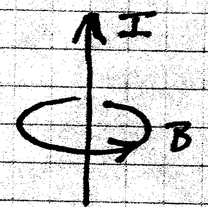
ECE 241 LECTURE 14 MAGNETIC FIELDS & CIRCUITS

MAGNETIC FIELDS

Moving charge $\longrightarrow$ magnetic field

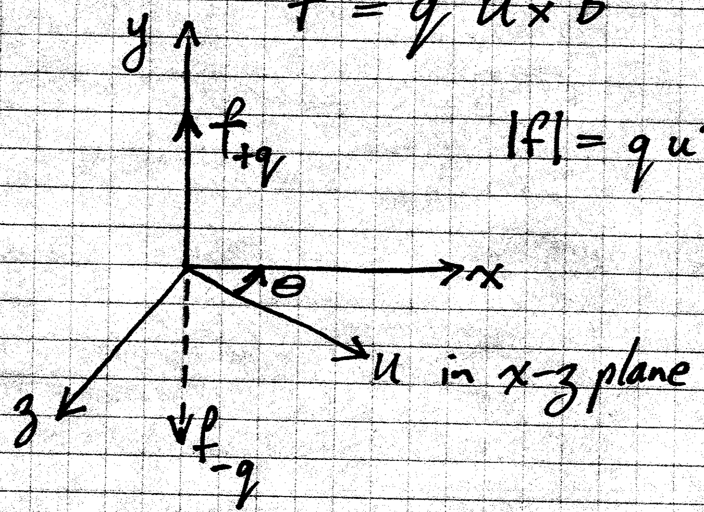


Moving charge in magnetic field experiences force $\longrightarrow$ used to define magnetic field units

$$\bar{f} = q\, \tilde{u} \times \tilde{B}$$

Charge 1C moving 1m/s in field 1 Tesla (1T) experiences force 1N

$$|f| = q u B \sin\theta$$



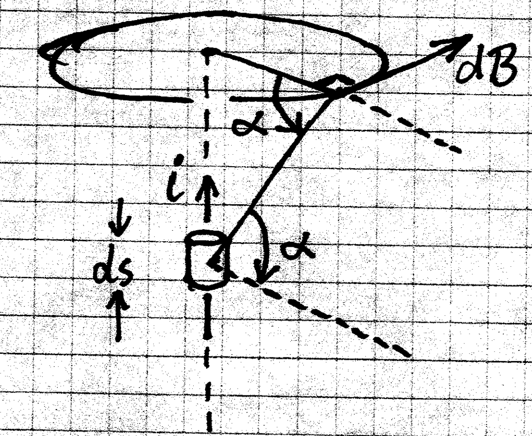
Magnetic Flux (webers) $\phi = \int B.dA$

$\uparrow$ magnetic field (Teslas)

Represent magnetic flux density by lines (lines of magnetic force.) Tangents to flux density vector $\tilde{B}$.

Flux entering any closed surface = flux leaving, i.e. $\oint B.dA = 0$

FIELD DUE TO CURRENT



dB due to element ds carrying current i

$$i = \frac{dq}{dt} \qquad dq \text{ moves } ds \text{ in time } dt$$

$$\therefore u = \frac{ds}{dt}$$

$$\therefore i\,ds = \frac{dq}{dt}\cdot(u\,dt) = u\,dq$$

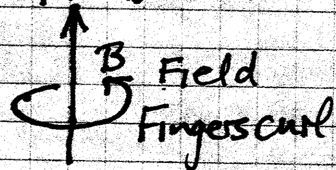
$$dB = \mu \frac{i\,ds\cos\alpha}{4\pi r^2}$$

