

## ECE 241L Fundamentals of Electrical Engineering

### Experiment 7 Frequency Response

NAME \_\_\_\_\_

PARTNER \_\_\_\_\_

#### A. Objectives:

- I. Measure frequency response of RC low-pass and CR high-pass filters
- II. Measure frequency response of an RLC frequency selective filter
- III. Measure uncalibrated inductance and capacitance

#### B. Equipment:

Breadboard, resistor(s), potentiometer, wire (student lab kit), wire stripper  
Digital Volt-Ohm Meter (DVM): Fluke 189 (or equivalent)  
Oscilloscope: Tektronix TDS3043 Digital Storage Scope  
Function Generator: Tektronix AFG310/320 Arbitrary Function Generator  
Inductor: 100 mH nominal  
Capacitors: 0.001 and 0.1  $\mu\text{F}$  nominal

#### C. Introductory Notes:

Filters: Electric circuits can be used as filters to pass certain frequencies and stop others. For example the tone control on a radio is used to vary the amount of low frequencies (bass) and high frequencies (treble) in playing music. Frequency selectable filters are used to tune in a desired station and reject other stations. This experiment will demonstrate three standard types of filters: low-pass, high-pass, and band select. Simple filters can be designed using R, C, and L in voltage divider circuits.

#### 1. LOW-PASS FILTER

A low-pass filter can be constructed as a voltage divider with a resistance R and a capacitance C in series, with the output voltage across the capacitor.

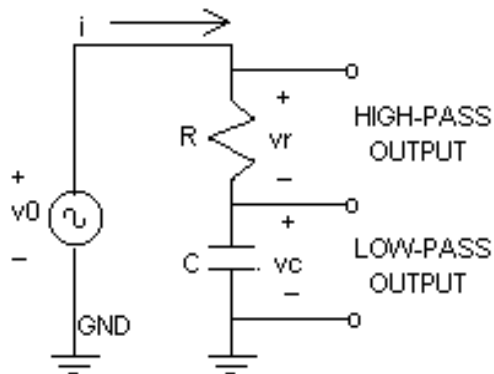


Figure E4-1 Voltage divider circuit to form an RC filter

In this circuit, the low-pass filter frequency response  $|H(f)|$  is given by the (dimensionless) ratio of the voltage across the capacitor divided by the input voltage.

$$|H| = [V_c]/[V_o]$$

From the voltage divider formula and Kirchhoff's Voltage Law for a-c circuits

$$[V_c] = [V_o] X_c / (R^2 + X_c^2)^{0.5}$$

From the reactance of a capacitor

$$X_c = 1/2\pi fC$$

$$|H| = 1 / [1 + (f / F_c)^2]^{0.5}$$

$$F_c = 1/2\pi RC$$

**FREQUENCY RESPONSE OF LOW-PASS FILTER  
FILTER CUTOFF FREQUENCY (Hz)**

The graph of  $|H|$  as a function of frequency is sketched in Figure E4-2

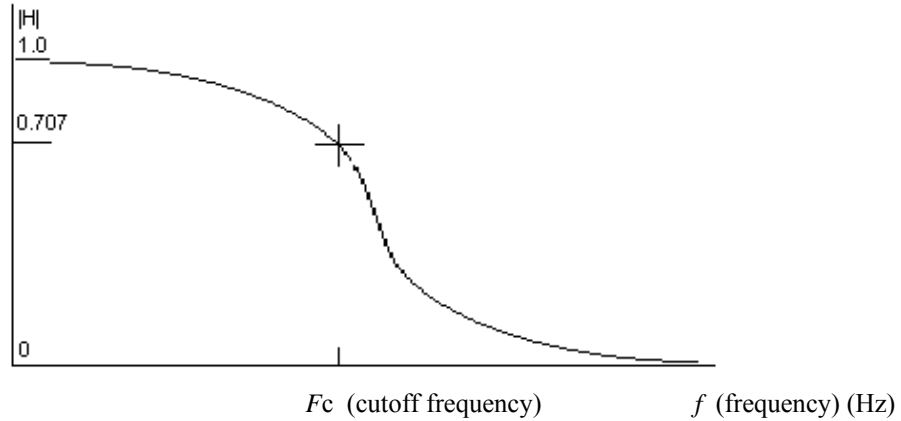


Figure E4-2 Frequency response of an RC low-pass filter.

