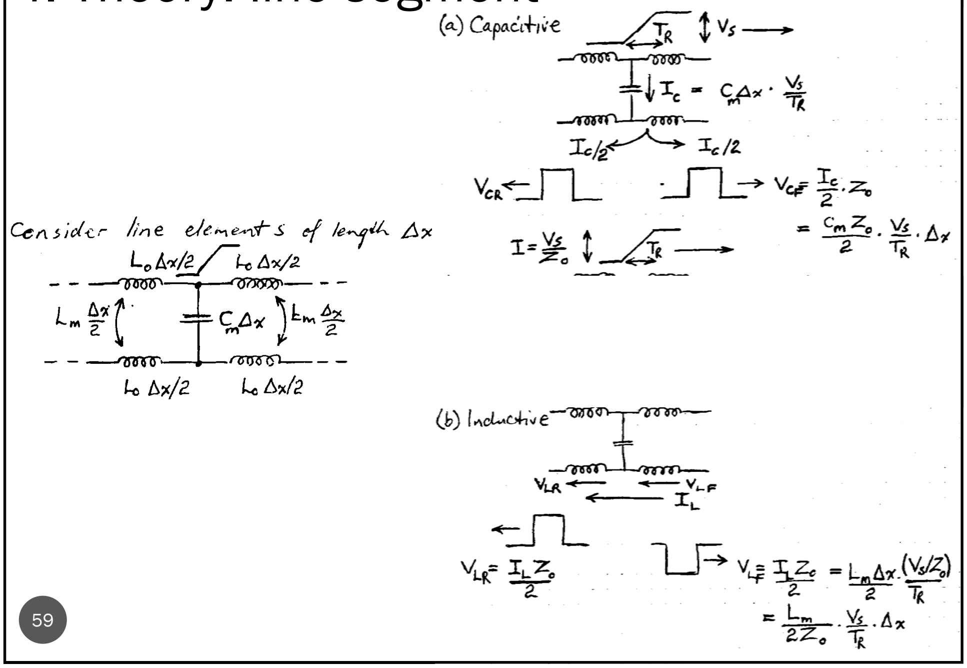
2. Forward & backward noise



3. Near end & far end



4. Theory: line segment



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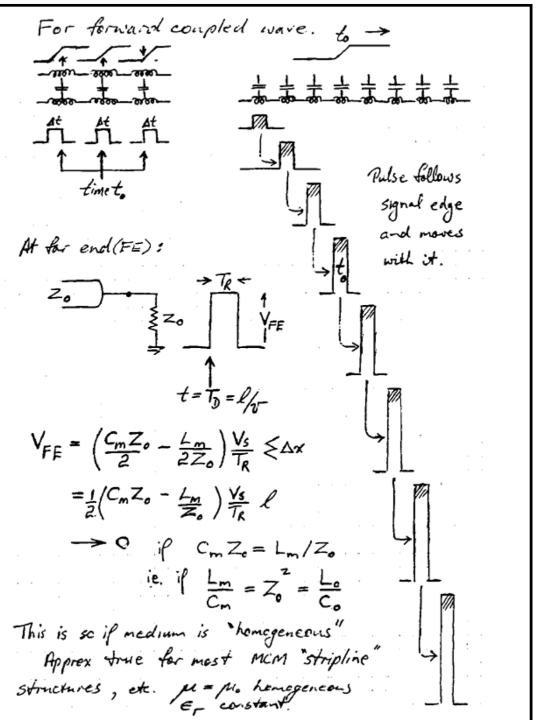
Far end overview



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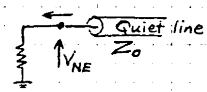
Backward wave model

