ECE 241 Assignment #8 Model Answers

21.5 Inductance coil : N turns High permeability core with air gap; c/s area A, length l_i, length l_a

(a) Derive approx inductance :

"High permeability core" $\Rightarrow$ assume $\mathcal{R}_{core}$ is negligible

$$\therefore \mathcal{F} \approx \frac{B}{\mu_a} l_a = NI \quad \text{gives} \quad B = \mu_0 \frac{NI}{l_a}$$

$$\& \quad L = N\phi/I = NBA/I = \frac{\mu_0 N^2 I A}{I l_a} = \mu_0 \frac{N^2 A}{l_a}$$

(b) $N = 500 \quad A = 2\,\text{cm}^2 \quad l_i = 10\,\text{cm} \quad l_a = 2\,\text{mm} \quad I = 10\,\text{mA}$

$$\therefore L = 4\pi \times 10^{-7} \times (500)^2\ 2\times10^{-4}/2\times10^{-3} = 31.4\ \text{mH}$$

(c) Cast steel core, $I = 4A$ $\quad H = NI/l \longrightarrow \dfrac{500\times4}{10\times10^{-2}}$ for cast steel if all mmf dropped in steel.

$= 20{,}000$

Extrapolating from Fig 20.16, $B \sim 1.3T$ and core is saturated

i.e. $\dfrac{\Delta B}{\Delta H} \sim \mu_0$

For air gap, mmf $= H_a l_a = \dfrac{B}{\mu_0} 2\times10^{-3} \longrightarrow \dfrac{1.3\times2\times10^{-3}}{4\pi\times10^{-7}} = 2.07\times10^3$ A.t

For core $\quad$ mmf $= H_i l_i = 2\times10^4 \cdot 10\times10^{-2} = 2\times10^3$ A.t

i.e. comparable.

$$\text{so} \quad \mathcal{F} = NI = \phi\mathcal{R} = \phi\left(\frac{1}{\mu_0}\frac{l_a}{A} + \frac{1}{\mu_{eff}}\frac{l_i}{A}\right) = \frac{\phi}{A}(\quad)$$

$$\& \quad L = N\phi/I = \frac{N}{I}\frac{NI}{\left(\frac{1}{\mu_0}\frac{l_a}{A} + \frac{H_i}{B_i}\frac{l_i}{A}\right)} = \frac{N^2 A}{\left(l_a/\mu_0 + l_i H_i/B_i\right)}$$

using $\mu_{eff} = \dfrac{B_i}{H_i}$ from above.

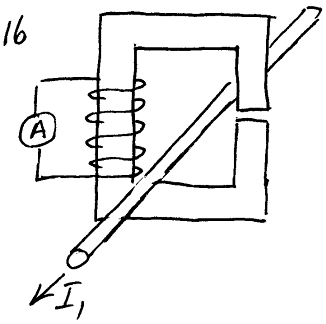
$$= \frac{500^2\ 2\times10^{-4}}{\frac{2\times10^{-3}}{4\pi\times10^{-7}} + 10^{-1}\frac{2\times10^4}{1.3}} = \frac{50}{1592 + 1200}$$

$$\approx 17.9\ \text{mH}$$

This suggests ~½ mmf dropped in the iron, so $H \sim$ ½ estimated value, $B \sim$ same,

so $\mu_{eff} \sim 2\times$ value used. Iterative process would give a better value OR linear ...

21.16



Single turn primary of the transformer

For negligible flux linkage

$I_{conduct} = I_{N} = \left(\frac{N_2}{N_1}\right) I = 20 \times 1.5 = 30A$

21.35. Transformer: 30 KVA 12,000 : 4,000

OC test: 4000 V, 1.1 A, 2000W

SC test: 300 V, 25A, 2800W

(a) $V_{OC} = 4000V \therefore V_{OC} = V_{LV}$ and OC test done with low voltage side OC

$I_{SC} = 25A = \frac{300\,KVA}{12{,}000V} = \frac{\#\,KVA}{V_{HV}}$ and SC test done with high voltage side SC.

