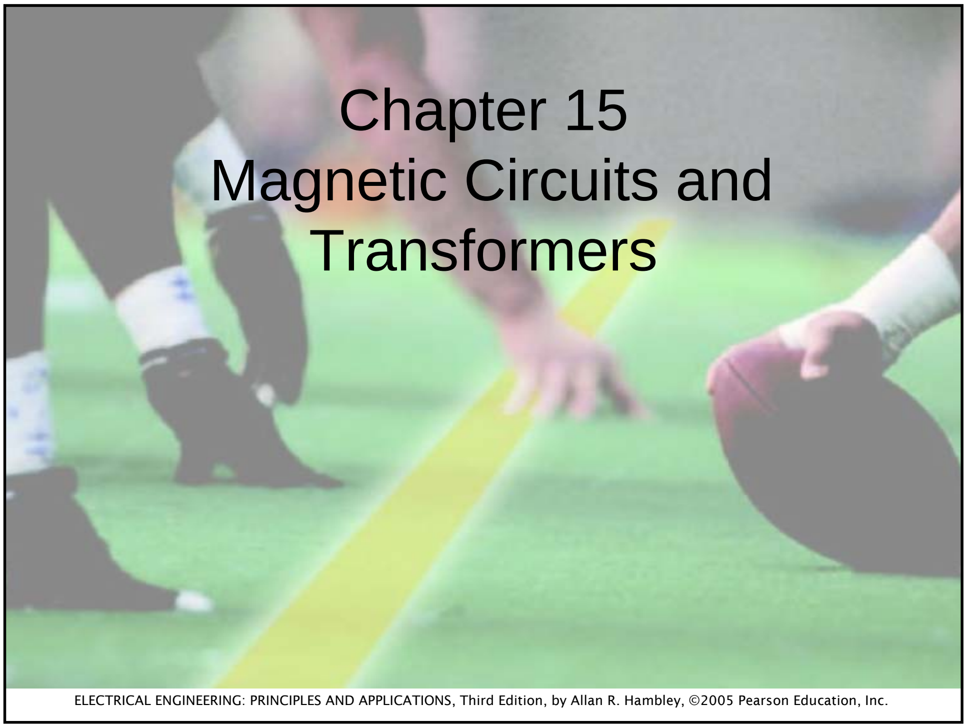


Chapter 15
Magnetic Circuits and Transformers

Chapter 15
Magnetic Circuits and Transformers

1. Understand magnetic fields and their interactio with moving charges.
2. Use the right-hand rule to determine the directi of the magnetic field around a current-carrying wire or coil.

3. Calculate forces on moving charges and current carrying wires due to magnetic fields.

4. Calculate the voltage induced in a coil by a changing magnetic flux or in a conductor cutting through a magnetic field.

5. Use Lenz’s law to determine the polarities of induced voltages.

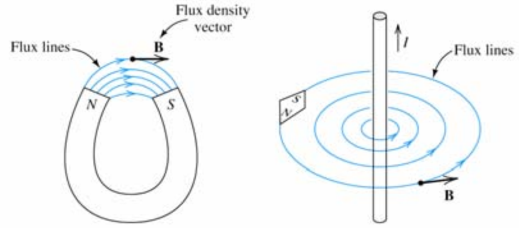
6. Apply magnetic-circuit concepts to determine the magnetic fields in practical devices.

7. Determine the inductance and mutual inductance of coils given their physical parameters.

8. Understand hysteresis, saturation, core loss, and eddy currents in cores composed of magnetic materials such as iron.

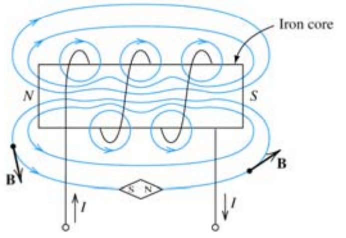
9. Understand ideal transformers and solve circuits that include transformers.

10. Use the equivalent circuits of real transformers to determine their regulations and power efficiencies.



(a) Permanent magnet

(b) Field around a straight wire carrying current I



(c) Field for a coil of wire

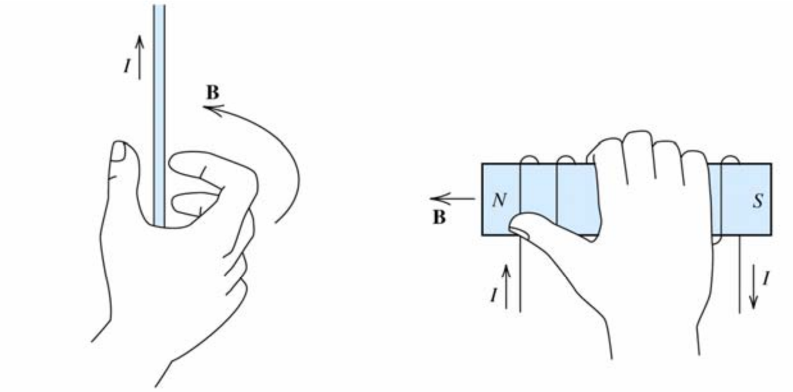
Figure 15.1 Magnetic fields can be visualized as lines of flux that form closed paths. Using a compass, we can determine the direction of the flux lines at any point. Note that the flux density vector **B** is tangent to the lines of flux.

MAGNETIC FIELDS

Magnetic flux lines form closed paths that are close together where the field is strong and farther apart where the field is weak.

Flux lines leave the north-seeking end of a magnet and enter the south-seeking end.

When placed in a magnetic field, a compass indicates north in the direction of the flux lines.