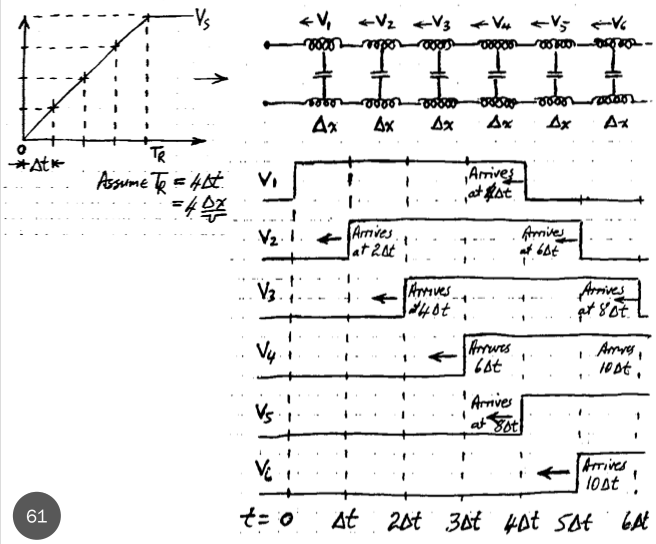
For reverse coupled wave

Consider the near end (NE) as pulse starts:-

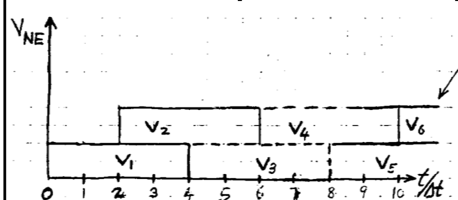
Each pulse

$$\Delta V = \left(\frac{C_m Z_o}{2} + \frac{L_m}{2Z_o}\right)\frac{V_s}{T_R}\Delta x$$





Near end pulse shape



Saturation at

$$V_{NE_{sat}} = 2\left(\frac{C_m Z_o}{2} + \frac{L_m}{2Z_o}\right)\frac{V_s}{T_R}\Delta x$$

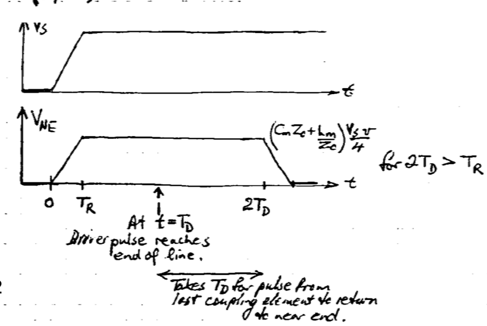
$$= \frac{1}{2}\left(\frac{C_m Z_o}{2} + \frac{L_m}{2Z_o}\right)\frac{V_s}{T_R} T_R v$$

$$= \frac{1}{2}\left(\frac{C_m Z_o}{2} + \frac{L_m}{2Z_o}\right) V_s v$$

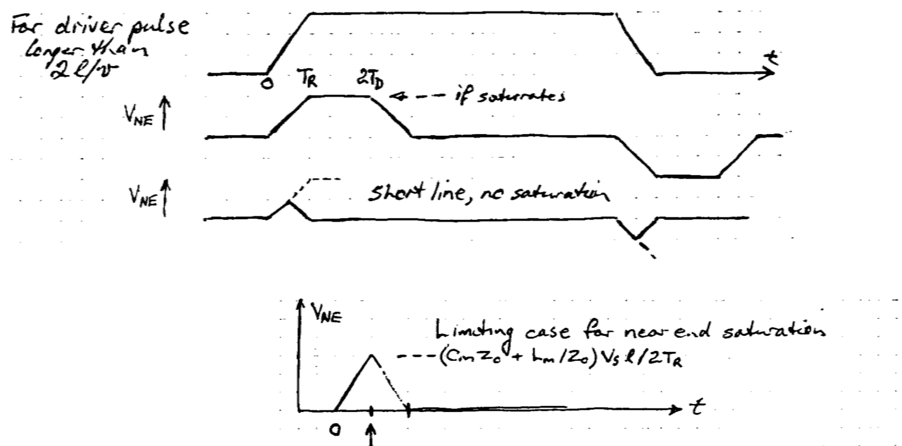
Note: Independent of T_R !!