(b) For R_L load $P_L = 300kW \rightarrow \eta_1 = \frac{P_L}{P_L + P_{cu} + P_{FE}} = \frac{300kW}{300kW + 2kW + 2.8kW}$

$= 98.4\%$

(c) For Z_L load 300KVA, pF = 0.6 lagging $= \frac{300 \times 0.6}{300 \times 0.6 + 2 + 2.8} = 97.4\%$

(d) To find voltage regulation for part (c) above:

Need Regulation $= \frac{\text{No load V} - \text{full load V}}{\text{full load V}}$

$= \frac{V_1 - V_2/a}{V_2/a}$ for the model of Fig 21.18(b)

where $\tilde{V}_1 = \hat{V}_2/a + a\hat{I}_2 \tilde{Z}_{E1}$ from Fig 21.18b and find R_E, X_{E1}

(e) $a = 4000/12000 = 1/3$ $\quad R_{E1} = \frac{P_{SC}}{I_{1S}^2} = \frac{2800}{(25)^2} = 4.48\ \Omega$

$Z_{E1} = V_{1S}/I_{1S} = \frac{300}{25} = 12\Omega$ so $X_{E1} = (Z_{E1}^2 - R_{E1}^2)^{1/2} = (12^2 - 4.5^2)^{1/2}$

$= 11.13\ \Omega$

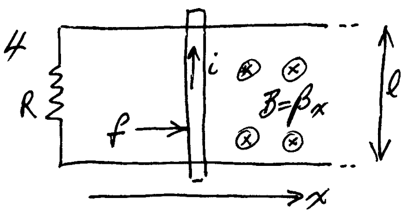
300KVA & 0.6pf lagging $\therefore a\vec{I}_2 = I_{1SC}(0.6 - j0.8) = 25(0.6 - j0.8)$

$= 15 - j20\ A$

$\vec{V}_1 = \vec{V}_2/a + \vec{I}_2 \vec{Z}_{E1} = 12{,}000 + (15 - j20)(4.48 + j11.13) = 12{,}289\ \angle 0.36^\circ\ V$

$\therefore$ Voltage Regulation $= (12{,}289 - 12{,}000)/12{,}000 = 0.024 = 2.4\%$

22.4



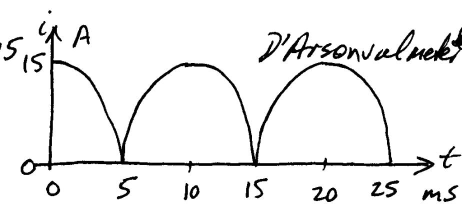
For bar, velocity $= u$
position $x = ut$

(a) $e(t) = \dfrac{d\lambda}{dt} = B\ell u$

$\therefore i(t) = \dfrac{e(t)}{R} = \dfrac{B\ell u}{R} = \dfrac{(\beta x)\ell u}{R} = \dfrac{B(ut)\ell u}{R}$
$= \beta \dfrac{u^2 \ell t}{R}$ A

(b) $f(t) = f_d = B\ell i = (\beta x)\ell\, i(t) = \beta(ut)\ell\, \beta \dfrac{u^2 \ell t}{R} = \beta^2 \dfrac{u^3 \ell^2}{R} t^2$ N

22.15

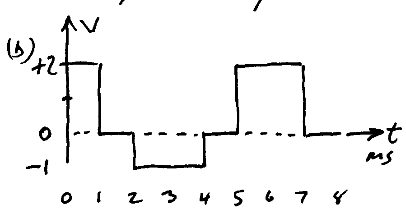


D'Arsonval meter & FW Rectification system (calibrated in RMS) are used to measure currents shown.

(a) D'Arsonval meter reads $I_{AV} = 2\, I_{PK}/\pi = 2\times 15/\pi = 9.55$A

Rectification system reads $I_{rms} = 1.11\, I_{av} = 1.11 \times 9.55 = 10.6$A

(b)



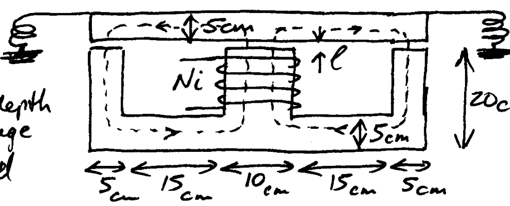
D'Arsonval: $V_{AV} = \dfrac{2\times 1 - 1\times 2 + 2\times 2}{8}$
$= 0.5$v

(note: Interpretation of where the cycle ends — possibly 6 ms period
$V_{AV} \rightarrow 2/6 = 1/3$ v)

For rectification instrument $V_{rms} = 1.11 V'_{AV} = 1.11\left(\dfrac{2\times 1 + 1\times 2 + 2\times 2}{8}\right) = 1.11$v

(Note formula on page 731.)

