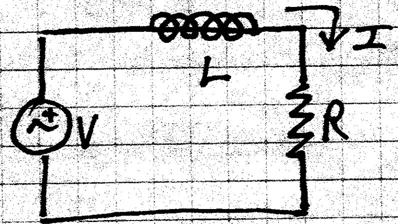
FREQUENCY RESPONSE



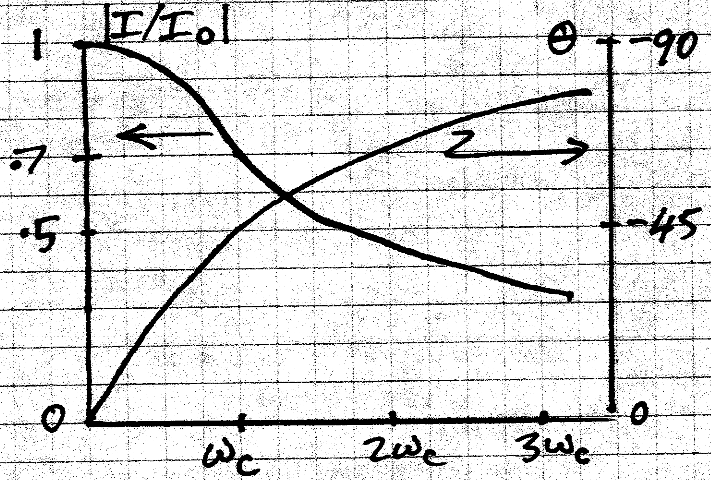
$$I(\omega) = \frac{\hat{V}}{Z} = \frac{\hat{V}}{R + j\omega L} = \frac{\hat{V}/R}{1 + j\omega \frac{L}{R}} \xrightarrow[\substack{DC \\ \omega = 0}]{} \hat{I}_0 = \hat{V}/R$$

$$\therefore \frac{\hat{I}}{\hat{I}_0} = \frac{1}{1 + j\omega \frac{L}{R}} = \frac{1 - j\omega L/R}{1 + \left(\omega \frac{L}{R}\right)^2} = \frac{\left[1 + (\omega L/R)^2\right]^{1/2}}{1 + (\omega L/R)^2} \angle \arctan -\omega \frac{L}{R}$$

$$= \frac{1}{\sqrt{1 + \left(\omega \frac{L}{R}\right)^2}} \angle \arctan -\omega L/R = \frac{I}{I_0} \angle \theta$$

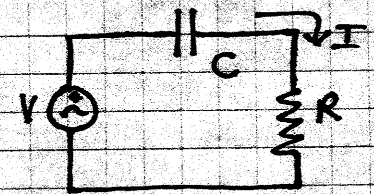
At $\omega_c = R/L$ $\quad \frac{\hat{I}}{\hat{I}_0} = \frac{1}{\sqrt{2}} \angle \arctan -1 = 0.707 \angle -45^\circ$

$I^2 R = \frac{1}{2} I_0^2 R$ i.e. this is the "half-power" freq



Low pass filter

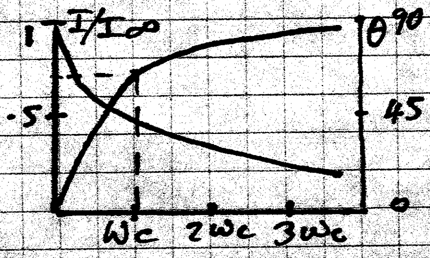
Similarly



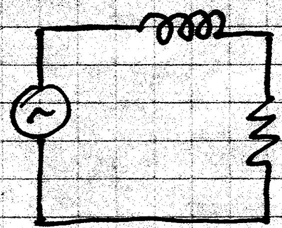
$$\hat{I} = \frac{\hat{V}}{Z} = \frac{\hat{V}}{R - j/\omega C} = \frac{\hat{V}/R}{1 - \frac{j}{\omega RC}} \xrightarrow[\substack{HF \\ \omega \to \infty}]{} \hat{I}_\infty = \frac{\hat{V}}{R}$$

$$\frac{\hat{I}}{\hat{I}_\infty} = \frac{1}{1 - j/\omega RC} = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} \angle \tan^{-1} 1/\omega RC$$

