ECE 241 LECTURE 11: AC POWER (& FREQUENCY RESPONSE)

From past work: $p = vi = i^2R = v^2/R$ for resistance R

For sinusoidal voltage/current, power varies sinusoidally too.

In practice, usually want AVERAGE power P

$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t\, R\,dt = \frac{1}{T}\int_0^T I_m^2 R\,\frac{1}{2}(1-\cos 2\omega t)\,dt$$

$$= \frac{1}{T}\cdot\frac{I_m^2}{2}R\,.\,T = I_{RMS}^2 R \quad \text{where } I_{RMS} = \frac{I_m}{\sqrt{2}}$$

$$= V_{RMS}^2/R$$

typically written as I^2R or V^2/R

Power measurement by "wattmeter":

v applied to "voltage coil" $\Rightarrow$ creates current $\propto v$, & hence magnetic field $\propto v$

i applied to "current/field coil" $\Rightarrow$ produces torque in magnetic field $\propto vi$

Torque $\rightarrow$ deflection, averaged by inertia (e.g. cannot follow 60Hz)

For v, i out of phase by θ $\Rightarrow$ reading $= VI\cos\theta$

Note: For 110v 60Hz, V_{rms} actually 120v

& for $v = \sqrt{2}\,120\text{v}.\sin\omega t$, $\sqrt{2}\times 120\text{v} \approx 170\text{v}$

REACTIVE POWER

$p_R = i^2 R = v^2/R$ i^2, v^2 positive even when $i, v < 0$
Hence power dissipated throughout cycle.

For

Inductor

Energy stored as current increases to max $I_m = \frac{1}{2} L I_m^2$

then returned to circuit as current decreases to zero

Capacitor

Energy stored as voltage increases to max $V_m = \frac{1}{2} C V_m^2$

then returned to circuit as voltage decreases to zero.

Power positive when absorbed (ie. being stored), negative when returned.

Inductor:

$$i_L = I_m \cos(\omega t - \pi/2) = \sqrt{2} I \sin\omega t$$

$$\therefore v_L = L\frac{di}{dt} = \sqrt{2} I L \cos\omega t \cdot \omega$$

$$\& \; p_L = i_L v_L = 2I^2 \omega L . \sin\omega t \cos\omega t$$

$$= I^2 \omega L . \sin 2\omega t$$

$$= I^2 X_L . \sin 2\omega t$$

Capacitor:

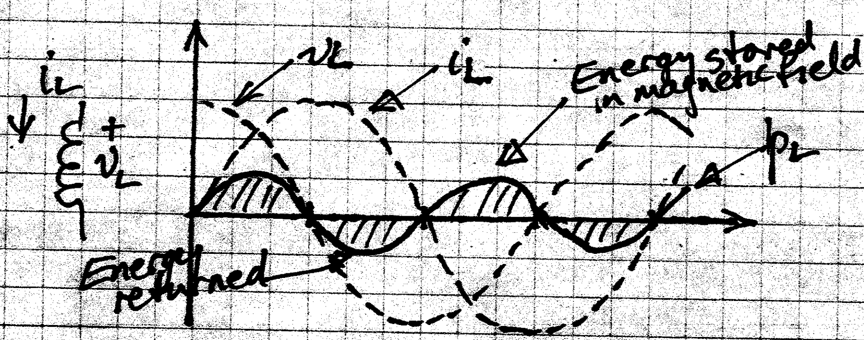
$$i_c = I_m \cos(\omega t - \pi/2) = \sqrt{2} I \sin\omega t$$

