ECE 241 LECTURE 13 NETWORK THEORY

ONE-PORT NETWORKS:

Generalize earlier results for one-port resistive networks to AC.

Equivalence: 2 one-ports are equivalent if they have the same I-V characteristics.
(For ac networks, equivalence is generally only at one frequency)

Network Reduction:

$$\bar{Z}_{EQ} = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3 + \cdots \bar{Z}_n$$

$$\bar{Y}_{EQ} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \cdots \bar{Y}_n$$

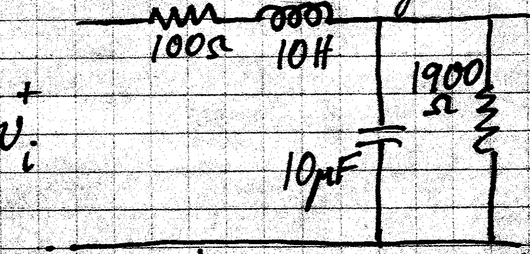
Dividers: Voltage Divider $\longrightarrow$ $\bar{V}_2 = \dfrac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}\bar{V}$

Current Divider $\longrightarrow$ $\bar{I}_2 = \dfrac{\bar{Y}_2}{\bar{Y}_1 + \bar{Y}_2}\bar{I} = \dfrac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2}\bar{I}$

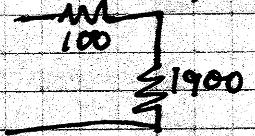
Linearity & Superposition: Linear $\Longrightarrow$ $f(kx) = kf(x)$

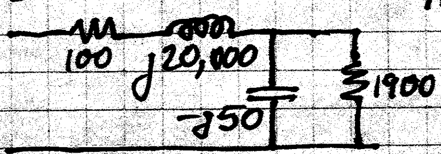
$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

Exercise 8.1 Signal 50V dc + 50V (peak) ac at 2000 rad/s at input. Predict output.



DC





AC

$X_C = \dfrac{1}{2000 \times 10^{-5}} = 50\Omega$, $X_L = 2000 \times 10 = 20{,}000\Omega$

$v_o)_{DC} = \dfrac{1900}{100 + 1900} 50v = 47.5V$

AC: Neglect R_s $100\Omega \ll |j20{,}000|$
$1900 \gg |-j50|$

$\therefore v_o)_{AC} \approx \dfrac{-j50}{+j20000 - j50} 50 \approx 0.125V pk$

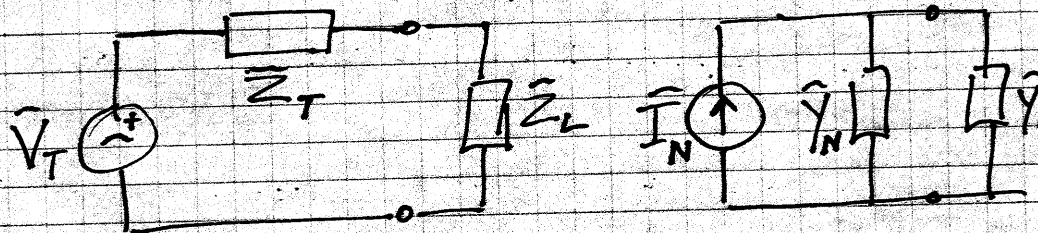
$\therefore v_o = 47.5 - 0.125 \cos 2000t$

ACTIVE NETWORKS: (Norton/Thevenin)

From the point of view of the load, any one-port network of linear elements and sources can be replaced by:

Thevenin → Either: A series combination of ideal voltage source and linear impedance

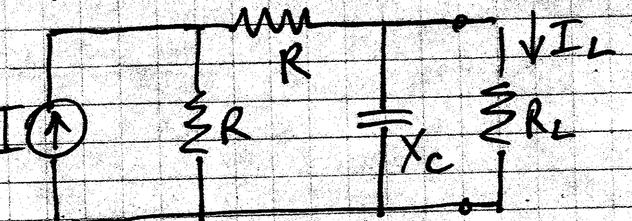
Norton → Or: A parallel combination of ideal current source and linear admittance.



