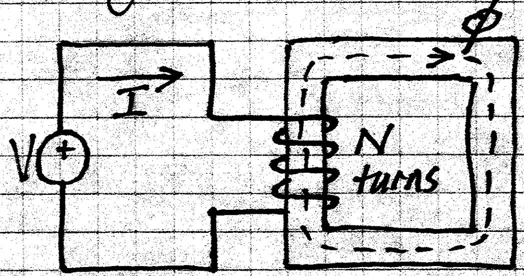
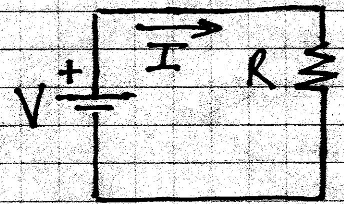
ECE 241 Lecture 15 TRANSFORMERS

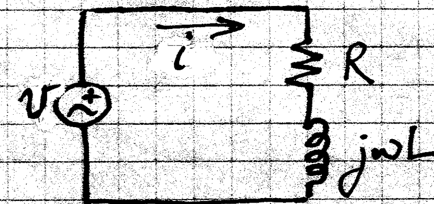
MAGNETIC CIRCUITS: AC



DC circuit



AC circuit



$I = V/R$ (windings)

$\phi_{DC} = NI/\mathcal{R}$

$= \mu NIA/l$

(constant)

Typically $R \ll \omega L$

$\phi_{AC} \longrightarrow$ induced voltage

$\lesssim v \approx j\omega L i$

INDUCTANCE

Induced voltage in coil linked to changing field $v = N\frac{d\phi}{dt} = \frac{d\lambda}{dt}$

(λ = number of flux linkages ... weber turns ... $N\phi$)

LENZ'S LAW: Induced voltage direction opposes the change in flux linkage that produces it.

ie. if flux ϕ is decreasing, the induced voltage direction ($v = N\frac{d\phi}{dt}$) will cause current to increase ϕ

Assuming constant μ (linear B-H curve):

For $\phi = \Phi_m \sin \omega t$

Induced voltage $v = N \dfrac{d\phi}{dt} = N\omega\Phi_m \cos\omega t$

i.e. $V_{rms} = \dfrac{V_M}{\sqrt{2}} = \dfrac{N\omega\Phi_M}{\sqrt{2}} = \dfrac{2\pi}{\sqrt{2}} Nf\Phi_m = 4{\cdot}44\, Nf\Phi_m$

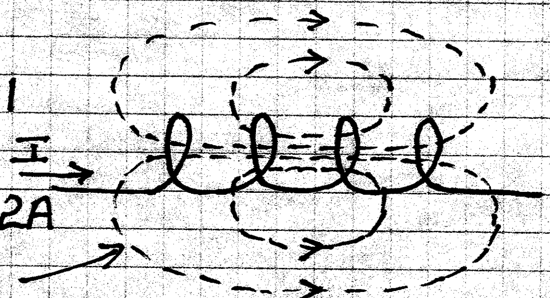
$v = L\dfrac{di}{dt} = N\dfrac{d\phi}{dt} \left(= \dfrac{d\lambda}{dt}\right)$ i.e. $L = N\dfrac{d\phi}{dt}\left(=\dfrac{d\lambda}{dt}\right)$

Magnetic Circuit

& for $\phi \propto i$

$L = N\dfrac{\phi}{i} \left(= \dfrac{\lambda}{i}\right)$

Example 1



Each line represents 1mWb flux

Find λ and L:

2 turns link 4 mWb
other 2 turns link 2 mWb

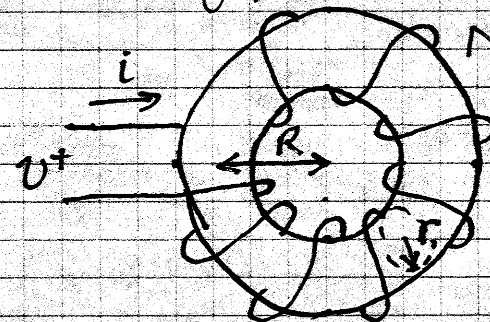
$\therefore \lambda = 2\times4\times10^{-3} + 2\times2\times10^{-3} = 12$ mWb.t

$\therefore L = \lambda/i = 12\times10^{-3}/2 = 6$ mH

Note "leakage" effect of field lines linking few turns

$\left.\begin{array}{l}\longrightarrow \text{high permeability core} \\ \longrightarrow \text{tightly wound coils}\end{array}\right\} \longrightarrow$ high L