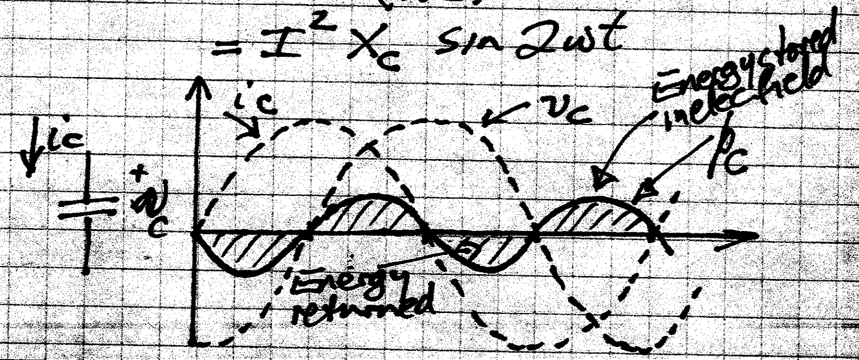
$$\therefore v_C = \frac{1}{C}\int i_c \, dt + V_0 = -\sqrt{2}\frac{I}{C}\cos\omega t . \frac{1}{\omega}$$

$$\& \; p_C = i_c v_c = -2I^2\left(\frac{1}{\omega C}\right)\sin\omega t \cos\omega t$$

$$= 2I^2\left(\frac{-1}{\omega C}\right)\sin\omega t \cos\omega t$$

$$= I^2 X_C \sin 2\omega t$$





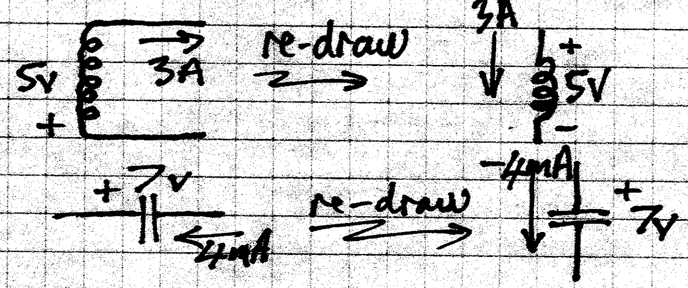
Note: No net power consumption by L or C

Periodic storage/return of energy

Power $p_X = I^2 X \sin 2\omega t$, Amplitude $I^2 X$

Define <u>Reactive Power</u> $P_X = I^2 X = \frac{V^2}{X} = Q$ ← convention (but used for other things too)

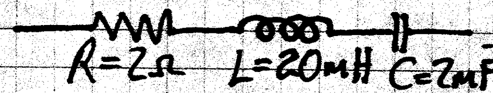
Exercise 7.1



$p_L = 3A \times 5v = 15w$ <u>stored</u>

$p_C = 7v \times (-4mA) = -28mW$
$= 28mW$ <u>returned</u>

Exercise 7.2



$i = 10\sqrt{2} \cos 120\pi t$ A

Max. Instantaneous powers

$p_R)_{max} = I_M^2 R = (10\sqrt{2})^2 \times 2 = 400w$

$p_L)_{max} = I^2 X_L = 100 . 120\pi\, 20 \times 10^{-3} = 240\pi = 754w$

$p_C)_{max} = I^2 X_C = 100 . \frac{1}{120\pi . 2 \times 10^{-3}} = 132.6w$

Note p_L and $p_C = I^2 X \sin 2\omega t \xrightarrow[max]{} I^2 X$ when $\sin 2\omega t = 1$

Average power in series combination $= I^2 R = 10^2 \times 2 = 200w$
since L, C consume no average power