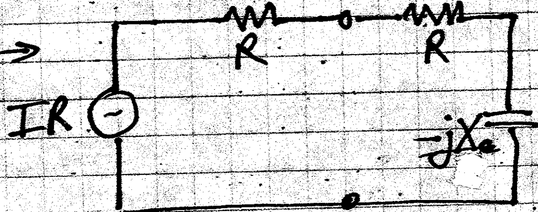
$$\hat{Y}_N = \frac{1}{\hat{Z}_T}$$

See Example 2 (Bridge circuit)

Exercise 8.2: Find I_L

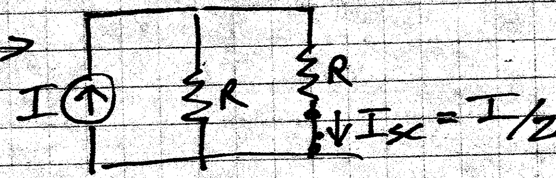


V_{OC} ⇒



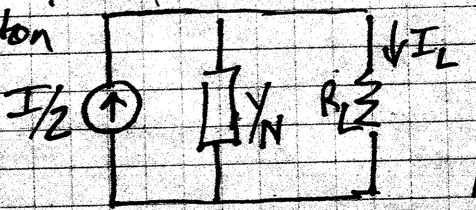
$$V_{OC} = \frac{-jX_c}{2R - jX_c}$$

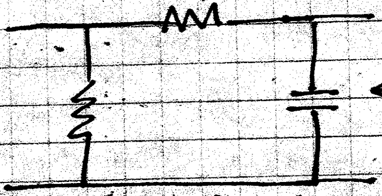
I_{SC} ⇒



Use any 2 of V_{OC}, I_{SC}, Y_N (or Z_T)

For Norton

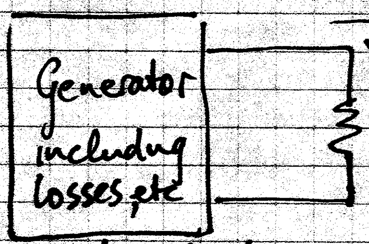




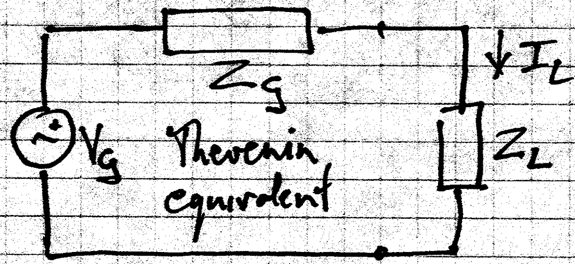
$$\leftarrow Y_N = \frac{1}{Z_T} = \frac{-j}{X_c} + \frac{1}{2R}$$

Hence $I_L = \dfrac{1/R_L}{Y_N + 1/R_L} \cdot \dfrac{I}{2} = \dfrac{I}{2(1 + Y_N R_L)}$

MAXIMUM POWER TRANSFER (Recall P_{max} when $R_L = R_g$ for DC)



Fixed characteristics



$$I_L = \frac{V_g}{|\bar{Z}_g + \bar{Z}_L|} = \frac{V_g}{\sqrt{(R_g+R_L)^2 + (X_g+X_L)^2}}$$

Real power transferred to load $= P_L = I_L^2 R_L = \dfrac{V_g^2 R_L}{(R_g+R_L)^2 + (X_g+X_L)^2}$