Also independent of ϵ_r $C_m Z_o v \propto \epsilon_r \epsilon_r^{-1/2} \epsilon_r^{-1/2}$

$v/Z_o \propto \epsilon_r^{-1/2}(\epsilon_r^{-1/2})^{-1}$



Near End Saturation

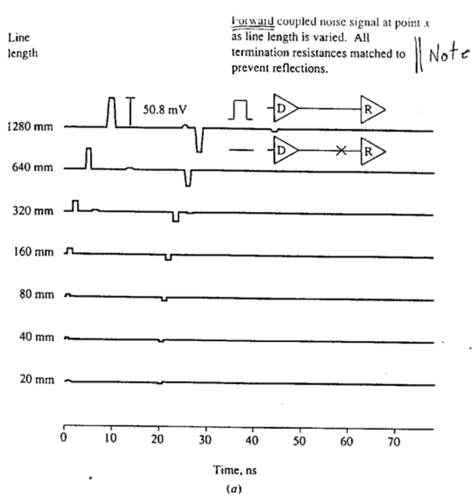


$T_R = 2T_D$ i.e. $2\frac{l}{v} = T_R$, $l = \frac{1}{2} v T_R$

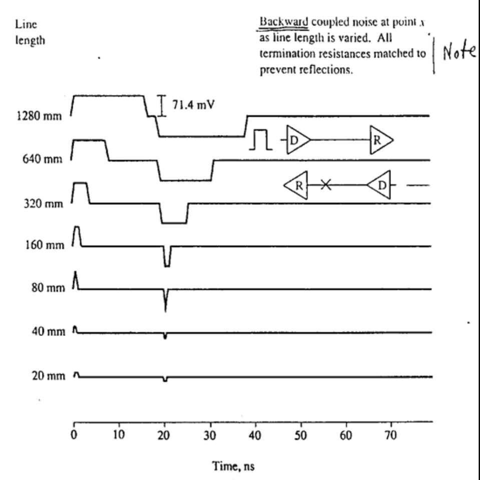
If need to reduce $V_{NE})_{max}$, need to increase T_R &/or decrease l (i.e. decrease l/T_R) so cannot saturate.

5. Pulse crosstalk noise examples

Drive pulse: 0.5 volt, t_{width}=20ns, t_{rise}=t_{fall}=1ns



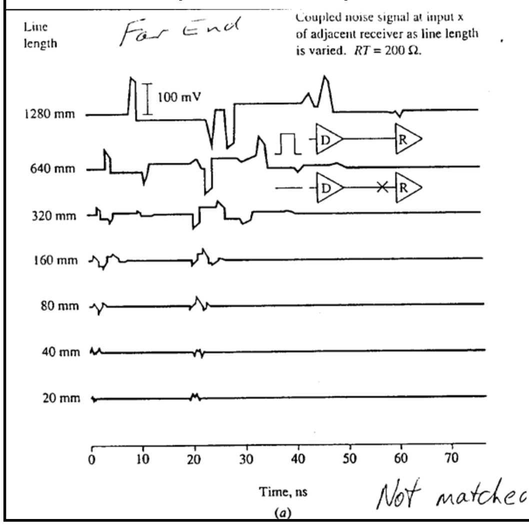
Forward noise: matched



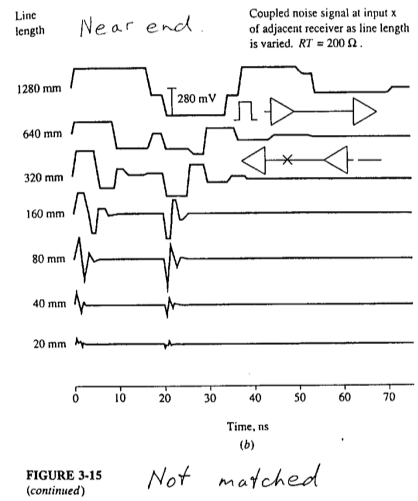
Backward: matched

Not matched

Forward(Far end)



