<u>**ECE 241L Fundamentals of Electrical Engineering**</u>

NAME

Experiment 5 Transient Response

PARTNER

A. Objectives:

I. Learn how to use the function generator and oscilloscope
II. Measure step response of RC and CR series circuits
III. Measure step response of an RLC series circuit

B. Equipment:

Breadboard, resistor(s), potentiometer, wire (student lab kit), wire stripper
Digital Volt-Ohm Meter (DVM): Fluke 189 (or equivalent)
Oscilloscope: Tektronix TDS3043 Digital Storage Scope
Function Generator: Tektronix AFG310/320 Arbitrary Function Generator
Inductor: 100 mH nominal
Capacitors: 0.001 and 0.1 μF nominal

C. Introductory Notes:

1. Step response of a first order RC circuit

The response of a series RC circuit to a sudden, constant d-c voltage input (called a "step function") is obtained from the solution of the differential equation corresponding to Kirchhoff's Voltage Law. The input can be represented by a switch that instantaneously moves from ground to a constant voltage source Vo, as shown for an RC series circuit in Figure E2-1.

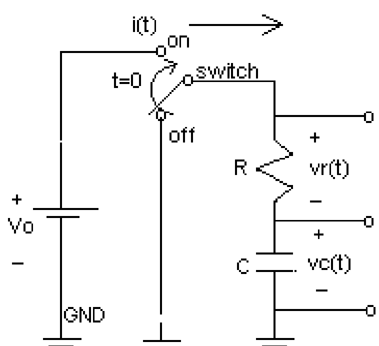

Figure E2-1 Voltage divider circuit to form an RC series circuit with step (d-c) input

The current i(t) through the capacitor is given in terms of the capacitor voltage Vc(t) by the derivative.

$$i(t) = C\ dV_c / dt$$

The voltage across R is given by Ohm's Law

$$V_r(t) = R\ i(t)$$

For the series circuit in Figure E2-1, the KVL equation is

$$V_o = RC\ dV_c/dt + V_c$$

The solution of this equation, the "charging curve", can be obtained by several methods (LaPlace Transform is the easiest). The voltage Vc(t) rises from zero to an equilibrium value of Vo as 0 --> t --> ∞.

$$V_c(t) = V_o\,[\,1 - \exp(-t/\tau)] \quad \text{for} \quad t > 0 \qquad \textbf{CHARGING CURVE}$$

There is a characteristic time *T* called the "**time constant**"equal to RC seconds.

$$\tau = RC \qquad \textbf{TIME CONSTANT}$$

The initial rate of change of capacitor voltage when the switch is operated is given by the derivative.

$$dV_c/dt \ (\text{at } t = 0) = V_o/\tau \qquad \text{volts/second}$$

It can also be shown that if the switch is returned to ground (after a long enough time for the capacitor to charge) the voltage across the capacitor is given by the "discharging curve". The time is now measured from when the switch returns to the short.

$V_c(t) = V_o \exp(-t/RC)$ DISCHARGING CURVE

In each case above, there is the same time constant τ = RC seconds. The time constant can be identified from the curve at the 63% level charging, and the 37% level discharging. The levels are calculated by setting t = τ in the equations.