22.35



10cm depth into page
Silicon steel

(a) Find N, i for 60N force on bar at $\ell = 1$cm

Consider 2 fluxes in parallel:

Gap area for each $= 10 \times 5 \times 10^{-4} = 5 \times 10^{-3}\ m^2$

Tot area $= 4 \times 5 \times 10^{-3} = 2 \times 10^{-2}\ m^2$

$$f = \frac{B^2 A}{2\mu_0} \quad (\text{eq}^n\ 22.40) \rightarrow B = \sqrt{\frac{2 f \mu_0}{A_{tot}}} = \sqrt{\frac{2 \times 60 \times 4\pi \times 10^{-7}}{2 \times 10^{-2}}}$$

assuming $R_{air} \gg R_{iron}$

$$= (75.4 \times 10^{-6})^{1/2} = 8.7 \times 10^{-3}\ Wb/m^2$$

Fig 20.16 $\rightarrow B = 8.7\ mWb/m^2$

$$H \approx 8.7\ 10^{-3} \frac{50}{.37}\ At/m = 1.18\ At/m.$$

$$\ell_i = 2 \times (20+20) \times 10^{-2} \text{ for each loop} \quad \therefore \mathcal{F}_i = H_i \ell_i = 0.8 \times 1.18$$
$$= 0.8\ m \qquad\qquad = .944\ A.t. \text{ for each loop}$$

i.e. 1.89 At total.

$$\& \ \mathcal{F}_a = H_a \ell_a = \frac{B}{\mu_0} \ell_a = \frac{8.7 \times 10^{-3}\ 2 \times 10^{-2}}{4\pi \times 10^{-7}} = 138.5\ A.t.$$

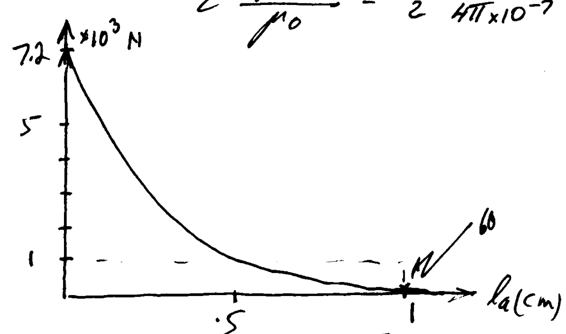
$\therefore\ Ni = (138.5 + 1.9)\ A.t\ /\ \text{loop}$

i.e. $= 2 \times 140.4\ A.t. = 288.8\ A.t.$ total.

(b) If $\ell_a \rightarrow 0$ $\quad H_i = \frac{Ni}{\ell_i} = \frac{140.4}{0.8}\ At/m = 175.5\ At/m$ each loop.

i.e. Fig 20.16 $\rightarrow B_i = 0.95\ T$

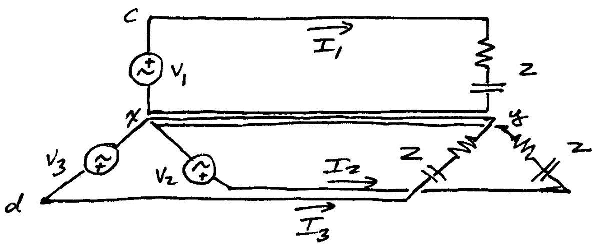
$$f = \frac{1}{2} \frac{B_i^2 A}{\mu_0} = \frac{1}{2} \frac{.95^2}{4\pi \times 10^{-7}} 2 \times 10^{-2} \text{ total} = 7.2 \times 10^3\ N$$



$f \propto B^2 \propto \frac{1}{\ell_a^2}$

for $\ell_a \gg \frac{\mu_i}{\mu_0} \ell_i$

7.36



$V_1 = 240 \angle 90^\circ$ V

$V_2 = 240 \angle -30^\circ$ V

$V_3 = 240 \angle -150^\circ$ V

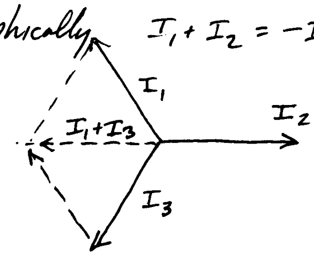
$Z = 40 \angle -30^\circ \ \Omega$

(a) I_1, I_2, I_3

$$I_1 = \frac{240 \angle 90^\circ}{40 \angle -30^\circ} = 6 \angle 120^\circ$$ Similarly $I_2 = 6 \angle 0^\circ$, $I_3 = 6 \angle -120^\circ$