At $\omega_c = 1/RC$ $\quad \left|\frac{I}{I_\infty}\right| \to \frac{1}{\sqrt{2}}$

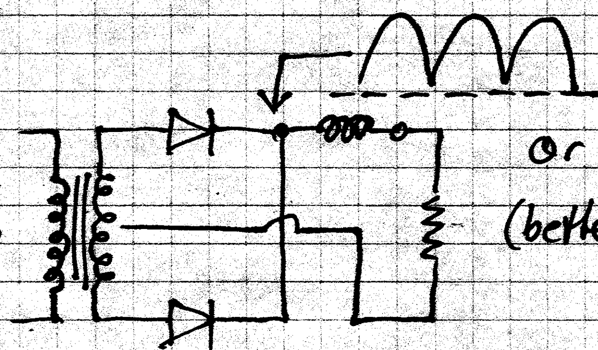


L-SECTION FILTERS

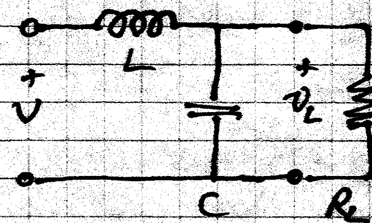


Low Pass Filter.

Possible use in powersupply filtering if $\omega L \gtrless R_L/3$ →



Or (better)



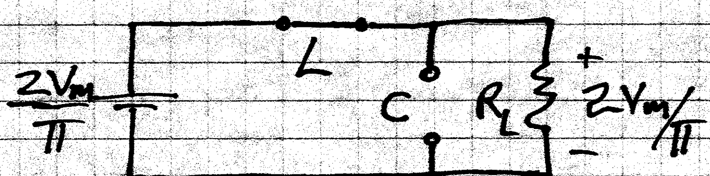
"L-section"

Full wave rectifier ⟶ half-sinewave series of pulses

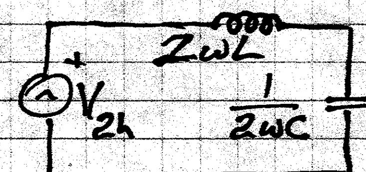
Fourier Series ⟶ $v = \frac{2}{\pi} V_m \left(1 - \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t - \frac{2}{35}\cos 6\omega t - \ldots\right)$

DC term ⟶ 1 ; 2nd harmonic ⟶ $\frac{2}{3}\cos 2\omega t$; Higher frequency harmonics ⟶ $\frac{2}{15}\cos 4\omega t$, $\frac{2}{35}\cos 6\omega t$, …

Consider L-section for DC



For 2nd harmonic



$$V_{ac} \approx \frac{1/2\omega C}{2\omega L} V_{2h} = \frac{1}{4\omega^2 LC}\,\frac{4V_m}{3\pi\sqrt{2}}$$

rms

$$V_{2h} = -\frac{4}{3\pi} V_m \cos 2\omega t$$

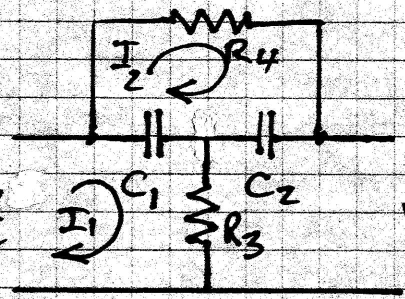
Assume $R_L \gg \frac{1}{2\omega C}$ & $2\omega L \gg \frac{1}{2\omega C}$ for effective smoothing