μ = permeability
$= \mu_r \mu_0$
μ_0 ← freespace
μ_r ← relative permeability

{units Wb/A-m or H/m}

$\mu_0 = 4\pi \times 10^{-7}$ H/m

RH Rule



MAGNETIC FIELD INTENSITY $\bar{H} = \frac{\bar{B}}{\mu}$ in A/m

$\hat{H}$ is "tendency" of moving charge to produce a field

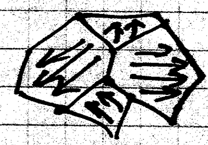
$\hat{B}$ is the field actually produced, which depends on medium

e.g. higher in ferromagnetic materials high μ_r

FERROMAGNETISM

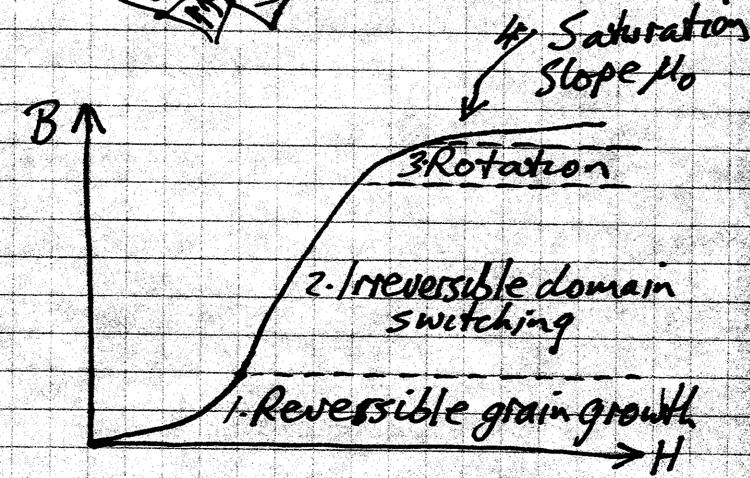
Magnetic domains → aligned electron spins

Randomly oriented → no external field



Apply external field to align domains

1. ~Aligned domains grow
2. Misaligned domains switch polarization
3. Domain orientations rotate
4. Eventually all domains aligned
   → saturation



$$\tilde{B} = \mu_0 (\tilde{H} + \tilde{M}) \longrightarrow \mu_0 \left(1 + \frac{\tilde{M}}{\tilde{H}}\right)\tilde{H} = \mu_0 \mu_r \tilde{H} = \mu \tilde{H}$$

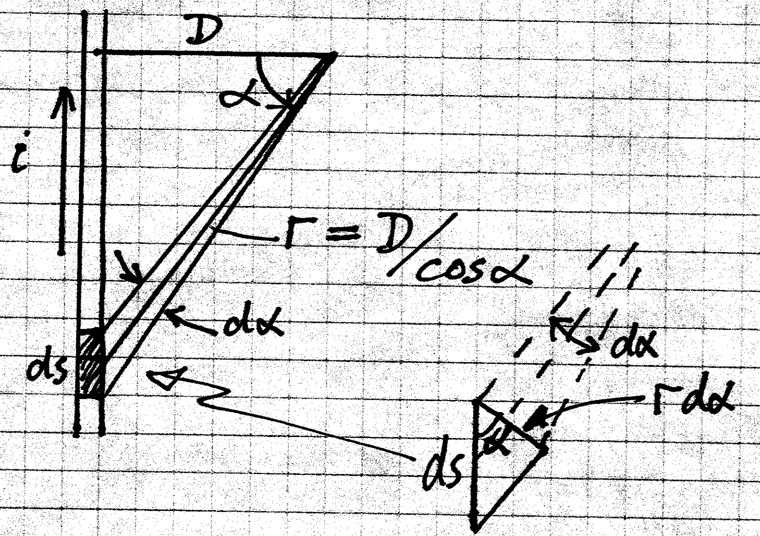
Applied field intensity

Magnetic Polarization

μ_r not constant

Often assumed ~ constant.