At the cutoff frequency  $F_c$  the output voltage is 70.7% of the input. As frequency is increased above the cutoff (in the stop band) the voltage is less and less. For this filter the output voltage in the stop band is proportional to  $1/f$ , the reciprocal of the frequency. At all lower frequencies the ratio  $|H|$  approaches 1.0 (in the pass band). At the cutoff frequency the output power (proportional to the square of the voltage) is 0.5 times the input power, so this frequency is also referred to as the "half-power" frequency.

**2. HIGH-PASS FILTER**

By interchanging the R and C in the voltage divider the circuit can be made to stop low frequencies and pass high frequencies. The output  $V_r$  appears across the resistor.

$$[V_r] = [V_o] R / (R^2 + X_c^2)^{0.5}$$

$$|H| = (f / F_c) / [1 + (f / F_c)^2]^{0.5}$$

**FREQUENCY RESPONSE OF HIGH-PASS FILTER**

Again at  $f = F_c$  the output voltage is 70.0% of the input voltage, but now the output is less and less at frequencies below  $F_c$  (in the stop band), going to zero at  $f = 0$ . The output voltage in the stop band is linearly proportional to frequency. At all higher frequencies the ratio  $|H|$  approaches 1.0 (in the pass band). The graph of  $|H|$  is sketched in Figure E4-3.

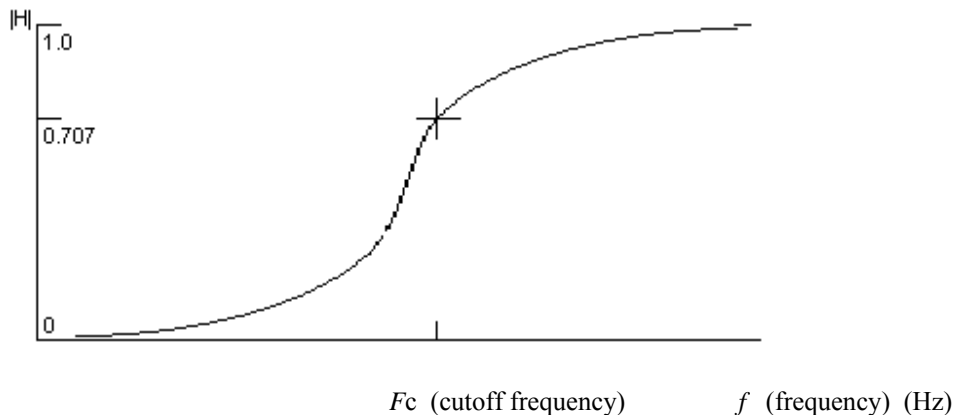


Figure E4-3 Frequency response of a CR high-pass filter.

### 3. FREQUENCY SELECTIVE FILTER

A band-pass filter can be made by connecting a low-pass filter to the output of a high-pass filter, or vice-versa. However, such a filter is not very frequency selective, with a rather wide pass band. A narrow-band, frequency selective filter can be designed using the phenomenon of "resonance". A series circuit with inductance L and capacitance C has the unique property that the capacitive reactance is of opposite (imaginary) sign from the inductance, and at a particular frequency the reactance of the inductance will cancel the reactance of the capacitance. In fact the impedance of the series combination would approach zero at the resonant frequency  $F_0$ .

$$2\pi F_0 L = 1/2\pi F_0 C$$

$$F_0 = 1/2\pi (LC)^{0.5}$$

**RESONANT FREQUENCY OF LC CIRCUIT**

If it were not for the fact that the inductor always has finite resistance in addition to inductance, the LC series combination would cause a short circuit across the source at resonance. For practical inductors, the resistance cannot be separated from the inductance. However, in the voltage divider circuit in Figure E4-4 the resistance is shown explicitly in the circuit diagram even though the node between the resistance and the inductance is not accessible.

