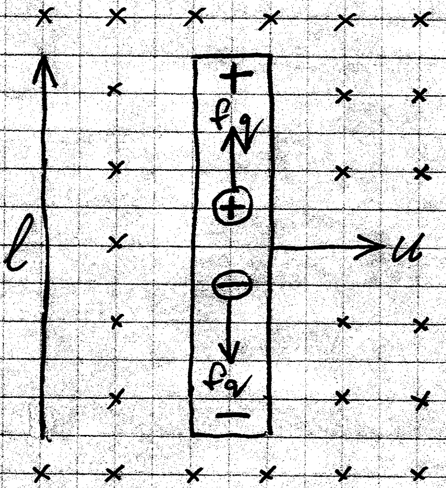
ECE 241 Lecture 16 ELECTRO-MECHANICS

TRANSLATIONAL TRANSDUCERS: (1) EMF

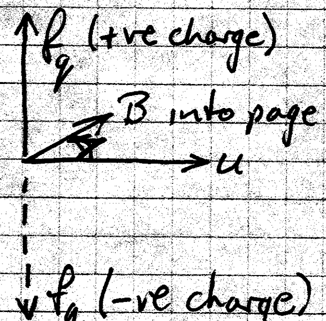


Conductor moving velocity $\hat{u}$

$\hat{B}$ into page

Force on each charged particle in the bar of length ℓ

$\hat{F} = q(\hat{u} \times \hat{B})$



At equilibrium:

Force on charge due to magnetic field = Force on charge due to electric field

i.e. $q\mathcal{E} = quB$

$\therefore \frac{e}{\ell} = Bu$ & emf: $e = B\ell u$

ℓu = area "swept"/unit time $\therefore e$ = rate at which flux $\phi = B(\ell \cdot ut)$ is "cut"

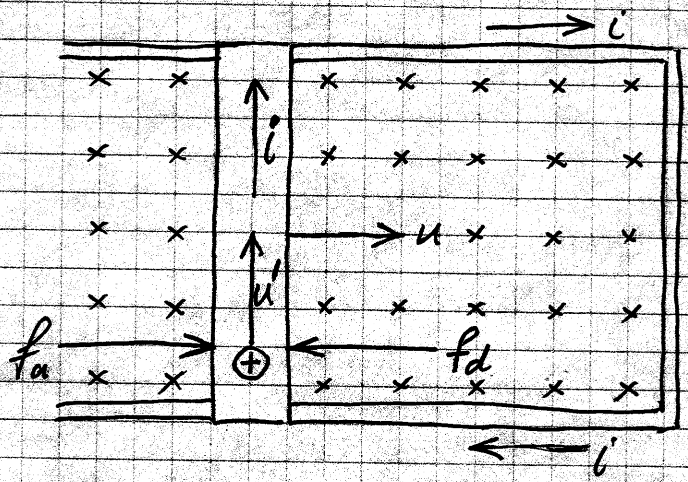
$$\therefore e = \hat{B}\cdot(\hat{\ell}\times\hat{u}) = B\ell u \cos\alpha \sin\beta \longrightarrow B\ell u$$

α: Angle B to normal to $\hat{\ell}$–$\hat{u}$ plane (0° here)

β: Angle $\hat{\ell}$ to $\hat{u}$ (90° here)

Note: $e = B\ell u = B\underbrace{\frac{dA}{dt}}_{\text{"sweep rate"}} = \frac{d\phi}{dt} = \underbrace{\frac{d\lambda}{dt}}_{(N=1)}$ rate of change of flux linkage.

TRANSLATIONAL TRANSDUCERS : (2) CURRENT FLOW



Bar is free to move on rails.

Charge velocity $u' \rightarrow i$

$$i\,dl = \frac{dq}{dt}(u'dt) = u'dq$$

i.e. $il = qu'$

Charges moving in the bar experience force:

"Developed force" $f_d = q(\hat{u}' \times \hat{B}) = i(\hat{l} \times \hat{B})$

on charges transferred to the bar.