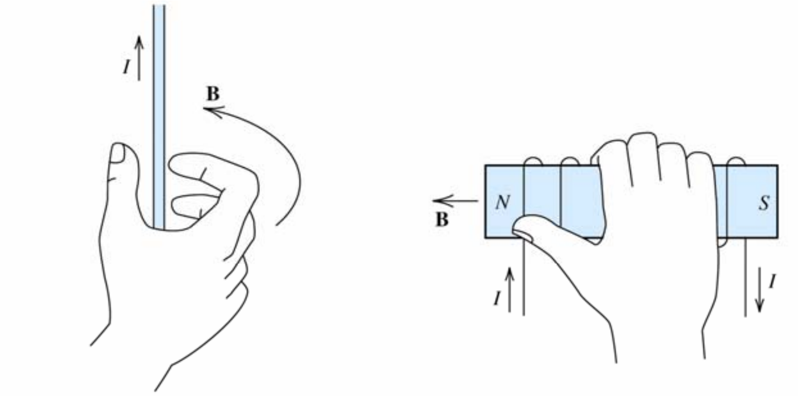


(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Figure 15.2 Illustrations of the right-hand rule.

Right-Hand Rule



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

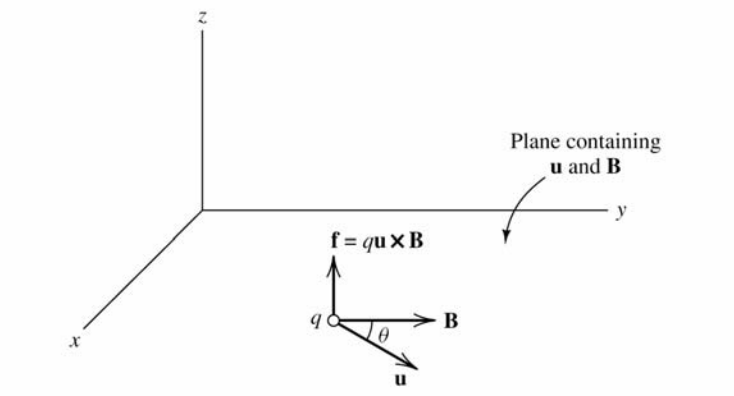


Figure 15.3 A charge moving through a magnetic field experiences a force **f** perpendicular to both the velocity **u** and flux density **B**.

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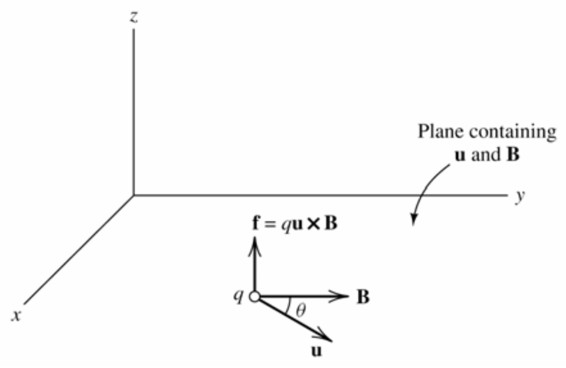
Forces on Charges Moving in Magnetic Fields



$$\mathbf{f} = q\mathbf{u} \times \mathbf{B}$$

$$f = quB\sin(\theta)$$

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Forces on Current-Carrying Wires

$$d\mathbf{f} = i d\mathbf{l} \times \mathbf{B}$$

$$f = ilB \sin(\theta)$$

Flux Linkages and Faraday's Law

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

$$\lambda = N\phi$$

Faraday's law of magnetic induction:

$$e = \frac{d\lambda}{dt}$$

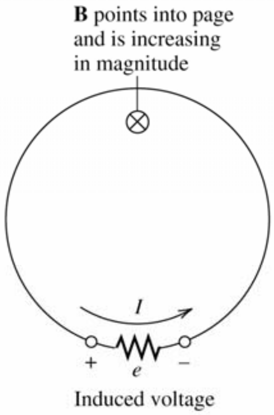


Figure 15.4 When the flux linking a coil changes, a voltage is induced in the coil. The polarity of the voltage is such that if a circuit is formed by placing a resistance across the coil terminals, the resulting current produces a field that tends to oppose the original change in the field.

Lenz’s Law

Lenz’s law states that the polarity of the induced voltage is such that the voltage would produce a current (through an external resistance) that opposes the original change in flux linkages.

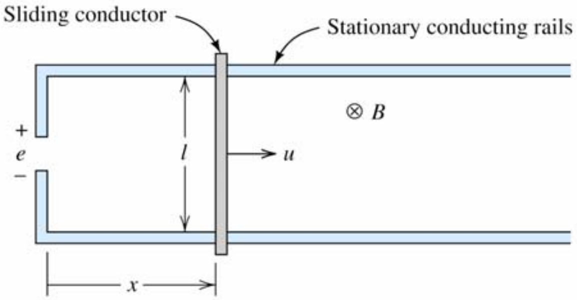
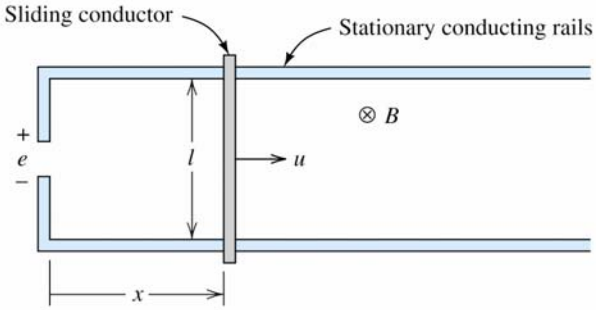
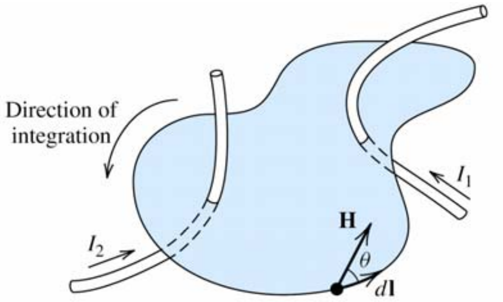


Figure 15.5 A voltage is induced in a conductor moving so as to cut through magnetic flux lines.