Ripple factor $r = \frac{I_{ac}}{I_{dc}} = \frac{V_{ac}}{V_{dc}} = \frac{\text{rms AC amplitude}}{\text{DC amplitude}} \Longrightarrow \frac{4V_m}{4\omega^2 LC\, 3\pi\sqrt{2}} \cdot \frac{\pi}{2V_m}$ (2nd harmonic only)

$$= \frac{\sqrt{2}/3}{4\omega^2 LC}$$

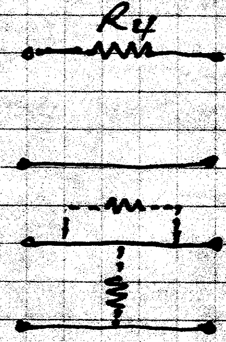
Note: $4\omega^2$ term justifies 2nd harmonic approx!
Next terms $5 \times 16\omega^2$, $11\frac{2}{3} \times 36\omega^2$, etc

BRIDGED-T FILTERS



Low freq: C_1, C_2 O.C.

High freq: C_1, C_2 S.C.



For Loops

$$① \rightarrow V_i = Z_1(I_1 - I_2) + Z_3 I_1$$

$$② \rightarrow I_2 Z_4 = (I_1 - I_2)Z_1 - I_2 Z_2$$

Need $V_o = V_i - I_2 R_4$ $\therefore$ Eliminate $I_1 = \frac{Z_1 + Z_2 + Z_4}{Z_1} I_2$ from ②

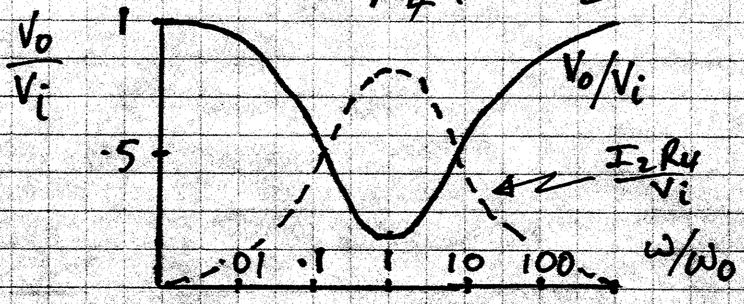
$$① \longrightarrow I_2 = \frac{1}{Z_1}(Z_1 + Z_3)\frac{Z_1 + Z_2 + Z_4}{Z_1} I_2 - \frac{V_i}{Z_1}$$

$$= \frac{V_i/Z_1}{\left(1 + \frac{Z_3}{Z_1}\right)\left(1 + \frac{Z_2}{Z_1} + \frac{Z_4}{Z_1}\right) - 1} = \frac{V_i}{(Z_1 + Z_3)(Z_1 + Z_2 + Z_4) - Z_1^2} Z_1$$

$$= \frac{V_i Z_1}{Z_1 Z_2 + Z_1 Z_4 + Z_1 Z_3 + Z_2 Z_3 + Z_3 Z_4}$$

So $$\frac{V_o}{V_i} = 1 - \frac{1}{\frac{Z_2}{Z_4} + 1 + \frac{Z_3}{Z_4} + \frac{Z_2 Z_3}{Z_1 Z_4} + \frac{Z_3}{Z_1}} = 1 - \frac{1}{1 + \frac{1/j\omega C_2}{R_4} + \frac{R_3}{R_4} + \frac{R_3}{1/j\omega C_1} + \frac{(1/j\omega C_2)R_3}{(1/j\omega C_1)R_4}}$$

$$= 1 - \frac{1}{1 + \frac{R_3}{R_4}\left(1 + \frac{C_1}{C_2}\right) + j\omega R_3 C_1 - j\frac{1}{\omega R_4 C_2}} = 1 - \frac{1}{1 + \frac{R_3}{R_4}\left(1 + \frac{C_1}{C_2}\right) + j\frac{1}{\omega R_4 C_2}\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]}$$

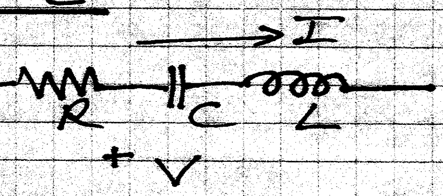


where $\frac{1}{\omega_0^2} = R_4 C_2 R_3 C_1$

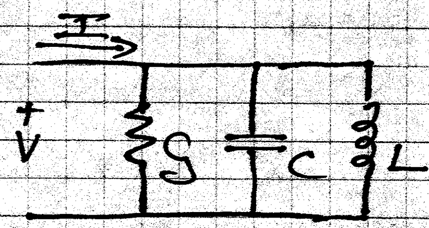
Band-reject filter.