For any value of R_L (R_g, X_g, V_g fixed),

P_L is max when $X_L = -X_g$

Hence $P_L = \dfrac{V_g^2 R_L}{(R_g+R_L)^2}$, $\dfrac{dP_L}{dR_L} = V_g^2 \cdot \dfrac{(R_g+R_L)^2 - R_L(2)(R_g+R_L)}{(R_g+R_L)^4} = 0$ for max

when $(R_g+R_L)^2 = 2R_L(R_g+R_L)$

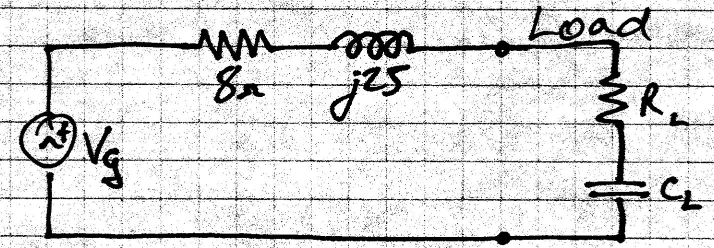
$$R_g + R_L = 2R_L$$

$$R_L = R_g$$

Hence max power transfer when $R_L = R_g$ and $X_L = -X_g$

$$\bar{Z}_L = R_L + jX_L = R_g - jX_g = \bar{Z}_g^* \text{ (complex conjugate)}$$

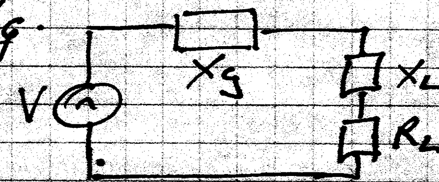
Exercise 8.3



Find R_L, C_L for max power delivered to load.
$\omega = 4000$ rad/s

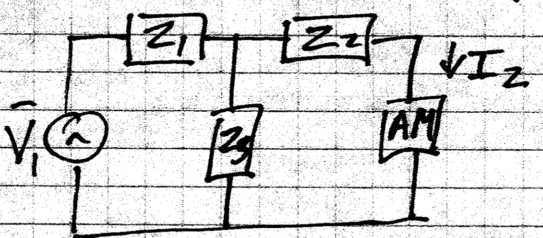
$R_L = R_g = 8\Omega \qquad X_L = \frac{-1}{\omega C_L} = -X_g = -j25 \qquad \therefore C_L = \frac{1}{25\omega} = 10^{-5} = 10\mu F$

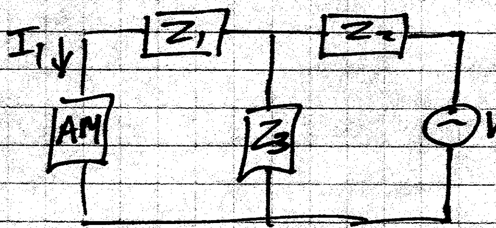
Note: Max power transfer condition is NOT maximum efficiency. (50%)
Max efficiency → minimize losses → minimize R_g.



Max power when $X_L = -X_g$.

RECIPROCITY THEOREM: In any passive linear network, if a voltage $\bar{V}$ in branch 1 causes current $\bar{I}$ in branch 2, then voltage $\bar{V}$ in branch 2 will cause current $\bar{I}$ in branch 1.





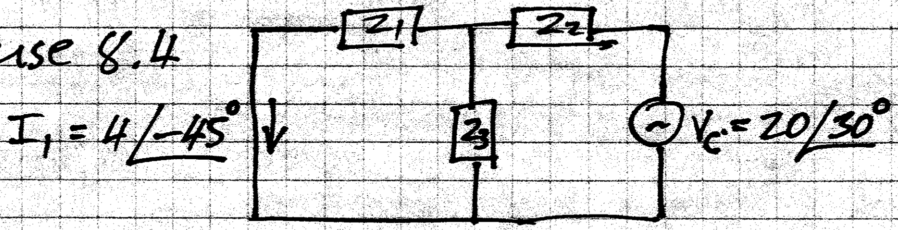
Define "transfer impedance"
$\bar{Z}_{12} = \bar{V}_1 / \bar{I}_2$