Voltages Induced in Field-Cutting Conductors



$$e = Blu$$



$$\oint \mathbf{H} \bullet d\mathbf{l} = I_1 + I_2$$

Figure 15.6 Ampère's law states that the line integral of magnetic field intensity around a closed path is equal to the sum of the currents flowing through the surface bounded by the path.

Magnetic Field Intensity and Ampère's Law

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb}/\text{Am}$$

$$\mu_r = \frac{\mu}{\mu_0}$$

Ampère's Law:

$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i$$

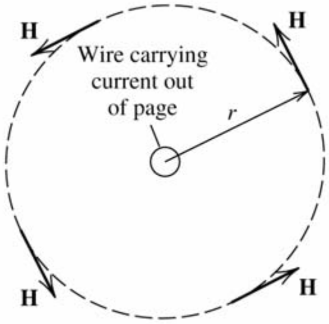
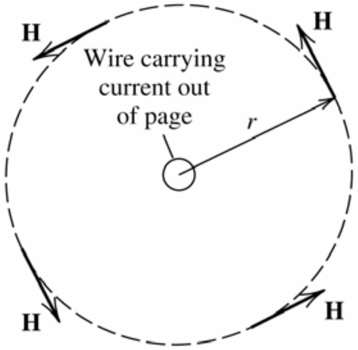


Figure 15.7 The magnetic field around a long straight wire carrying a current can be determined with Ampère's law aided by considerations of symmetry.

Magnetic Field Around a Long Straight Wire



$$B = \mu H = \frac{\mu I}{2\pi r}$$

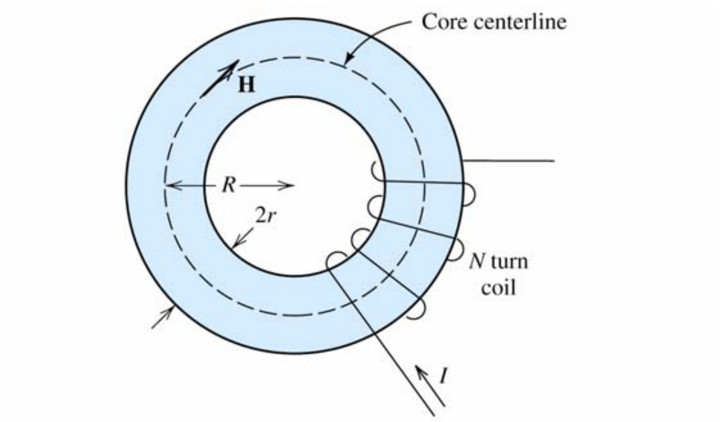
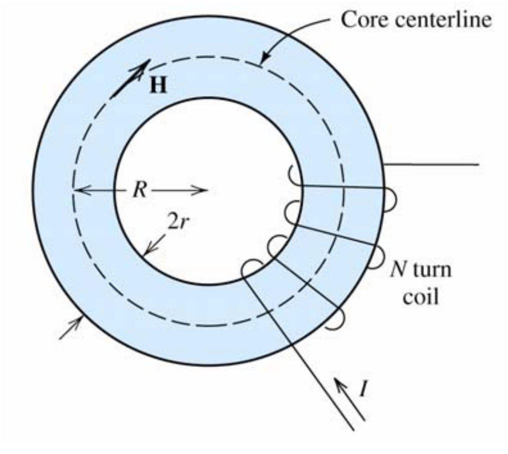


Figure 15.8 Toriodal coil analyzed in Examples 15.2, 15.3, and 15.4.

Flux Density in a Toroidal Core



$$B = \frac{\mu N I}{2\pi R}$$

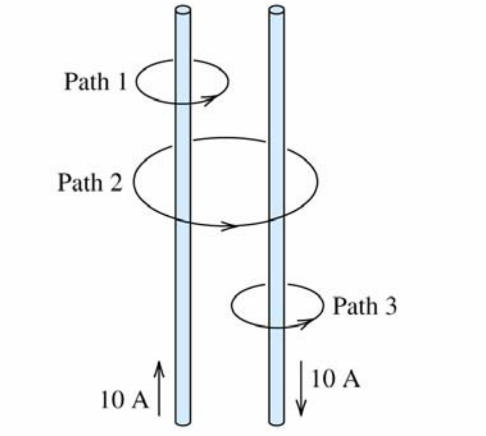
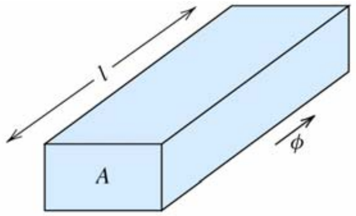


Figure 15.9 See Exercises 15.7 and 15.8.



$$\mathcal{R} = \frac{l}{\mu A}$$

Figure 15.10 The reluctance $\mathcal{R}$ of a magnetic path depends on the mean length l, the area A, and the permeability μ of the material.

MAGNETIC CIRCUITS

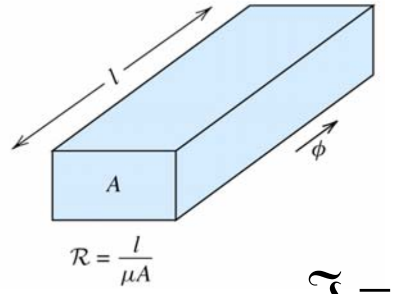
In many engineering applications, we need to compute the magnetic fields for structures that lack sufficient symmetry for straight-forward application of Ampère’s law. Then, we use an approximate method known as magnetic-circuit analysis.

