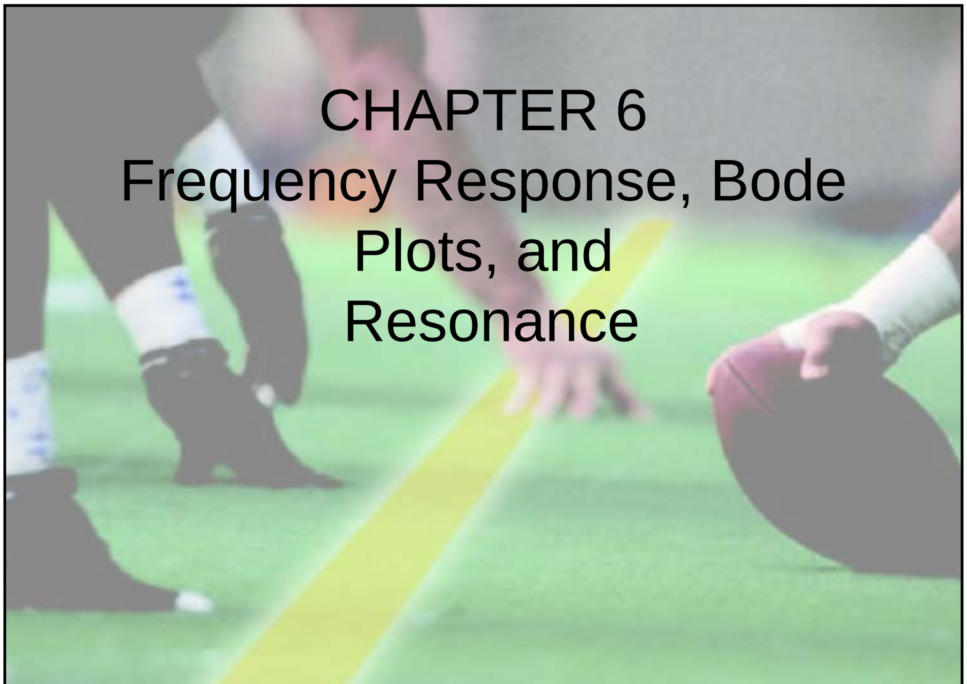

CHAPTER 6 Frequency Response, Bode Plots, and Resonance

CHAPTER 6 Frequency Response, Bode Plots, and Resonance

1. State the fundamental concepts of Fourier analysis.

2. Determine the output of a filter for a given input consisting of sinusoidal components using the filter’s transfer function.

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3. Use circuit analysis to determine the
transfer functions of simple circuits.

4. Draw first-order lowpass or highpass filter
circuits and sketch their transfer functions.

5. Understand decibels, logarithmic frequency
scales, and Bode plots.

6. Draw the Bode plots for transfer functions of
first-order filters.

7. Use software to produce Bode plots for more
complex *RLC* filters.

8. Calculate parameters for series and parallel
resonant circuits.

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9. Select and design simple filter circuits.

10. Design simple digital signal-processing systems.

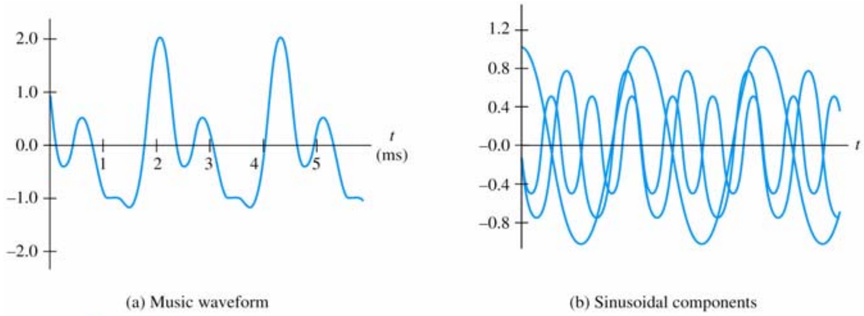

Figure 6.1 The short segment of a music waveform shown in (a) is the sum of the sinusoidal components shown in (b).

Fourier Analysis

All real-world signals are sums of sinusoidal components having various frequencies, amplitudes, and phases.

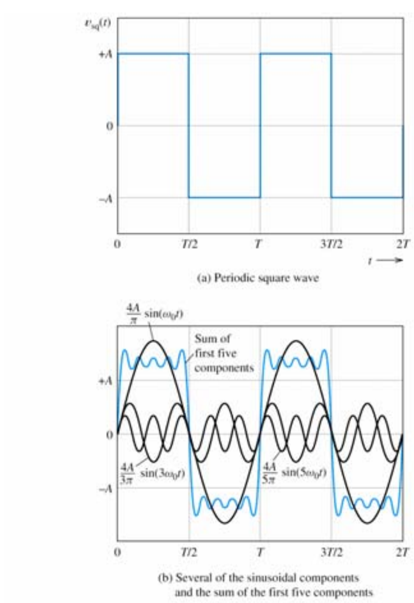

Figure 6.2 A square wave and some of its components.

Table 6.1. Frequency Ranges of Selected Signals

Electrocardiogram	0.05 to 100 Hz
Audible sounds	20 Hz to 15 kHz
AM radio broadcasting	540 to 1600 kHz
Video signals (U.S. standards)	Dc to 4.2 MHz
Channel 6 television	82 to 88 MHz
FM radio broadcasting	88 to 108 MHz
Cellular radio	824 to 891.5 MHz
Satellite television downlinks (C-band)	3.7 to 4.2 GHz
Digital satellite television	12.2 to 12.7 GHz

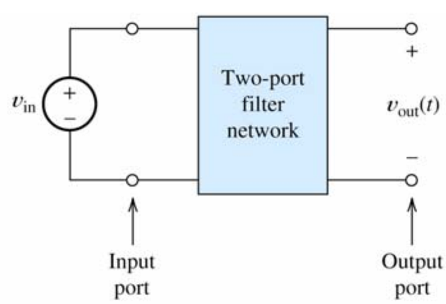

Figure 6.3 When an input signal $v_{in}(t)$ is applied to the input port of a filter, some components are passed to the output port while others are not, depending on their frequencies. Thus, $v_{out}(t)$ contains some of the components of $v_{in}(t)$ but not others. Usually, the amplitudes and phases of the components are altered in passing through the filter.

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Filters

Filters process the sinusoid components of an input
signal differently depending of the frequency of each component. Often, the goal of the filter is to retain the components in certain frequency ranges and to reject components in other ranges.

Transfer Functions

The **transfer function** *H(f)* of the two-port filter is defined to be the ratio of the phasor output voltage to the phasor input voltage as a function of frequency:

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}}$$

The magnitude of the transfer function shows how the amplitude of each frequency component is affected by the filter. Similarly, the phase of the transfer function shows how the phase of each
frequency component is affected by the filter.