$f_d = Bli \sin\gamma$

$\uparrow$ Angle $\hat{l}$ to $\hat{B}$ (90° here)

$= Bil$ for $\gamma = 90°$

For constant velocity $\hat{u}$, need force f_a applied equal to f_d

(Second order effect: Current i generates magnetic field, a perturbation of field B.
Assume negligibly small compared with $\hat{B}$.)

Note: This is a simple generator:–

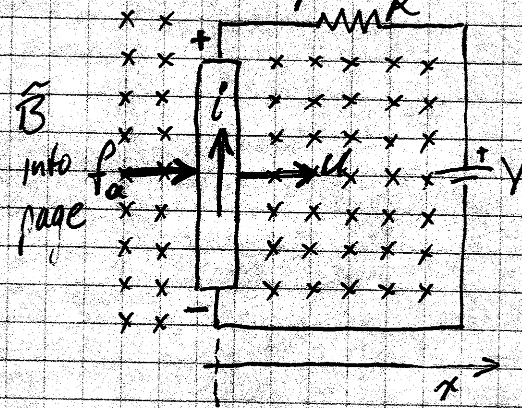
Mechanical energy (f_a) converts to electrical (i)

BILATERAL TRANSDUCER

For the Translational Transducer : Electrical power out $= P_E = ei = Blu \cdot i$

Mechanical power in $= P_M = f_a u = Bil \cdot u$

Circuit example:



$$i = \frac{e - V}{R}$$

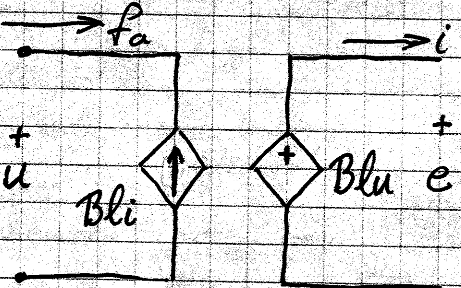
If $e = Blu > V$, $i > 0 \Rightarrow$ generator

" " $< V$ $i < 0 \Rightarrow f_d$ reverses

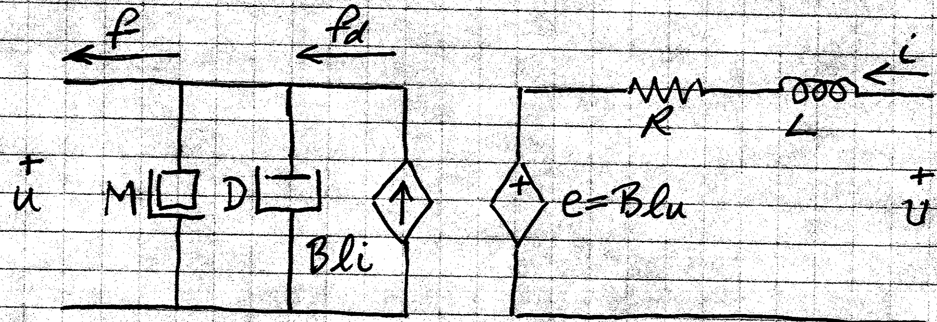
Conductor moved to right by $f_d = Bil$

Mechanical power input $< 0 \Rightarrow$ motor

ie. Motor/generator $\longrightarrow$ bilateral transducer



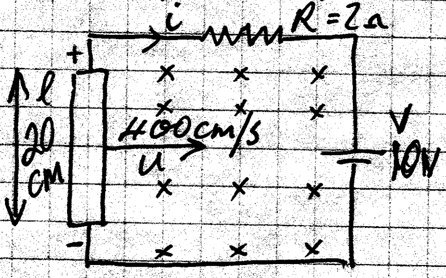
Add real effects $\longrightarrow$



$$f_d = Bli = f + Du + M\frac{du}{dt} \qquad e = Blu = v - Ri - L\frac{di}{dt}$$

Developed force f_d must overcome inertia (mass) and friction to drive motion u with force f.