Backward (Near end)



6. Ground coupled crosstalk

$$\text{Crosstalk} \approx \frac{K}{1+(D/H)^2}$$

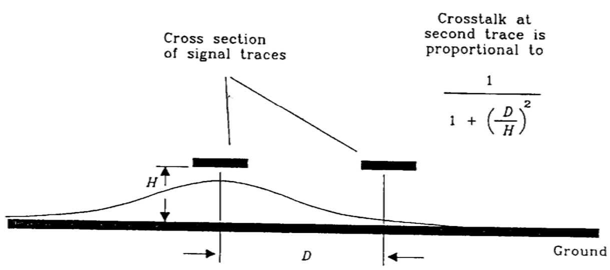


Figure 5.4 Cross section of two traces s

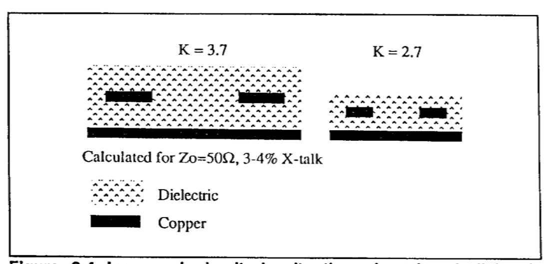


Figure 8.4 Increased circuit density through reduced dielectric constant.

7. Mutual capacitances

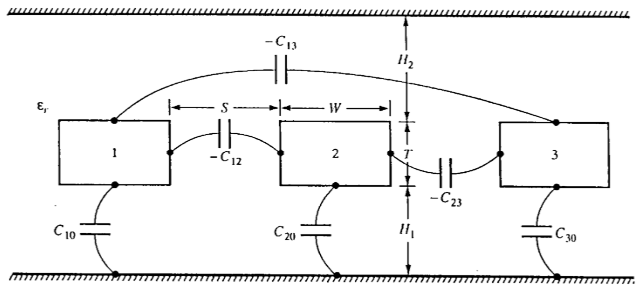
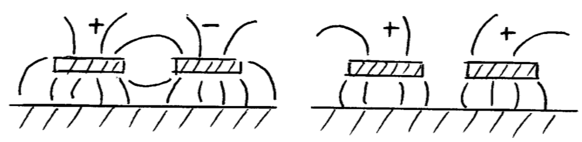


FIGURE 4-3
Cross section of three coupled lines in a homogeneous medium.

Even/odd modes



$C_{mo} > C_{me}$

Lecture topics E: Electromagnetic Modeling

Microstrip

Transmission line

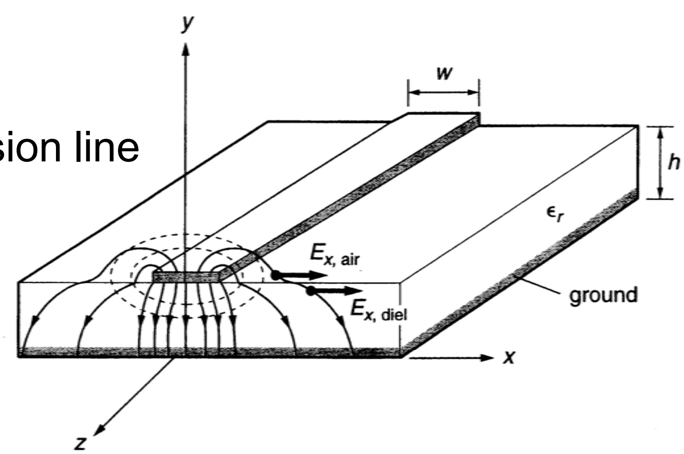
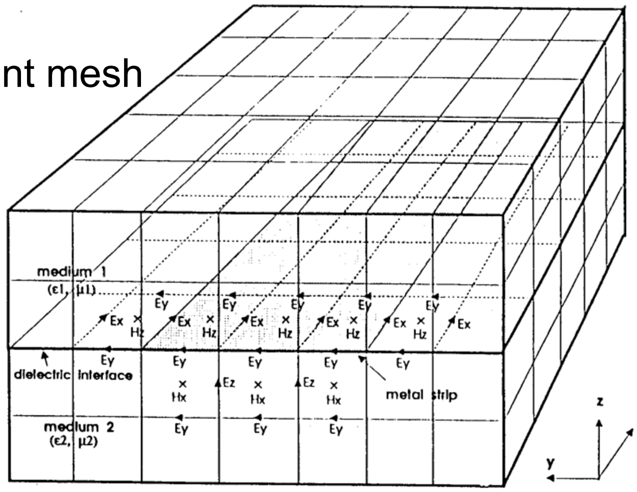