POWER FACTOR

General case of a complex impedance

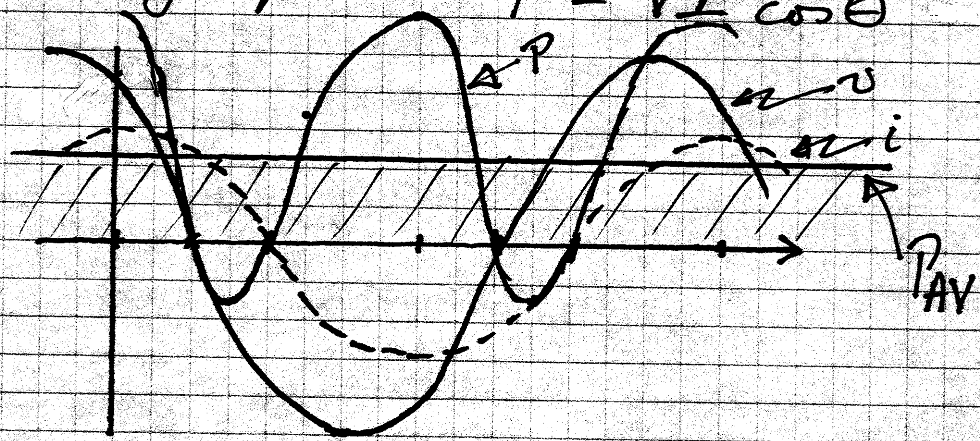
$$\bar{V} = \bar{Z}\bar{I} = (Z\angle\theta)\bar{I}$$

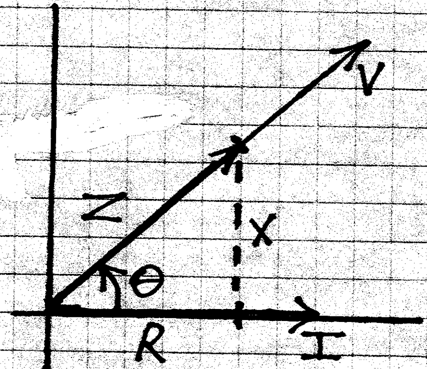
$$V = ZI \quad \text{magnitudes}$$

$$p = vi = \sqrt{2}\,V\cos(\omega t+\theta)\cdot\sqrt{2}\,I\cos\omega t = 2VI\cos(\omega t+\theta)\cos\omega t$$

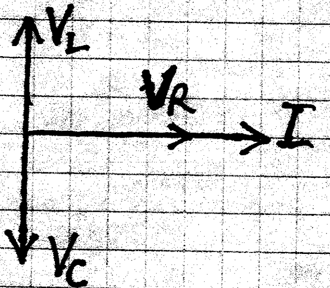
$$= VI\cos\theta + VI\cos(2\omega t+\theta)$$

Hence average power $P = VI\cos\theta$





Definition: Power Factor $pf = \cos\theta = \frac{P}{VI}$



Inductance: Current lags voltage $\rightarrow$ lagging power factor

Capacitance: Current leads voltage $\rightarrow$ leading power factor

$X = Z\sin\theta$ (above) so $Q = P_X = I^2X = I^2Z\sin\theta = VI\sin\theta$

$\sin\theta$ = "reactive factor"

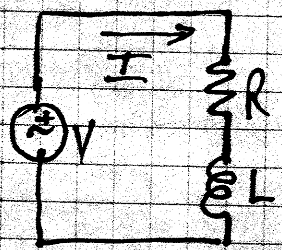
$Q = P_X$ in volt-amps reactive $\rightarrow$ VARs