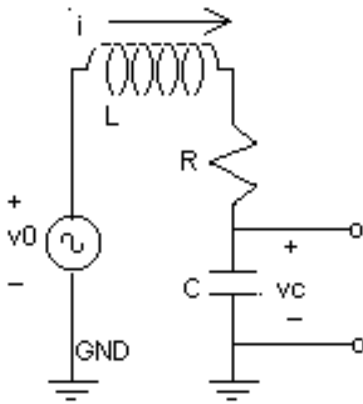


Figure E4-4 Voltage divider circuit to form an LC frequency selective filter

At the resonant frequency the inductive reactance and capacitive reactance cancel each other and only the resistance remains effective to limit the current. At resonance the current is equal to  $[V_0]/R$ . At resonance a remarkable phenomenon occurs, in that the voltage across the capacitance will be much greater than the input voltage. The voltage across the inductance is equal to that across the capacitance, but is in opposite phase. However, the voltage across the inductor coil includes the voltage across both the inductive reactance and the resistance.

In this experiment you will take data to plot the frequency response by measuring the output voltage  $[V_c]$  across the capacitance, and the input voltage  $[V_0]$ . The response at each frequency is the ratio of the output voltage divided by the input voltage. Since  $|H|$  is a dimensionless ratio the voltage can be expressed either as the sinusoidal amplitude or as the RMS voltage, as long as the output and input are expressed in the same way. It will be assumed that all a-c voltages are expressed as RMS.

$$|H| = [V_c]/[V_0] = X_c / (R^2 + |X|^2)^{0.5} \quad \text{FREQUENCY RESPONSE, SELECTIVE FILTER}$$

$$|X|^2 = (2\pi f L - 1/2\pi f C)^2$$

At low frequencies  $X_c \rightarrow \infty$ ,  $|X| \gg R$  and  $|X| \rightarrow X_c$  so that  $|H| \rightarrow 1$ . At high frequencies  $X_c \rightarrow 0$  so that  $|H| \rightarrow 0$ . However at resonance  $|X| = 0$  and

$$|H| = X_c / R = 1/2 \pi F_0 C R = (L/C)^{0.5} / R \quad \text{RESPONSE OF FILTER AT RESONANCE}$$

$$(L/C)^{0.5} / R = Q_0 \quad \text{QUALITY FACTOR OF SELECTIVE FILTER}$$

This dimensionless ratio is also designated as the "quality factor"  $Q_0$ . If the resistance  $R$  is small compared to the numerator term  $(L/C)^{0.5}$ , the output voltage can be much greater than the input. It is also possible to derive the fact that the current is 70.7% of its maximum at two frequencies, one slightly above and the other slightly below resonance. These "half-power" cutoff frequencies are located at

$$F_c(\text{upper}) = F_0 + F_0/2Q_0$$

$$F_c(\text{lower}) = F_0 - F_0/2Q_0$$

The frequency separation between the cutoff frequencies is called the "bandwidth"  $\Delta f$ .

$$\Delta f = F_0/Q_0$$

#### BANDWIDTH OF SELECTIVE FILTER

The output voltage at the cutoff frequencies is also approximately at 70.7% of the maximum that occurs at the resonant frequency. The pass band is located between them. For high- $Q$  circuits this filter will be very selective, with very small response at any frequency other than resonance. However, for this circuit there is a finite response at zero frequency since  $X_c \rightarrow \infty$  so that  $[V_c] = [V_o]$  as the capacitive reactance becomes an open circuit.

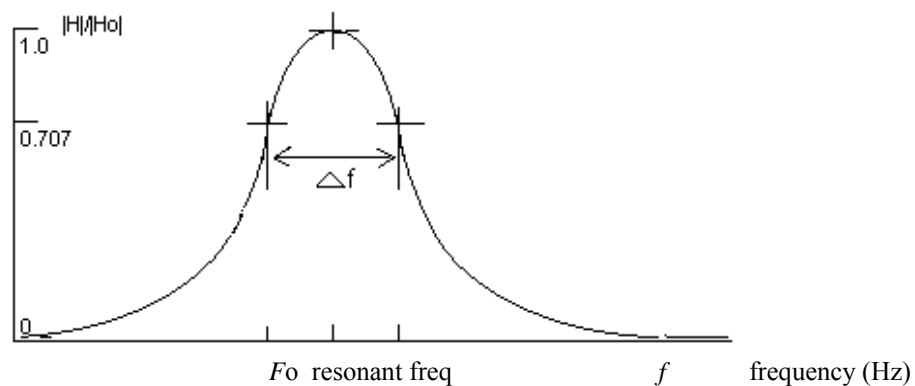


Figure E4-5 Frequency response of an LCR selective filter.