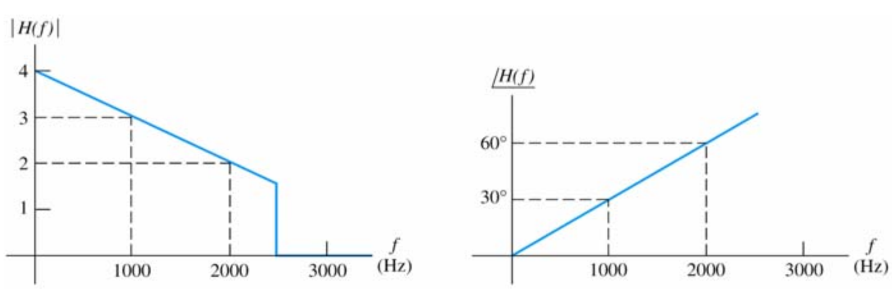

Figure 6.4 The transfer function of a filter. See Examples 6.1 and 6.2.

Determining the output of a filter for an input with multiple components:

1. Determine the frequency and phasor representation for each input component.

2. Determine the (complex) value of the transfer function for each component.

3. Obtain the phasor for each output component by multiplying the phasor for each input component by the corresponding transfer-function value.

4. Convert the phasors for the output components into time functions of various frequencies. Add these time functions to produce the output.

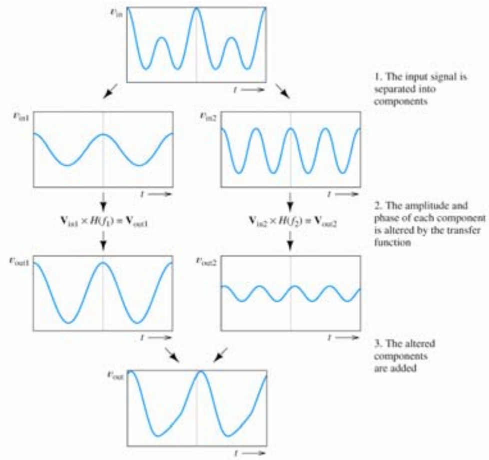

Figure 6.5 Filters behave as if they separate the input into components, modify the amplitudes and phases of the components, and add the altered components to produce the output.

Linear circuits behave as if they:

1. Separate the input signal into components having various frequencies.

2. Alter the amplitude and phase of each component depending on its frequency.

3. Add the altered components to produce the output signal.

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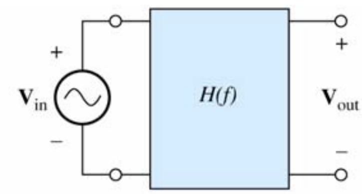


Figure 6.6 To measure the transfer function, we apply a sinusoidal input signal, measure the amplitudes and phases of input and output in steady state, and then divide the phasor output by the phasor input. The procedure is repeated for each frequency of interest.

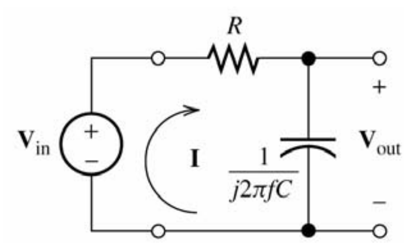
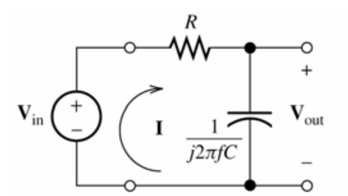


Figure 6.7 A first-order lowpass filter.

FIRST-ORDER LOWPASS FILTERS

$$f_B = \frac{1}{2\pi RC} \qquad \left|H(f)\right| = \frac{1}{\sqrt{1+\left(f/f_B\right)^2}}$$

$$H(f) = \frac{1}{1+j\left(f/f_B\right)} \qquad \angle H(f) = -\arctan\left(\frac{f}{f_B}\right)$$



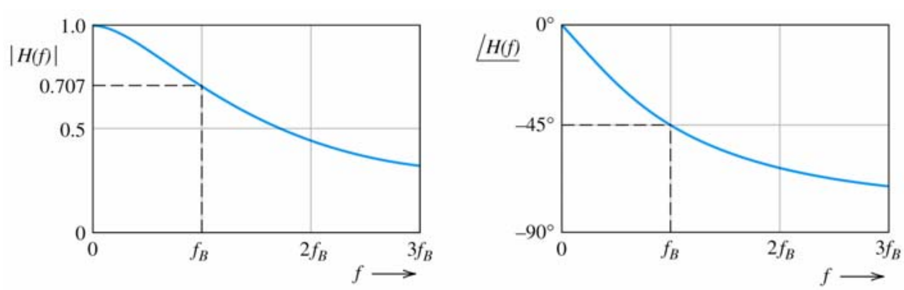


Figure 6.8 Magnitude and phase of the first-order lowpass transfer function versus frequency.

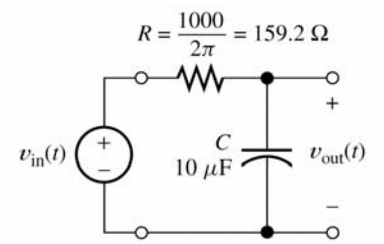


Figure 6.9 Circuit of Example 6.3. The resistance has been picked so the break frequency turns out to be a convenient value.

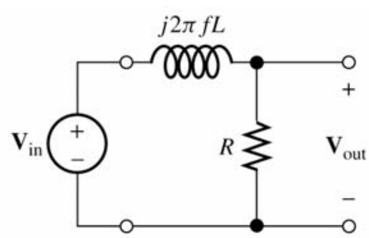


Figure 6.10 Another first-order lowpass filter; see Exercise 6.4.

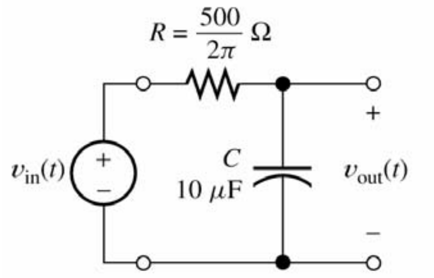


Figure 6.11 Circuit for Exercise 6.5.

DECIBELS, THE CASCADE CONNECTION, AND LOGARITHMIC FREQUENCY SCALES

$$\left|H(f)\right|_{\text{dB}} = 20\log\left|H(f)\right|$$

Table 6.2. Transfer-Function Magnitudes and Their Decibel Equivalents

$\|H(f)\|$	$\|H(f)\|_{dB}$
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	−3
1/2	−6
0.1	−20
0.01	−40

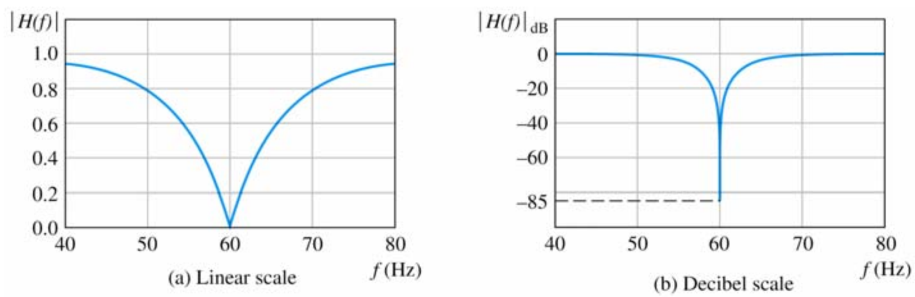


Figure 6.12 Transfer-function magnitude of a notch filter used to reduce hum in audio signals.

