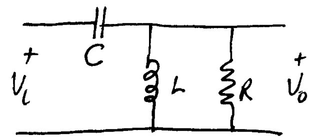
ECE 241 ASSIGNMENT #6 MODEL ANSWERS

7.16



Input components $\omega_1 = 10$, $\omega_2 = 100$, $\omega_3 = 1000$, $\omega_4 = 10{,}000$ rad/s

(a) $$V_o/V_i = \frac{R /\!/ j\omega L}{R /\!/ j\omega L + \frac{1}{j\omega C}} = \frac{\frac{j\omega L R}{j\omega L + R}}{\frac{j\omega L R}{R + j\omega L} \cdot -\frac{j}{\omega C}} = \frac{j\omega L R\, \omega C}{j\omega L R\, \omega C - j(R + j\omega L)}$$

$$= \frac{\omega^2 LRC}{\omega^2 LRC - R - j\omega L} = \frac{-\omega^2 RLC}{R(1-\omega^2 LC) + j\omega L}$$

OR $$= \frac{\omega^2 RLC}{[R^2(1-\omega^2 LC)^2 + (\omega L)^2]^{1/2}} \angle -\tan^{-1}\frac{\omega L}{R(1-\omega^2 LC)}$$

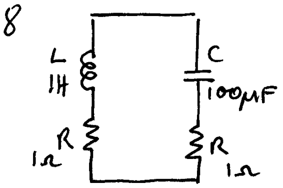
(b) For $C = 10\mu F$ $L = 1H$ $R = 20k\Omega$

$$\left|\frac{V_o}{V_i}\right| = \frac{20\times10^3 \times 1 \times 10\times10^{-6}\,\omega^2}{[(2\times10^4)^2(1-\omega^2 10^{-5})^2 + (\omega)^2]^{1/2}} = \frac{0.2\,\omega^2}{\sqrt{(2\times10^4 - 0.2\omega^2)^2 + \omega^2}}$$

$\omega =$	10	100	1000	10,000
$\left\|\frac{V_o}{V_i}\right\| =$	$\frac{20}{\sqrt{(2\times10^4-20)^2+100}}$	$\frac{2\times10^3}{\sqrt{(2\times10^4-2\times10^3)^2+10^4}}$	$\frac{2\times10^5}{\sqrt{(2\times10^4-2\times10^5)^2+10^6}}$	$\frac{2\times10^7}{\sqrt{(2\times10^4-2\times10^7)^2+10^8}}$
$\approx$	$\frac{20}{2\times10^4}$	$= \frac{2\times10^3}{\sqrt{3.24\times10^8+10^4}}$	$\frac{2\times10^5}{\sqrt{3.24\times10^{10}+10^6}}$	$\frac{2\times10^7}{\sqrt{4\times10^{14}}}$
$=$	$.001$	$\approx .111$	≈ 1.011	≈ 1

$\therefore$ High pass filter.

7.28



(a) $$Z = \frac{\left(R - \frac{j}{\omega C}\right)(R + j\omega L)}{R - j/\omega C + R + j\omega L} = \frac{\left(R^2 + \frac{L}{C}\right) + j\left(\omega L - \frac{1}{\omega C}\right)R}{2R + j(\omega L - 1/\omega C)}$$

$$= \frac{\left[\left(R^2 + L/C\right) + jR(\omega L - 1/\omega C)\right]\left[2R - j(\omega L - 1/\omega C)\right]}{4R^2 + (\omega L - 1/\omega C)^2}$$

Resonance when Z purely real, i.e. when $\rightarrow$

$$= \frac{\left[2R(R^2 + L/C) + (\omega L - 1/\omega C)^2 R\right] + j\left[2R^2(\omega L - 1/\omega C) - (\omega L - 1/\omega C)(R^2 + 1/LC)\right]}{\text{den}}$$

$\left(\omega L - \frac{1}{\omega C}\right)\left(R^2 - \frac{L}{LC}\right) = 0$ so $\omega_0^2 = \frac{1}{LC} = \frac{1}{1 \times 10^{-4}}$ $\therefore \omega_0 = 100$ rad/s