## D. PROCEDURE:

### I. Low-pass and High-pass Filter

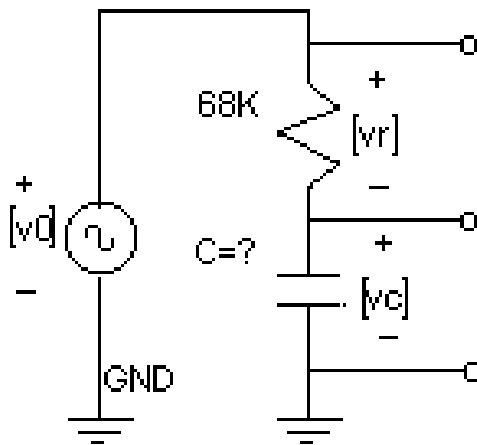


Figure E4-5 RC Low-pass ( $[V_c]/[V_0]$ ) and High-pass ( $[V_r]/[V_0]$ ) filters

1. Build the circuit in Figure E4-5 on your breadboard, with  $C = 1nF$ . Mount a coaxial TEE connector and  $50\Omega$  termination to the output ( $V_0$ ) of the Function Generator. Connect the circuit to one branch of the TEE using coaxial cable with clip leads. Be sure the clip lead from the shield of the coax is connected to GND on the circuit. The other clip lead connects to the resistor. Connect CH1 of the scope in parallel with the Function Generator  $[V_0]$ , using the BNC micro-clip cable. Connect CH2 of the scope to the capacitor  $[V_c]$  using clip leads. Note that the outer shields of both scope probes are connected to GND on the circuit. However, the leads from the voltmeter AC VOLTS are to be connected across the resistor  $[V_r]$  where neither point is at ground.

On the Function Generator, select the SINE WAVE function. On the scope use trigger [MODE] (AUTOLEVEL).

2. Temporarily set the input voltage ( $V_0$ ) by observing the display on the scope. The period at 1000 Hz is 1.0 millisecond. Adjust the Function Generator to obtain  $V_{pp} = 2.8$  volts on the scope, corresponding to  $[V_0] = 1.0$  volt RMS. In order to get better accuracy in measurement of frequency, you will use the scope to measure the period of the a-c signal instead of relying on the dial of the Function Generator. The frequency will be varied over a wide range in this experiment.

3. The frequency will be varied in "octave" steps beginning at 200 Hz. Octave steps are obtained by multiplying by  $2^n$  at each step  $n$ , starting from 200 Hz at  $n = 0$ . (The term "octave" is from music, where each octave corresponds to 8 "white keys" on a music keyboard. For example there are seven octaves on a piano.) The frequency steps are then at 200, 400, 800, 1.6K, 3.2K, 6.4K, 12.8K, 25.6K (Hz). Before starting to make measurements, note the period corresponding to each frequency as tabulated. At each step, measure RMS voltages  $[V_c]$  on the scope and  $[V_r]$  on the voltmeter. As you make changes in the frequency recheck the input voltage and adjust the ATTN control on the Function Generator whenever necessary to maintain  $[V_0] = 1.0$  volts.

The frequency response will be plotted in units of **decibels** referred to one volt,  $dB_v = 20 \log([V]/[1.0])$ . Since  $|H|$  is a dimensionless fraction with a value equal to or less than unity, the response in decibels will be negative numbers. The frequency scale is plotted with **equal size** increments on the horizontal axis, creating a logarithmic scale, proportional to  $\log_2(f)$ . This type of plot is called a "Bode" diagram, (pronounced BO-DEE). Note: You will also encounter theoretical analysis of filters in which the

horizontal frequency scale is in "decades", proportional to  $10^n$  increments. The graph looks the same in either case.

For a low-pass filter the data should plot as a horizontal line for frequencies well below cutoff in the pass band, and as a line slanting downward for frequencies well above cutoff in the stop band. The slope of the line can be derived theoretically as -6 dB/octave. On the other hand, for a high-pass filter the data should plot as a line slanting upward at +6 dB/octave in the stop band and as a horizontal line in the pass band. At the cutoff frequency  $F_c$  the data should curve away from the straight lines and pass through a point -3dB below the intersection of the horizontal and sloping straight lines. Plot both sets of data on the same graph.