Exercise 22.2



$B = 0.5T$

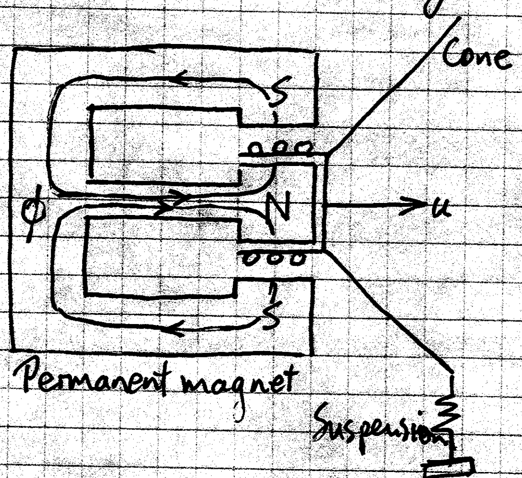
(a) $\frac{d\lambda}{dt} = Blu = 0.5 \times 0.2 \times 4 = e = 0.4V$

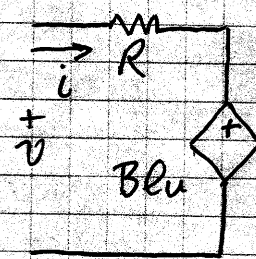
(b) $V = 10V$, $R = 2\Omega$ $\therefore i = \frac{0.4 - 10}{2} = -4.8A$

(c) i.e. operating as motor $\longrightarrow$ 10V source driving motion

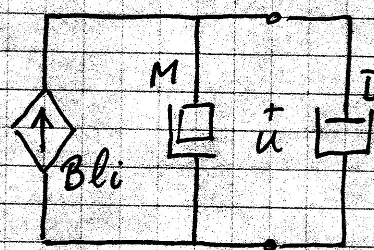
(d) $f_d = Bli = 0.5 \times 0.2 \times (-4.8) = -0.48N$
i.e. drives bar to right.

Exercise 22.3 Dynamic Loudspeaker (moving coil)





Elec



Mech

(See also Example 1)

0.6T across annular air gap, radius 1cm
length 2mm, depth 1cm ($\vec{u}$ dir^n)

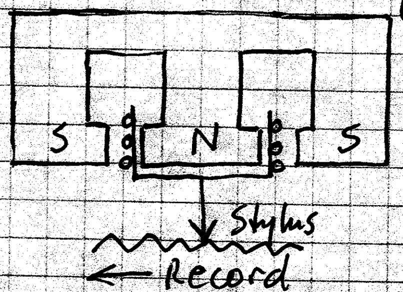
Max force 1N required from current $I_m \cos \omega t$. Find I_m

$f_d = Bli \longrightarrow NBli$
for N turns

$= NB(2\pi r)I = 1$ newton

$\therefore I = \frac{1}{100 \times 0.6 \times 2\pi \times 10^{-2}} = 0.265A$

Exercise 22.4 Dynamic Pickup (moving coil)



N turns, 1cm length/turn
$B = 0.2T$
Maximum displacement 0.02mm

Find N for 10mV out at 1000Hz

Displacement $x = A \sin \omega t = 2 \times 10^{-5} \sin 2\pi 10^3 t$

$\therefore u = \frac{dx}{dt} = (2 \times 10^{-5})(2\pi \times 10^3) \cos 2\pi 10^3 t$
$= 4\pi \times 10^{-2} \cos \omega t$

Effective $l = N \times 10^{-2}$ m

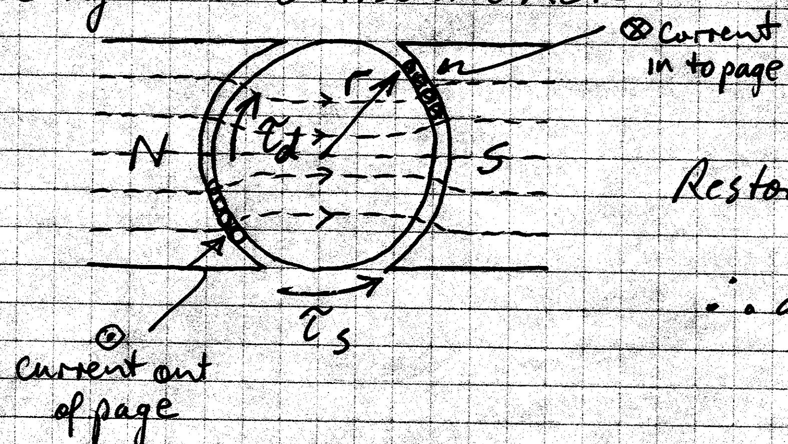
Voltage out $\simeq e = Blu = 0.2 \times 10^{-2} N \times 4\pi \times 10^{-2} \cos \omega t$
$= 8\pi N \times 10^{-5} \cos \omega t$