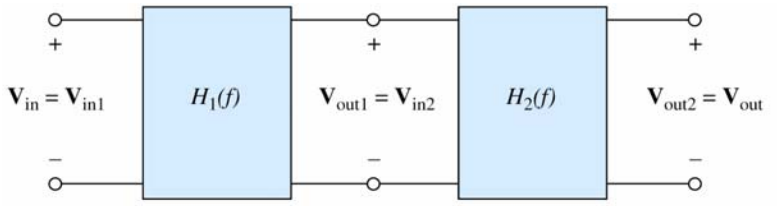
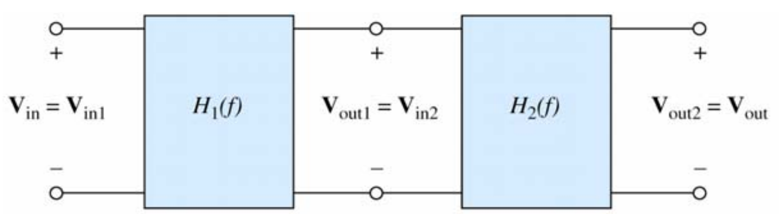


Figure 6.13 Cascade connection of two two-port circuits.

Cascaded Two-Port Networks

$$H(f) = H_1(f) \times H_2(f)$$

$$\left|H(f)\right|_{\mathrm{dB}} = \left|H_1(f)\right|_{\mathrm{dB}} + \left|H_2(f)\right|_{\mathrm{dB}}$$



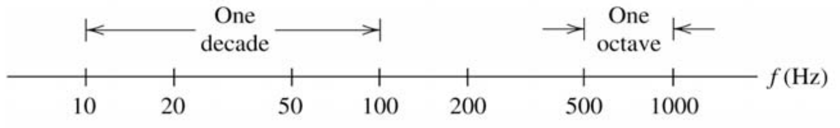


Figure 6.14 Logarithmic frequency scale.

Logarithmic Frequency Scales

On a logarithmic scale, the variable is multiplied by a given factor for equal increments of length along the

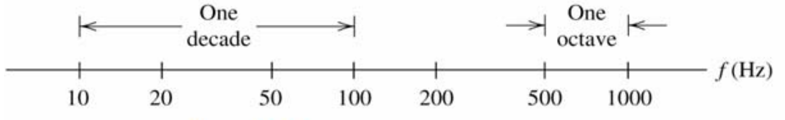


Figure 6.14 Logarithmic frequency scale.

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A **decade** is a range of frequencies for which the ratio of the highest frequency to the lowest is 10.

$$\text{number of decades} = \log\left(\frac{f_2}{f_1}\right)$$

An **octave** is a two-to-one change in frequenc

$$\text{number of octaves} = \log_2\left(\frac{f_2}{f_1}\right) = \left(\frac{\log(f_2/f_1)}{\log(2)}\right)$$

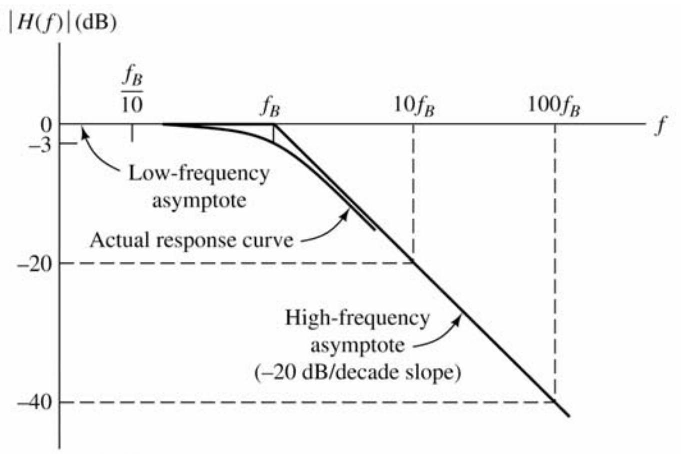

Figure 6.15 Magnitude Bode plot for the first-order lowpass filter.

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BODE PLOTS

A Bode plot shows the magnitude of a network function in decibels versus frequency using a logarithmic scale for frequency.

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

$$|H(f)|_{\mathrm{dB}} = -10\log\left[1 + \left(\frac{f}{f_B}\right)^2\right]$$

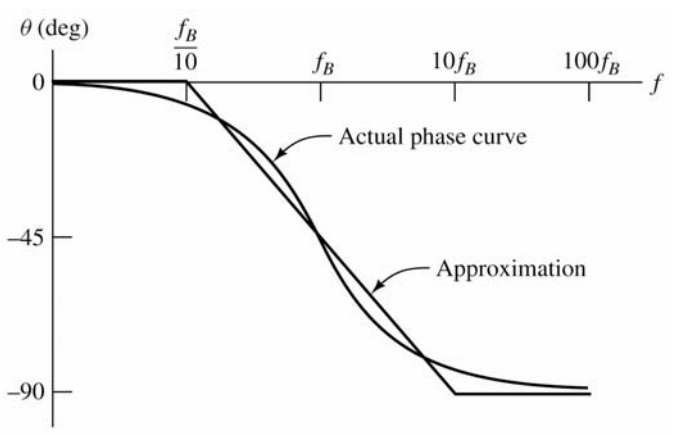
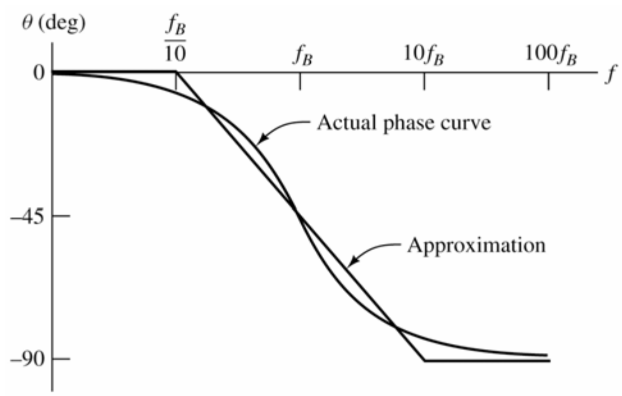


Figure 6.16 Phase Bode plot for the first-order lowpass filter.



1. A horizontal line at zero for $f < f_B/10$.

2. A sloping line from zero phase at $f_B/10$ to –90° at 10

3. A horizontal line at –90° for $f > 10f_B$.

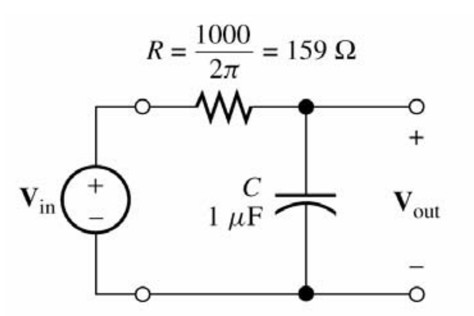


Figure 6.17 Circuit for Exercise 6.11.