Freq (Hz)	Period (msec)	[Vc] (RMS)	20log([Vc]/[Vo]) (decibels)	[Vr] (RMS)	20log([Vr]/[Vo]) (decibels)
200	5.0				
400	2.5				
800	1.25				
1.6K	0.62				
3.2K	0.32				
6.4K	0.16				
12.8K	0.08				
25.6K	0.04				

Plot data on the graph on the following page

4. Go back and find the cutoff frequency  $F_c$  from the frequency where  $[V_r] = [V_c] = 0.707 [V_o]$ .

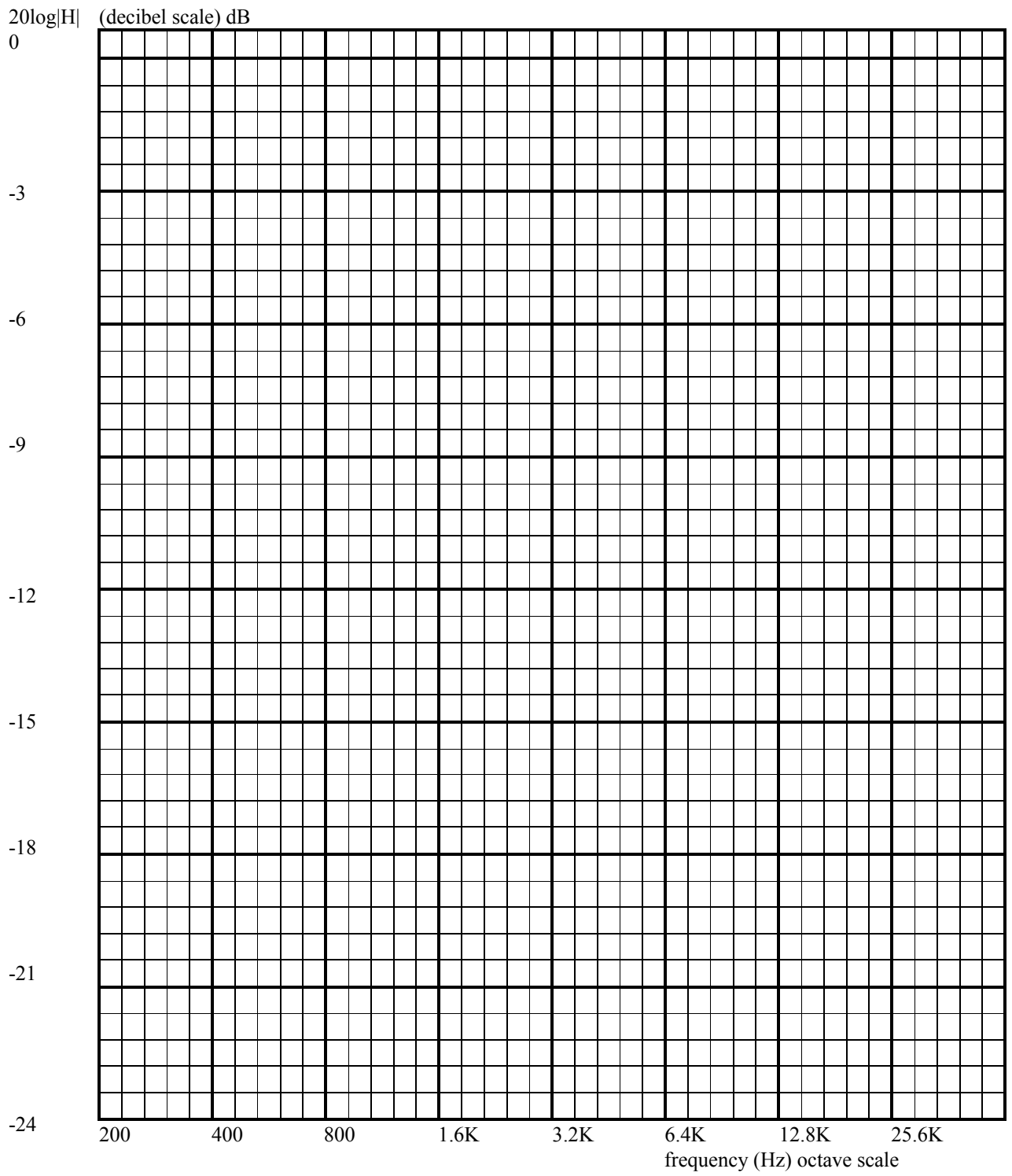
$$F_c = \text{_____} \text{ (Hz)}$$

From  $F_c$  and the known (measured) value of R, calculate the value of the (nominal) 1nF capacitor.