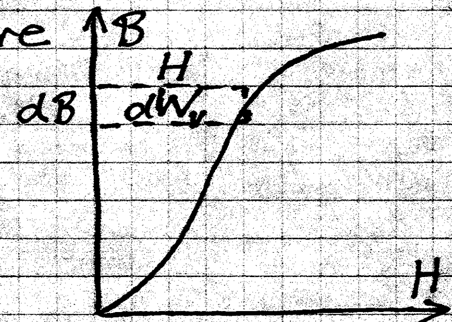
ENERGY STORAGE



$$\mathcal{F} = Ni = H\overset{\approx 2\pi R}{l}$$

$\phi = BA$ for $r \ll R$, approx uniform B, H

For initially unmagnetized core



$$W = \int_0^t vi\,dt = \int_0^t N\frac{d\phi}{dt}\cdot i\,dt$$

$$= \int_0^\phi Ni\,d\phi = \int_0^B (Hl)(A\,dB)$$

$$= \underbrace{(lA)}_{\text{volume}}\int_0^B H\,dB$$

i.e. Energy density $W_V = \frac{W}{vol} = \int_0^B H\,dB$

(see above for non-linear interpretation)

For linear $B = \mu H$, $W_V = \int_0^B \frac{B}{\mu}\cdot dB = \frac{1}{2}\frac{B^2}{\mu}$

$$= \int_0^H H.\mu\,dH = \frac{1}{2}\mu H^2$$

Note also: $W = (lA)\int_0^B H\,dB = \frac{(lA)}{2\mu}B^2 = \frac{(lA)}{2\mu}\cdot\frac{\phi}{A}\cdot\frac{\phi}{A} = \frac{(lA)}{2\mu}\cdot\frac{\phi}{A}\cdot\frac{Ni}{A.\frac{1}{\mu}\frac{l}{A}} = \frac{1}{2}N\phi i$

$$= \frac{1}{2}\,\frac{N\phi}{i}\,i^2 = \frac{1}{2}Li^2$$

Ex 21.2 Thin toroid $r \ll R$ with air gap length l_a. Toroid "length" $l_i \approx 2\pi R$

Toroid permeability $\mu_r = 1800$ Find $\frac{l_a}{l_i}$ for $W_a = 0.9\, W_i$

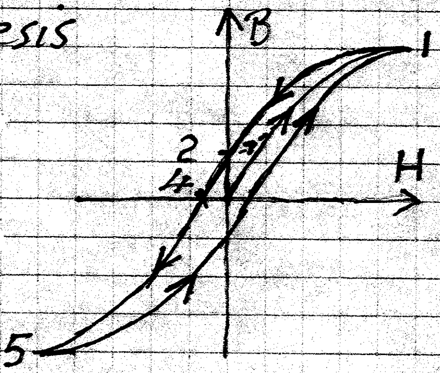
$$\frac{W_a}{W_i} = \frac{l_a A_a}{l_i A_i} \, \frac{B_a^2/\mu_0}{B_i^2/\mu_r \mu_0} \longrightarrow \mu_r \frac{l_a}{l_i}$$

assuming negligible fringing in the gap & hence that $A_a = A_i$, $B_a = B_i$

$$\text{So } \frac{l_a}{l_i} = \frac{1}{\mu_r} 0.9 = \frac{0.9}{1800} = 5 \times 10^{-4} \Longrightarrow 0.05\%$$

PHYSICAL ENERGY LOSS MECHANISMS

(a) Hysteresis



1: Saturation $\rightarrow dB = \mu_0 dH$

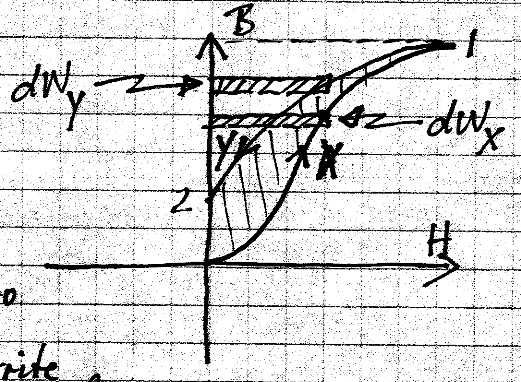
2: Residual magnetism

(If re-apply $H > 0$, path $2 \to 1$)