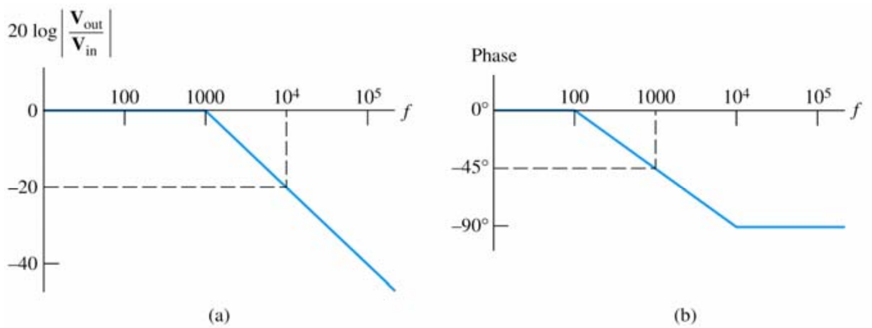


Figure 6.18 Answers for Exercise 6.11.

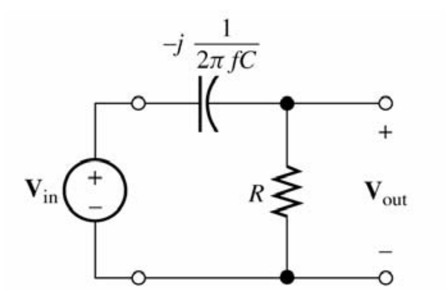


Figure 6.19 First-order highpass filter.

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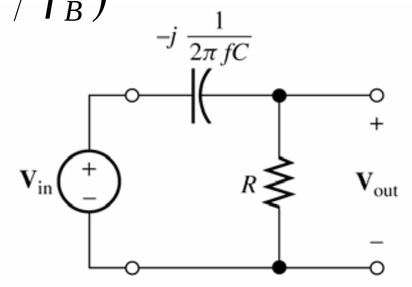
FIRST-ORDER HIGHPASS FILTERS

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

$$f_B = \frac{1}{2\pi RC}$$



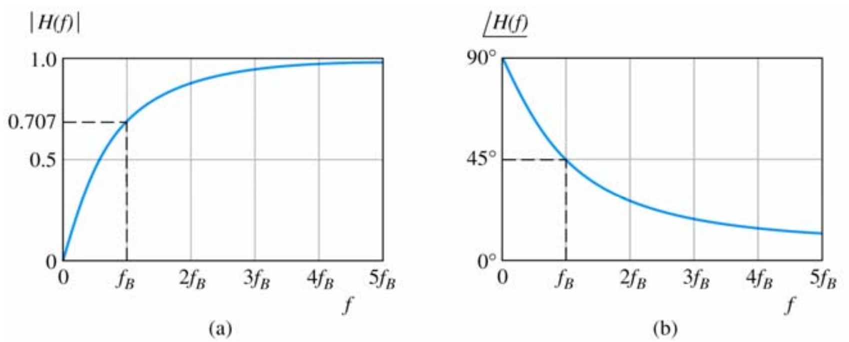


Figure 6.20 Magnitude and phase for the first-order highpass transfer function.

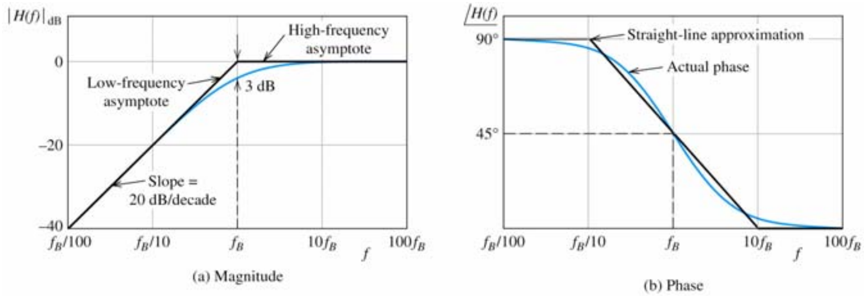


Figure 6.21 Bode plots for the first-order highpass filter.

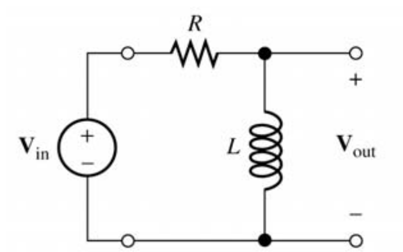
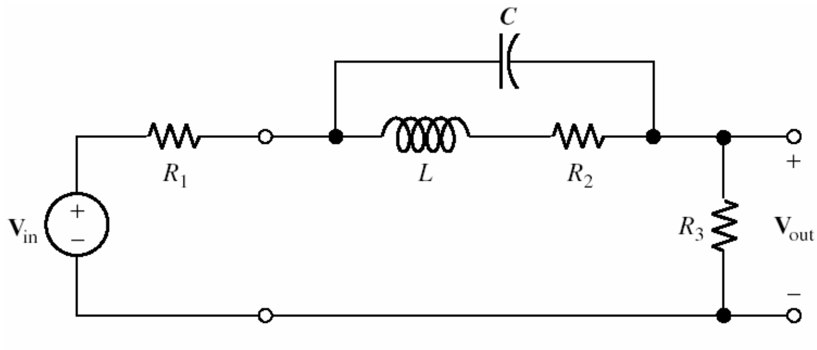


Figure 6.22 Circuit for Exercise 6.13.

Computer-Generated Bode Plot



$R_1 = 90\ \Omega$ $\quad L = 100$ mH

$R_2 = 10\ \Omega$ $\quad C = 0.1\ \mu$F

$R_3 = 100\ \Omega$

$$H(0) = \frac{R_3}{R_1 + R_2 + R_3} = 0.5$$

$$H_{\mathrm{dB}}(0) = 20\log(0.5) = -6\ \mathrm{dB}$$

$$H(\infty) = \frac{R_3}{R_1 + R_3} = 0.5263$$

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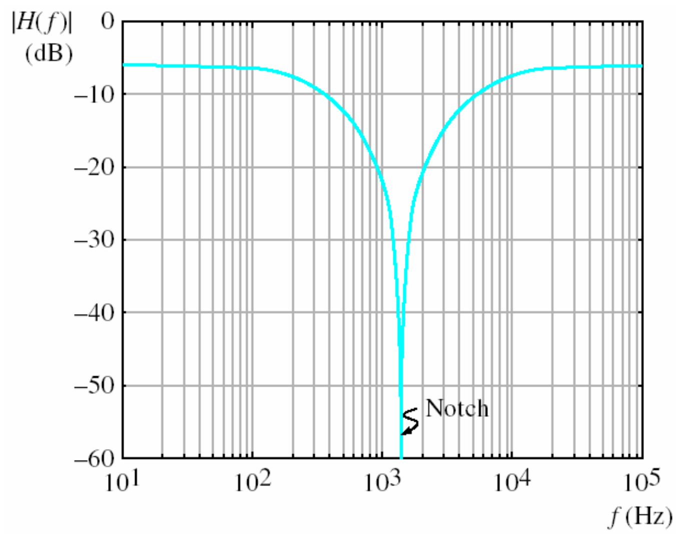
$$H_{\text{dB}}(\infty) = 20\log(0.5263) = -5.575\,\text{dB}$$

$$H(f) = \frac{R_3}{R_1 + R_3 + 1/\left[j\omega C + 1/(R_2 + j\omega L)\right]}$$


```
R1 = 90
R2 = 10
R3 = 100
L = 0.1
C = 1e-7
logf = 1:0.001:5;
f = 10.^logf;
w = 2*pi*f;
H = R3./(R1+R3+1./(j*w*C + 1./(R2 + j*w*L)));
semilogx(f,20*log10(abs(H)))
```



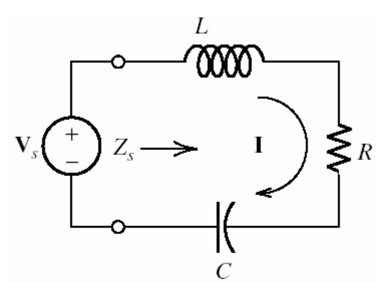


Figure 6.25 The series resonant circuit.