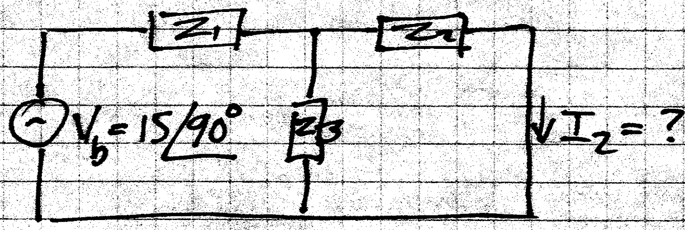
In passive linear networks
$\bar{Z}_{12} = \bar{Z}_{21}$

$$I_2 = \frac{Z_3}{Z_2+Z_3} \cdot \frac{V_1}{Z_1 + \frac{Z_2 Z_3}{Z_2+Z_3}} = \frac{Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} V_1$$

$$I_1 = \frac{Z_3}{Z+Z_3} \cdot \frac{V_1}{Z_2 + \frac{Z_1 Z_3}{Z_1+Z_3}} = \frac{Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} V_1 = I_2$$

Exercise 8.4





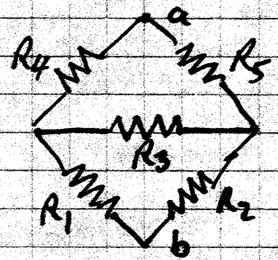
$V_b = 20\angle 30^\circ$ would yield $I_2 = 4\angle{-45^\circ}$

$V_b = 15\angle 30^\circ$ " " $\therefore$ $I_2 = \frac{15}{20} 4\angle{-45^\circ} = 3\angle{-45^\circ}$

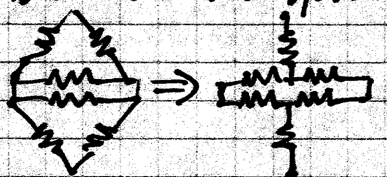
$V_b = 15\angle 90^\circ$ " " $I_2 = 3\angle{-45^\circ + 60^\circ} = 3\angle 15^\circ$

TWO PORT NETWORKS

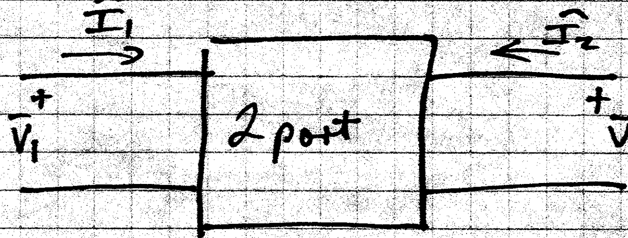
Problem with finding R_{ab} by network reduction



But can transform



In general, for a 2-port network



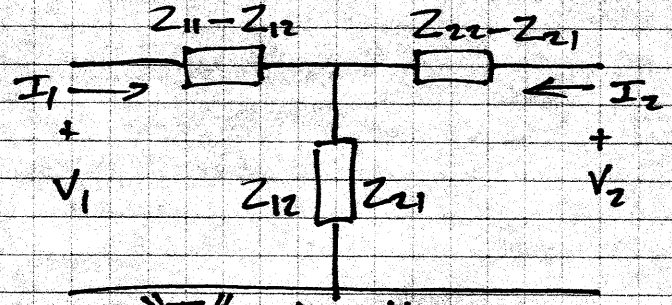
$$\bar{V}_1 = \bar{Z}_{11}\bar{I}_1 + \bar{Z}_{12}\bar{I}_2 \qquad \bar{I}_1 = \bar{Y}_{11}\bar{V}_1 + \bar{Y}_{12}\bar{V}_2$$
$$\bar{V}_2 = \bar{Z}_{21}\bar{I}_1 + \bar{Z}_{22}\bar{I}_2 \quad \text{or} \quad \bar{I}_2 = \bar{Y}_{21}\bar{V}_1 + \bar{Y}_{22}\bar{V}_2$$

where $\bar{Z}_{12} = \bar{Z}_{21}$ or $\bar{Y}_{12} = \bar{Y}_{21}$

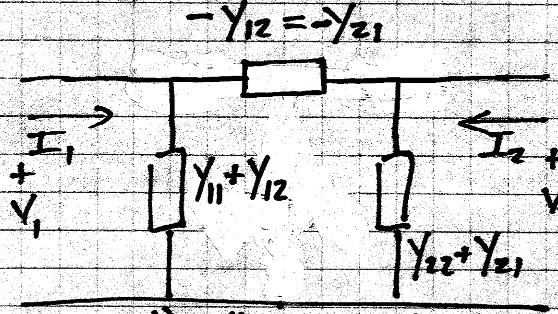
by the Reciprocity Theorem.

i.e. any 2-port linear network can be characterized by 3 parameters

For the circuits shown.