Cyclic path $1 \to 2 \to 4 \to 5 \to 1$

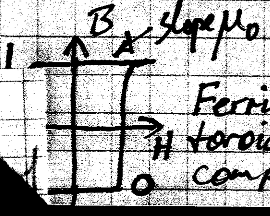
Area enclosed $\rightarrow$ hysteresis loss

From 2, negative H (coercive force) needed for $B = 0$ (pt 4)



Energy absorbed, path X $\int_0^{B_1} H_X dB_X$

Energy returned, path Y $\int_{B_1}^{B_2} H_Y dB_Y <$ path X



Ferrite toroids for computer memory

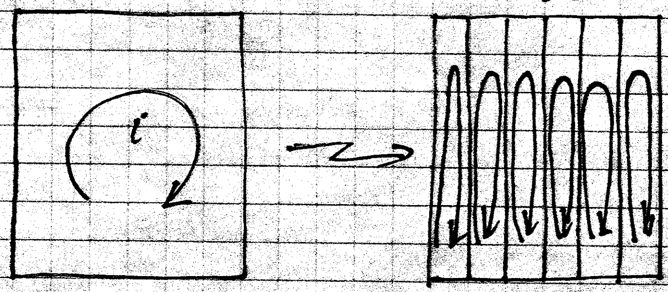
Energy absorbed $\rightarrow$ heats up, inelastic domain rotation etc

$P_H = K_H f B_m^n$ (n typ 1.6)

(b) Eddy currents

Conductive core material (eg iron), changing ϕ induces v_{core} in core

$v_{core} \rightarrow i_{core}$ & $i_{core}^2 R$ Losses $\rightarrow$ Joule heating.



Solid core

Laminated (eg. varnish)

Laminations reduce induced v_{core} and increase path resistance.

Losses (Joule heating) $P_E = K_E \underbrace{f^2 B_m^2}$

$v_{induced} \propto f B_m$, $P \propto v_{induced}^2$

(c) Copper losses — winding resistances.

TRANSFORMER

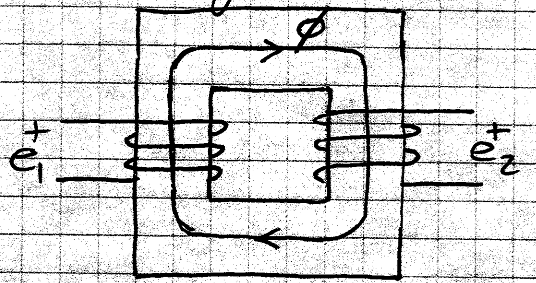
Primary voltage $\rightarrow$ primary current $\rightarrow$ generates core flux $\rightarrow$ generates secondary induced voltage $\rightarrow$ drives secondary load current.

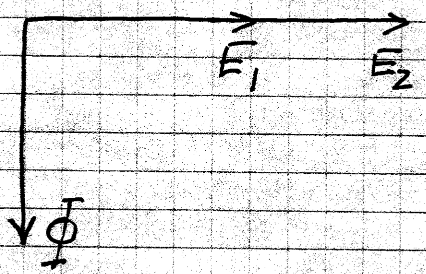
See Fig 2.19 for construction examples. (Also toroid, etc)

VOLTAGE & CURRENT RELATIONS

Assuming negligible flux leakage, tight coupling $\longrightarrow$ i.e. both coils link same flux

Voltage induced $\longrightarrow$ electromotive force (emf)





For $\phi = \Phi_m \sin\omega t$

$$e_1 = N_1 \frac{d\phi}{dt} = N_1 \omega \Phi_m \cos\omega t$$
$$= \sqrt{2}\, E_1 \cos\omega t$$
$$e_2 = N_2 \frac{d\phi}{dt} = N_2 \omega \Phi_m \cos\omega t$$
$$= \sqrt{2}\, E_2 \cos\omega t$$

$\sin\omega t = \cos(\omega t - 90°)$

$\therefore$ flux phasors lags the emf phasors by $\pi/2$

$$e_2/e_1 = E_2/E_1 = N_2/N_1$$

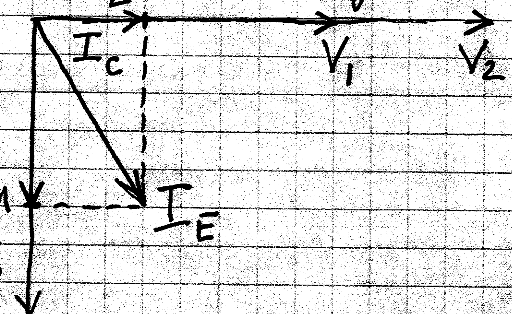
Terminal voltages $\neq$ emfs.