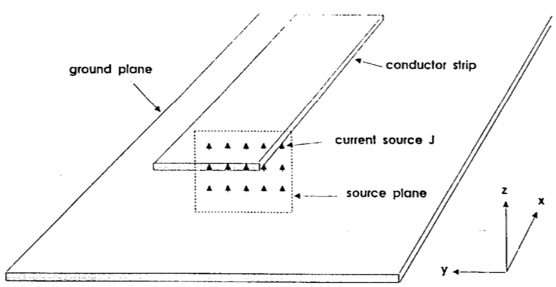
Figure 23.8 Sketch of a microstrip line. The electric field lines are sketched in solid line, and the magnetic field lines in dashed line.

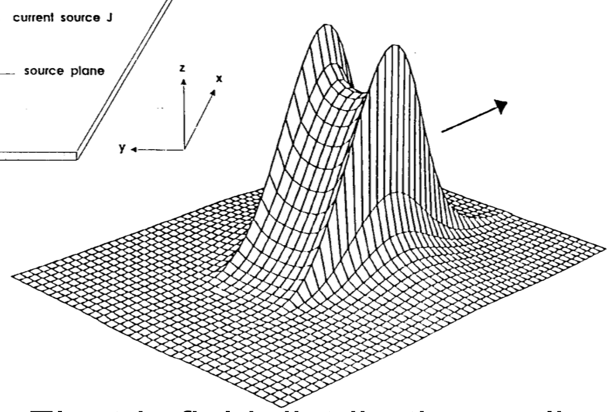
3. Electromagnetic Modeling (Fang)

