ECE 241L Fundamentals of Electrical Engineering

NAME

Experiment 6 AC Circuits

PARTNER

A. Objectives:

Objectives:
I. Calculate amplitude and phase angles of a-c voltages and impedances
II. Calculate the reactance and capacitance of a capacitor

B. Equipment:

Breadboard, wire stripper, resistors, capacitors
Oscilloscope: Tektronix TDS3043 Digital Storage Scope
Function Generator: Tektronix AFG310/320 Arbitrary Function Generator
Digital Volt-Ohm Meter (DVM): Fluke 189 (or equivalent)

C. Introductory Notes:

Frequency

Direct measurements of a-c waveforms are routinely made from the scope display. The period **T** is the time duration of a complete cycle, where usually several cycles are visible on the screen.

$v(t) = A \sin(2\pi\ t/T)$ voltage waveform
T = period (seconds)

In the laboratory the frequency f is calculated from the number of periods (complete cycles) per second, measured in units of Hertz.

$f = 1/T$ frequency (cycles per second = Hz)

In theoretical calculations the frequency is usually measured as the RADIAN RATE ω in units of radians/second, which is also equal to the number of periods in 2π seconds.

$\omega = 2\pi/T = 2\pi f$ radian rate (radians/sec)

RMS Voltage

Measurements of voltage are also made from the scope display of a-c waveforms. In theoretical equations the voltage is expressed as the peak amplitude A of the sinusoidal wave.

$v(t) = A \sin(2\pi\ t/T)$ voltage waveform
A = peak amplitude (volts)

In the laboratory, a-c voltage is most conveniently measured on a scope as the distance between the positive peak +A and the negative peak -A, called the **peak-to-peak voltage Vp-p= 2A**. However a-c voltmeters are always calibrated in units of **RMS volts [v] = 0.707A** , in order to simplify calculations of average power dissipated in a circuit.

RMS voltage will be expressed using **square brackets [v]**.

In this experiment the RMS voltage will be calculated from the scope in order to be compatible with measurements using an a-c voltmeter.

[v] = 0.707 A a-c voltage (volts RMS)
[v] = 2A / 2.83 volts RMS, calculated from V p-p

A-C Current

Current is also expressed as RMS and can be calculated in the usual way from Ohms Law

[i] = [v]/R a-c current RMS

A-C Electrical Power

For sinusoidal a-c voltages, the average electrical power can be calculated using the same Joule's Law formula as for d-c voltage when RMS units are used.

$\mathbf{P_{ac} = [v][i] = [v]^2 / R = [i]^2 R}$ **a-c electric power**

A-C Circuits, Reactance and Impedance

The most important characteristic of a-c circuits for the student to learn is the **phasor** addition of voltage, which is closely related to the concepts of **reactance** and **impedance**. The voltage across a capacitor or inductor is related to the current by a differential equation. However, in the middle 1800s the self-taught English mathematical genius Oliver Heaviside introduced a way of converting differential equations to algebraic equations called "operational calculus", now called the Laplace Transform. His method was extended by the great electrical engineer Charles Proteus Steinmetz at the General Electric Research Laboratory in Schenectady, New York in the early 1900s to make it possible to relate voltage and current in a form similar to Ohm's Law, using the algebra of Complex numbers. Calculation of a-c circuits is greatly simplified using phasor notation instead of trigonometric functions.

Note: Engineers use **Complex numbers called "phasors"** that are expressed as **pairs of numbers** in either "rectangular" (*Re* + j*Im*), or "polar" (Magnitude x Angle) form. **Student should become proficient in the use of the "rectangular-to-polar" conversion function on a pocket calculator, and its inverse.** The addition of complex quantities requires the rectangular form, and multiplication requires the polar form. Therefore, application of the parallel resistance formula or the voltage divider formula requires both operations. (The method of "multiplication by complex conjugates" presented in some math courses is to be discouraged as unwieldy and with a high probability of error in computation).

Phase Angle

Another way of measuring time for a sinusoidal wave is in terms of an angle, called the phase angle.