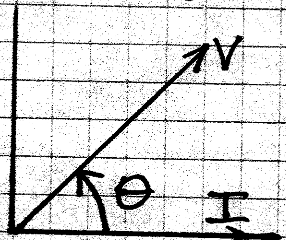
COMPLEX POWER

AC power → $VI\cos\theta$

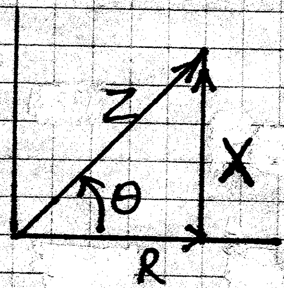
Reactive power → $VI\sin\theta$

For inductive load eg. motor

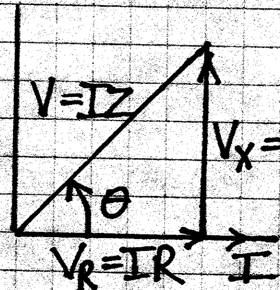




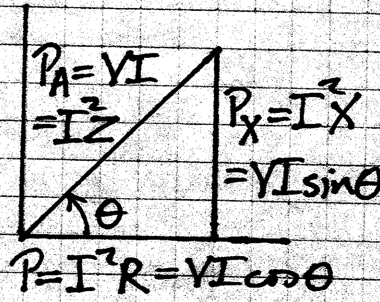
I, V phasors



Impedance Triangle



Phasor Diagram



Power Triangle

$$\text{Apparent Power} = \tilde{P}_A = VI\cos\theta + jVI\sin\theta = VI\angle\theta$$
$$= P + jP_X \;(= P + jQ) = P_A\angle\theta$$

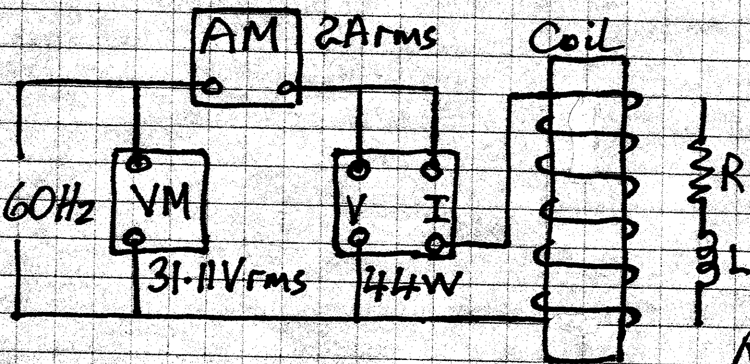
Units of apparent power → volt-amps (VA)

Phase angle: $\theta > 0$ & $P_X = VI\sin\theta > 0$ for current lags voltage (inductive)

$\theta < 0$ & $P_X = VI\sin\theta < 0$ for current leads voltage (capacitive)

Real power $VI\cos\theta > 0$ for $-90° < \theta < +90°$

Exercise 7.3 Find L & R for meter readings shown



Real $P = I^2 R$ $\therefore R = \frac{P}{I^2} = \frac{44W}{(2A)^2} = 11\Omega$

pf $\cos\theta = \frac{P}{VI} = \frac{44}{2 \times 31.11} = 0.707$

$\therefore \theta = 45°$

Reactive $P_X = VI \sin\theta = 2 \times 31.11 \sin 45° = 44$ VAR

$\therefore X_L = \frac{P_X}{I^2} = \frac{44}{2^2} = 11\Omega = \omega L$ $\therefore L = \frac{11}{2\pi 60} = 0.029H$

Exercise 7.4 For a load: $v = 10\sqrt{2}\cos(25t + \pi/8)V$ & $i = 3.5\sqrt{2}\sin(25t + 3\pi/4)A$

(a) Calculate pf

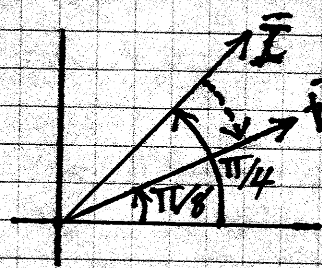
First need i as cos $\Rightarrow i = 3.5\sqrt{2}\cos(25t + \frac{\pi}{4})$

$= 3.5\angle\pi/4$

& $v = 10\angle\pi/8$

$\therefore \theta = \pi/8 - \pi/4$

$= -\pi/8$



θ = angle by which $\bar{V}$ leads $\bar{I}$

So pf $= \cos(-\pi/8) = \cos 22.5°$

$= 0.9239$

(b) Calculate load average power and reactive power

$P = VI\cos\theta = 10 \times 3.5 \times 0.924 = 32.3$ watts

$Q = P_X = VI\sin\theta = 10 \times 3.5 \times \sin(-22.5°) = -13.4$ VARs

Exercise 7.5 20KW Load heating (real) + 150 kVA @ 0.6 lagging pf (induction motors)

Supply is 4000V.