For the charging curve

$V_c(t) = 1 - \exp(-1) = 0.63$ 63% when t = τ CHARGING TIME CONSTANT

For the discharging curve

$V_c(t) = \exp(-1) = 0.37$ 37% when t = τ DISCHARGING TIME CONSTANT

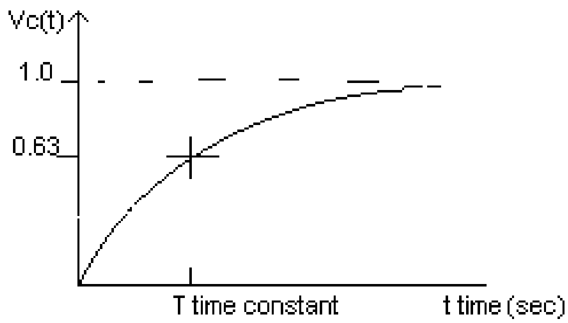

Figure E2-2 Charging curve, step response of RC series circuit

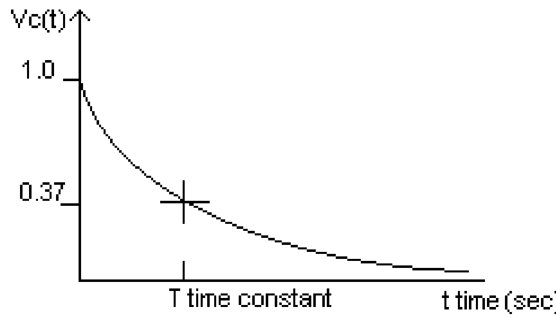

Figure E2-3 Discharging curve, step response of RC series circuit

It can be shown that the charging current i(t) is given by

$i(t) = [V_o/R] \exp(-t/\tau)$ CHARGING CURRENT

The current i(t) therefore suddenly rises from zero to Vo/R, then falls to zero as 0 --> t --> ∞. The current stops when the capacitor is charged. The discharge current is the same, but in the reverse direction.

$i(t) = - [V_o/R] \exp(-t/T)$ DISCHARGING CURRENT

2. Step response of a second order RLC circuit

If the resistor is replaced by an inductor, (a coil of wire,) another phenomenon can be observed called "transient oscillation". In this case the series circuit contains both inductance and capacitance. When shocked by a sudden step function input, the inductance and capacitance will exchange energy with each other in a periodic, sinusoidal oscillation also known as "simple harmonic motion". The oscillation decays away after a characteristic time, similar to the response of an RC circuit. The rate of decay depends on the amount of resistance in the circuit, compared to the inductance and capacitance. The series LCR circuit is sketched in Figure E2-4. The resistance is inherent in the coil, rather than being a separate resistor, and hence only the voltage across the combination of resistance and inductance can be measured.

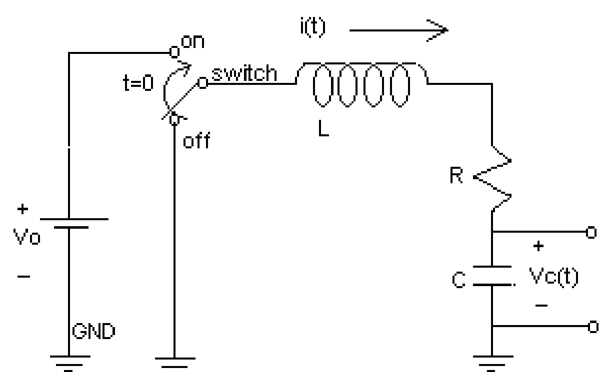

Figure E2-4 Series LCR circuit to display decaying transient oscillation.

The voltage across the inductance is related to the current by the derivative:

$$v_L(t) = L\ di/dt$$

The KVL equation then becomes, after again substituting $i(t) = C\ dVc/dt$

$$Vo = RC\ dVc/dt + LC\ d^2Vc/dt^2 + Vc$$

The form of the solution for this equation depends critically on the relative size of RC and LC. However, if $RC >> (LC)^{0.5}$ the circuit has a strong oscillatory response and the solution can be approximated by

$$Vc(t) = Vo\ [\ 1 - \exp(-t/\tau) \cos(2\pi t/\tau o)] \qquad \textbf{RESPONSE OF LCR SERIES CIRCUIT}$$

The period of the transient oscillation is given by

$$\tau o = 2\pi(LC)^{0.5} \text{ seconds} \qquad \textbf{OSCILLATION PERIOD}$$

and the decay time constant τ is given by

$$T = 2L/R \qquad \text{seconds} \qquad \textbf{TIME CONSTANT}$$

There is another measure of transient oscillation, the "quality factor" Q_o given by

$$Q_o = \pi\ \tau/\tau_o \qquad \textbf{QUALITY FACTOR}$$

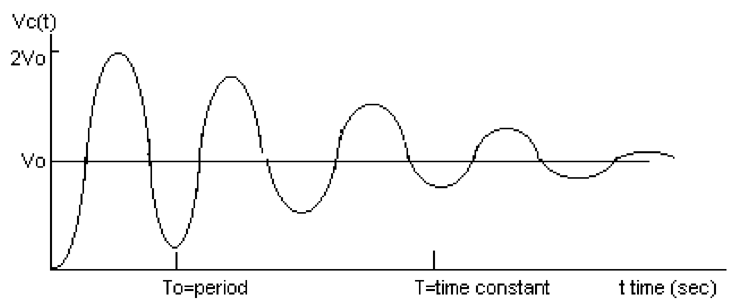

Figure E2-5 Step response of LCR series circuit, showing decaying sinusoidal oscillation

