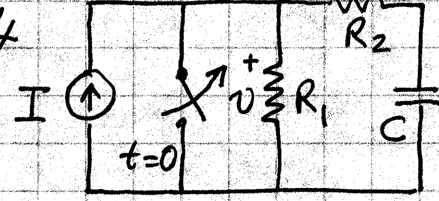
ECE 241 Assignment #5 Model Answers

Problems P6.14, 6.16, 6.20, 6.22, 7.3, 7.5, 7.9, 7.10

P6.14



(a) $Y(s) = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{sC}} = \frac{1}{R_1} + \frac{sC}{1 + sR_2C}$

$= \frac{1 + sR_2C + sR_1C}{R_1(1 + sR_2C)} = \frac{1}{R_1}\frac{1 + s(R_1+R_2)C}{1 + sR_2C}$

(b) Forced response: DC, $\therefore s=0$ $Y(0) = 1/R_1$ $v_F = I/Y(0) = IR_1$

(c) Natural response: Voltage from zeros of Y $s = -\frac{1}{(R_1+R_2)C}$

$\therefore v_N = A e^{-t/(R_1+R_2)C}$

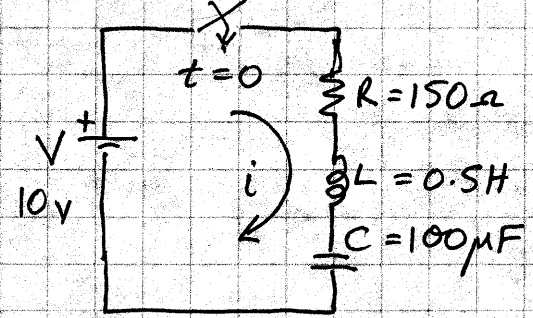
(d) Complete response $v = v_F + v_N = IR_1 + Ae^{-t/(R_1+R_2)C}$

Initially $v_C(0) = 0$ $\therefore v(0^+) = I(R_1 /\!/ R_2) = IR_1 + A$

$\therefore A = I\left(\frac{R_1R_2}{R_1+R_2} - R_1\right) = -I\frac{R_1^2}{R_1+R_2}$

$\therefore v(t) = IR_1\left(1 - \frac{R_1}{R_1+R_2}\exp - \frac{t}{(R_1+R_2)C}\right)$

P6.16



Find complete current response

$$Z(s) = 150 + 0.5s + \frac{1}{10^{-4}s}$$

$$= \frac{0.5s^2 + 150s + 10^4}{s}$$

Pole of Z(s) at s=0

Forced (DC) response $i_F = 0$

Zeros of $Z(s) = 0.5\,\frac{s^2 + 300s + 2\times10^4}{s}$ at $s = -150 \pm \sqrt{150^2 - 2\times10^4}$

$= -150 \pm 50$

$= -100, -200$

$\therefore$ Natural response $i_N = Ae^{-100t} + Be^{-200t}$

& Complete response $i = i_F + i_N = Ae^{-100t} + Be^{-200t}$

At $t=0^+$ $\quad i(0^+) = 0 \quad \therefore A = -B$

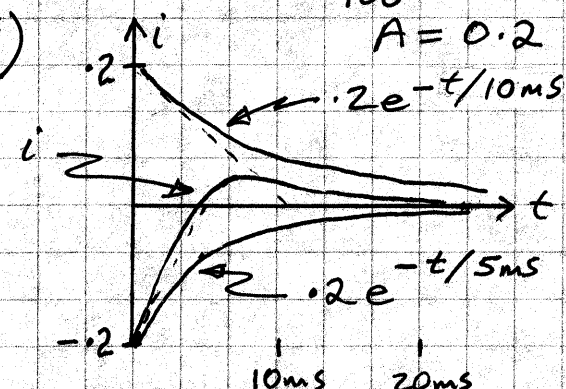
$v_C(0^+) = 0$

and $v_R(0^+) = 0$ $\quad \therefore V = L\left.\frac{di}{dt}\right|_{t=0^+} \quad 10 = (0.5)\begin{bmatrix}-100A \\ -200B\end{bmatrix}$