Determine total current and overall pf.

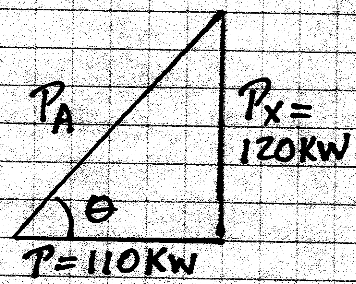
$$P_H = 20\text{ kW} \qquad P_M = 150 \times 0.6 = 90\text{ kW}$$

$$P_{XH} = 0 \qquad P_{XM} = 150 \sin\theta = 150(1-\cos^2\theta)^{1/2} = 150(1-0.6^2)^{1/2} = 150\sqrt{0.64}$$

$$= 150 \times 0.8 = 120\text{ kW}$$

$\therefore$ Total real power $= 20 + 90 = 110\text{ kW}$

Total reactive power $= 0 + 120 = 120\text{ kW}$



$$pf = \cos\theta = \cos\left(\arctan\tfrac{120}{110}\right) = 0.676$$

$$P_A = (110^2 + 120^2)^{1/2} \angle \arctan\tfrac{120}{110} = 162.79 \angle 47.49^\circ \text{ KVA}$$

& Total current $= \dfrac{P_A}{V} = \dfrac{162.8 \times 10^3\text{ VA}}{4000\text{ V}} = 40.7\text{A}$

Note that to supply this load requires wire to carry 40.7A and a 163KVA transformer.

Much of this requirement is due to the <u>unnecessary</u> reactive power.

If $pf = 1$, ie. $\theta = 0$, would only need 110 KVA transformer and $\dfrac{110{,}000}{4{,}000} = 27.5$A wire, and also less I^2R losses in wire etc.

Also, power costs are by apparent KVA.

Usually industrial equipment is inductive $\Rightarrow$ lagging pf.

Add C to move $pf \Rightarrow 1$

EXERCISE 7.6

Plant Loads specified in the table. Specify equipment to correct to 0.95 pf lagging.

Load	P_A (KVA)	θ (°)	pf	P (KW)	P_X (KVAR)	V (volts)
Initial	170	45	0.71	120		4000
Required			0.95	120		4000

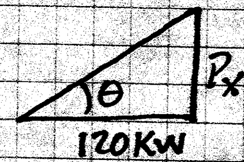
θ reqd $= \cos^{-1} 0.95$

$= 18.2°$

$\therefore$ Reduce reactive power to

$P_X = 120 \tan 18.2°$

$= 39.44$ KVAR



Initial $P_X = 120 \tan 45° = 120$ KVAR

i.e. Correction needed $= 39.44 - 120 = -80.56$ KVAR

Negative correction needs capacitor C with $P_{XC} = \dfrac{V^2}{X_C} = V^2(-\omega C)$

i.e. $C = \dfrac{P_{XC}}{-\omega V^2} = \dfrac{-80.56 \times 10^3}{-(2\pi 60)(4000)^2} = 13.4\,\mu F$

So, complete table

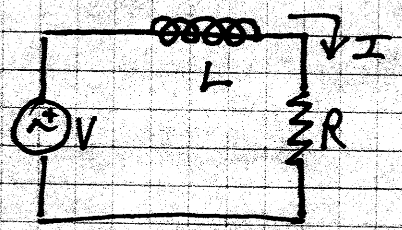
Load	P_A (KVA)	θ (°)	pf	P (KW)	P_X (KVAR)
Initial	170	45	0.71	120	120
Final	126.3	18.2	0.95	120	39.44
Correction	-43.7			0	-80.56