D. PROCEDURE:

I. RC series circuit. The symbol [_|¯¯] represents a square wave input voltage with peak value ±Vo.

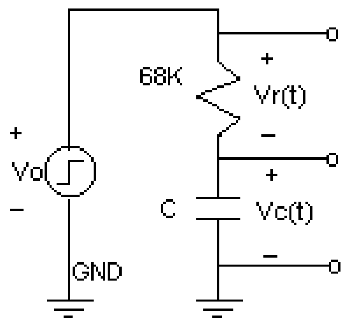

Figure E2-6 Series RC circuit to observe the time constant.

1. Measure the actual value of the 68K resistor with the ohmmeter. R = ______________.

2. Build the circuit in Figure E2-6 on your breadboard, with C = 0.001µF = 1nF = 1000pF. (What is the color code for this capacitor? _________ ________ x ________ (Note: Not all capacitors are made with color codes. There are three in the kit, which may be marked CM102, CM103, CM104, plus an electrolytic. Check with the instructor to identify the proper capacitor.) Mount a coaxial TEE connector to the output Vo of the Function Generator. Connect the circuit to one branch of the TEE using coaxial cable with clip leads. Be sure the clip lead from the shield of the coax is connected to GND on the circuit. The other clip lead connects to the resistor. Connect the 50Ω terminator to the other side of the TEE. (This ensures that the FG calibrations are correct.) Connect CH1 of the scope to the resistor, and CH2 of the scope to the capacitor Vc(t) using clip leads. Note that the outer shields of both scope probes are connected to GND on the circuit.

3. For this experiment, the SQUARE WAVE output of the Function Generator will be used. Set the square wave voltage to Vo = Vpp = 2.0 volts (goes from +1 volt to -1 volt) at 1000 Hz. The period at 1000 Hz is 1.0 millisecond. Measure the period of the square wave signal on the scope, instead of relying on the dial of the Function Generator. The period will be maintained at a value sufficiently long compared to the time constant to allow the capacitor to charge almost completely during the first half-period, and discharge during the second half-period.

4. Select the appropriate horizontal scale with the TIME/DIV control on the scope to make as accurate a measurement of the charging time constant as possible. The half-period of the square wave should be at least 5τ seconds in order for the charging curve to level off before the discharging curve starts. (Hint: Switch the Trigger control to NEGATIVE SLOPE in order to get a better measurement of the discharge time constant, then reset to POSITIVE SLOPE.)

5. Measure the resistor and capacitor charge and discharge time constants. In order to measure Vr(t), use the Math [+/-] capability of the scope to subtract CH2 from CH1. Use the [CURSORS] on the scope to measure the time constants. Theoretically they should all be the same. Average the measured values.

τ (V_c charge)	τ (V_c dischg)	τ (V_r charge)	τ (V_r dischg)	τ (average)

From the measured value of the RC time constant and the measured value of R calculate the value of the capacitor, and compare to nominal (1000pF).

C = τ / R = _______________ pF, Difference = ____________ %

II. Series RLC circuit

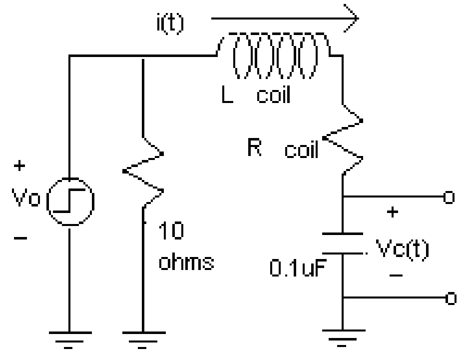

Figure E2-7 Series RLC circuit to observe decaying transient oscillation

1. Measure the DC resistance of the 100 mH inductor/coil with the Ohmeter function of the DVM. Replace the 68K resistor in your circuit with the inductor/coil, and replace the 1nF capacitor with the 0.1μF value.

2. (You MAY need to connect a 10 ohm resistor in parallel with the input to the circuit as shown in the diagram above. The purpose of this resistor is to reduce the effective Thevenin resistance of the Function Generator source, but the 50Ω termination on the TEE should be sufficient. If the effective source Thevenin resistance is too high, it would have the undesired effect of decreasing the oscillation decay time of the circuit.) Note that the resistance R_{coil} is not a separate resistor, but is due to the resistance of the copper wire from which the coil is made.