RESONANCE

Series



Parallel



$$Z = R + j\omega L - \frac{j}{\omega C}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$
$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1} \frac{\omega L - 1/\omega C}{R}$$

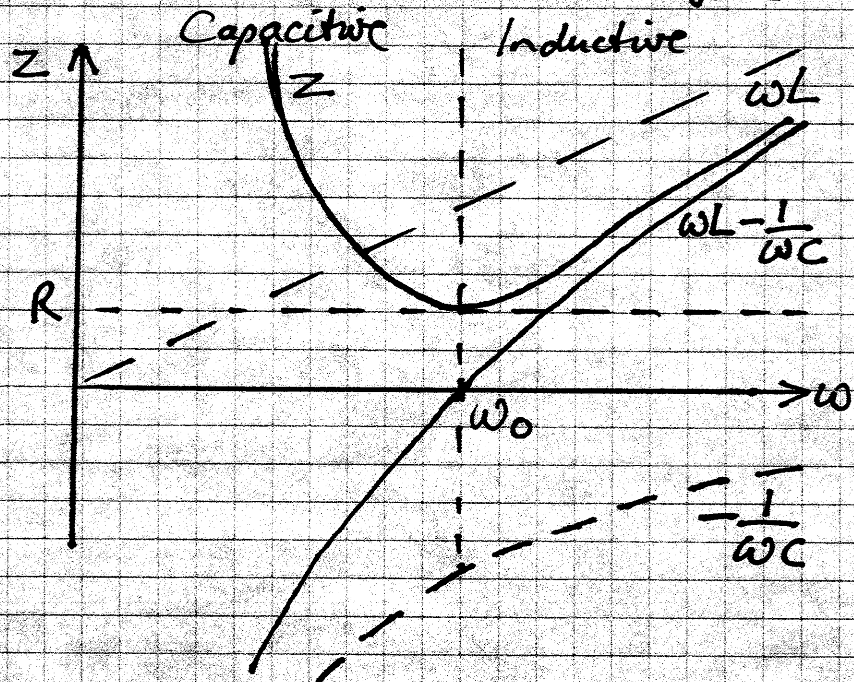
$\longrightarrow R\angle 0^\circ$ when $\omega_0 L = \frac{1}{\omega_0 C}$
at $\omega = \omega_0$

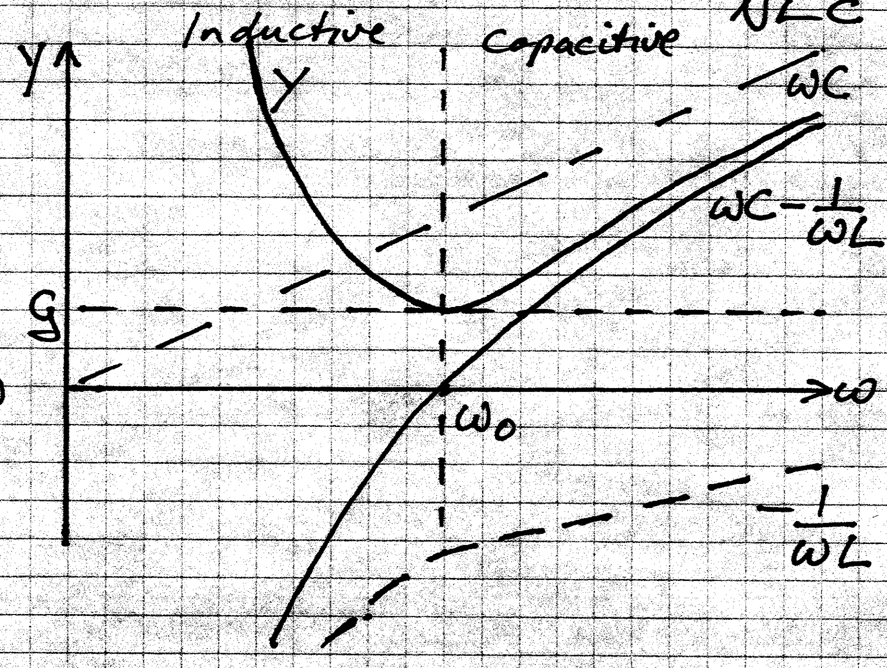
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y = G + j\omega C - j/\omega L$$
$$= G + j\left(\omega C - \frac{1}{\omega L}\right)$$
$$= \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} \angle \tan^{-1} \frac{\omega C - 1/\omega L}{G}$$