$P_A^2 = P^2 + P_X^2$

$= 120^2 + 39.44^2$

$P_A = 126.3$ KVA

FREQUENCY RESPONSE



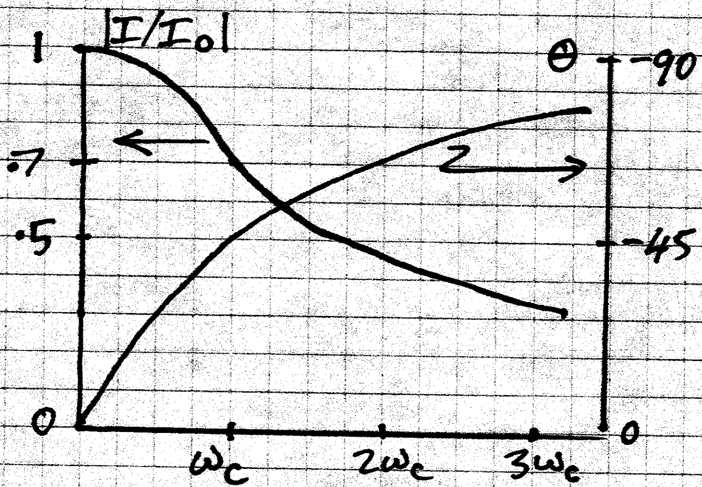
$$I(\omega) = \frac{\hat{V}}{Z} = \frac{\hat{V}}{R + j\omega L} = \frac{\hat{V}/R}{1 + j\omega \frac{L}{R}} \xrightarrow[\substack{DC \\ \omega = 0}]{} \hat{I}_0 = \hat{V}/R$$

$$\therefore \frac{\hat{I}}{\hat{I}_0} = \frac{1}{1 + j\omega \frac{L}{R}} = \frac{1 - j\omega L/R}{1 + \left(\omega \frac{L}{R}\right)^2} = \frac{\left[1 + (\omega L/R)^2\right]^{1/2}}{1 + (\omega L/R)^2} \angle \arctan \frac{-\omega L}{R}$$

$$= \frac{1}{\sqrt{1 + \left(\omega \frac{L}{R}\right)^2}} \angle \arctan - \omega L/R = \frac{I}{I_0} \angle \theta$$

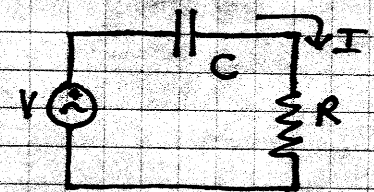
At $\omega_c = R/L$ $\quad \frac{\hat{I}}{\hat{I}_0} = \frac{1}{\sqrt{2}} \angle \arctan -1 = 0.707 \angle -45^\circ$

$I^2 R = \frac{1}{2} I_0^2 R$ i.e. this is the "half-power" freq



Low pass filter

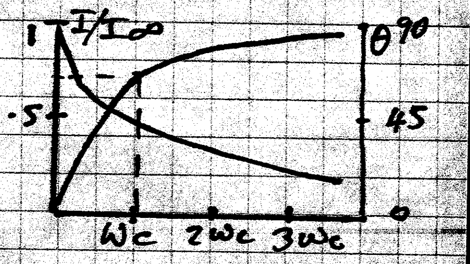
Similarly



$$\hat{I} = \frac{\hat{V}}{Z} = \frac{\hat{V}}{R - j/\omega C} = \frac{\hat{V}/R}{1 - \frac{j}{\omega RC}} \xrightarrow[\omega \to \infty]{HF} \hat{I}_\infty = \frac{\hat{V}}{R}$$

$$\frac{\hat{I}}{\hat{I}_\infty} = \frac{1}{1 - j/\omega RC} = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} \angle \tan^{-1} 1/\omega RC$$

At $\omega_c = 1/RC$ $\quad \left|\frac{I}{I_\infty}\right| \to \frac{1}{\sqrt{2}}$



ASSIGNMENT #5

P 6.14

6.16

6.20

6.22

7.3

7.5

7.9

7.10