$v(t) = A \sin(\theta)$ $\qquad \theta = 2\pi f t$ $\qquad f = 1/T$

The "argument" of a sinusoidal function is an angle $\theta = 2\pi f t$ for which one complete cycle is equal to 2π radians. When comparing two sine waves of the same frequency, their relative position on the time axis can be expressed in terms of relative phase angle ϕ.

$$x(t) = A \sin(2\pi f t)$$
$$y(t) = B \sin(2\pi f t - \phi)$$

For example, if $\phi = \pi/2$, then y(t) is at a negative maximum when t = 0, while x(t) is at a zero going positive as sketched in Figure E3-1

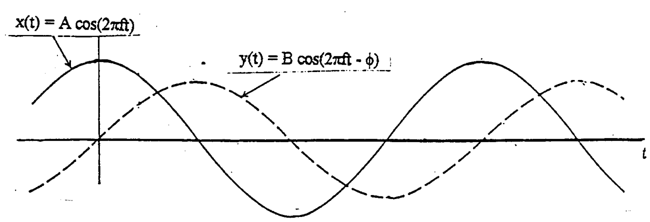


Figure E3-1 Comparison of two sine waves as would be observed on a dual trace scope

In this case, y(t) is said to be "lagging" x(t) by $\pi/2$ radians. In other words, y(t) is at a zero going positive at a later time than x(t), designated by an angle $\pi/2$ radians.

Rather than drawing the entire graph of a sine wave, the same information can be expressed more compactly using a s**pecial form of vector called a phasor**. It is conventional to draw a phasor with the angle measured counterclockwise (rotating left) from the positive horizontal axis. The previous example is sketched in Figure E3-2.

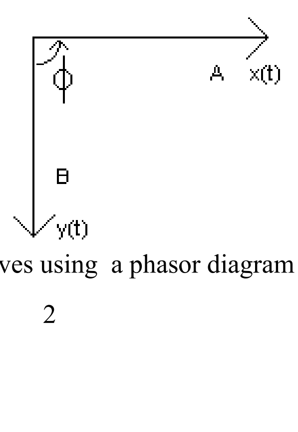


Figure E3-2 Comparison of two sine waves using a phasor diagram

The value of sin(θ) can be obtained from the phasor by the "projection" on the vertical axis (like the length of its shadow). Likewise the value of cos(θ) can be obtained from the projection on the horizontal axis. Theoretically, a phasor is expressed in terms of an exponential function with amplitude A and angle θ as

$$x(t) = A \exp(j\,\theta)$$

IMPORTANT: NOTE THAT THE ARGUMENT OF THE EXPONENTIAL IS IMAGINARY. The function exp(±θ) is quite different! The relationship to the trigonometric form is given by the Euler (pronounced "oiler", not you-ler) formula.

$$\exp(j\theta) = \cos(\theta) + j \sin(\theta) \qquad \text{EULER FORMULA}$$

In the laboratory the voltage (amplitude) is often expressed in terms of the RMS value (length of the phasor) and the phase angle in terms of the angle in DEGREES. For the example above, the relationship could be written in these terms as:

x(t) = [A] /0 degrees	y(t) = [B] /-90 degrees
[x] = 0.707 A	[y] = 0.707 B

Phasors

The phase is accounted for by the angle of a vector compared to the angle of another vector. The amplitude can be indicated with the length of a vector equal to the RMS value [A]. The relative phase angle between two a-c voltages can be displayed on a dual trace scope. By setting the horizontal time scale, one complete period T corresponding to 360 degrees of phase on the horizontal axis can be calibrated so the relative horizontal location of one wave with respect to another can be determined.

A-C Circuits

For **a-c circuits**, the Heaviside-Steinmetz equivalent to Ohm's Law should be **memorized by the student**.

V = Z I **PHASOR OHM'S LAW**

Z = impedance

Note that this is a vector equation, in which all quantities are Complex

V = [V] /V	**polar form of voltage**
I = [I] /I	**polar form of current**
Z = |Z| /Z	**polar form of impedance**
|Z| = [V] / [I]	**absolute magnitude of impedance**
/Z = /V - /I	**phase angle of impedance**
R + jX	**rectangular form of impedance**

Capacitive Reactance

For a capacitance, the impedance Z is mathematically negative (-j *Im*) , with an angle /Z = -90 degrees, meaning that the instantaneous a-c voltage reaches a maximum when the instantaneous current through the capacitance is zero going positive. The voltage is said to "lag" the current by a phase angle of -90 degrees. The ratio of the amplitude of the voltage to the amplitude of the current is the **capacitive reactance,** usually written $\mathbf{-j\,X_C}$. The Imaginary coefficient -j is another way of expressing the phase angle -90 degrees, since that is the way it would be plotted as a phasor (like a vector) on a rectangular coordinate graph of *Re* vs. j*Im* quantities. For a capacitance, the impedance (in this case a negative reactance) can be expressed in terms of the capacitance and the frequency.

$$\mathbf{V/I = Z = -jX_C}$$

$$\mathbf{-jX_C = 1/\,j\omega C = -j/2\,\pi f\, C}$$

$-jX_C$ = Capacitive **Reactance** (units Ohms) C = capacitance (units Farad)

Inductive Reactance

Similarly for inductance the impedance Z is mathematically positive Imaginary (+j*Im*), with angle /Z = +90 degrees. The voltage is said to "lead" the current by 90 degrees.