(b) $Q_L = \frac{\omega L}{R} = \frac{\omega_0 L}{R}$ at resonance $= \frac{100 \times 1}{1} = 100$

Similarly $Q_C = \frac{1}{\omega RC} = \frac{1}{\omega_0 RC}$ at resonance $= \frac{1}{100 \times 1 \times 10^{-4}} = 100$

Complete circuit $Q. = \frac{\omega_0 L}{2R} = 50$

(c) For $V = 20$V rms at ω_0 $\quad Z = R\,\frac{\left(2(R^2 + L/C) + (\omega_0 L - 1/\omega_0 C)^2\right)}{4R^2 + (\omega_0 L - 1/\omega_0 C)^2}$

$$= \frac{2R(R^2 + L/C)}{4R^2} = \frac{1}{2}(R + L/RC)$$

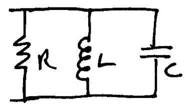
$$= \frac{1}{2}\left(1 + \frac{1}{1}10^4\right) = 5{,}000.5 = Z_0$$

$I \approx \frac{V}{5000} = \frac{20}{5000} = 4$mA

$I_L = \frac{V}{R + j\omega L} = \frac{20}{1 + j100} \approx \frac{20\angle 0^\circ}{100\angle \tan^{-1} 100} = 0.2\angle -89.4^\circ = 200\text{mA}\angle -89.4^\circ$

$I_C = \frac{V}{R - j/\omega_0 C} = \frac{20}{1 - j100} \approx \frac{20\angle 0^\circ}{100\angle -89.4^\circ} = 0.2\angle 89.4^\circ = 200\text{mA}\angle 89.4^\circ$

7.33



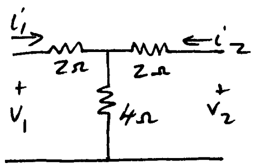
$Z_0 = 50{,}000\,\Omega = R$

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 2\pi \times 2\times10^6$

$Q_p = \omega_0 RC = 80 \quad \therefore\ C = \dfrac{80}{5\times10^4\ \ 2\pi\times2\times10^6} = 127.3\ pF$

$\therefore\ L = \dfrac{1}{\omega_0^2 C} = \dfrac{\omega_0 R}{\omega_0^2\ 80} = \dfrac{R}{\omega_0 Q_p} = \dfrac{5\times10^4}{2\pi\times2\times10^6\times80} = 49.7\,\mu H$

8.19



$* \quad v_1 = 2i_1 + 4(i_1+i_2) = 6i_1 + 4i_2$

$* \quad v_2 = 2i_2 + 4(i_1+i_2) = 4i_1 + 6i_2$

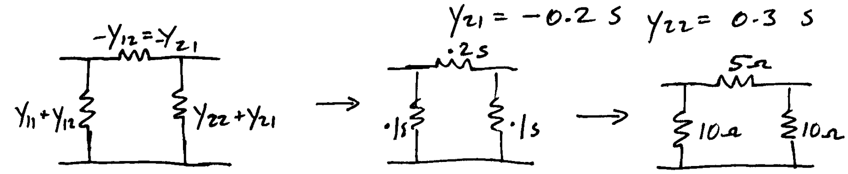
So $\left.\begin{aligned} 3v_2 &= 12i_1 + 18i_2 \\ 2v_1 &= 12i_1 + 8i_2 \end{aligned}\right\}\ 3v_2 - 2v_1 = 10i_2$

& $\left.\begin{aligned} 3v_1 &= 18i_1 + 12i_2 \\ 2v_2 &= 8i_1 + 12i_2 \end{aligned}\right\}\ 3v_1 - 2v_2 = 10i_1$

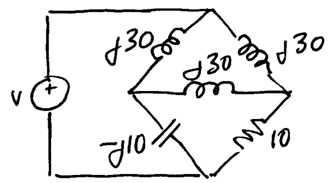
$\left.\begin{aligned} i_1 &= 0.3v_1 - 0.2v_2 \\ i_2 &= -0.2v_1 + 0.3v_2 \end{aligned}\right\}\ \therefore\ \begin{aligned} k_5 &= 0.3\,S \quad k_6 = -0.2\,S \\ k_7 &= -0.2\,S \quad k_8 = 0.3\,S \end{aligned}$