EXCITING CURRENT

Open circuit secondary $\longrightarrow$ v_2 at terminals 2 $= e_2 =$ induced emf$_2$

Ideally $I_1 = 0$ if $I_2 = 0$, but in practice current must flow to generate ϕ.

so $I_1\big]_{I_2=0} = I_E$ (exciting current) $= I_M$ (magnetizing current) $+ I_C$ (core loss current)



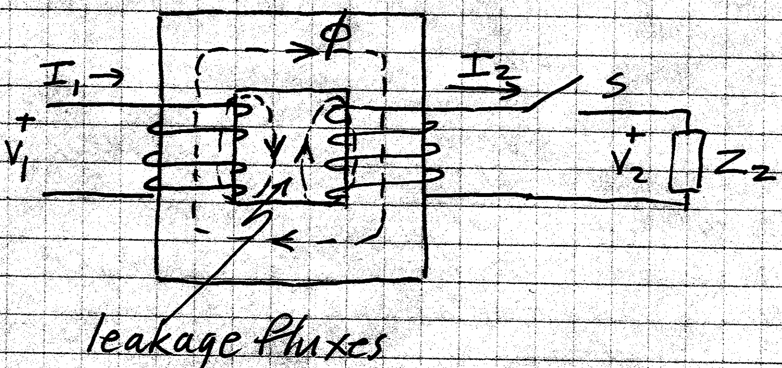
Magnetic circuit $\longrightarrow$ I_M

$\phi \propto I_M$

Hysteresis/eddy currents

(typically/ideally small $\ll I_m$)

PRIMARY & SECONDARY CURRENTS WITH LOAD



Assume $V_2 = E_2$ $V_1 = E_1$ $I_c = 0$ so $I_E = I_M$

With S closed $I_2 = V_2/Z_2$ flows

$I_2 \longrightarrow$ mmf $N_2 I_2$ which

<u>Opposes</u> ϕ (Check by RH Rule as drawn)

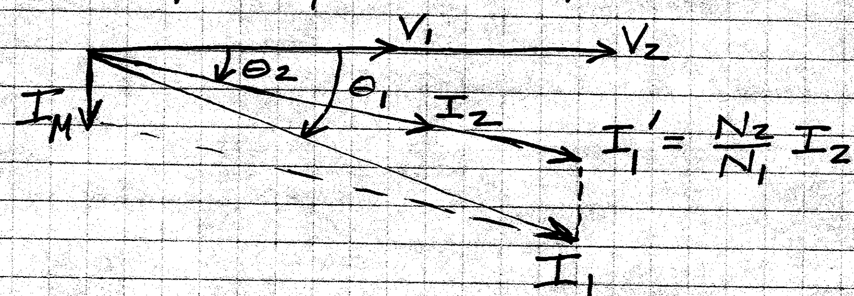
Additional I_1 current must flow to offset the potential reduction in ϕ.

New net mmf = mmf due to I_m (no load)

$$N_1 I_M - N_2 I_2 + N_1 I_1' = N_1 I_M$$

added primary current for finite Z_2, I_2

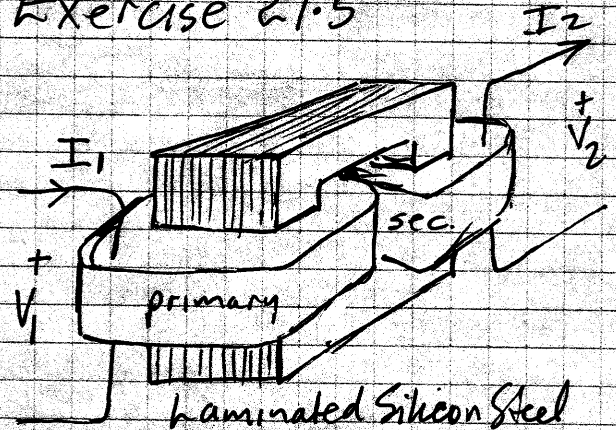
$$\therefore I_1' = \frac{N_2}{N_1} I_2$$