$$C = 1/2\pi R F_c = \text{_____} \text{ (Farad)}$$

Express C in SI prefix units

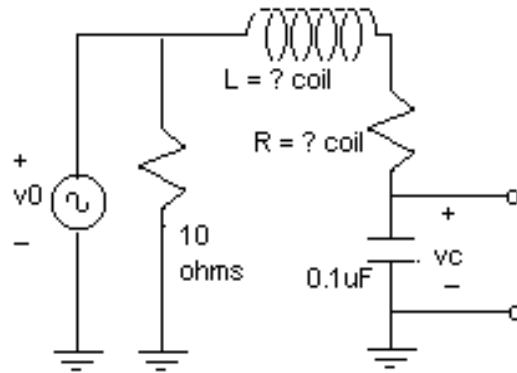
$$C = \text{_____} \mu\text{F} = \text{_____} \text{ nF} = \text{_____} \text{ pF}$$





II. Frequency Selective Filter

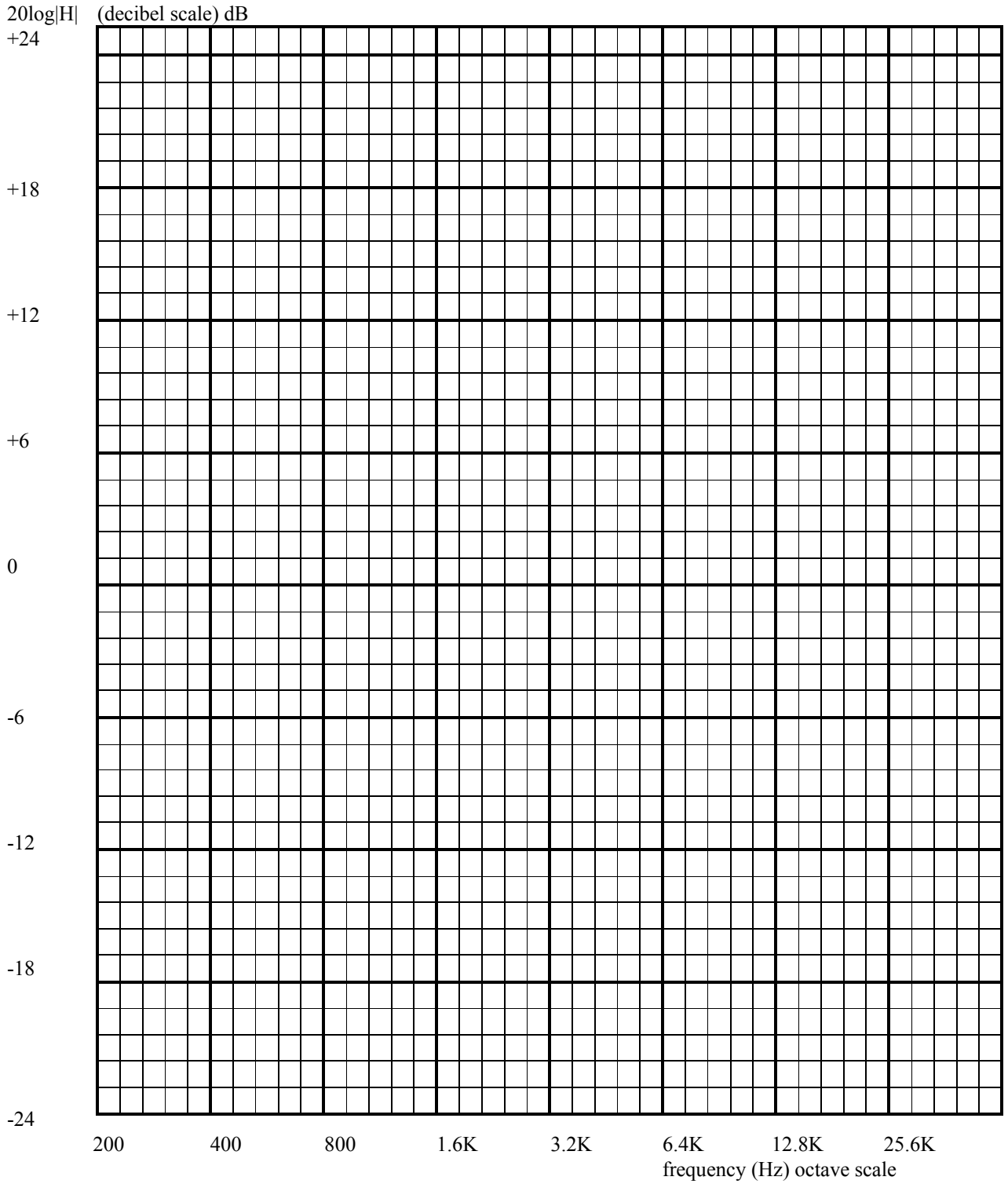
1. Replace the 68K resistor in your circuit with the 100mH coil. Replace the 1nF capacitor with a 0.1uF capacitor.
2. (You may need to connect a 10 ohm resistor in parallel with the series LC circuit as shown in the diagram below. The purpose of the 10 ohm resistor is to reduce the Thevenin resistance of the Function Generator to the much lower value of 10 ohms, but the 50Ω termination should be sufficient. Otherwise the Thevenin resistance of the generator would have the undesired effect of reducing the  $Q_0$  of the circuit and loading it down so much that it would not be resonant.) Note that  $R_{coil}$  is the inherent resistance of the copper wire in the coil, not a separate component. Measure  $R_{coil}$  with the DVM ohmmeter.



