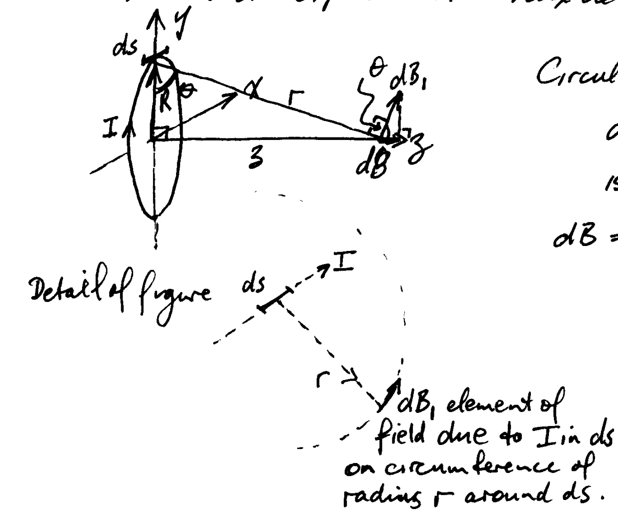
ECE 241 ASSIGNMENT #7

20.5 Circular coil, radius R, current I in x-y plane, center (0,0)
Derive an expression for flux density $\vec{B}$ as a function of z.



Circular symmetry: only the axial component of $d\vec{B}_1$ is important i.e. $d\vec{B}$

$$dB = dB_1 \cos\theta \quad \text{due to } ds$$

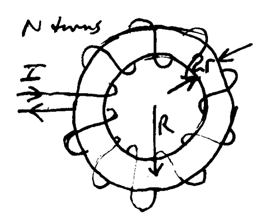
$$= \frac{\mu I\, ds}{4\pi r^2}\cos\theta = \frac{\mu I R}{4\pi r^3} ds$$

$$\therefore B = \frac{\mu I R}{4\pi r^3}\int ds$$

$$= \frac{\mu I R}{4\pi r^3}\cdot 2\pi R = \frac{\mu I R^2}{r^3}$$

$$= \frac{\mu I R^2}{(R^2+z^2)^{3/2}}$$

20.12 Toroidal memory element (0/1 depends on magnetization direction)



Ferrite $\rightarrow \mu_r = 1100 \quad R = \cdot 1\text{ cm} \quad r = \cdot 1\text{ cm}$

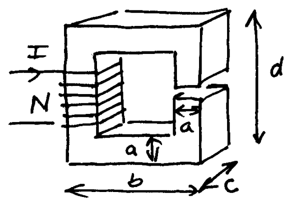
Find mmf for flux density $1\cdot 5\text{ mT}$

$$\mathcal{F}_{mmf} = \frac{1}{\mu}\frac{l}{A}\phi = \frac{1}{\mu}\frac{l}{A}BA = \frac{Bl}{\mu}$$

$$\therefore \mathcal{F} = NI = \frac{B(2\pi R)}{\mu_r \mu_0} = \frac{1\cdot 5\times 10^{-3}\times 2\pi\times 0.1\times 10^{-2}}{1100\times 4\pi\times 10^{-7}}$$

$$= 0.0068 = 6\cdot 8\times 10^{-3}\text{ A.t}$$

20.21



Armco iron core $N = 200$

$a = 5cm$ $b = d = 20cm$ $c = 10cm$

Find I for $\phi = 6.5 mWb$, for

(a) 5mm air gap

(b) No air gap

(a) $B = \phi / A = \dfrac{6.5 \times 10^{-3}}{(5 \times 10^{-2})(10 \times 10^{-2})} = 1.3T$ assuming uniform area $A = ca$

i.e. neglecting fringing.

For airgap: $\mathcal{F}_{gap} = H_{gap} l_{gap} = \left(\dfrac{B}{\mu_0}\right) l_{gap} = \dfrac{1.3}{4\pi \times 10^{-7}} 5 \times 10^{-3}$

$= 5.17 \times 10^3 \ A.t$

For the iron: Average $l_i = 2(d-a) + 2(b-a) - 5mm$

$= 2(20-5) + 2(20-5) - 0.5 \ cm$

$= 0.55m$

From Fig 20.16 for Armco iron:

$B_i = 1.3T \longrightarrow H_i \approx 250 A.t/m$

$\therefore \mathcal{F}_i = H_i l_i = 250 \times 0.55 = 137.5 \ A.t$

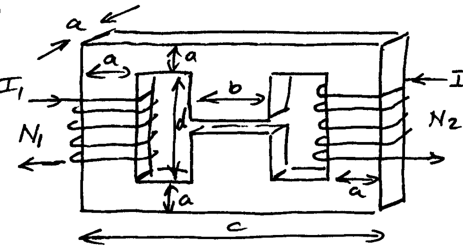
$\therefore I = (\mathcal{F}_{gap} + \mathcal{F}_i)/N = \dfrac{5170 + 137.5}{200} = \dfrac{5307.5}{200} = 26.54A$

(b) No air gap $\mathcal{F} = \mathcal{F}_i = H_i l_i = 250 \times 0.6 = 150 A.t$

$\therefore I = \mathcal{F}/N = 150/200 = 0.75A$

Compare 0.75A for no gap with 26.54A with gap.