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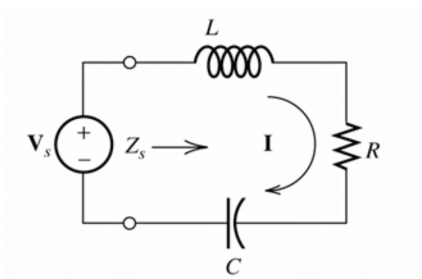
SERIES RESONANCE

Resonance is a phenomenon that can be observed in mechanical systems and
electrical circuits.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad Q_s = \frac{1}{2\pi f_0 CR}$$

$$Q_s = \frac{2\pi f_0 L}{R} \qquad Z_s(f) = R\left[1 + jQ_s\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right]$$



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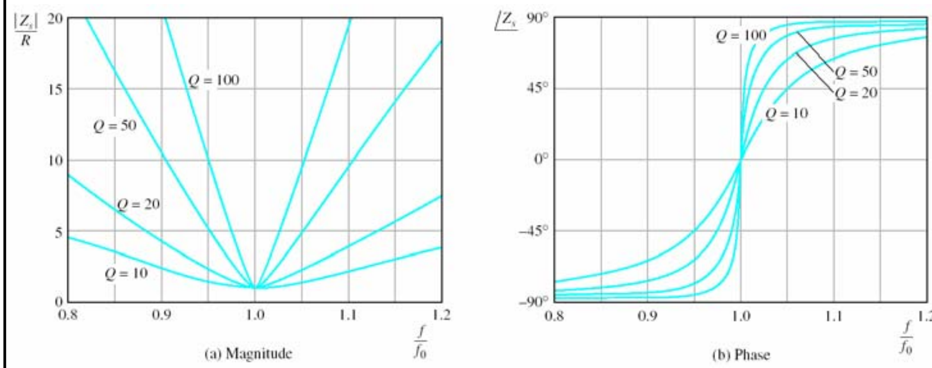


Figure 6.26 Plots of normalized magnitude and phase for the impedance of the series resonant circuit versus frequency.

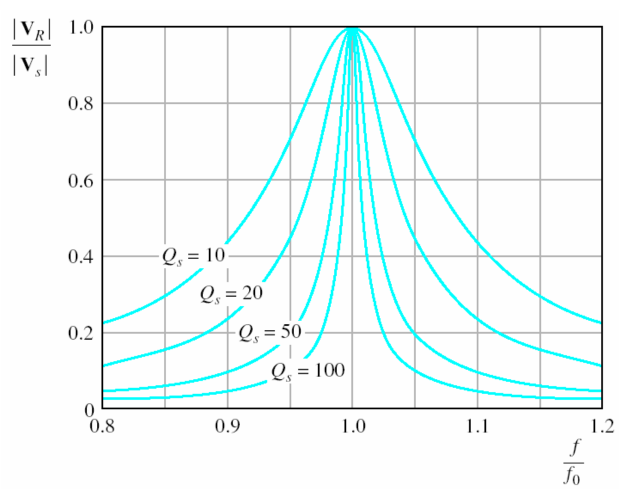
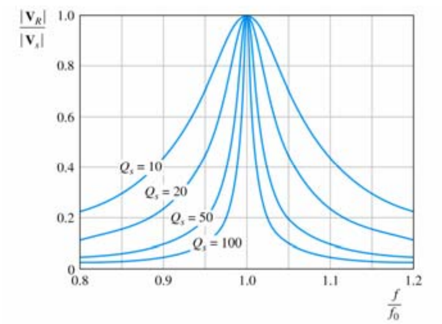


Figure 6.27 Plots of the transfer-function magnitude $|\mathbf{V}_R/\mathbf{V}_s|$ for the series resonant bandpass-filter circuit.

Series Resonant Circuit as a Bandpass Filter

$$\frac{\mathbf{V}_R}{\mathbf{V}_s} = \frac{1}{1 + jQ_s\left(f/f_0 - f_0/f\right)}$$



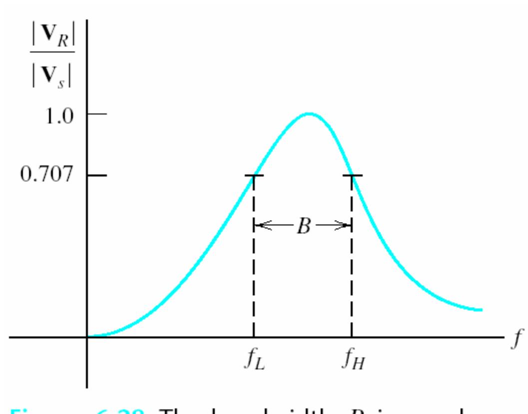


Figure 6.28 The bandwidth B is equal to the difference between the half-power frequencies.

$$B = f_H - f_L \qquad f_H \cong f_0 + \frac{B}{2}$$

$$B = \frac{f_0}{Q_s} \qquad f_L \cong f_0 - \frac{B}{2}$$

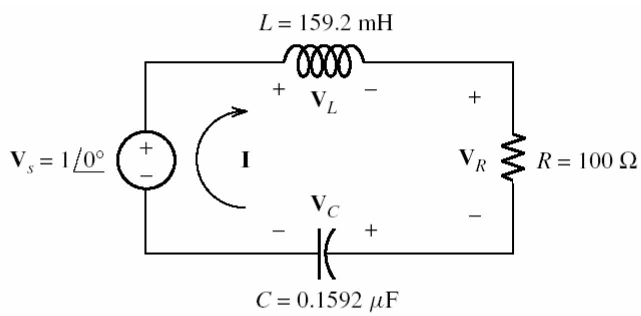


Figure 6.29 Series resonant circuit of Example 6.6. (The component values have been selected so the resonant frequency and Q_s turn out to be round numbers.)

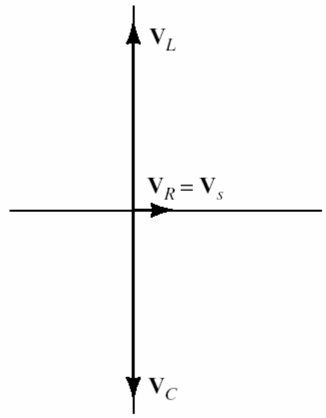


Figure 6.30 Phasor diagram for Example 6.6.