---

magnetomotive force (mmf) of an *N*-turn current-carrying coil

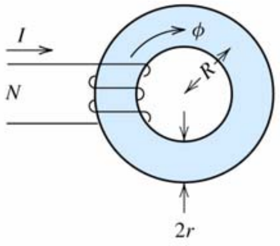
$$\mathfrak{I} = N\ I$$

reluctance of a path for magnetic flux

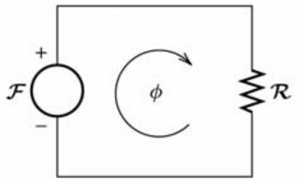


$$\Re = \frac{\ell}{\mu A}$$

$$\mathfrak{I} = \Re\phi$$



(a) Coil on a toriodal iron core

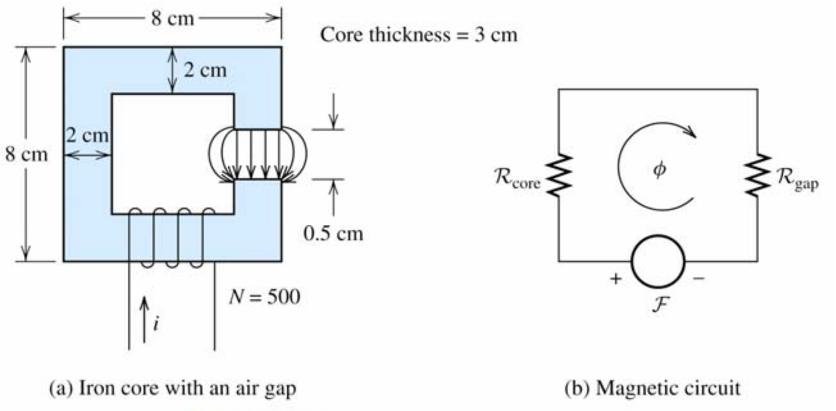


(b) Magnetic circuit

Figure 15.11 The magnetic circuit for the toriodal coil.

Advantage of the Magnetic-Circuit Approach

The advantage of the magnetic-circuit approach is that it can be applied to unsymmetrical magnetic cores with multiple coils.



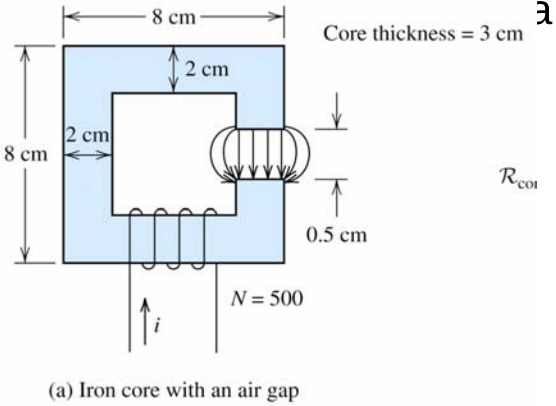
(a) Iron core with an air gap

(b) Magnetic circuit

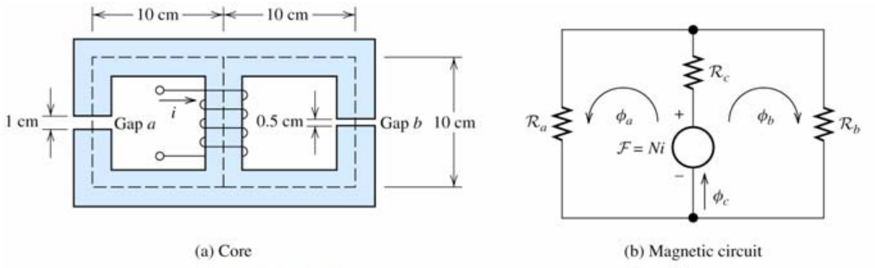
Figure 15.12 Magnetic circuit of Example 15.5.

Fringing

We approximately account for fringing by adding the length of the gap to the depth and width in [illegible] a.



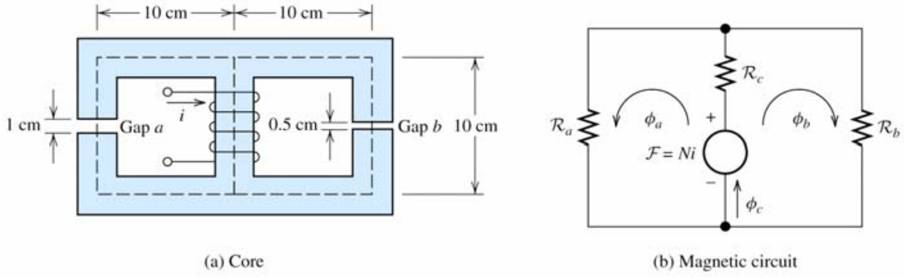
(a) Iron core with an air gap



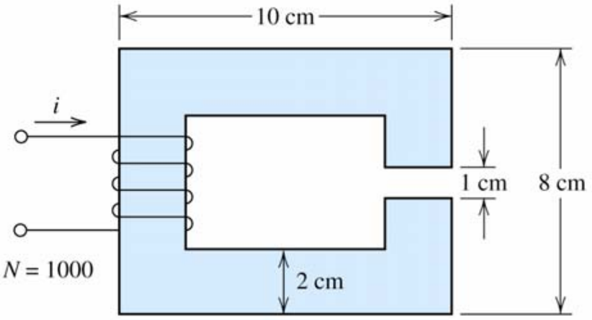
(a) Core (b) Magnetic circuit

Figure 15.13 Magnetic circuit of Example 15.6.

A Magnetic Circuit with Reluctances in Series and Parallel



(a) Core (b) Magnetic circuit



Core thickness = 2 cm

$\mu_{\text{core}} = 5000\mu_0$

Figure 15.14 Magnetic circuit of Exercise 15.9.

INDUCTANCE AND MUTUAL INDUCTANCE

$$L = \frac{\lambda}{i} \qquad L = \frac{N^2}{\Re}$$

$$e = L\frac{di}{dt}$$

Mutual Inductance

$$L_1 = \frac{\lambda_{11}}{i_1} \qquad\qquad L_2 = \frac{\lambda_{22}}{i_2}$$

$$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$$

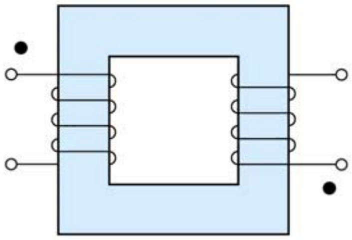
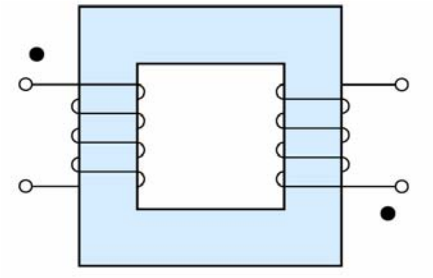


Figure 15.15 According to convention, currents entering the dotted terminals produce aiding fluxes.