$$\mathbf{V/I = Z = +jX_L}$$

$$\mathbf{+jX_L = j\omega L = +j\,2\pi f\, L}$$

$+jX_L$ = Inductive **Reactance** (units Ohms) L = inductance (units Henry)

Complex Impedance

Resistances are always mathematically Real impedances. **If a reactance is connected in series with a resistance, the Complex sum is called the impedance of the circuit Z.** The magnitude of the impedance is given by the Pythagorean Theorem, since the resistance and reactance form the sides of a right triangle.

$$|Z|^2 = R^2 + X^2 \qquad \textbf{Pythagorean theorem}$$

$$\underline{/Z} = \arctan(-X / R) \qquad \textbf{arctan angle (capacitive)}$$

Example: Given a series RC circuit connected to a voltage source. Kirchhoff's Voltage Law gives

$$\underline{Vo} = \underline{Vr} + \underline{Vc} \qquad \textbf{THIS IS A VECTOR EQUATION !}$$

Note that Vc is at a phase angle of -90 degrees with respect to Vr . Hence by the Pythagorean Theorem the **square** of the RMS voltages add.

$$[V]^2 = [Vr]^2 + [Vc]^2$$

The phase angle between the voltages is given by the arctan function, which can also be expressed in terms of RMS voltage. However, the negative sign of the phase (for capacitive reactance) must still be taken into account.

$$\underline{/V}o = -\arctan([Vc] / [Vr])$$

It is important to note that if the **frequency is increased** in this **capacitive** circuit, the **reactance decreases** and therefore in the circuit the **voltages and current change**.

$$-jX_C = -j/2\pi f C$$

In the case of an inductive circuit, if the frequency is increased, the reactance also increases in proportion.

$$+jX_C = +j\omega L = +j\, 2\pi f L$$

D. PROCEDURE:

1. Measure the actual value of a 6.8K ohm resistor. Identify the 0.010 μF capacitor. Build the series circuit in Figure E3-3.

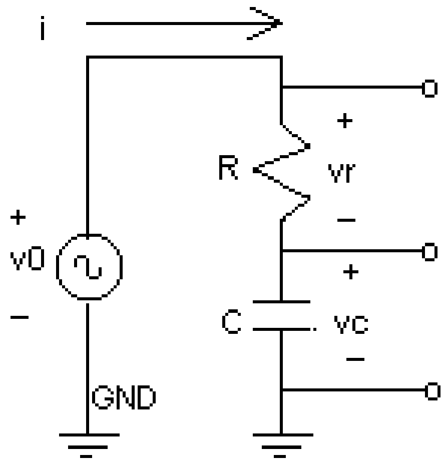


Figure E3-3 Series circuit for measurement of a-c voltage and impedance.

(a) Connect CH1 of the scope to Vo using a coax-to-clip lead cable. Select trigger source CH1. Be sure that the shield (black clip) is connected to the ground point in your circuit. Connect CH2 to Vc in the same way. Set the voltmeter to AC VOLTS and connect across the resistor to measure [Vr], and also to calculate the current [I]. (Use Vo = 5Vrms at around 1kHz, (but less than f_{max} for the voltmeter.)

[Vr] = __________________ (volts RMS)

[I] = [Vr]/R =______________ (milliAmps RMS)

(b) On the scope, press [VOLTS] and select Vrms softkey to measure [Vo] and [Vc] .

[Vo] = _______________ [Vc] = __________________

(c) Calculate the capacitive reactance

Xc = [Vc]/[I] = ________________

and the absolute magnitude of the Complex impedance of the series circuit

|Z| = [Vo]/[I] = __________________

Check that KVL and the Pythagorean Theorem is satisfied by the a-c voltages

$[Vo]^2$ = _______________ = $[Vc]^2 + [Vr]^2$ = ____________________

Check that the impedance is given by the Pythagorean theorem from the reactance and the resistance

Z^2 = ___________________ = $X^2 + R^2$ = _______________________

(d) Measure the phase angle between Vo and Vc by observing the relative position of the traces on CH1 and CH2 on the scope. The observed period of the wave **T** (seconds) on CH1 can easily be interpreted as **360 degrees** on the screen. Measure the time difference between the point where the wave is passing through zero going positive on CH2 and the point where the trace is passing through zero going positive on CH1. Convert your reading to degrees. Use MEASURE (Time) to measure the period T. Use MEASURE (Cursor) to measure the delay (t2 - t1) = Δt.