LONG STRAIGHT CONDUCTOR



$$dB = \frac{\mu i \, ds \cos\alpha}{4\pi r^2}$$

$$\therefore B = \frac{\mu i}{4\pi}\int_{-\infty}^{\infty} \frac{\cos\alpha}{r^2}\, ds$$

$$= \frac{\mu i}{4\pi D}\int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha$$

$$= \frac{\mu i}{4\pi D}\Big[\sin\alpha\Big]_{-\pi/2}^{\pi/2} = \frac{\mu i}{4\pi D}\big[1-(-1)\big] = \frac{\mu i}{2\pi D}$$

$$ds \cos\alpha = r\, d\alpha$$

$$\frac{ds\cos\alpha}{r^2} = \frac{r\,d\alpha}{r^2} = \frac{d\alpha}{r} = \frac{\cos\alpha . d\alpha}{D}$$

Note also: $\oint \hat{H}.d\hat{l} = \oint \frac{B}{\mu}\, dl = \frac{i}{2\pi D}\cdot\oint dl = \frac{i}{2\pi D}\cdot 2\pi D = i$

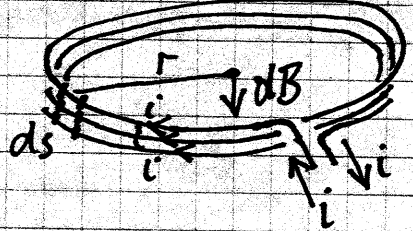
around closed
loop radius D

Line integral of field intensity $\hat{H}$ along any closed path = current linked by the path.

$\int H.dl$ = magnetomotive force
(mmf)

Similar effect in magnetic circuits to emf in electric circuits.

COIL FLUX DENSITY



B at center of flat coil, N-turns

$$dB = \frac{\mu i\, ds \cos\alpha}{4\pi r^2} \xrightarrow[\cos\alpha = 1]{\alpha = 0} \frac{\mu i\, ds}{4\pi r^2}$$

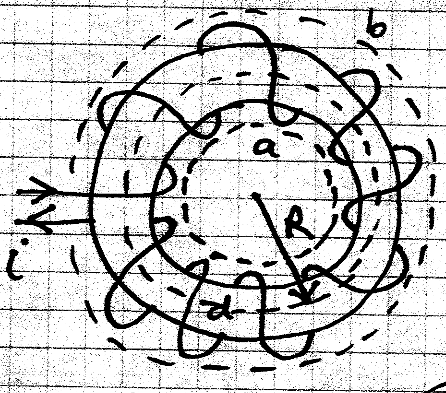
B for one turn $= \int dB = \dfrac{\mu i}{4\pi r^2}\displaystyle\int_0^{2\pi r} ds = \dfrac{\mu i}{2r}$

B for N turns $= \dfrac{\mu N i}{2r}$

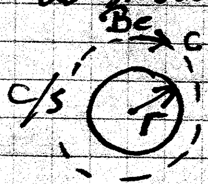
eg. Exercise 20.2   10 turn coil, $r = 10\text{ cm}$, $i = 0.1\text{A} \longrightarrow B = \dfrac{4\pi\times 10^{-7}\times 10\times 0.1}{2\times 10^{-1}}$

$= 2\pi\ \mu T$

TOROID FLUX DENSITY



Paths a, b, c, d as shown.



Path a links no current. $\therefore \oint_a H.dl = 0$

$\therefore \hat{H}_a = 0$

Path b links no current (net current in $= 0$)

$\therefore \hat{H}_b = 0$

Path c links i $\therefore \oint H\,dl = i$ $\therefore$ Find H around toroid, into center

$H.\,2\pi r_c = i$, $H = \dfrac{i}{2\pi r_c}$

Path d links i N times

$\therefore \oint \hat{H}.dl = H.\,2\pi R = Ni$ Amp-turns

$B = \mu H = \dfrac{\mu N i}{2\pi R}$

For approx uniform flux density in core ($r \ll R$)

$\phi = BA = \dfrac{\mu N i}{2\pi R}\pi r^2 = \dfrac{\mu N i r^2}{2R}$

Note: Flux $\phi = \mu \frac{\pi r^2}{2\pi R} Ni = \mu \frac{A}{l} \mathcal{F} \longleftarrow$ Basis of magnetic circuit

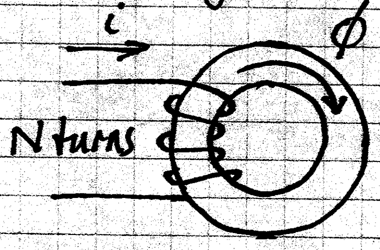
Magnetic field summary:

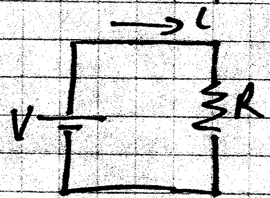
$$\bar{f} = q\hat{u} \times \hat{B} \qquad dB = \frac{\mu i \, ds \cos\alpha}{4\pi r^2}$$

$$\phi = \int \hat{B} \cdot d\hat{A} \qquad \oint \hat{B} \cdot d\hat{A} = 0$$

$$\hat{B} = \mu \hat{H} \qquad \oint \hat{H} \cdot d\hat{l} = Ni$$

MAGNETIC CIRCUITS





Toroid:

Showed $\phi = BA = \mu N i \frac{\pi r^2}{2\pi R} = \mu \frac{A}{l} \mathcal{F}$

(above)

ie. $\frac{\mathcal{F}}{\phi} = \frac{1}{\mu} \frac{l}{A} = \mathcal{R}$ (reluctance) Amp-turns/Wb

$\mathcal{F} = Ni$ — mmf magnetomotive force

Compare $\frac{V}{I} = \frac{1}{\sigma} \frac{l}{A} = R$

Analogs:

Electric		Magnetic	
Current density	J	Magnetic flux density	B
Current	I	Magnetic flux	ϕ
Electric field intensity	E	Magnetic field intensity	H
Voltage (emf)	V	Magnetomotive force	$\mathcal{F}$
Conductivity	σ	Permeability	μ
Resistance	R	Reluctance	$\mathcal{R}$

$$El = V = IR = \frac{J}{\sigma} l$$

$$Hl = \mathcal{F} = \phi\mathcal{R} = \frac{B}{\mu} l$$

Exercise 20.4: To establish flux 0.5 mWb in a toroidal iron core with 400 turns and $\mu_r = 1000$, c/s area $= 10^{-3}\ m^2$ & $R = 0.04\ m$, find reluctance $\mathcal{R}$, mmf, and current required.

$$\text{Reluctance } \mathcal{R} = \frac{1}{\mu}\frac{l}{A} = \frac{1}{10^3 \times 4\pi \times 10^{-7}} \cdot \frac{2\pi(0.04)}{10^{-3}} = 2\times 10^5\ A.t/Wb$$

turns

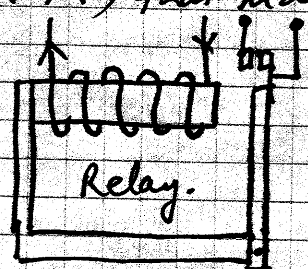
$\mu = \mu_r \mu_0$

$$\underset{mmf}{\mathcal{F}} = \mathcal{R}\phi = 2\times 10^5\ A.t/Wb \times 0.5\times 10^{-3}\ Wb = 100\ A.t$$

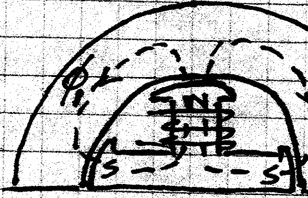
$$\therefore\ i = \frac{\mathcal{F}}{N} = \frac{100\ A.t}{400\ t} = 0.25 A$$

Where there is a winding on a high permeability material, flux is largely confined to (or concentrated in) that material.

Examples



Magnetic path around loop, across gap.



1/4 4-pole generator

Flux concentrated in ferromagnetic (iron) core --- plus air gap.