"T" circuit



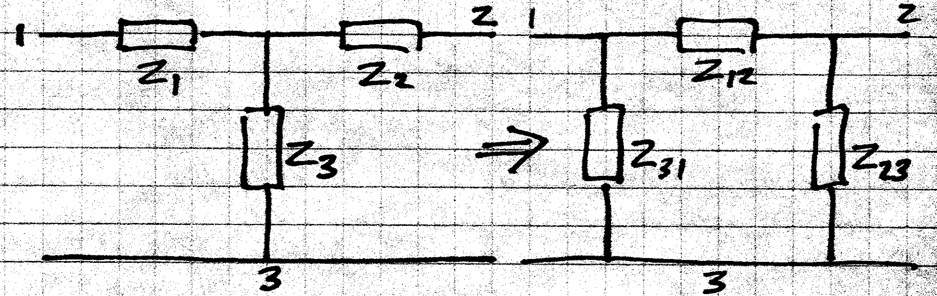
"Π" circuit

$$V_1 = (Z_{11}-Z_{12})I_1 + Z_{12}(I_1+I_2)$$
$$= Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = (Z_{22}-Z_{21})I_2 + Z_{21}(I_1+I_2)$$
$$= Z_{21}I_1 + Z_{22}I_2$$

$$I_1 = (Y_{11}+Y_{12})V_1 - Y_{12}(V_1-V_2)$$
$$= Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = (Y_{22}+Y_{21})V_2 - Y_{21}(V_2-V_1)$$
$$= Y_{21}V_1 + Y_{22}V_2$$

T-to-Π TRANSFORMATION

(Three terminal circuits)



Equating impedances (with 3rd terminal open)

$$Z_3 + Z_1 = Z_{31} \parallel (Z_{12}+Z_{23})$$
$$Z_2 + Z_3 = Z_{23} \parallel (Z_{31}+Z_{12}) \qquad \text{Elim } Z_3 \quad Z_1 - Z_2 = Z_{31}\parallel(Z_{12}+Z_{23}) - Z_{23}\parallel(Z_{31}+Z_{12})$$
$$Z_1 + Z_2 = Z_{12} \parallel (Z_{23}+Z_{31}) \qquad \text{Elim } Z_2 \quad 2Z_1 = Z_{12}\parallel(Z_{23}+Z_{31}) - Z_{23}\parallel(Z_{31}+Z_{12}) + Z_{31}\parallel(Z_{12}+Z_{23})$$
$$= \frac{Z_{12}(Z_{23}+Z_{31}) - Z_{23}(Z_{31}+Z_{12}) + Z_{31}(Z_{12}+Z_{23})}{Z_{12}+Z_{23}+Z_{31}}$$
$$= \frac{Z_{12}\cancel{Z_{23}} + Z_{12}Z_{31} - \cancel{Z_{23}}Z_{31} - Z_{12}\cancel{Z_{23}} + Z_{31}Z_{12} + \cancel{Z_{31}Z_{23}}}{Z_{12}+Z_{23}+Z_{31}}$$

$$Z_2 = \frac{Z_{12}Z_{23}}{Z_{12}+Z_{23}+Z_{31}} \quad Z_3 = \frac{Z_{23}Z_{31}}{Z_{12}+Z_{23}+Z_{31}} \xleftarrow{\text{\& similarly}} Z_1 = \frac{Z_{31}Z_{12}}{Z_{12}+Z_{23}+Z_{31}}$$