I_2 phase θ_2 depends on Z_2

$P_{IN} = I_1 V_1 \cos\theta_1$

Exercise 21.5



Laminated Silicon Steel
Core length 0.6m
C/S $0.005\,m^2$

$N_1 = 200 \quad N_2 = 800$ Find I_1 for (a) OC secondary

$V_1 = 200\,V_{rms}$ at 60 Hz (b) Secondary load = $200\,\Omega$

(a) Magnetizing flux $= \Phi_m = \dfrac{\sqrt{2}\,E_1}{N_1\,\omega} = \dfrac{\sqrt{2}\;200}{200\times 2\pi\times 60}$

$= 3.75\times 10^{-3}\,\text{wb}$

$B_m = \Phi_m/A = 3.75\times10^{-3}/5\times10^{-3} = 0.75\,T$

For Si steel $B_m = 0.75T \longrightarrow H = 130\,A.t/m$

Magnetizing current $I_m = \dfrac{1}{\sqrt{2}}\dfrac{\mathcal{F}}{N_1} = \dfrac{Hl}{\sqrt{2}\,N_1} = \dfrac{130\times 0.6}{\sqrt{2}\times 200}$

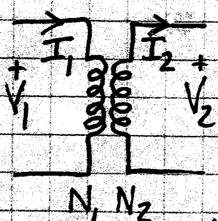
$= 0.276\,A = 276\,mA$

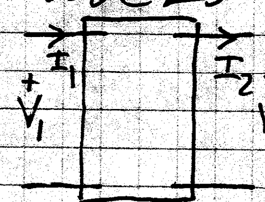
(b) Assuming $V_2 = \dfrac{N_2}{N_1}V_1 = \dfrac{800}{200}\,200 = 800V.$

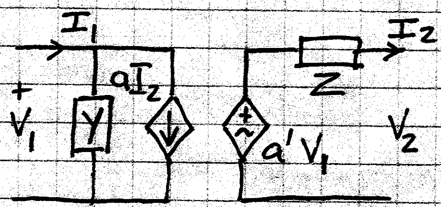
With $R_L = 200\,\Omega$ connected $I_2 = V_2/R_2 = 800/200 = 4A$

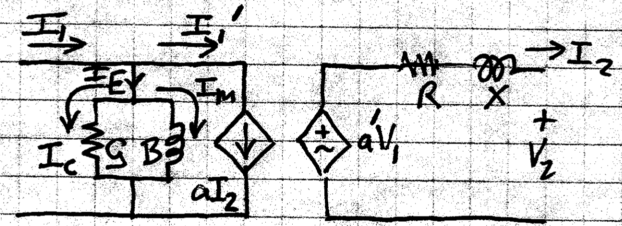
$\therefore I_1' = \dfrac{N_2}{N_1}I_2 = \dfrac{800}{200}\times 4 = 16A$

LINEAR MODELS









Hybrid parameters $I_1 = YV_1 + aI_2$

$V_2 = a'V_1 - ZI_2$

$I_2 = 0 \longrightarrow I_1 = I_E \therefore Y = I_E/V_1 = \dfrac{I_C}{V_1} - j\dfrac{I_M}{V_1}$

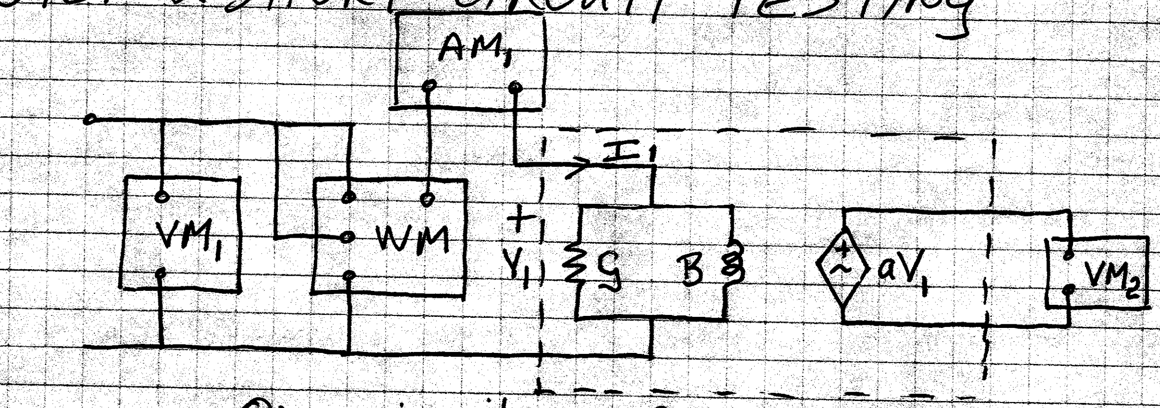
$a' = N_2/N_1$

$= G + jB$

$I_2 \neq 0 \quad I_1' = \dfrac{N_2}{N_1}I_2$ added to $I_1 \therefore a = \dfrac{N_2}{N_1} = a'$