By analogy with electric circuits:

Series elements: $\phi = \phi_1 = \phi_2 = \phi_3 = \dots \phi_n$

$\mathcal{F} = \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 + \dots \mathcal{F}_n$

Parallel elements: $\phi = \phi_1 + \phi_2 + \phi_3 + \dots \phi_n$

$\mathcal{F} = \mathcal{F}_1 = \mathcal{F}_2 = \mathcal{F}_3 = \dots \mathcal{F}_n$

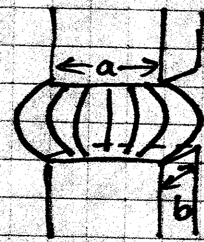
SERIES MAGNETIC CIRCUITS

Determine flux density $B = \Phi/A$ in each element & corresponding field intensity $H = B/\mu$

Calculate the mmf drop $\mathcal{F} = Hl$ for each element

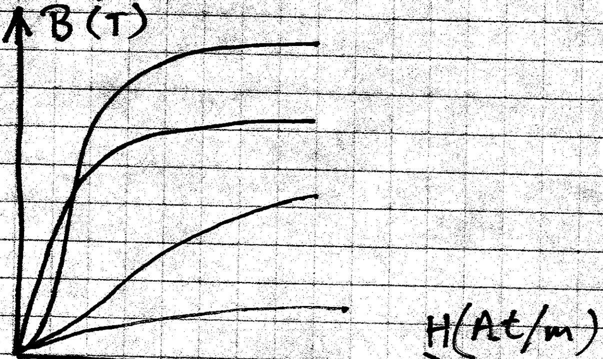
Calculate total mmf $= Ni$ and required current.

Air gap: $\mu \rightarrow \mu_0$ in air Account for fringing



1. Assume constant area $\rightarrow$ increases mmf due to gap

2. Account for fringing $\rightarrow$ effective c/s area $= (a+l) \times (b+l)$



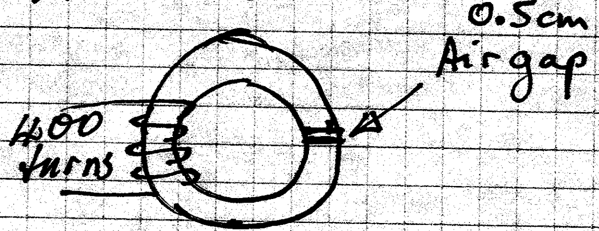
Find μ_r from magnetization curves

OR can often neglect $\mathcal{R}_{ferro}$

since ~~thy~~ air gap dominates $\mathcal{R}$.

See Examples 4 & 5. $\leftarrow$ Show that error in neglecting ferro $\mathcal{R} \sim 2.5\%$

Exercise 20-5



$R = 0.04$ m

C/s area $= \pi r^2 = 10^{-3}\,m^2$

$\phi = 10^{-3}$ Wb

$\mu_r = 1000$

Determine $\mathcal{R}_{tot}$ & i for $\phi = 10^{-3}$ Wb

Account for fringing.

$$\mathcal{R} = \frac{1}{\mu_r \mu_0}\,\frac{2\pi R - 5\times 10^{-3}}{A} + \frac{1}{\mu_0}\,\frac{5\times 10^{-3}}{A_{air}}$$

Rect. core: $(a+l)(b+l)$

$$= \underbrace{ab}_{A = 10^{-3}\,m^2} + \underbrace{(a+b)l}_{a=?\ b=?} + \underbrace{l^2}_{25\times 10^{-6}\,m^2}$$

Circular core: $\pi r^2 = 10^{-3}\,m^2$

$r = (10^{-3}/\pi)^{1/2} = 0.018$ m

$\therefore$ effective diameter $= 2\times 0.018 + 0.005$
$= 0.043$ m

$\therefore$ effective area for fringing
$= \pi(.0215)^2$

$\gg 5\times 10^{-3}$

$$\mathcal{R} = \frac{2\pi(0.04)}{10^3\, 4\pi 10^{-7} \times 10^{-3}} + \frac{1}{4\pi 10^{-7}}\,\frac{5\times 10^{-3}}{\pi(.0215)^2}$$