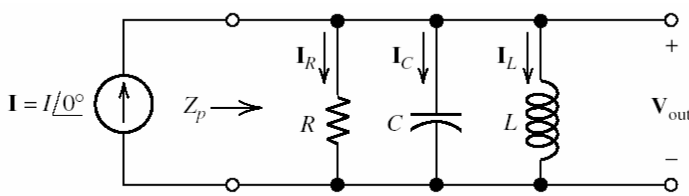


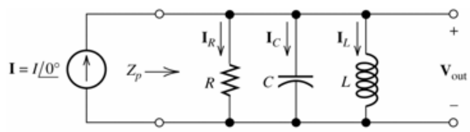
Figure 6.31 The parallel resonant circuit.

PARALLEL RESONANCE

$$Z_p = \frac{1}{(1/R) + j2\pi fC - j(1/2\pi fL)}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad Q_p = \frac{R}{2\pi f_0 L} \qquad Q_p = 2\pi f_0 CR$$

$$Z_p = \frac{R}{1 + jQ_p(f/f_0 - f_0/f)}$$



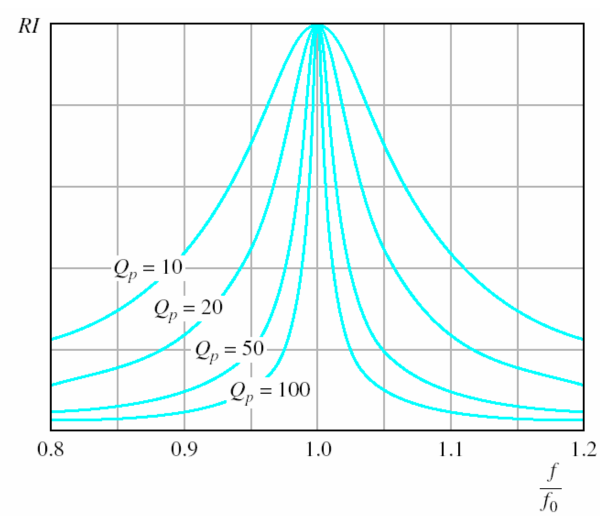


Figure 6.32 Voltage across the parallel resonant circuit for a constant-amplitude variable-frequency current source.

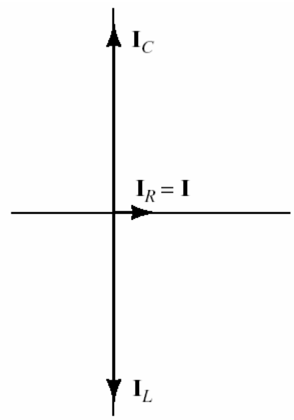


Figure 6.33 Phasor diagram for Example 6.7.

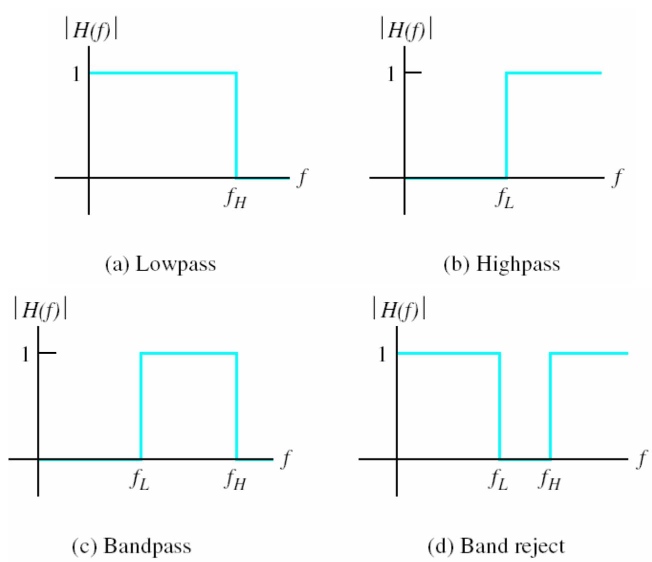


Figure 6.34 Transfer functions of ideal filters.

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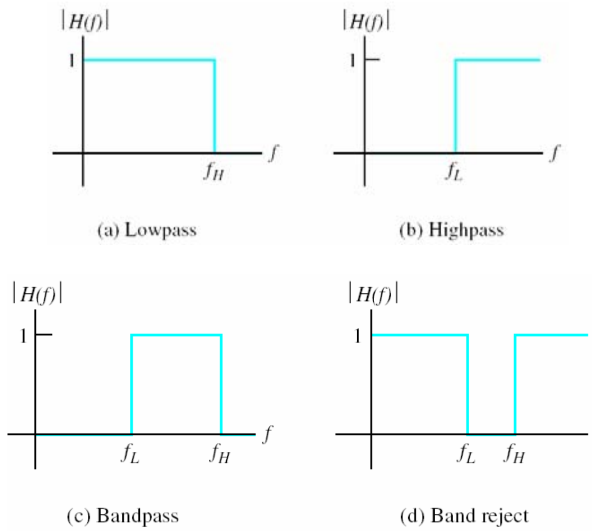
Ideal Filters



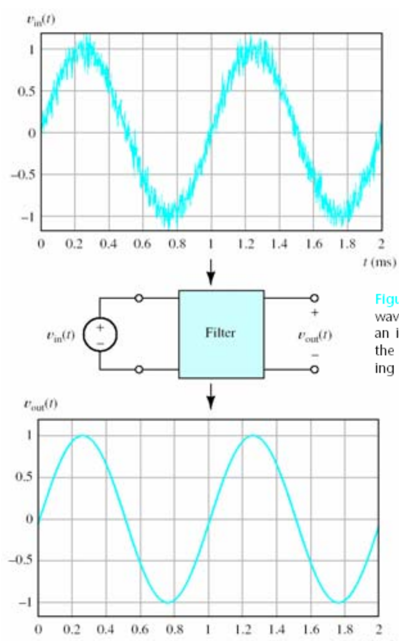


Figure 6.35 The input signal v_{in} consists of a 1-kHz sine wave plus high-frequency noise. By passing v_{in} through an ideal lowpass filter with the proper cutoff frequency, the sine wave is passed and the noise is rejected, resulting in a clean output signal.

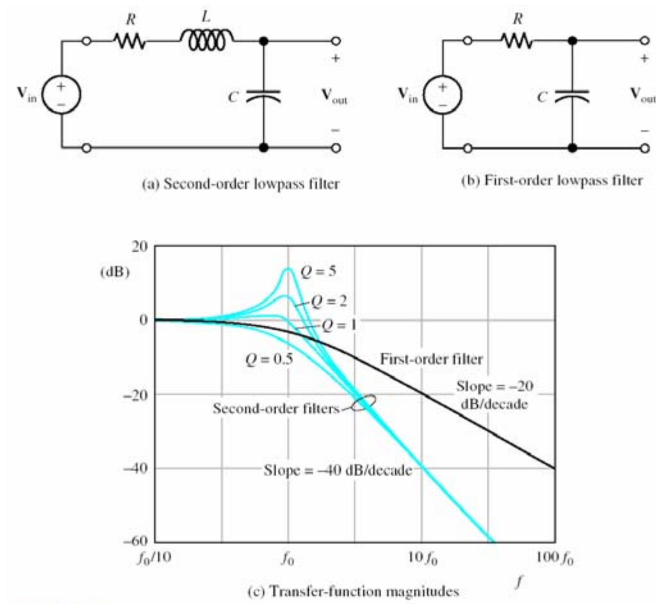


Figure 6.36 Lowpass filter circuits and their transfer-function magnitudes versus frequency.