RMS V $\rightarrow 8\pi N \times 10^{-5} / \sqrt{2} = 10^{-2}$ V

$\therefore N = \sqrt{2} \times 10^3 / 8\pi = 56.3 \Rightarrow 57$ turns

ROTATIONAL TRANSDUCERS & METERS

See Fig 22·7 D'Arsonval meter



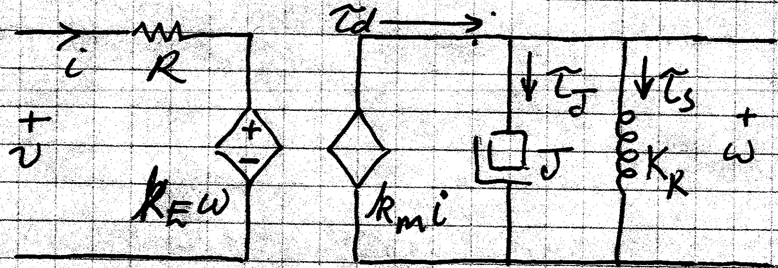
$2l$/turn, N turns, l length $\perp B$ into page

$$\tau_d = f_d \cdot r = B(2Nl)i \cdot r$$
$$= 2NBlr\,i = k_m i$$

Restoring torque τ_s due to springs, $\tau_s = \frac{\theta}{K_R}$ ← deflection, ← Compliance

∴ at equilibrium (meter reading)
$\tau_s = \tau_d$ and $\theta = K_R K_M i = K_\theta i$
$= K_R\, 2NBlr.\, i$

Meter Response :—



$$k_E = e/\omega$$
$$= \frac{2N.Blu}{\omega}$$
$$= 2NBlr = k_M$$
(since $u = \omega r$)

$$\tau_d = 2NBlr\,i = k_m i \longrightarrow \frac{T_d(s)}{I(s)} = k_M$$

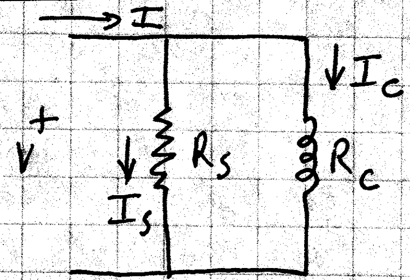
$$\tau_d = J\frac{d\omega}{dt} + \frac{1}{K_R}\int \omega\, dt \longrightarrow \frac{T_d(s)}{\Omega(s)} = sJ + \frac{1}{sK_R}$$

$$\frac{\Omega(s)}{I(s)} = \frac{T_d(s)}{I(s)} \cdot \frac{\Omega(s)}{T_d(s)} = \frac{k_M}{sJ + 1/sK_R} = \frac{sk_M K_R}{s^2 J K_R + 1} \longrightarrow \text{Imaginary poles}$$

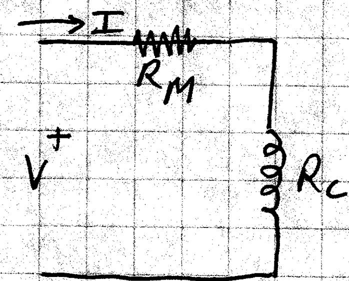
∴ undamped sinusoidal response of θ to i step