$= 2\times 10^5 + 27.4\times 10^5 = 2.94\times 10^6$ A.t/Wb

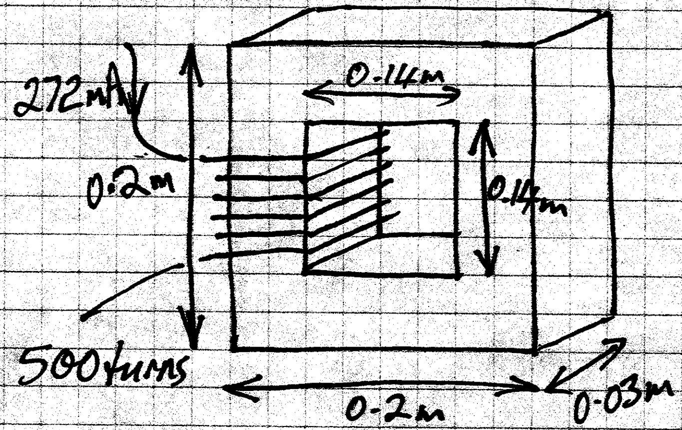
$\therefore \mathcal{F} = Ni = \phi\mathcal{R} = 2.94\times 10^3$ A.t

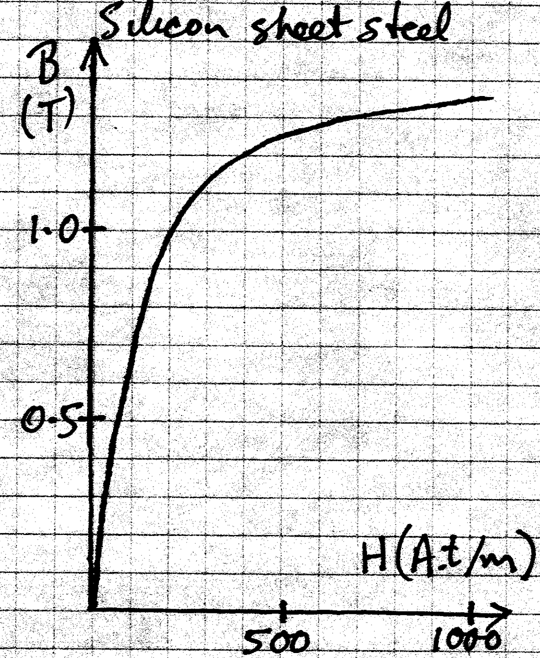
$\therefore i = \frac{\mathcal{F}}{N} = \frac{2.94\times 10^3}{400} = 7.35$ A

(Different answers in text)

Exercise 20.6

Use Fig 20.16 to find (a) H (b) B (c) ϕ





(a) $\mathcal{F} = Ni = 500 \times 272 \times 10^{-3}$

$= 136 \text{ A.t}$

Core length:

Take central path

$4 \times (0.14 + .03) = 0.68\text{m}$

$\mathcal{F} = Hl \quad \therefore H = \dfrac{\mathcal{F}}{l}$

$= \dfrac{136 \text{ A.t}}{.68 \text{ m}}$

$= 200 \text{ A.t/m}$

(b) For $H = 200$ A.t/m, $B = 1T$ (from the graph)

(c) $\phi = B.A = 1T \times (0.03)^2 = 9 \times 10^{-4} = 0.9 \text{ mWb}$

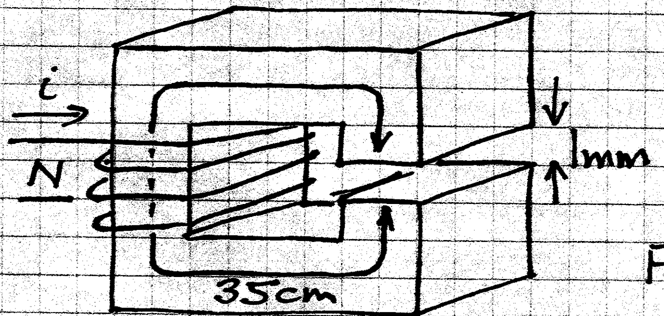
Exercise 20.7

Introduce 3mm air gap & estimate flux density B for $i = 5.3A$

Assume all reluctance (mmf drop) in air gap

$$\therefore B = \mu \frac{Ni}{l} = \frac{4\pi \times 10^{-7} . 500 \times 5.3}{3 \times 10^{-3}} = 1.11T$$

Example 6



Cast steel core

c/s area = 10 cm^2

N = 200 turns

i = 3A

Find ϕ across gap

First assume all $\mathcal{R}$ in gap.

$$B = \mu H = \frac{\mu_0 \mathcal{F}}{l} = \frac{\mu_0 N i}{l}$$

$$= \frac{4\pi \times 10^{-7}\ 200\ 3}{10^{-3}}$$

$$= 0.754T$$

Note: fringing has been ignored.