Dot Convention

Aiding fluxes are produced by currents entering like marked terminals.



Circuit Equations for Mutual Inductance

$$\lambda_1 = L_1 i_1 \pm M i_2$$

$$\lambda_2 = \pm M i_1 + L_2 i_2$$

$$e_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

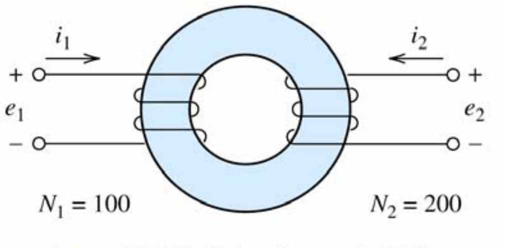


Figure 15.16 Coils of Example 15.8.

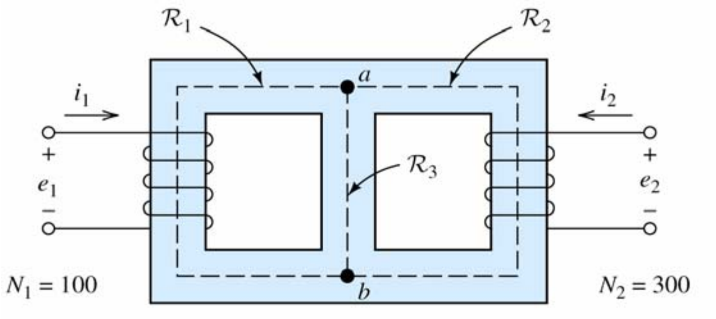
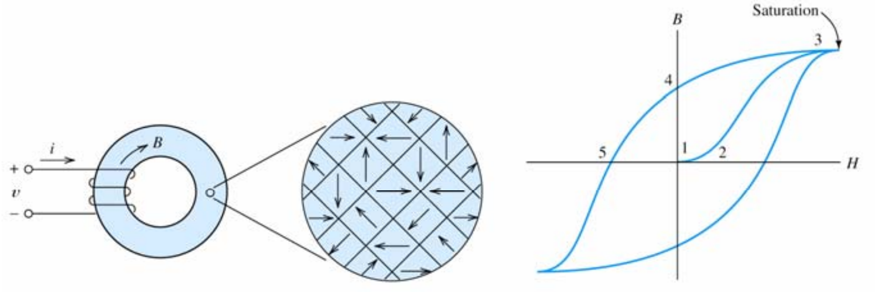


Figure 15.17 Magnetic circuit of Exercise 15.13.

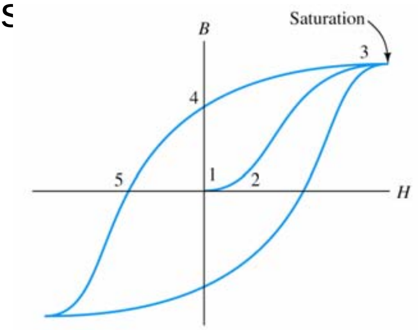


(a) Sample and coil for applying H (b) Magnetic domains (c) Hysteresis loop in the $B-H$ plane

Figure 15.18 Materials such as iron display a $B-H$ relationship with hysteresis and saturation.

MAGNETIC MATERIALS

The relationship between *B* and *H* is not linear for the types of iron used in motors and transformers



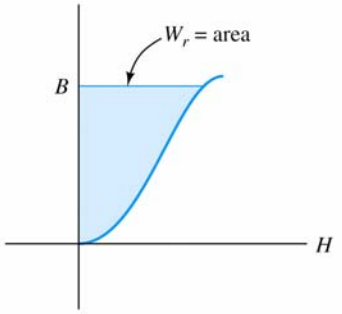
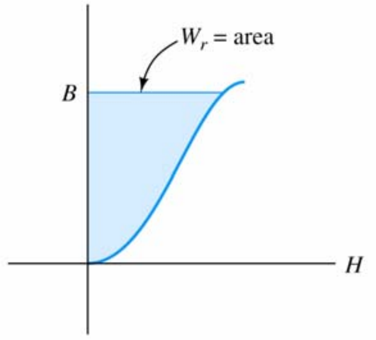


Figure 15.19 The area between the $B\text{–}H$ curve and the B axis represents the volumetric energy supplied to the core.

Energy Considerations



$$W_v = \frac{W}{Al} = \int_0^B H\,dB$$

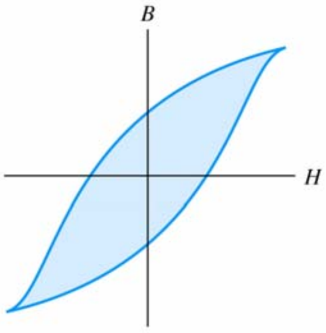
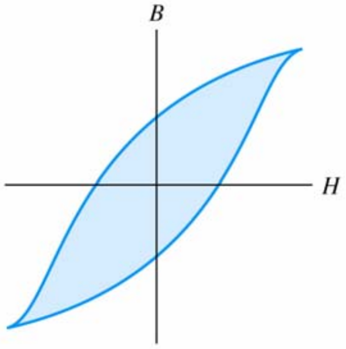


Figure 15.20 The area of the hysteresis loop is the volumetric energy converted to heat per cycle.

Core Loss

Power loss due to hysteresis is proportional to frequency, assuming constant peak flux.