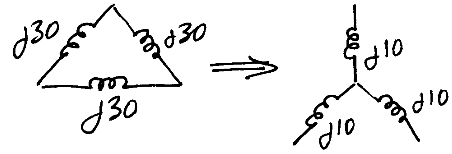
$\therefore\ \ y_{11} = 0.3\,S \quad y_{12} = -0.2\,S$

$\quad\ \ y_{21} = -0.2\,S \quad y_{22} = 0.3\,S$



8.23

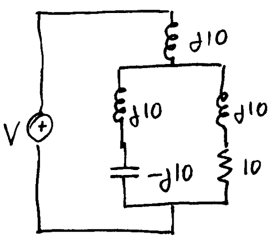




$V = 60\angle 0°$ V

From Example 4 $Z_Y = \frac{1}{3} Z_\Delta$

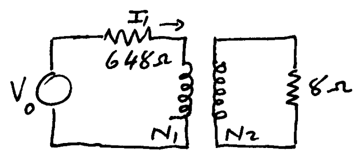
∴



$$I = \frac{V}{j10 + 0 /\!/ 10(1+j)} = \frac{60}{j10} = 6\angle -90°$$

L–C arm is SC ∴ no current in the resistor. $I_R = 0$

8.30



(a) Max power transfer

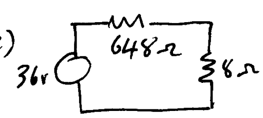
$648 = \left(\frac{N_1}{N_2}\right)^2 8\Omega \qquad \frac{N_1}{N_2} = \sqrt{\frac{648}{8}} = 9$

(b) $V_0 = 36\text{v} \quad I_1 = \frac{V_0}{2\times 648\Omega} = \frac{36}{2\times 648}$ ∴ Total power consumption $P = \frac{(36)^2}{2\times 648} = 1\text{W}$

½ power in load (8Ω) and ½ in 648Ω

∴ $P_{Lx} = \frac{1}{2}$ W

(c)



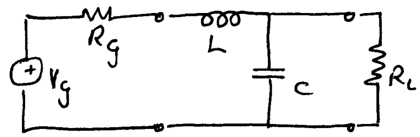
$P_{Ls} = I^2R = \left(\frac{36}{648+8}\right)^2 8 = 24\text{ mW}$

w/ transformer

$$\therefore \frac{P_{Lx}}{P_{Ls}} = \frac{1/2\text{ W}}{.024\text{ W}} = 20.8\dot{3}$$

series

AP 8.7



$R_g = 500\,\Omega$

$R_L = 5000\,\Omega$

Find L, C for max power transfer at $\omega = 10^4$ rad/s

Need $Z_{RC} = 500 - jX_1 = R_L \parallel -jX_c = \dfrac{-jX_c R_L}{R_L - jX_c} = \dfrac{R_L X_c^2 - j R_L^2 X_c}{R_L^2 + X_c^2}$

$\therefore \dfrac{R_L X_c^2}{R_L^2 + X_c^2} = 500 \longrightarrow 5000 X_c^2 = (5000^2 + X_c^2)500$

$4500 X_c^2 = \underset{\times 500}{5000^2} \rightarrow X_c = 5000\sqrt{\dfrac{500}{4500}}$

$= \dfrac{5000}{3} = 1.67\,\text{k}\Omega$

$\therefore C = \dfrac{1}{\omega X_c} = \dfrac{3}{5000 \times 10^4} = 0.06\,\mu\text{F}$

Now need $X_L = X_1 = \dfrac{R_L^2 X_c}{R_L^2 + X_c^2} = \dfrac{(5000)^2\, 5000/3}{(5000)^2 + \left(\frac{5000}{3}\right)^2}$

$= \dfrac{(5000)^2\, 5000 \times 3}{(5000)^2\,(9+1)} = 1500\,\Omega$

$\therefore L = \dfrac{X_L}{\omega} = \dfrac{1500}{10^4} = 150\,\text{mH}$