More accurately —— See example: Guess 0.6T since $\mathcal{R}_{core}$ reduces B.

Read H from Fig 20.16 curve for cast steel

$H = 400$ A.t/m $\therefore (Hl)_{core} = 400 \times 35 \times 10^{-2} = 140$ A.t

& $(Hl)_{gap} = \left(\frac{Bl}{\mu_0}\right)_{gap} = \frac{10^{-3} \times 0.6}{4\pi \times 10^{-7}} = 478$ A.t

$\therefore$ Total mmf required $= 140 + 478 = 618$ A.t

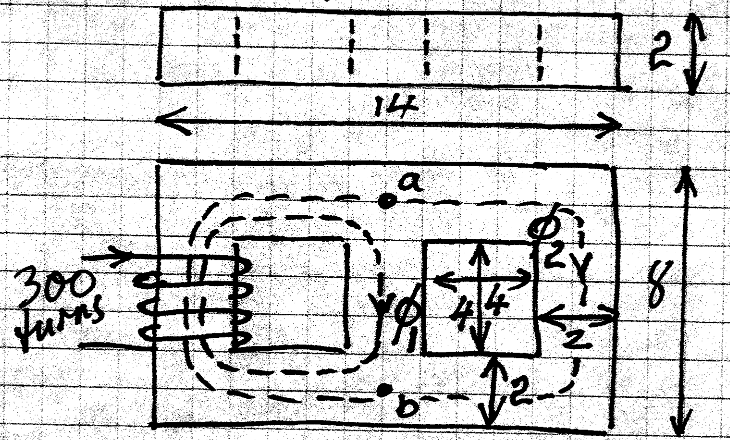
Note: A bit too high ($Ni = 600$ A.t specified)

Try $\frac{600}{618}$ 0.6T for new B.

Note: Problem is the non-linearity in Fig 20.16

$\rightarrow \mu_{core}$ varies.

Example 7



Dimensions in cm

Need $\phi_1 = 3.6\times10^{-4}$ wb in center leg

Sheet steel core Find i

① Flux density in center leg $= B_1 = \dfrac{\phi_1}{A_1} = \dfrac{3.6\times10^{-4}}{.02\times.02}$

$= 0.9T$

Fig 20.16 → For $B_1 = 0.9T$ $H_1 = 150$ A.t/m

so $\mathcal{F}_{ab} = H_1 l_1 = 150\times 0.06 = 9$ A.t

② Similarly $H_2 = \mathcal{F}_{ab}/l_2 = \dfrac{9}{(6+6+6)10^{-2}} = 50$ A.t/m

From Fig 20.16 for $H_2 = 50$ At/m $B_2 = 0.35T$ so $\phi_2 = B_2 A_2$

$= 0.35\times 4\times10^{-4}$

$= 1.4\times10^{-4}$ wb

③ Left of ab, Total flux $\phi = \phi_1 + \phi_2 = (3.6+1.4)\times10^{-4}$ wb $= 5\times10^{-4}$ wb

& $B = \dfrac{\phi}{A} = \dfrac{5\times10^{-4}}{4\times10^{-4}} = 1.25T$

From Fig 20.16 for $B = 1.25T$, $H = 500$ A.t/m

& $\mathcal{F} = H.l = 500\times \underbrace{0.18}_{3\times.06m} = 90$ A.t

$\therefore i = \dfrac{\mathcal{F} + \mathcal{F}_{ab}}{N} = \dfrac{90+9}{300} = 0.33A$