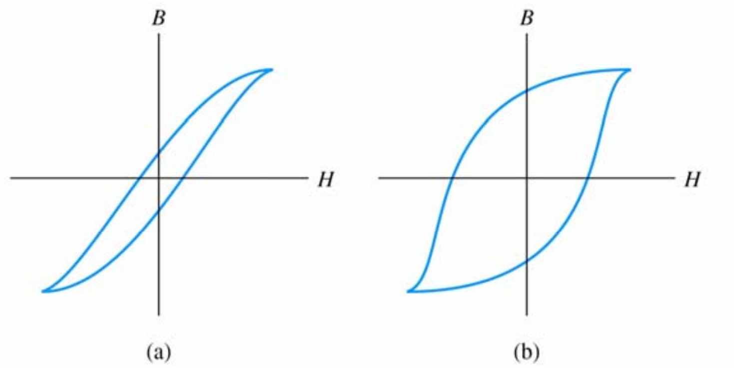


Figure 15.21 When we want to minimize core loss (as in a transformer or motor), we choose a material having a thin hysteresis loop. On the other hand, for a permanent magnet, we should choose a material with a wide loop.

Eddy-Current Loss

Power loss due to eddy currents is proportional

to the square of frequency, assuming constant

peak flux.

Energy Stored in the Magnetic Field

$$W_v = \int_0^B \frac{B}{\mu} dB = \frac{B^2}{2\mu}$$

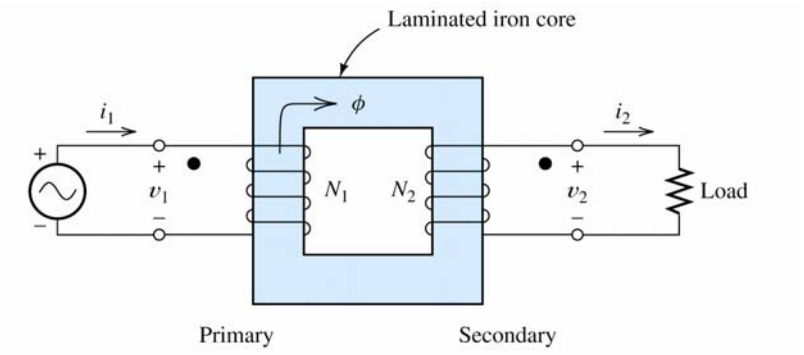
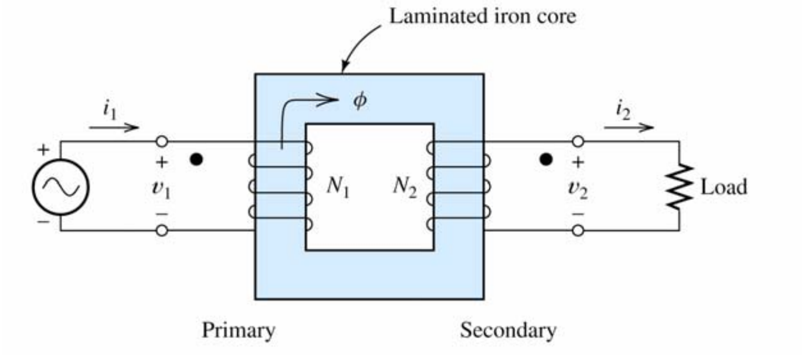


Figure 15.22 A transformer consists of several coils wound on a common core.

IDEAL TRANSFORMERS



$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

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$$I_{2\text{rms}} = \frac{N_1}{N_2} I_{1\text{rms}}$$

$$p_2(t) = p_1(t)$$

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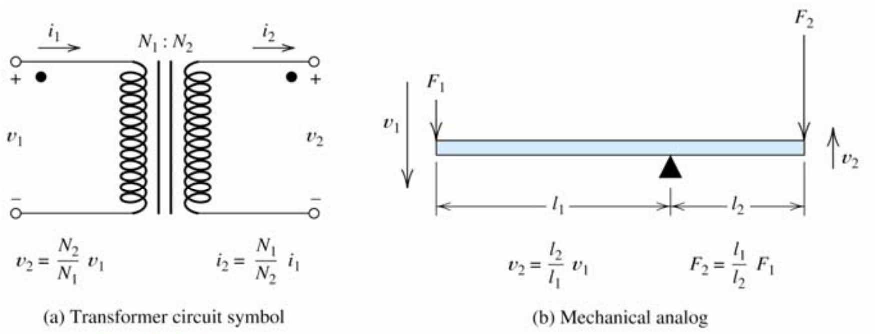


(a) Transformer circuit symbol

(b) Mechanical analog

Figure 15.23 The circuit symbol for a transformer and its mechanical analog.

Transformer Summary

1. We assumed that all of the flux links all of the windings of both coils and that the resistance of the coils is zero. Thus, the voltage across each coil is proportional to the number of turns on the coil.

$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

2**.** We assumed that the reluctance of the core is negligible, so the total mmf of both coils is zero.

$$i_2(t) = \frac{N_1}{N_2} i_2(t)$$

3**.** A consequence of the voltage and current relationships is that all of the power delivered to an ideal transformer by the source is transferred to the load.

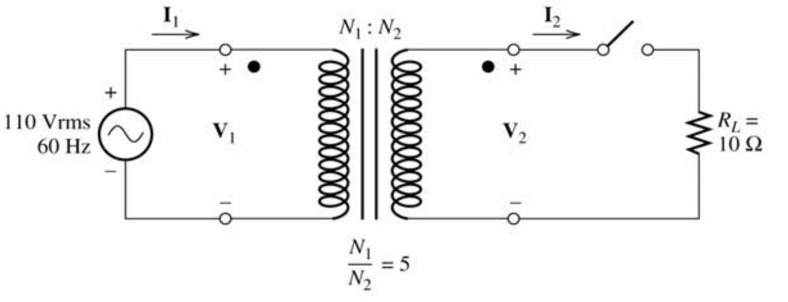
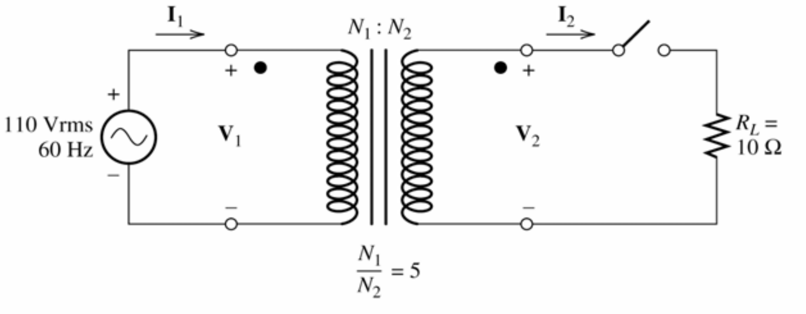


Figure 15.24 Circuit of Example 15.10.