Second-Order Lowpass Filter

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{-jQ_s(f_0/f)}{1 + jQ_s(f/f_0 - f_0/f)}$$

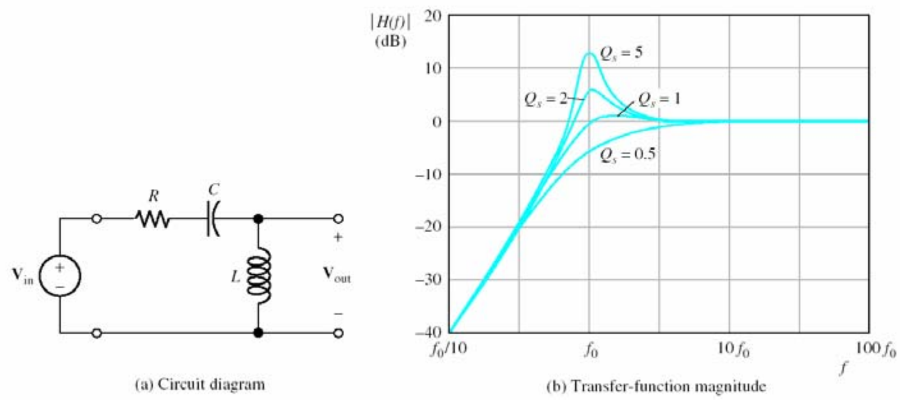


Figure 6.37 Second-order highpass filter and its transfer-function magnitude versus frequency for several values of Q_s.

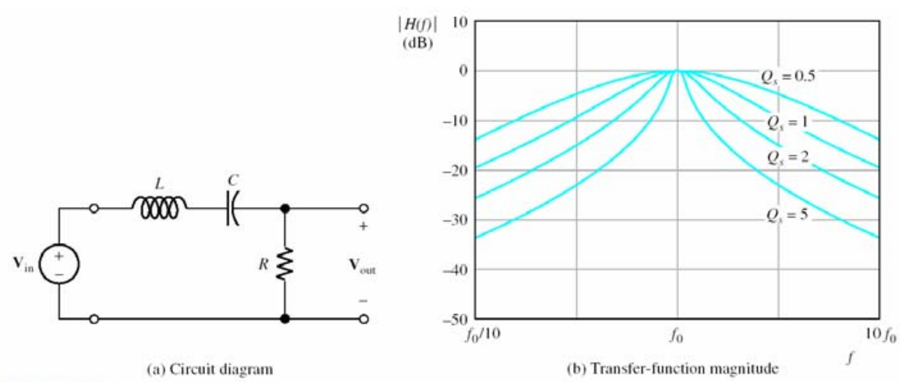


Figure 6.38 Second-order bandpass filter and its transfer-function magnitude versus frequency for several values of Q_s.

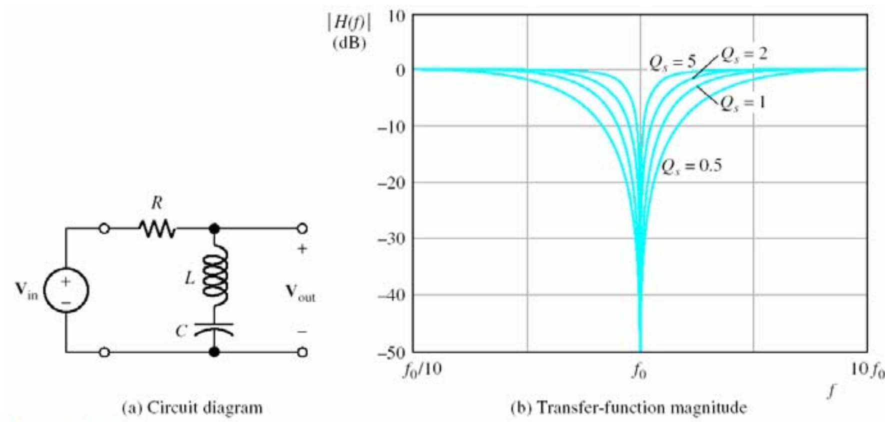


Figure 6.39 Second-order band-reject filter and its transfer-function magnitude versus frequency for several values of Q_s.

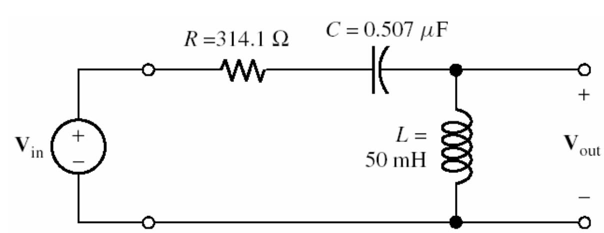


Figure 6.40 Filter designed in Example 6.8.

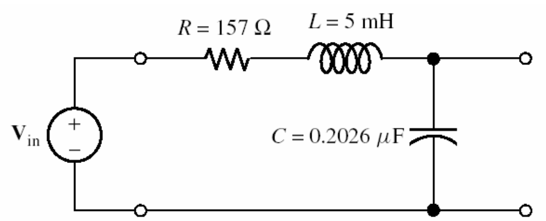


Figure 6.41 Answer for Exercise 6.20.

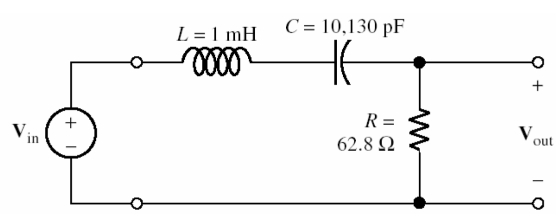


Figure 6.42 Answer for Exercise 6.21.

Figure 6.43 Generic block diagram of a digital signal-processing (DSP) system.

DIGITAL SIGNAL PROCESSING

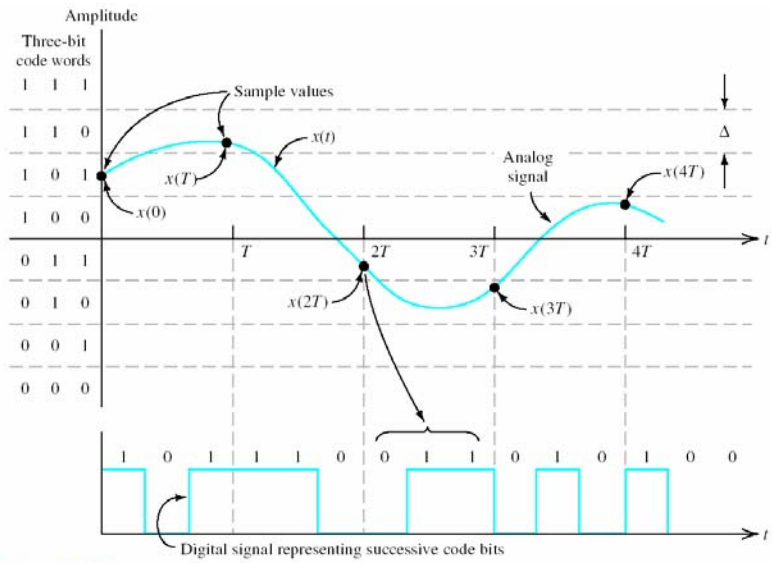


Figure 6.44 An analog signal is converted to an approximate digital equivalent by sampling. Each sample value is represented by a three-bit code word. (Practical converters use longer code words, and the width Δ of each amplitude zone is much smaller.)