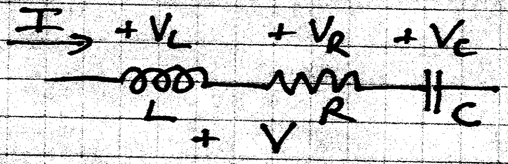
$\longrightarrow G\angle 0^\circ$ when $\omega_0 L = \frac{1}{\omega_0 C}$
at $\omega = \omega_0$

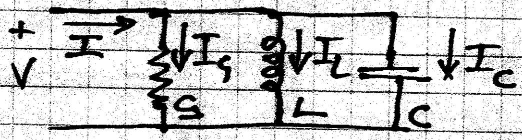
$$\omega_0 = \frac{1}{\sqrt{LC}}$$





PHASOR VIEW OF RESONANCE





Common element: I as reference

Common element: V as reference

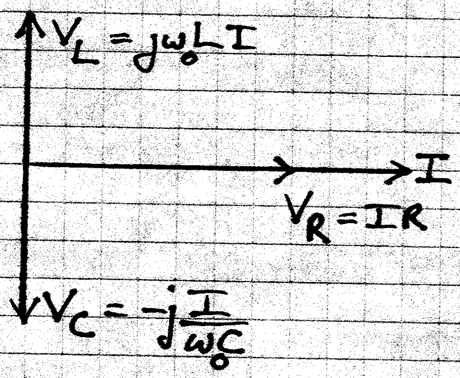
At resonance:

$\omega = \omega_0$

$V_C + V_L = 0$

$|V_C| = |V_L|$

Resistive

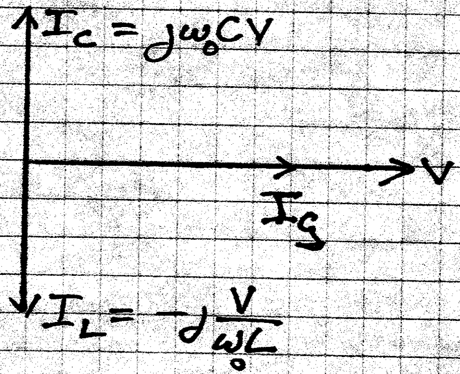


$\omega = \omega_0$

$I_C + I_L = 0$

$|I_C| = |I_L|$

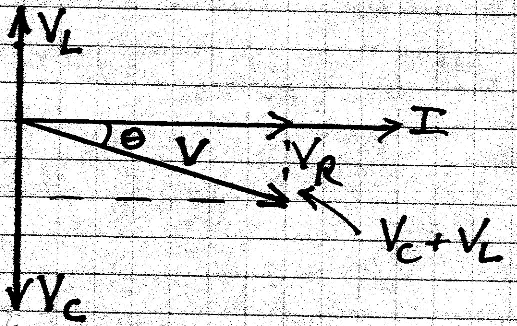
Resistive



$\omega < \omega_0$

$|V_C| > |V_L|$

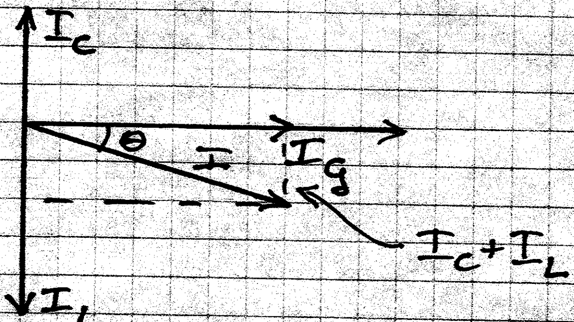
Capacitive



$\omega < \omega_0$

$|I_L| > |I_C|$

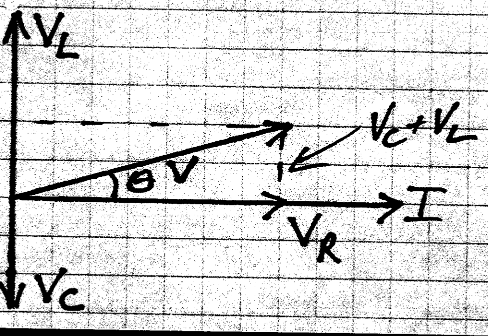
Inductive



$\omega > \omega_0$

$|V_L| > |V_C|$

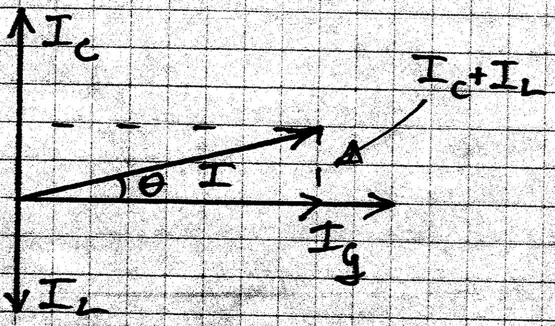
Inductive



$\omega > \omega_0$

$|I_C| > |I_L|$

Capacitive



QUALITY FACTOR, $Q = 2\pi \dfrac{\text{Maximum Energy Stored}}{\text{Energy Dissipated / Cycle}}$

Series

Common element is current

$i = \sqrt{2}\, I \cos\omega t$

$W_{diss/cycle} = P \times T_0 = \dfrac{I^2 R}{f_0}$

Max $W_{stored} = \dfrac{1}{2} L I_M^2 = L I^2$

$\therefore Q_S = 2\pi \dfrac{L I^2}{I^2 R / f_0} = \dfrac{\omega_0 L}{R}$

Parallel

Common element is voltage

$v = \sqrt{2}\, V \cos\omega t$

$W_{diss/cycle} = P \times T_0 = \dfrac{G V^2}{f_0}$

Max $W_{stored} = \dfrac{1}{2} C V_M^2 = C V^2$

$\therefore Q_P = 2\pi \dfrac{C V^2}{G V^2 / f_0} = \omega_0 \dfrac{C}{G}$

$= \omega_0 R C$

Note: Substitute for $\omega_0 = \dfrac{1}{\sqrt{LC}}$

$Q_S = \dfrac{L}{R} \dfrac{1}{\sqrt{LC}} = \sqrt{\dfrac{L}{C}} \cdot \dfrac{1}{R}$

$Q_P = RC \dfrac{1}{\sqrt{LC}} = \sqrt{\dfrac{C}{L}}\, R = \dfrac{1}{Q_S}$

At resonance $I_0 = V/R$ and $V_L = -V_C$

$$\left.\begin{aligned} \omega_0 L I_0 &= I_0 / \omega_0 C \\ \omega_0 L V/R &= V/\omega_0 RC \end{aligned}\right\} Q_S = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$V_0 = I/G$ and $I_L = -I_C$

$$\left.\begin{aligned} V_0/\omega_0 L &= \omega_0 C V \\ I_0 R/\omega_0 L &= \omega_0 C I_0 R \end{aligned}\right\} \begin{aligned} Q_P &= \omega_0 RC \\ &= R/\omega_0 L \end{aligned}$$

NORMALIZED RESONANCE CURVES

Series:

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = 1/Y$$

& at resonance $Z_0 = R = \frac{1}{Y_0}$

$$\frac{Y}{Y_0} = \frac{Z_0}{Z} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)}$$

$$= \frac{1}{1 + j\left(\frac{\omega_0 L}{R} \cdot \frac{\omega}{\omega_0} - \frac{1}{\omega_0 RC} \cdot \frac{\omega_0}{\omega}\right)}$$

$$= \frac{1}{1 + jQ_s\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Parallel:

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right) = \frac{1}{Z}$$

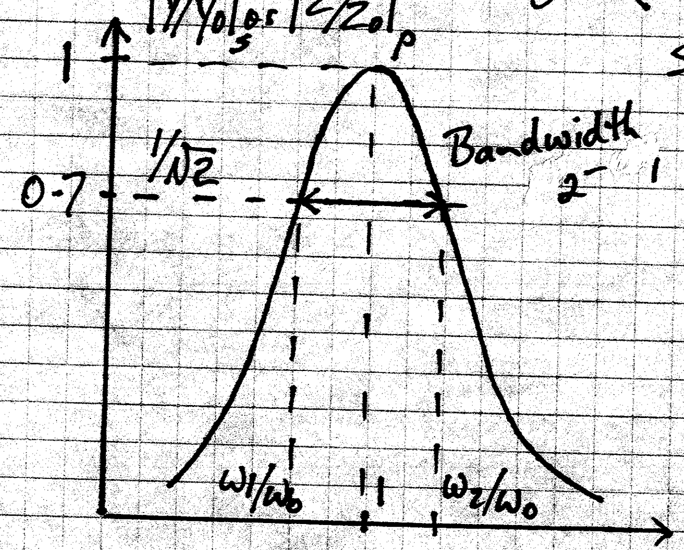
& at resonance $Y_0 = G = \frac{1}{Z_0}$

$$\frac{Z}{Z_0} = \frac{Y_0}{Y} = \frac{G}{G + j(\omega C - 1/\omega L)}$$

$$= \frac{1}{1 + j\left(\frac{\omega C}{G} - \frac{1}{\omega G L}\right)}$$

$$= \frac{1}{1 + j\left(\omega_0 RC \, \frac{\omega}{\omega_0} - \frac{R}{\omega_0 L} \frac{\omega_0}{\omega}\right)}$$

$$= \frac{1}{1 + jQ_p\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$



Series:

$$\left|Y/Y_0\right| = \frac{1}{\sqrt{2}} \text{ when } Y/Y_0 = \frac{1}{1 \pm j} = \frac{1}{\sqrt{2}} \angle \pm 45^\circ$$

when $\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \pm \frac{1}{Q_s}$

i.e. at $\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1} = -\frac{1}{Q_s}$ and $\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2} = +\frac{1}{Q_s}$

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q_s}\right)^2} - \frac{\omega_0}{2Q_s} \qquad \omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q_s}\right)^2} + \frac{\omega_0}{2Q_s}$$

Bandwidth $\omega_2 - \omega_1 = \omega_0/Q_s \rightarrow f_2 - f_1 = f_0/Q_s$

Approximation:

$$\text{If} \quad Q_s \gtrsim 10 \qquad \sqrt{1+\left(\frac{1}{2Q_s}\right)^2} \approx 1$$

so $\omega_1 \approx \omega_0 - \frac{\omega_0}{2Q_s}$ $\qquad \omega_2 \approx \omega_0 + \frac{\omega_0}{2Q_s}$

$$\frac{\omega_1}{\omega_0} \approx 1 - \frac{1}{2Q_s} \qquad \frac{\omega_2}{\omega_0} \approx 1 + \frac{1}{2Q_s}$$