3. Connect CH1 of the scope to Vo(t), connect CH2 to Vc(t). On the Function Generator increase the output voltage with the ATTEN control to set Vo(t) to Vpp = 200 millivolts at about 100 Hz. Use the procedure of part I to observe Vc(t). Vary the half-period of the square wave if necessary so that the transient oscillation mostly decays away at each step. Use [CURSORS] to measure the period of transient oscillation **τo** by measuring the time for several periods together and dividing by the total time, which gives better accuracy than measuring just one period. Estimate the time constant τ for which the amplitude of the oscillation has decayed to 37% of the first maximum after the step.

τ_o = ____________________ τ = ______________________

Calculate the "quality factor" of the circuit, a measure of the amount of energy stored compared to the energy dissipated in each oscillation period.

$Q_o = \pi \, \tau/\tau_o$ = ____________________

From measured period τo and known capacitance C, calculate the unknown inductance of the coil

$L_{coil} = (\tau_o/2\pi)^2/C$ = __________ (milliHenry)

Express inductance in SI prefix units: L = ______________ mH = _______________μH

From the value of L and τ calculate the effective resistance of the coil: $R_{coil} = 2L/\tau$= ______________

Record your measurement of the d-c coil resistance with the Ohmmeter. $R_{d\text{-}c}$ = ___________________

Calculate the percentage difference between the resistances obtained by different methods. % = ________

Turn off power to all equipment and disconnect all circuits before leaving the lab. Leave your lab station more clean and orderly than you found it!

SUMMARY:(To be completed at the end of lab)

Sketch the graph of the charging voltage curve Vc(t) of an RC series circuit. Label the sketch at the time constant.

Write the equation for the graph

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Sketch the graph of the charging current curve i(t) of an RC series circuit. Label the sketch at the time constant.

Write the equation for the graph

____

Sketch the graph of the step response of an LCR series circuit. Label the sketch to show the oscillation period To and the decay time constant *T*.

Write the equation for the graph

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Write the equation for the "quality factor" Q_o of an LCR series circuit in terms of the period τo and the decay time constant τ

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Write the equation for the time constant τ in terms of the inductance L and the resistance R.

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Comment on your observations. Were there any surprises?

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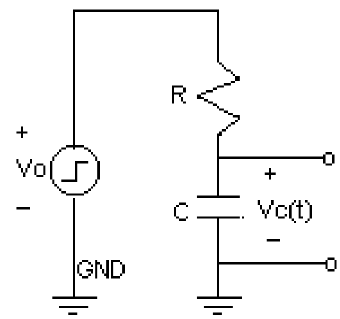
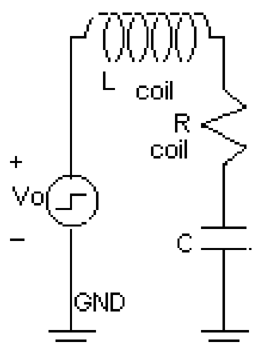
HOMEWORK

1. Given time constant T = 68 microseconds, and R = 68 K ohms, calculate C = ________________µF.
Given Vo = +5 volts, for the charging curve calculate the voltage Vc(t) at time t = 34 microseconds.



Vc(t = 34 µs) = ___________ volts

2. Given the sinusoidal period of transient oscillation To = 400 microsecond, and C = 0.1 µF. The decay time constant T = 1.2 millisecond.



(a) Calculate the inductance L = _______ milliHenry

(b) Calculate the quality factor Qo = ___________.

3. Given an inductor with inductance L = 50 milliHenry and resistance R = 60 ohms, in series with capacitance C = 0.1 uF.

(a) Calculate the period of transient oscillation: To = ______________ microseconds

(b) Calculate the decay time constant T of the step response. T = ________________ microseconds

4. An approximate formula for coil inductance is $L = \mu_o \mu_\rho n^2 Å/£$ (Henry)

where $Å = \pi r^2$ (m^2) where r = mean radius of coil
£ = thickness (length) of coil (m)
n = number of turns of wire in coil
μ_o = 1.2 microHenry per meter, permeability ("inductivity") of free space
μ_ρ = relative permeability

Use values of coil dimensions r = 0.02 meter, £ = 0.02 meter, n = 800 turns, to calculate the theoretical inductance of the coil.

L = ___________ milliHenry