(b) For x's, y's connected $\quad I_{yx} = I_1 + I_2 + I_3$
$= 6 \angle 120^\circ + 6 \angle 0^\circ + 6 \angle -120^\circ$

Graphically $\quad I_1 + I_2 = -I_3 \quad$ or



$= 6\cos 120^\circ + j6\sin 120^\circ$
$+ 6$
$+ 6\cos -120^\circ + j6\sin -120^\circ$

$$= \left(-\frac{6}{2} + j\frac{6\sqrt{3}}{2}\right) + 6 + \left(-\frac{6}{2} - j\frac{6\sqrt{3}}{2}\right)$$

$= 0$

(c) For x, x, x and y, y, y connected $\quad \vec{V}_{cd} = \vec{V}_1 - \vec{V}_3 = 240\angle 90^\circ - 240\angle -150^\circ$

$= 240 \angle 90^\circ + 240 \angle 30^\circ$

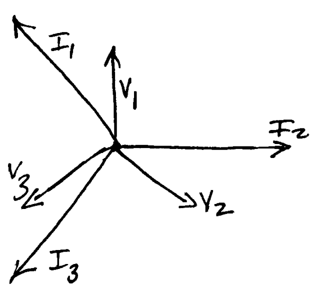
$= 240 \times 0 + j\,240 \times 1 + 240\frac{\sqrt{3}}{2} + j240 \cdot \frac{1}{2}$

$= 240\frac{\sqrt{3}}{2} + j\,240 \cdot \frac{3}{2}$

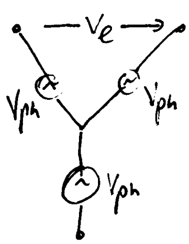
$= \frac{240}{2}\left(\sqrt{3}^2 + 3^2\right)^{1/2} \arctan \frac{3/2}{\sqrt{3}/2}$

$= 240\sqrt{3} \angle 60^\circ = 416 \angle 60^\circ$ V

(d)

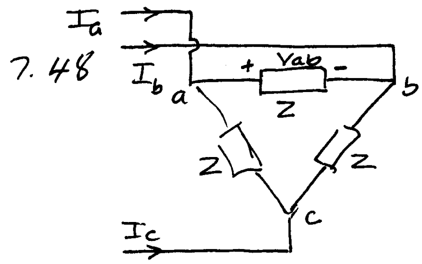


7.40



$V_{line} = \sqrt{3}\, V_{phase} \quad \therefore \ V_{phase} = V_{line}/\sqrt{3} = \frac{208}{\sqrt{3}}$
$= 120.09$ V

7.48



$|V_{ab}| = 300V$ Z → Capacitive Loads

$|I_a| = 20A$

$|V_{ab} \cdot I_a| = 4240W$

Find $\vec{Z}$, sketch phasors

$V_{ab} = 300\angle 0°$ (reference)

$I_a = 20\angle\theta$

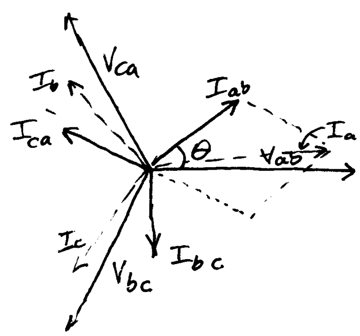
$4240 = V_{ab} I_a \cos\theta = 300 \times 20 \times \cos\theta \therefore \cos\theta = \frac{4240}{6000} = .707$
$= \frac{1}{\sqrt{2}} \therefore \theta = \pm 45°$

Phase current : Phase θ as shown for capacitor load.

$\therefore I_a = I_{ab} + I_{ac}$

$= I_{ab} - I_{ca}$.

as shown



I_{ab} phase $= I_a$ phase $+30°$ (Eqn 7.56)

$\therefore I_a$ phase $= \pm 45° - 30° > 0$ for capacitive load

$\therefore I_a = 20\angle 45°$

$I_{ab} = \frac{I_a}{\sqrt{3}}\angle 30° = \frac{20}{\sqrt{3}}\angle 30° + 45° = 11.5\angle 75°$ A

$\vec{Z} = \frac{\vec{V}_{ab}}{\vec{I}_{ab}} = \frac{300\angle 0°}{11.5\angle 75°} \approx 26\angle -75°$