∴ Need to introduce damping

METERS: Use of the basic meter movement R_C



Ammeter
with shunt R_S



Voltmeter
with series R_M

Exercise 22.6

If d'Arsonval meter movement with coil resistance $R_C = 25\Omega$ requires 0.8mA for full-scale deflection:

(a) coil voltage at FS deflection
$= 0.8\text{mA} \times 25\Omega = 0.02\text{V}$

(b) R_S for current meter $0 \rightarrow 1\text{A}$ FS

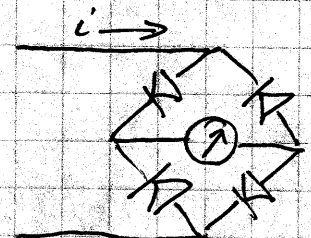
$\therefore$ at FS, $R_S = \dfrac{0.02\text{V}}{1 - 8\times10^{-4}} \approx 0.02\Omega$

(c) R_M for voltmeter $0 \rightarrow 10\text{V}$ FS

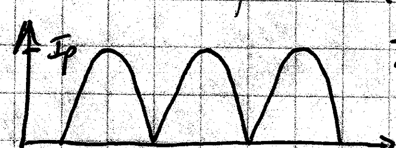
$\therefore$ at FS, $R_M = \dfrac{10\text{V} - 0.02\text{V}}{0.8\times10^{-3}} = 12.475\text{k}\Omega$

See also Ohmmeter

AC $\sim \longrightarrow$



Full wave rectification

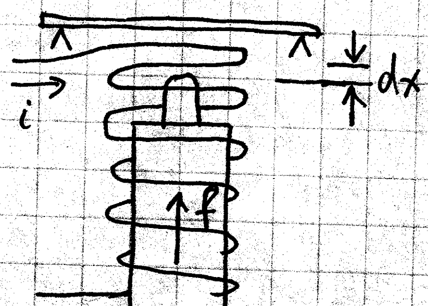


$I_{AV} = \dfrac{I_p}{\pi}$

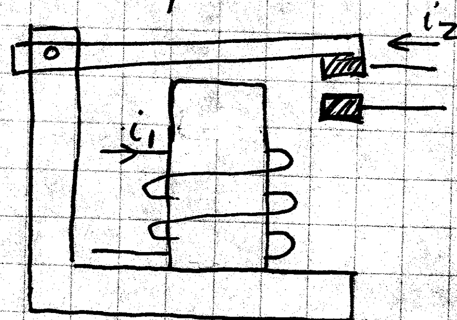
Scale reads RMS
i.e. Calibrated only for sine

MOVING IRON TRANSDUCERS (Virtual Work Method)

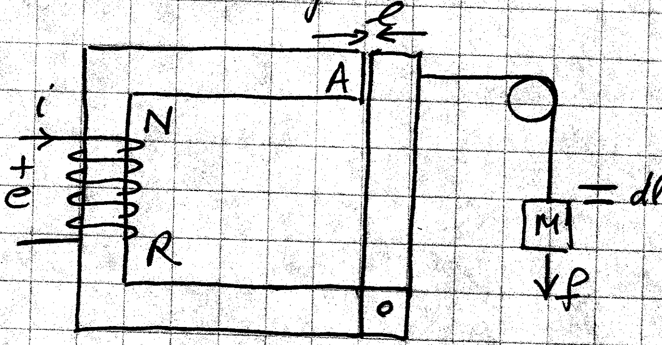
Door chime



Relay



Electromagnet



Virtual Work eg. Door chime

Mech Work done in movement $dx = f\,dx$

Energy stored $= \frac{1}{2}Li^2$ $\therefore$ change in energy storage $= \frac{i^2}{2}dL$ with constant i and geometry change

Electrical energy in $= dw = p\,dt = ei.dt$

$$= \left(N\frac{d\phi}{dt}\right)I\,dt = I\,d(N\phi) = I\,d(LI) = I^2 dL$$

$\therefore$ Elec energy in = Mech work done + increase in stored energy

$$I^2 dL = f\,dx + \tfrac{1}{2}I^2 dL \quad \therefore f = \frac{1}{2}I^2\frac{dL}{dx} \longleftarrow \text{general result}$$