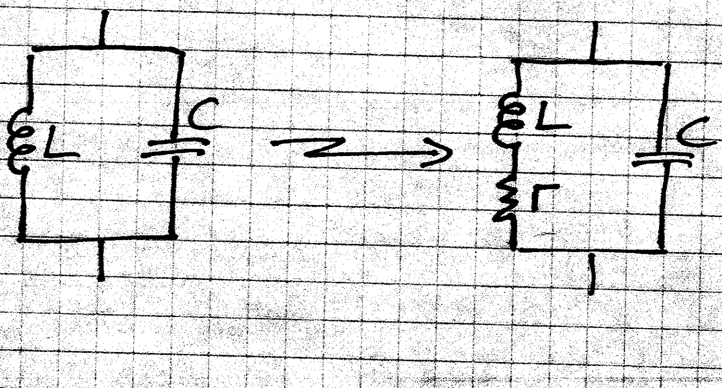
and $\frac{\omega_2}{\omega_0} - \frac{\omega_1}{\omega_0} = \frac{1}{Q_s}$ still (no approximation)

i.e. resonance curve is <u>approximately</u> symmetrical about ω_0 for <u>high Q_s</u>

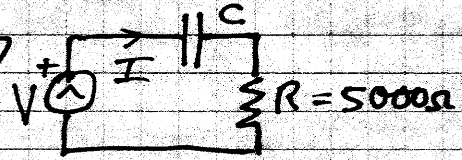
Parallel: All bandwidth results apply to the normalized parallel $|Z/Z_0|_p$ curve with Q_p substituted for Q_s.

Note: Practical resonant circuits:

Inductors typically have non-negligible series resistance so practical L-C parallel resonant "tank" circuits look like:

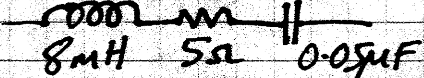


EXERCISES

7.7 

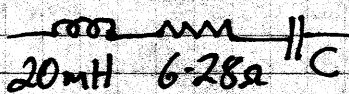
$\omega_{co} = 500\ \text{rad/sec}$ i.e. $f_{co} = 500/2\pi = 79.58\ Hz$

$$= \frac{1}{RC} = \frac{1}{5000C} \quad \therefore C = \frac{1}{(500)(5000)} = 0.4\ \mu F$$

7.8 

$$\omega_0 = \frac{1}{\sqrt{LC}} = (8\times10^{-3} \times .05\times10^{-6})^{-1/2} = (4\times10^{-10})^{-1/2} = 5\times10^{4}\ \text{rad/s}$$

At ω_0 $\quad X_L = \omega_0 L = 5\times10^4 \times 8\times10^{-3} = 400\Omega,\ X_c = (\omega_0 C)^{-1} = 2\times10^{-5} \times 20\times10^{6} = 400\Omega$

7.9 

$f_0 = 5000\,Hz$ (a) $2\pi f_0 = \frac{1}{\sqrt{LC}} \quad \therefore C = \frac{1}{L(2\pi f_0)^2} = \frac{1}{2\times10^{-2}\, 4\pi^2\, 25\times10^6} = 0.05\ \mu F$

(b) at f_0, $|V_c| = 5V \therefore |V_L| = 5v$ & $V_{tot} = V_R = IR = \frac{5V}{1/2\pi f_0 C} R = 5\times 2\pi \times 5000 \times 6.28 \times .05\times10^{-6}$

$$= 493\times10^{-3}\ V$$

(c) For $C' = 0.025\ \mu F$ $\quad f_0' = \frac{1}{2\pi\sqrt{LC'}} = f_0\sqrt{\frac{C}{C'}} = \sqrt{2} f_0 = 1.414 \times 5000 = 7070\ Hz$

$$Q_s' = \omega_0 L/R = 2\pi \times 7070 \times 20\times10^{-3}/6.28 = 14.14$$

7.10 

$Q_p = 60$, $Z_{max} = 10{,}000\Omega$ at 240,000 $\quad \therefore R = 10K\Omega$ and $f_0 = 240\,KHz$.

½ power frequencies ω_1, ω_2 ?

Since $Q_p = 60 > 10$, can use the approximation of symmetrical resonance

$$f_1, f_2 \approx f_0 \pm \frac{f_0}{2Q} = \left(240 \pm \frac{240}{2\times60}\right) KHz$$

$$\approx 240 \pm 2\ KHz$$

i.e. $\omega_1 \approx 238\ kHz \quad \omega_2 \approx 242\ kHz$