Analysis of a Circuit Containing an Ideal Transformer



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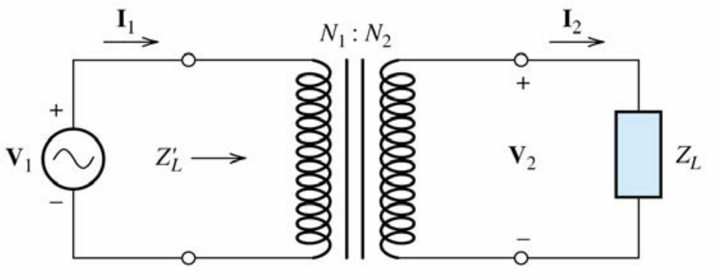

Figure 15.25 The impedance seen looking into the primary is $Z'_L = (N_1/N_2)^2 \times Z_L$.

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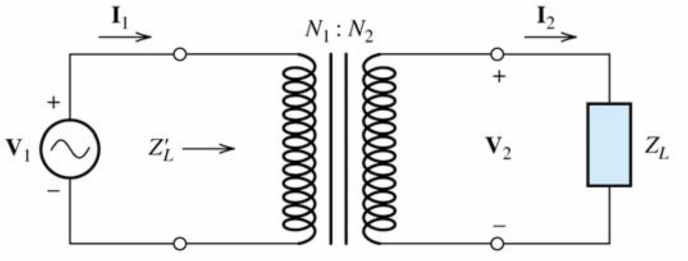
Impedance Transformations



$$Z'_L = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

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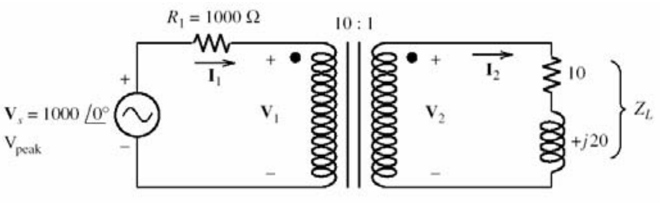
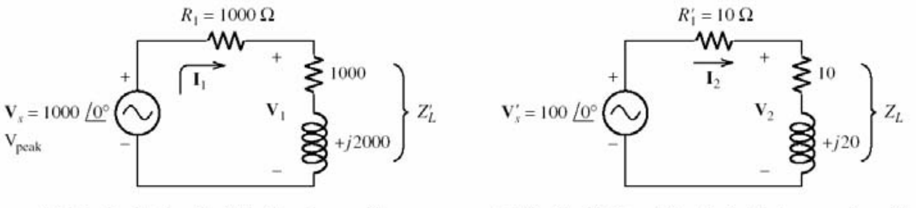


(a) Original circuit



(b) Circuit with Z_L reflected to the primary side

(c) Circuit with $\mathbf{V}_s$ and R_1 reflected to the secondary side

Figure 15.26 The circuit of Examples 15.11 and 15.12.

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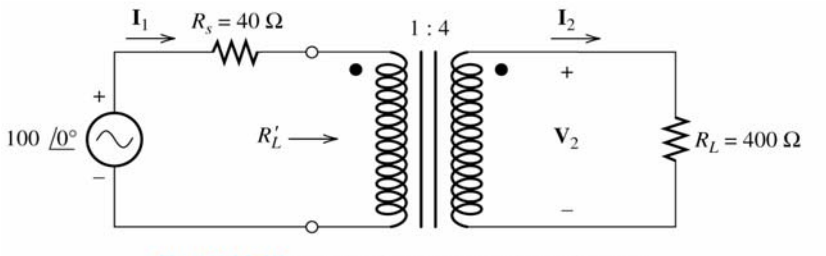


Figure 15.27 Circuit of Exercises 15.17 and 15.18.

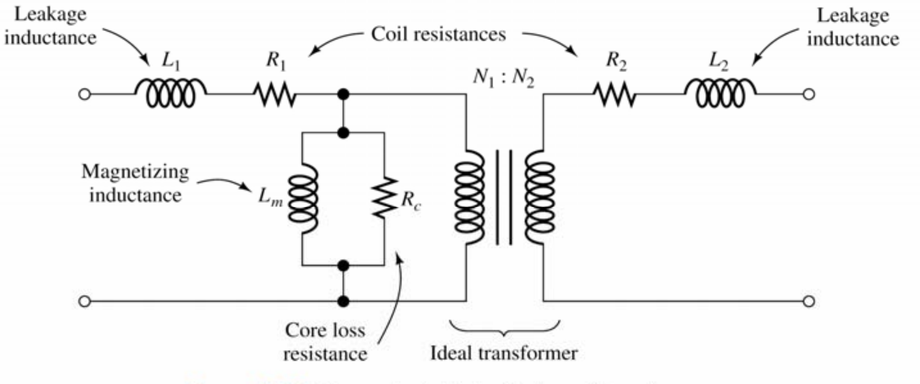
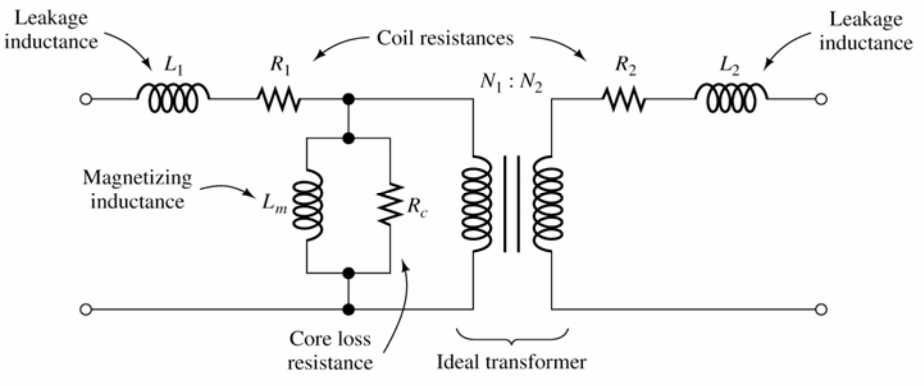