Similarly $\tau = \frac{1}{2}I^2\frac{dL}{d\theta}$ for rotational transducer

Energy Balance eg. Electromagnet

Mech input + Elec Input = Mech storage + magnetic storage + heat

For movement $d\ell$

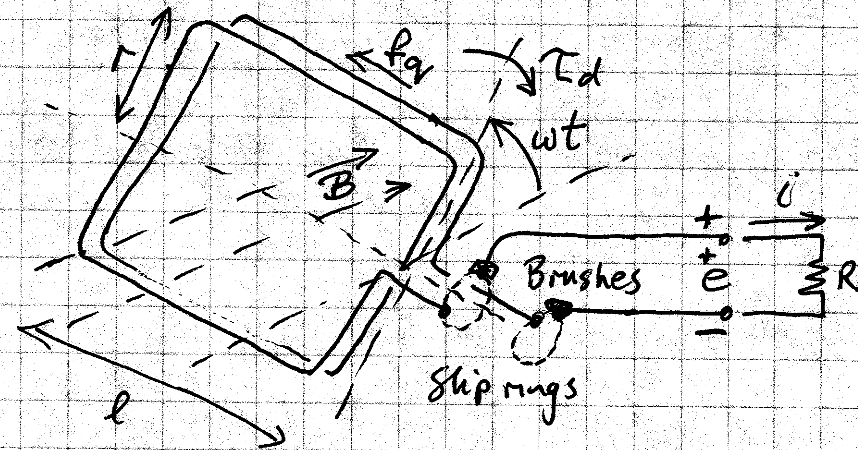
$$-f\,d\ell + i^2R\,dt = 0 - \frac{1}{2}\frac{B^2}{\mu_0}A\,d\ell + i^2R\,dt$$

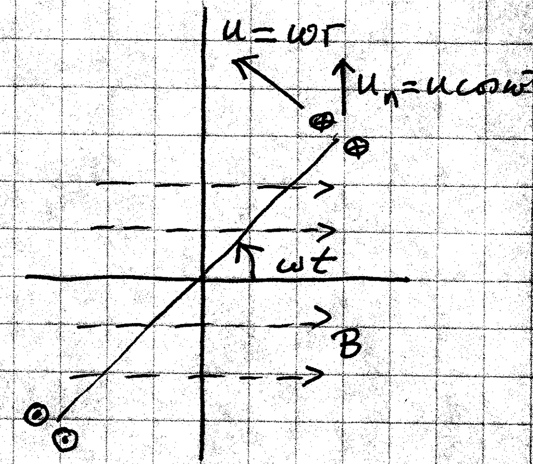
$$\text{i.e. } f = B^2A/2\mu_0$$

$$\& \; B = \frac{\phi}{A} \approx \frac{\mathcal{F}}{\mathcal{R}_{air}A} = \frac{Ni}{(\ell/\mu_0 A)A} = \frac{\mu_0 Ni}{\ell} \longrightarrow f = \mu_0 N^2 i^2 A/2\ell^2$$

DYNAMO

Generates sinusoidal emf.





2N conductors, length l, velocity u, field B

$$e = 2NBlu\cos\alpha\sin\beta \qquad \beta = 90^\circ \qquad 2rl = \text{Area } A$$

$$= 2NBlr\omega\cos\omega t = 2NBA\omega\cos\omega t$$

Similarly by flux linkage $e = \dfrac{d\lambda}{dt} = NB\dfrac{d(A\sin\omega t)}{dt} = NBA\omega\cos\omega t$

or by rate of flux "cut" $e = 2NBlu_n = 2NBlr\omega\cos\omega t = NBA\omega\cos\omega t.$

In general, $k_M = \dfrac{\tau_d}{i} = k_E = \dfrac{e}{\omega} = NBA$, so $e = k_E\,\omega = NBA\omega$

for rectangular coils in uniform field $\longrightarrow e = NBA\omega\cos\omega t$
for dynamo where BA seen by coils is not constant.

PROBLEMS 8(6) P22.4, P22.15, P22.35