3. Connect CH1 of the scope to  $V_o$ , connect CH2 to  $V_c$ . On the Function Generator increase the output voltage with the AMPLITUDE control to set  $[V_o]$  to 100 millivolts RMS at 1000 Hz. Vary the frequency from 200 Hz to 25 KHz and note the variation in voltage. Be sure the input voltage can be maintained constant at  $[V_o] = 100$  millivolts over the entire range of frequency by adjusting the attenuation knob on the generator. The voltmeter will not be used in this part. Use the procedure of part I to measure  $[V_c]$  RMS as a function of frequency, again in octave steps. At the higher frequencies,  $[V_c]$  will be so small that the scope display will be somewhat "fuzzy", so just estimate as well as you can, precise data is not needed. Calculate and plot  $[V_c]/[V_o]$  in deciBels =  $20 \log([V_c]/[V_o])$ .

Frequency (Hz)	Period (msec)	$[V_c]$ (RMS)	$20\log([V_c]/[V_o])$ (dB)
200	5.0		
400	2.5		
800	1.25		
1.6K	0.62		
3.2K	0.31		
6.4K	0.16		
12.8K	0.08		
25.6K	0.04		

Plot the data on the graph on the following page



4. Go back and find the frequency  $F_0$  at which  $[V_c]$  is a maximum.

$$F_0 = \underline{\hspace{2cm}}$$

Plot this point.

Carefully tune the Function Generator to frequencies on either side of  $F_0$  to locate the frequencies where the voltage is 70.7% of the maximum voltage.

$$\text{upper cutoff freq } (F_0 + \Delta f/2) = \underline{\hspace{2cm}} \quad \text{lower cutoff freq } (F_0 - \Delta f/2) = \underline{\hspace{2cm}}$$

Plot those points at -3dB below the peak. The difference in the frequencies is the bandwidth  $\Delta f$ .