Conversion of Signals from Analog to Digital Form

If a signal contains no components with frequencies higher than f_H, the signal can be exactly reconstructed from its samples, provided
that the sampling rate f_s is selected to be more than twice f_H.

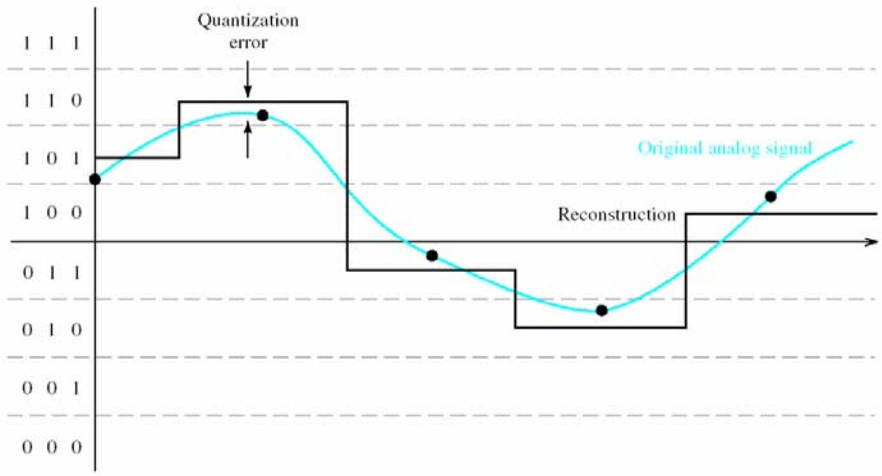


Figure 6.45 Quantization error occurs when an analog signal is reconstructed from its digital form.

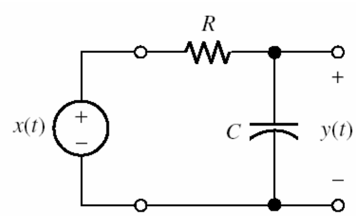


Figure 6.46 First-order RC lowpass filter.

Digital Lowpass Filter

$$y(n) = ay(n-1) + (1-a)x(n)$$

$$a = \frac{\tau/T}{1+\tau/T}$$

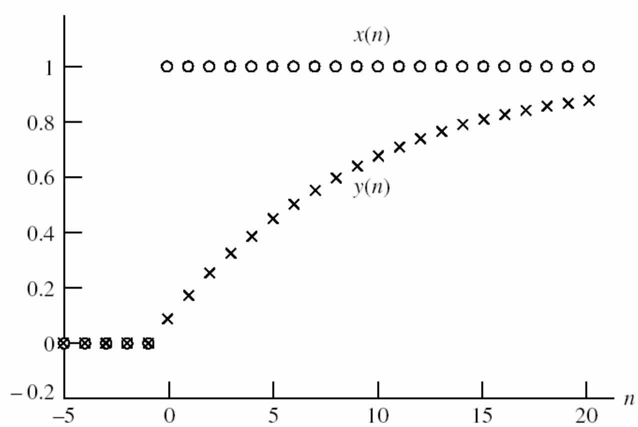


Figure 6.47 Step input and corresponding output of a first-order digital lowpass filter.

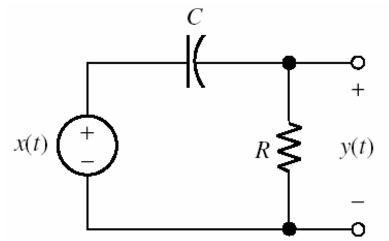


Figure 6.48 RC highpass filter. See Exercise 6.23.

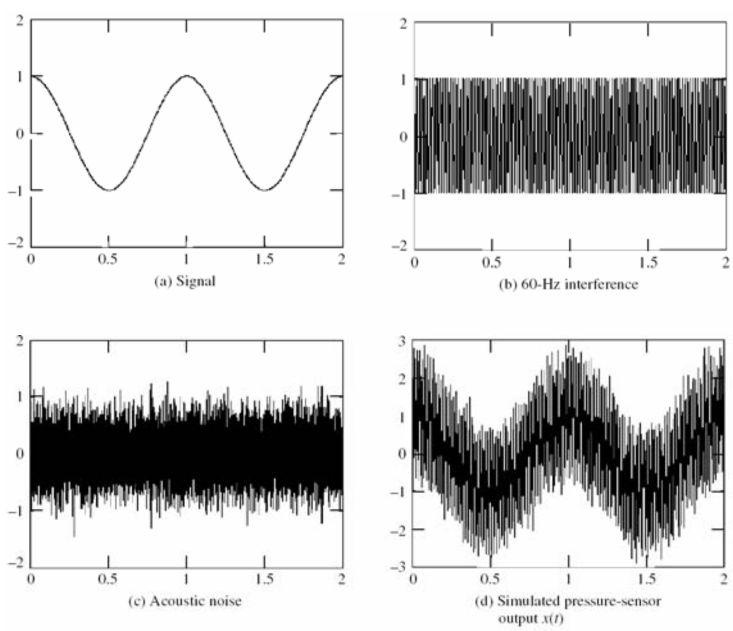


Figure 6.49 Simulated pressure-sensor output and its components.

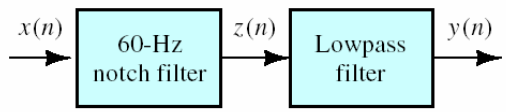


Figure 6.50 Digital filter.

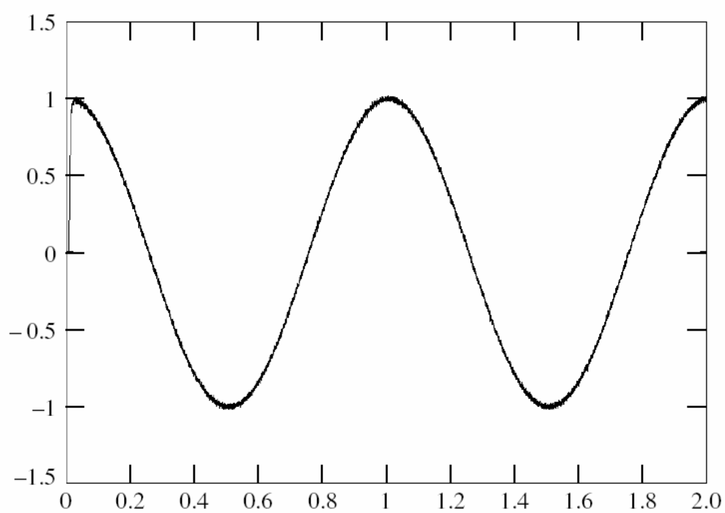


Figure 6.51 Output signal.