Figure 15.28 The equivalent circuit of a real transformer.

REAL TRANSFORMERS



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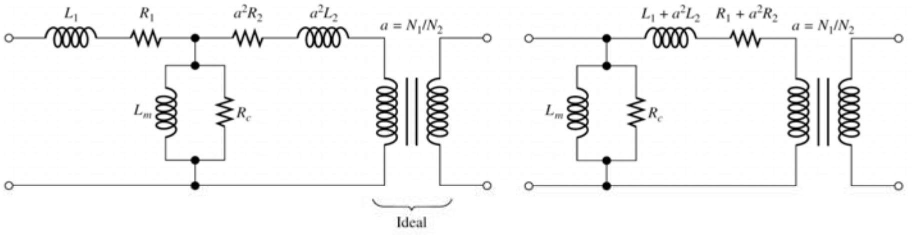

(a) All elements referred to the primary side

(b) Approximate equivalent circuit that is sometimes more convenient to use than that of part (a)

Figure 15.29 Variations of the transformer equivalent circuit. The circuit of (b) is not exactly equivalent to that of (a) but is sufficiently accurate for practical applications.

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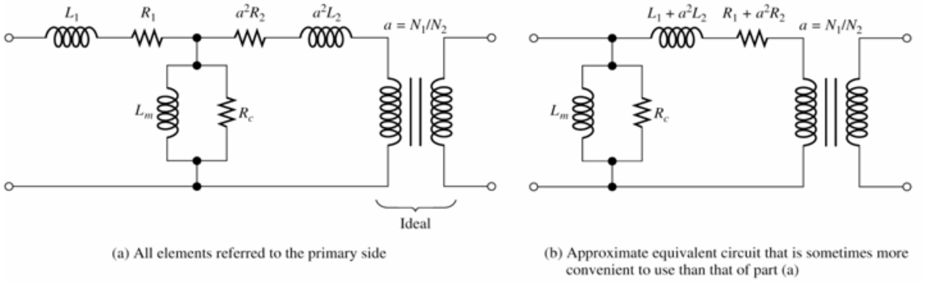
Variations of the Transformer Model



(a) All elements referred to the primary side

(b) Approximate equivalent circuit that is sometimes more convenient to use than that of part (a)

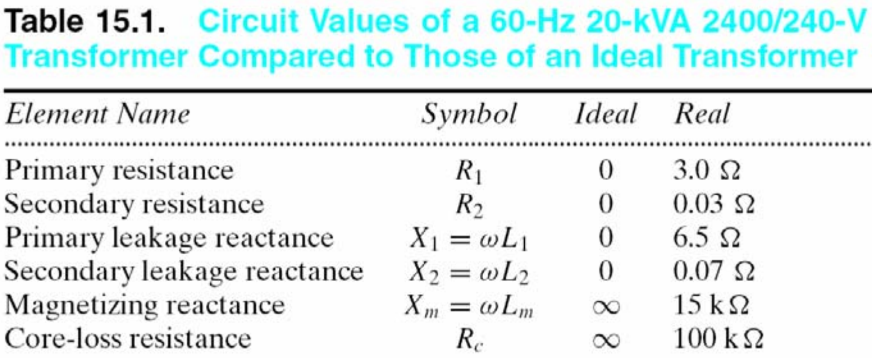


Table 15.1. Circuit Values of a 60-Hz 20-kVA 2400/240-V Transformer Compared to Those of an Ideal Transformer

Element Name	*Symbol*	*Ideal*	*Real*
Primary resistance	R_1	0	3.0 Ω
Secondary resistance	R_2	0	0.03 Ω
Primary leakage reactance	$X_1 = \omega L_1$	0	6.5 Ω
Secondary leakage reactance	$X_2 = \omega L_2$	0	0.07 Ω
Magnetizing reactance	$X_m = \omega L_m$	∞	15 kΩ
Core-loss resistance	R_c	∞	100 kΩ

Regulation and Efficiency

$$\text{percent regulation} = \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100\%$$

$$\text{power efficiency} = \frac{P_{\text{load}}}{P_{\text{in}}} \times 100\% = \left(1 - \frac{P_{\text{loss}}}{P_{\text{in}}}\right) \times 100\%$$

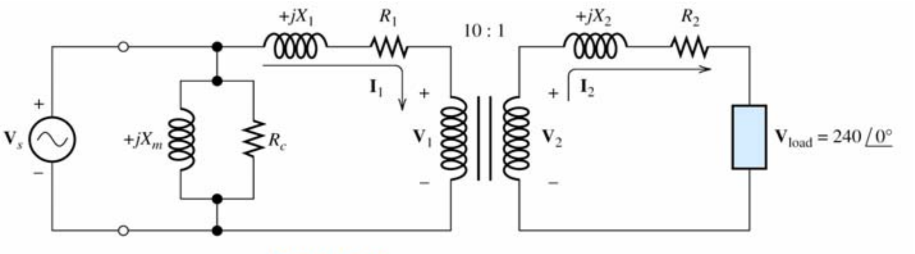


Figure 15.30 Circuit of Example 15.13.