20.27



Silicon sheet steel with 5 mm air gap

$a = 20\text{cm} \quad b = 40\text{cm}$

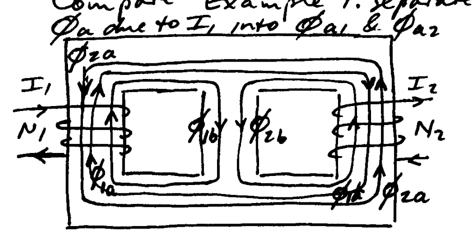
$c = 120\text{cm} \quad d = 60\text{cm}$

$N_1 = N_2 = 500$ turns

[Note: Hint about symmetry!]

Find $I_1 = I_2$ for 100 mWb in gap.

Compare Example 7: separate ϕ_a due to I_1 into ϕ_{a1} & ϕ_{a2}



Note directions of magnetic field due to I_1 and I_2 above. RH rule: both produce B "upwards" in coil.

ϕ_{1a}, ϕ_{2a} cancel in both outer legs, leaving only $\phi_{1b} + \phi_{2b}$ in center leg.

Assume $\mathcal{F}_{airgap} \gg \mathcal{F}_{iron}$

$$\therefore 2NI = N_1 I_1 + N_2 I_2 = H_{gap} l_{gap} = \frac{B_{gap}}{\mu_0} l_{gap} = \frac{\phi_{gap}}{A_{gap}} \frac{l_{gap}}{\mu_0}$$

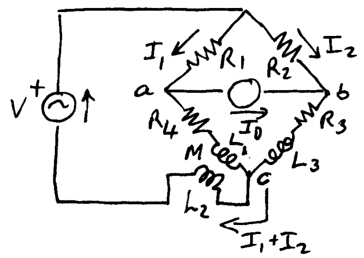
$$A_{gap} \approx ab = (20\times10^{-2} \times 40\times10^{-2})$$

$$\therefore I_1 = I_2 = \frac{1}{2N} \frac{\phi}{A_{gap}} \frac{l_{gap}}{\mu_0} = 10^{-3} \frac{100\times10^{-3}}{8\times10^{-2}} \frac{0.5\times10^{-2}}{4\pi\times10^{-7}} = 4.97A$$

(neglecting fringing.
Including fringing $A_{gap} \rightarrow 20.5 \times 40.5 \times 10^{-4} = 8.3025\times10^{-2}$
$I_1 = I_2 \longrightarrow 4.79A$)

For each winding, could consider the extra mmf required by the core flux but complicated by effective μ_{iron}! (Fig 20.16) and zero net flux-areas.

20.32



Find M (mutual inductance between coils 1 & 2) when $I_D = 0$

$I_D = 0 \therefore V_a = V_b$

$\therefore I_1 R_1 = I_2 R_2$

$\therefore I_2 = \frac{R_1}{R_2} I_1$

& $V_{ac} = V_{bc}$ $\quad \therefore I_1 (R_4 + j\omega L_1) + j\omega M(I_1 + I_2) = I_2 (R_3 + j\omega L_3)$

$$I_1 (R_4 + j\omega L_1) + j\omega M I_1 \left(1 + \frac{R_1}{R_2}\right) = \frac{R_1}{R_2} I_1 (R_3 + j\omega L_3)$$

$$R_4 + j\omega \left(L_1 + M + \frac{R_1}{R_2} M\right) = \frac{R_1 R_3}{R_2} + j\omega L_3 \frac{R_1}{R_2}$$

Equating real and imaginary parts : $R_4 = \frac{R_1 R_3}{R_2}$

and $\quad L_1 + M + \frac{R_1}{R_2} M = L_3 \frac{R_1}{R_2}$

gives $\quad M = \dfrac{L_3 \frac{R_1}{R_2} - L_1}{1 + R_1/R_2} = \dfrac{R_1 L_3 - R_2 L_1}{R_1 + R_2}$