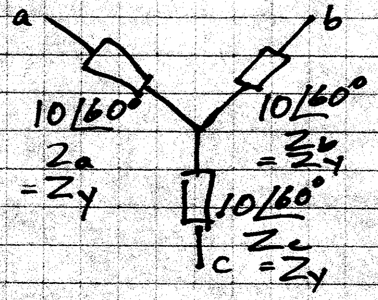
Similarly for Π-to-T (Δ-Y delta-wye)

$$Z_{12} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3} \quad \text{etc.}$$

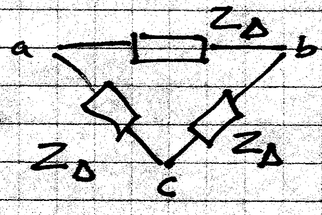
OR

$$Y_{12} = \frac{Y_1 Y_2}{Y_1 + Y_2 + Y_3} \quad \text{etc.}$$

Example 4:



⇒

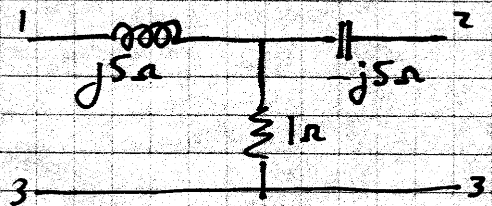


$$Z_\Delta = Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

$$= \frac{3(Z_y)^2}{Z_y} = 3Z_y = 30\angle 60^\circ$$

This is important for 3ϕ applications.

Exercise 8.5:



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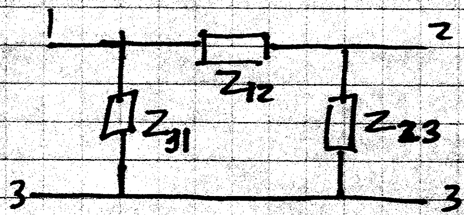
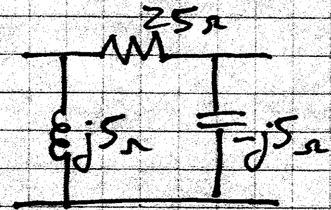


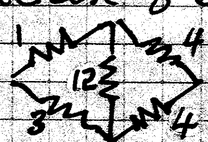
$$Z_{12} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3} = \frac{(j5)(-j5) + (-j5)1 + 1(j5)}{1} = 25\Omega$$

$$Z_{23} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1} = \frac{25\Omega}{j5} = -j5\Omega$$

$$Z_{31} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} = \frac{25\Omega}{-j5} = +j5\Omega$$



Exercise 8.6



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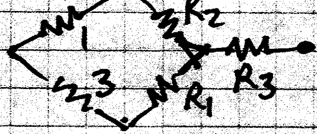
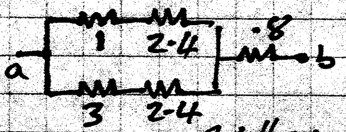
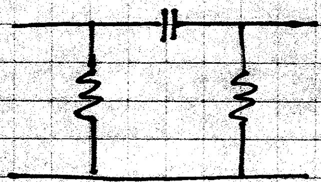


$$R_1 = \frac{12\times 4}{6+4+10} = \frac{48}{20} = 2.4\Omega \qquad R_2 = \frac{4\times 12}{20} = 2.4\Omega \qquad R_3 = \frac{4\times 4}{20} = 0.8\Omega$$

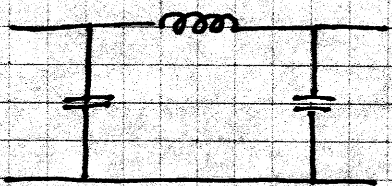


$$R_{ab} = .8 + 3.4 \| 5.4 = 2.89\Omega$$

COUPLING CIRCUITS

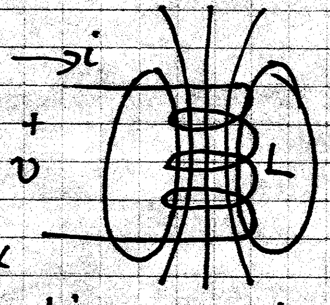


e.g. amplifier coupling from source or to load
blocks dc, passes signal.