$\phi = \underline{/Vc} - \underline{/Vo} = 360(\Delta t)/T =$ ____________ degrees (negative, since Vc lags Vo)

(e) Note : In order to read the voltage across the resistor with the scope, it is necessary to **subtract CH2 from CH1. Neither end of the resistor is at ground, and the shield of the cable is grounded to the scope.**

DO NOT CONNECT A COAXIAL CABLE FROM THE SCOPE TO THE RESISTOR.

In order to make the measurement, press [±] and select (1 - 2). Press [VOLTS] and select (Vrms) to read [Vr] and compare to the reading of AC VOLTS on the voltmeter.

[Vr] measured from scope = ______________

[Vr] measured on AC VOLTS = _______________

(f) Draw a phasor diagram of your measurements of a-c voltage for the series circuit. Label the length of the phasors [Vo], [Vr], and [Vc]. Also label the phase angle ϕ between Vo and Vc. Compare the phase angle between the voltages as obtained from the phasor diagram to the phase measured from the scope.

ϕ measured from diagram = ___________ ϕ measured from scope = _____________

(g) Calculate the value of the capacitance

$C = 1 / 2\pi f Xc =$ ___________________ (units Farad)

Compare to the nominal value of the capacitor 0.010 μF (units μF = microFarad) .

____

____

What would happen if a different capacitor were used? Calculate the theoretical values of voltage and impedance for the series circuit for capacitor C = 0.10 μF. Assume Vo stays the same. First calculate the new value of capacitive reactance $X_C = 1/2\pi f C$. Then calculate the magnitude of the impedance of the series RC circuit $|Z| = (R^2 + X^2)^{0.5}$. Now the current is

$[I] = [Vo]/|Z| =$ ____________________________________

and the voltages at right angles are

$[Vr] = [I]\ R =$ _____________________ $[Vc] = [I]\ X_C =$ ____________________________

Sketch the phasor diagram and label with the voltages and impedances.

Calculate theoretical voltage and impedance as above for C = 0.001 uF.   Sketch the phasor diagram and label with voltages and impedances.

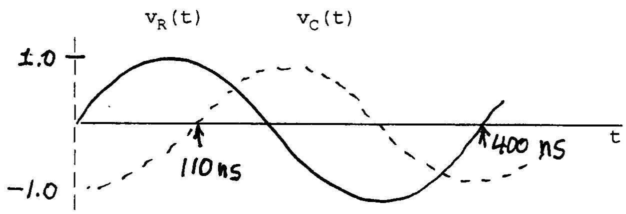
SUMMARY: (To be completed in lab)

Write a short summary of what you learned by doing this experiment. Point out any surprises; (that is when you learn the most!).

What formulas do you need to remember?

HOMEWORK:

1. Given the waveforms as observed on a scope



The voltage v_R is observed across a 1000 ohm resistor, and hence is equivalent to the current i(t) in milliAmps. The voltage v_C is the voltage across a capacitor in series with the resistor. Calculate the absolute magnitude and angle of the impedance of the combination. (Note: 110ns crossing should be at 100ns.)

$Z = |Z| \angle Z$ $\quad |Z| =$ _________ $\quad \angle Z =$ __________

Calculate the value of the resistor R = __________ = *Re* part of Z

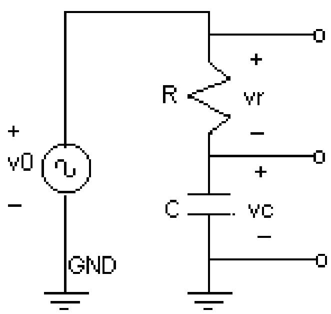
Calculate capacitive reactance $-jX_c =$ ______________ j*Im* part of Z

Sketch a phasor diagram for the voltages.

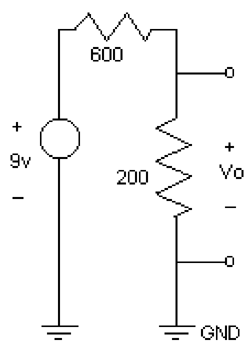
2. Given the a-c circuit, where R = 30K ohms, the capacitive reactance is -jXc = -j40K ohms. The supply voltage V_o = [5] volts RMS.

(a) Calculate [Vc] __________volts RMS [Vr] _____________volts RMS

(b) Calculate the angle ϕ between V_o and V_c ϕ = _____________

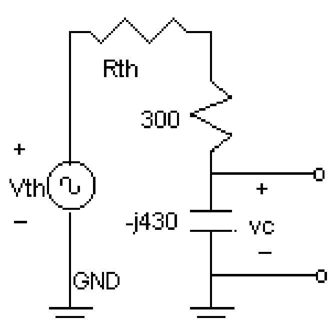


3. Given the circuit below, first find Vth and Rth, then connect to the a-c circuit and calculate Vc



Vth = ______________

Rth = ______________



Vc = ___________________