$R \rightarrow$ losses in both windings, $L \rightarrow$ leakage inductance

OPEN & SHORT-CIRCUIT TESTING



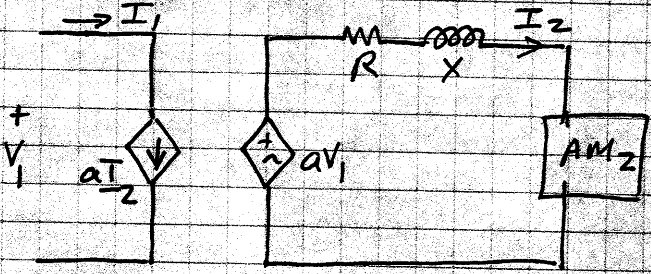
Open circuit — R, X in secondary no effect

$I_1' = aI_2 = 0$

$I_1 = I_E$

core loss

$a = \frac{V_{20}}{V_{10}}, \; G = \frac{P_{oc}}{V_{10}^2}$ & $Y = \frac{I_{10}}{V_{10}} \therefore B = -\sqrt{Y^2 - G^2}$



Short circuit

V_1 very small $\therefore$ Y negligible

$P_{SC} = I_{2S}^2 R = (I_{1S}/a)^2 R$

Copper loss

$\therefore R = a^2 \frac{P_{SC}}{I_{1S}^2}$

$Z = \frac{aV_{1S}}{I_{2S}} = a^2 \frac{V_{1S}}{I_{1S}}$ & $X = \sqrt{Z^2 - R^2}$

(AM_2 unnecessary, but useful check)

Exercise 21.6 Open circuit: $V_1 = 1000V$, $V_2 = 250V$, $I_1 = 0.3A$, $P = 20W$
Short circuit: $I_2 = 40A$, $V_1 = 128V$, $P = 100W$

$$OC \begin{cases} a = V_{20}/V_{10} = 250/1000 = 1/4 \\ Y = I_{10}/V_{10} = 0.3/1000 = 0.3mS = 300\mu S \end{cases}$$

$$G = \frac{P_{oc}}{V_{10}^2} = \frac{20}{1000^2} = 20\mu S$$

$$B = -\sqrt{Y^2 - G^2} = -\sqrt{300^2 - 20^2} \times 10^{-6} \approx -300\mu S$$

$$R = a^2 P_{sc}/I_{1S}^2 = \frac{1}{16} \frac{100}{(40 \times \frac{1}{4})^2} = \frac{1}{16}\Omega$$

$$X = \sqrt{Z^2 - R^2} = \sqrt{a^4 \left(\frac{V_{1S}}{I_{1S}}\right)^2 - R^2} = \sqrt{\left(\frac{1}{16} \cdot \frac{128}{10}\right)^2 - \left(\frac{1}{16}\right)^2} = 0.8\Omega$$

TRANSFORMER MISCELLANEOUS

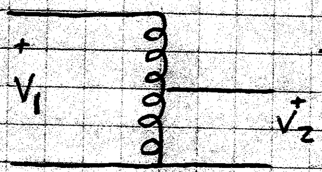
(a) Ratings   e.g. 4400 : 220 V   60 Hz   10 kVA

Turns ratio = 20 : 1   Either side primary

10 kVA rated "full" load → heating due to currents
∴ kVA rated rather than power kW

At 60 Hz, specified voltages bring magnetization up to knee of magnetization curve, but losses not excessive

(b) Autotransformer — some turns carry both primary & secondary currents



— no isolation

(c) Efficiency $= \dfrac{\text{output power}}{\text{input power}} = \dfrac{\text{output}}{\text{output + losses}} = 1 - \dfrac{\text{losses}}{\text{input}}$

(d) Voltage regulation $= \dfrac{\text{No-load} - \text{full-load voltages}}{\text{full load voltage}}$

ASSIGNMENT #8   Part A   Problems 21.5, 21.16, 21.35