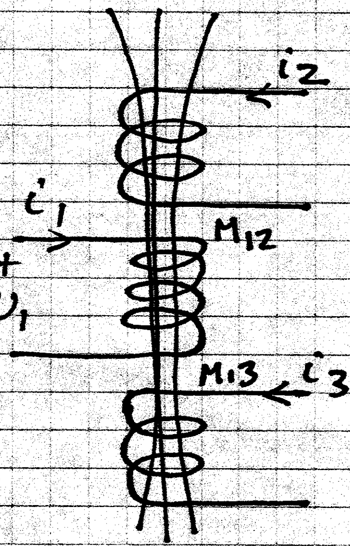
e.g. smoothing filter on power supply

MUTUAL INDUCTANCE

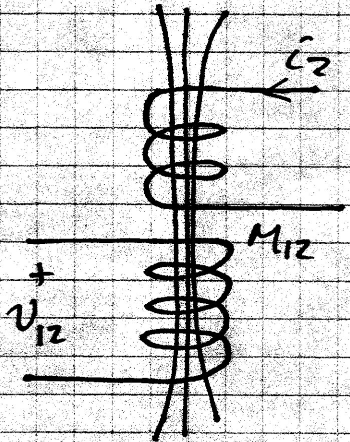


n turns
ϕ webers
magnetic flux

$$v = L\frac{di}{dt} = n\frac{d\phi}{dt}$$
$$= n\frac{d(kni)}{dt}$$
$$= kn^2\frac{di}{dt}$$



$$v_1 = v_{11} + v_{12} + v_{13}$$
$$= L\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} + M_{13}\frac{di_3}{dt}$$



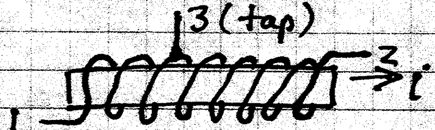
$$v_{12} = M_{12}\frac{di_2}{dt}$$

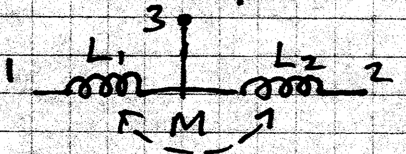
$\phi \propto ni$

L is "self-inductance"

Signs are important

Exercise 8.7





$$L_T = (L_1+M)+(L_2+M)$$
$$= L_1+L_2+2M$$

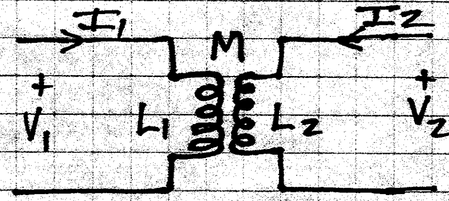
Exercise 8.8: 2 coils $N_2 = 2N_1$
$\therefore L_2 = 4L_1$

In series: $L_S = 8L_1$

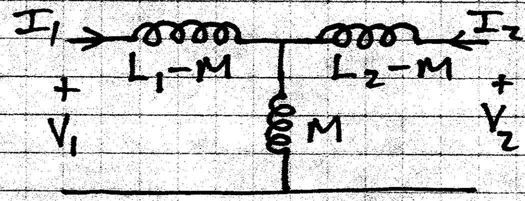
$= (L_1+M)+(4L_1+M)$

$\therefore 2M = 3L_1 \quad M = 1.5L_1$

INDUCTIVE COUPLING



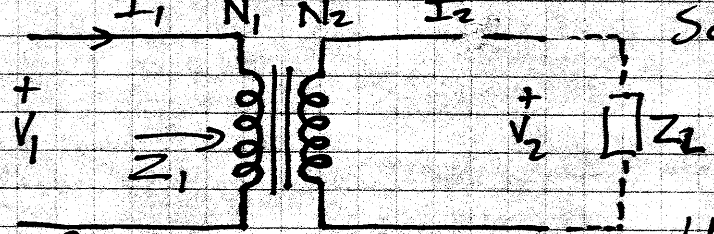
$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$
$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$