Finite element mesh



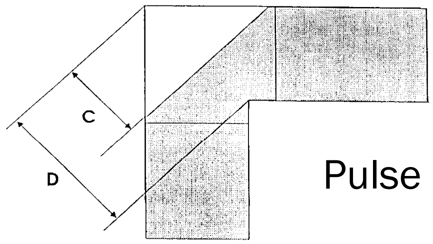
a. Current pulse excitation of line



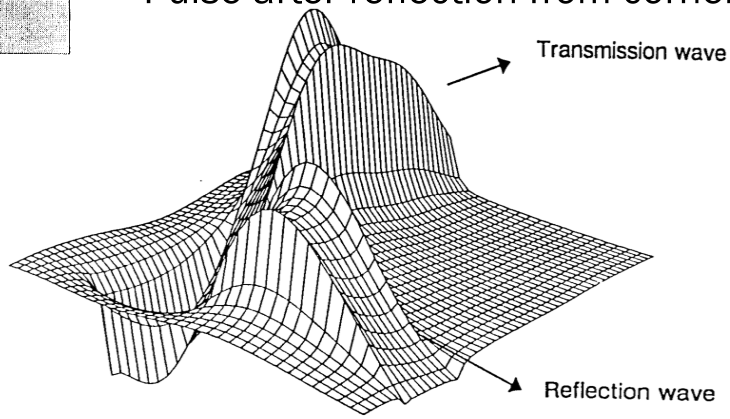


Electric field distribution on line

b. Chamfered right angle bend



Pulse after reflection from corner



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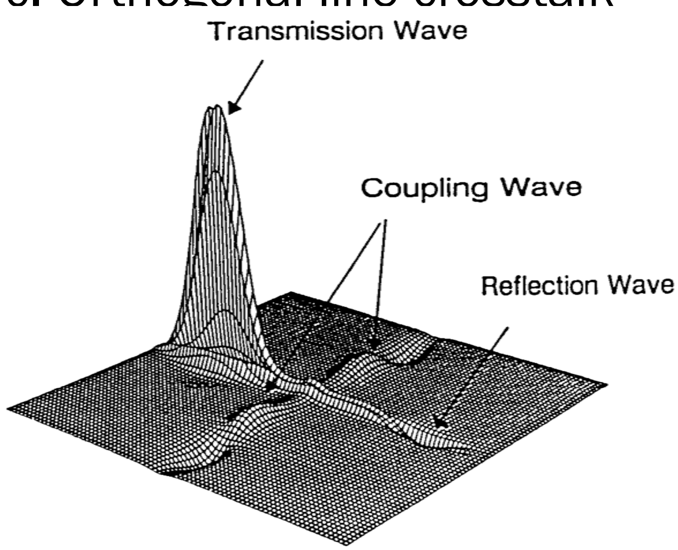
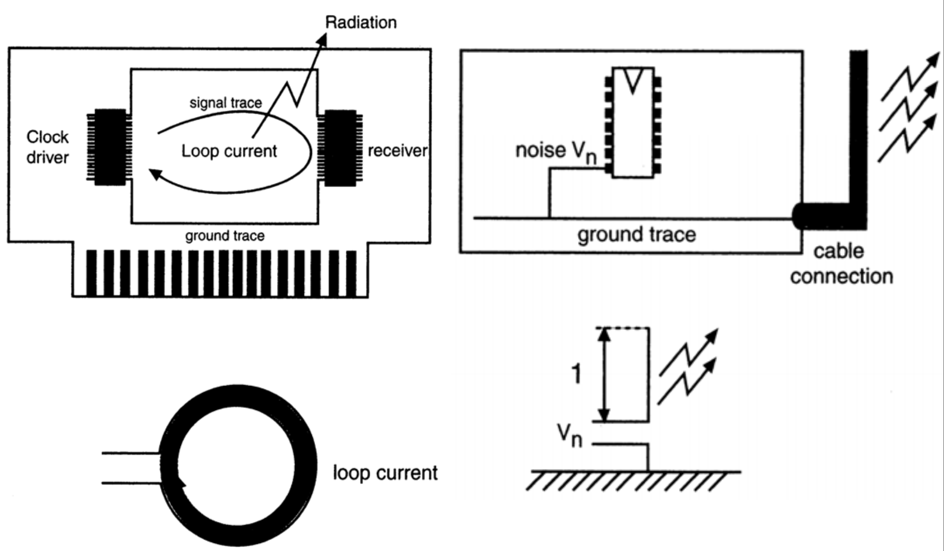
c. Orthogonal line crosstalk