$$\text{Measured bandwidth } \Delta f = \underline{\hspace{2cm}}$$

Calculate the quality factor

$$Q_0 = F_0/\Delta f \underline{\hspace{2cm}}$$

From the value of  $F_0$  and the capacitance  $C$ , calculate the inductance of the coil

$$L_{\text{coil}} = 1/[(2\pi F_0)^2 C] = \underline{\hspace{2cm}}$$

From the value of  $L_{\text{coil}}$ ,  $C$  and  $Q_0$  calculate the effective resistance of the coil

$$R_{\text{coil}} = [L_{\text{coil}}/C]^{0.5}/Q_0 \underline{\hspace{2cm}}$$

Correct for the signal generator, T-connector, &  $10\Omega$  resistor (if used)

$$R_{\text{source}} = 50\Omega(\text{SG}) \parallel 50\Omega(\text{T}) \parallel 10\Omega(\text{R}) = \underline{\hspace{2cm}}$$

$$\text{Corrected } R_{\text{coil}} = R_{\text{coil}} - R_{\text{source}} = \underline{\hspace{2cm}}$$

Measure the d-c resistance of the coil using the Ohmmeter.

$$R_{\text{d-c}} = \underline{\hspace{2cm}}$$

Compare to the (corrected) calculated resistance  $R_{\text{coil}}$ . What is the percentage difference?

$$\% \text{ diff} = \underline{\hspace{2cm}}$$

SUMMARY: (To be completed at the end of lab)

Sketch the graph of the frequency response of an RC low-pass filter. Label the sketch at the cutoff frequency. Write the equation for the graph.

Sketch the graph of the frequency response of a CR high-pass filter. Label the sketch at the cutoff frequency. Write the equation for the graph.

Sketch the graph of the frequency response of an LC frequency selective filter. Label the sketch at the resonant frequency and the two cutoff frequencies. Write the equation for the graph.

Write the equation for the "quality factor"  $Q_0$  of a frequency selective filter

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Write the equation for the bandwidth of a frequency selective filter, in terms of  $Q_0$ .

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Comment on the results of this experiment. Were there any surprises?

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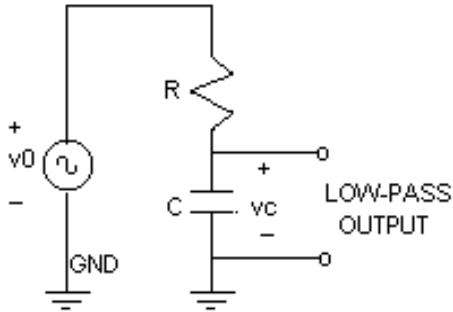
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HOMEWORK:

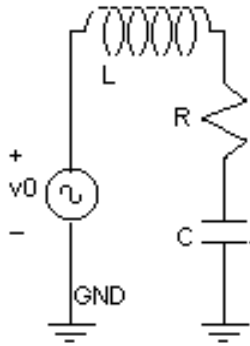
1. Given  $R = 47\text{K ohms}$ , and cutoff frequency  $F_c = 3200\text{ Hz}$ . Calculate  $C = \underline{\hspace{2cm}}$  uF (microFarad)

Calculate  $-jX_c$  at 3200 Hz =  $-j \underline{\hspace{2cm}}$  ohms.



2. Given resonant frequency  $F_o = 2500\text{ Hz}$  with capacitance  $C = 0.1\text{ uF}$ , calculate  $L = \underline{\hspace{2cm}}$  mH

Calculate inductive reactance  $jX_L$  at 2500 Hz =  $j \underline{\hspace{2cm}}$



3. Given an inductor with inductance  $L = 50\text{ milliHenry}$  and resistance  $R = 60\text{ ohms}$ , in series with capacitance  $C = 0.1\text{ uF}$ .

(a) Calculate the bandwidth  $\Delta f$  of the frequency selective circuit.  $\Delta f = \underline{\hspace{2cm}}$

(b) Calculate the quality factor  $Q_o = \underline{\hspace{2cm}}$

4. The formula for the capacitance of a parallel plate capacitor is:  $C = \epsilon_r \epsilon_0 \hat{A} / d$

where

$\hat{A}$  = area ( $\text{m}^2$ )

$d$  = separation of plates (m)

$\epsilon_0 = 8.85\text{ picoFarad per meter}$ , permittivity ("capacitivity") of free space

$\epsilon_r$  = dielectric constant of material between the plates (dimensionless)

Assume  $\hat{A} = 0.01\text{ m}^2$ ,  $d = 10^{-4}\text{ m}$ , Calculate the dielectric constant of the material separating the plates of a capacitor  $C = 0.001\text{ microFarad}$ .

$\epsilon_r = \underline{\hspace{2cm}}$