Equivalent circuits

$$V_1 = j\omega(L_1 - M)I_1 + j\omega M(I_1 + I_2)$$
$$= j\omega L_1 I_1 + j\omega M I_2$$
$$V_2 = j\omega(L_2 - M)I_2 + j\omega M(I_1 + I_2)$$
$$= j\omega M I_1 + j\omega L_2 I_2$$

TRANSFORMER COUPLING



Same flux ϕ links both coils

$$V = N\frac{d\phi}{dt} \longrightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Ideal transformer: No power loss $\therefore P_{IN} = P_{OUT}$

$$V_1 I_1 = V_2 I_2$$
$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

For impedance $Z_2 = \dfrac{V_2}{I_2}$ $\quad Z_1 = \dfrac{V_1}{I_1} = \dfrac{V_2\,(N_1/N_2)}{I_2\,(N_2/N_1)} = \left(\dfrac{N_1}{N_2}\right)^2 Z_2$

Z_1 is the "reflected" impedance "seen" at the primary, due to the secondary load Z_2

Exercise 8.9

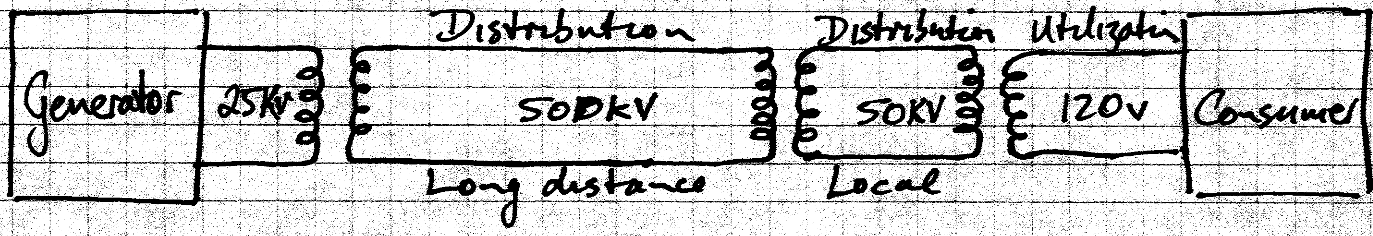
$$\left.\begin{aligned} N_1/N_2 &= 5 \\ Z_2 &= 10\,\Omega \\ I_1 &= 1\ A \end{aligned}\right\} \begin{aligned} &\therefore I_2 = 5 \times 1\ A = 5\ A \\ &\therefore V_2 = I_2 Z_2 = 50V \\ &\therefore V_1 = 5 \times 50 = 250V \end{aligned}$$

Note: Only changing flux induces voltage

$\therefore$ transformer acts on ac signals, not dc

Note use of transformers in power distribution

eg



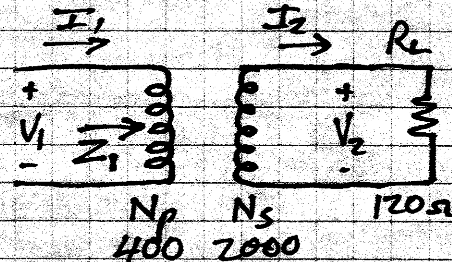
Impedance matching:

Exercise 8.10

$N_p = 400$ primary $\quad N_s = 2000$ secondary

$V_1 = 120V \quad R_L = 120\Omega$

Find V_2, I_1



$$V_2 = \frac{N_2}{N_1} V_1 = \frac{2000}{400} 120 = 600V$$

$$I_1 = \frac{N_2}{N_1} I_2 = \frac{N_2}{N_1} \frac{V_2}{R_L} = \frac{2000}{400} \cdot \frac{600}{120} = 25A$$

$$Z_1 = \frac{V_1}{I_1} = \frac{120v}{25A} = 4.8\Omega$$

Could use this transformer to couple current source to higher impedance load.

Assignment #6

P 7.16

P 7.28

P 7.33

P 8.19

P 8.23

P 8.30

AP 8.7