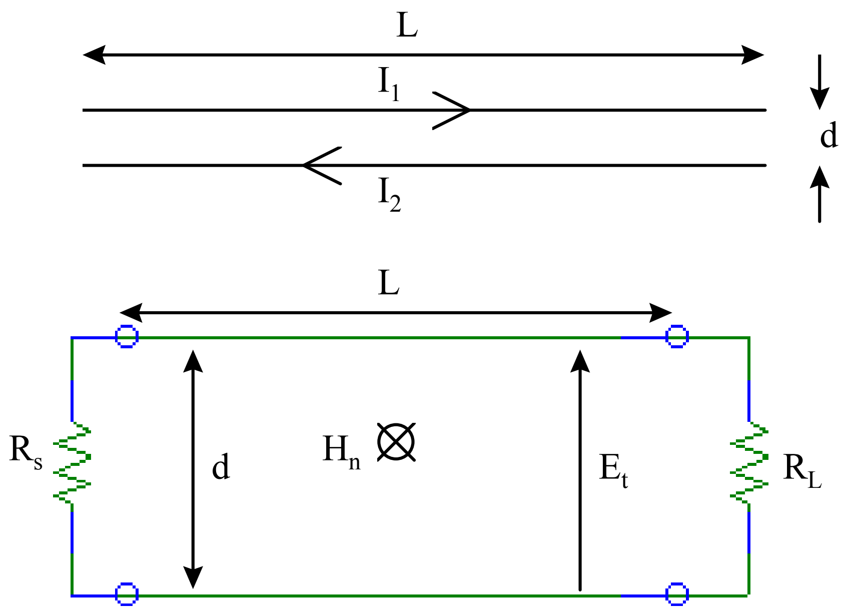


Lecture topics F: Electromagnetic Compatibility

1. Antennas
2. Emissions
3. Susceptibility

1. EMI/EMC Models: Loop/Dipole Antennas



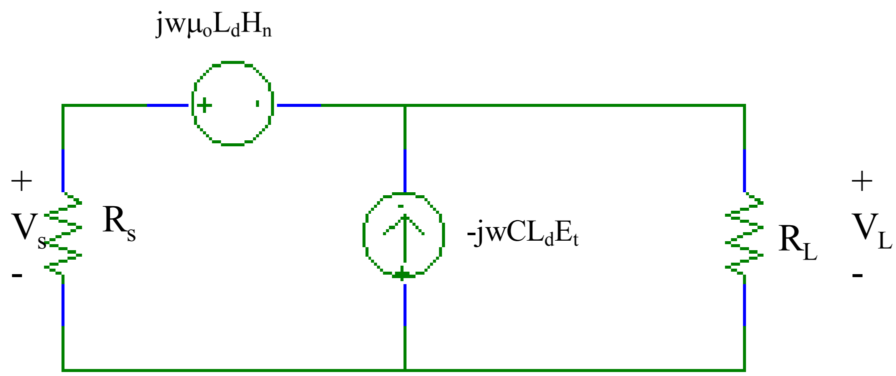


Electromagnetic compatibility/interference

2. EMI/EMC Models: Emissions

- At "far field" distance r ($\geq\lambda = c/f$) from the line of length L and area A = L.d, the radiated electric field strength is
  - $E = E_D + E_{CM}$
- where E_D due to the differential current I_D is
  - $E_D = 131.6 \times 10^{-16} f^2 A I_D / r$ V/m
- and E_{CM} due to the common mode current I_{CM} is (for $L\leq\lambda$)
  - $E_{CM} = 4\pi \times 10^{-7} f^2 L I_{CM} / r$ V/m
- where $I_D = (I_1 + I_2)/2$ and $I_{CM} = (I_1 - I_2)/2$. Common mode currents are ideally zero, but small values can lead to CM dominance over differential. For the differential current, the maximum value can be taken to be the supply current, but the user must specify a non-ideal common mode estimate. For digital systems, use $f = 2\pi/t_r$. For other geometries, other expressions for A are valid.
- There are many different standards for EM radiation limits, but for guidance the EU limit is $E\leq100\mu V/m$ at r = 10m (class A) or 3m (class B).

EMI/EMC Models: Susceptibility



odels

3. Susceptibility

- For the line shown, with capacitance C per unit length, e.g.
  - $C = \pi\, \varepsilon_r \varepsilon_0 / \ln(d/r_w)$
- for parallel wires, the induced voltages are
  - $V_S = -j\omega\, [R_L R_S/ (R_L + R_S)]\, [Ld]\, [C - (\mu_0/\eta_0)/R_L]$
  - $V_L = -j\omega\, [R_L R_S/ (R_L + R_S)]\, [Ld]\, [C + (\mu_0/\eta_0)/R_S]$
- where $E = E_t = \eta_0\, H_n$, and $\eta_0 = 120\pi = 377\Omega$.
- As an example of an EU standard, the device must function in a field of E = 3V/m.