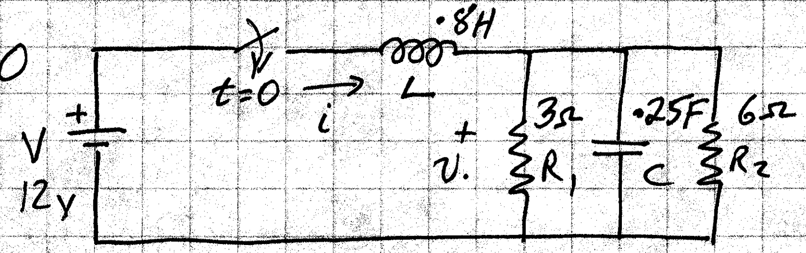
$\therefore \frac{20}{100} = -A + 2A$

$A = 0.2$

$$i(t) = 0.2\left(e^{-100t} - e^{-200t}\right)$$



P6.20



Find $i(t)$

$$Z(s) = sL + \frac{R_1 R_2 \, 1/sC}{R_1 R_2 + (R_1 + R_2) 1/sC}$$

$$= sL + \frac{1/sC}{1 + 1/sCR_p}$$

$$= sL + \frac{R_p}{1 + sR_pC}$$

$$= \frac{R_p + sL + s^2 R_p LC}{1 + sR_pC}$$

$$= \frac{2 + 0.8s + .4s^2}{1 + s(.5)}$$

$$= \frac{.4}{.5} \, \frac{s^2 + 2s + 5}{s + 2}$$

$$R_p = \frac{3 \times 6}{3 + 6} = 2\,\Omega$$

Forced (dc) response:

$$Z(0) = 0.8 \, \frac{5}{2} = 2\,\Omega$$

$$\therefore i_F = 12V/2\Omega = 6A$$

Zeros of $Z(s)$ for natural current response:

$$s_1, s_2 = -1 \pm \sqrt{1^2 - 5}$$

$$= -1 \pm j2$$

$\therefore$ Natural response $i_N(t) = Ae^{-t}\cos(2t + \theta)$

$\therefore$ Complete response $i = i_F + i_N = 6 + Ae^{-t}\cos(2t+\theta)$

At $t = 0^+$ $\quad i = 0 \quad \therefore 6 + A\cos\theta = 0 \quad A\cos\theta = -6$

$$L\frac{di}{dt} = V \quad \therefore \frac{12}{0.8} = A(\quad [-\sin\theta]2 + \quad (-1)\cos\theta)$$

$$A(\cos\theta + 2\sin\theta) = -15$$

$$-6 + 2A\sin\theta = -15$$

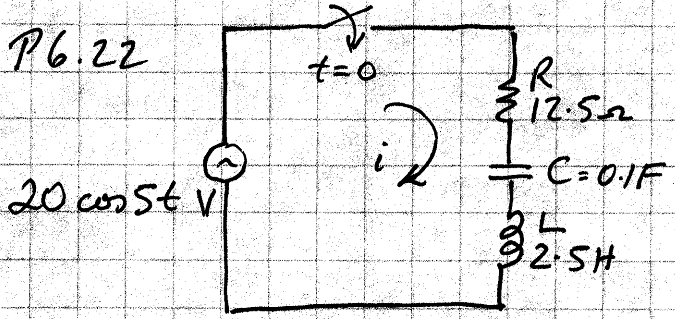
$$A\sin\theta = -9/2$$

$$\therefore \frac{A\sin\theta}{A\cos\theta} = \frac{-9/2}{-6} = \frac{3}{4} = \tan\theta \quad \therefore \theta = 36.87^\circ$$

$$A = -6/\cos 36.87^\circ = -7.5$$

$$\therefore i(t) = 6 - 7.5\,e^{-t}\cos(2t + 36.87^\circ)$$

P6.22



$$Z(s) = R + sL + \frac{1}{sC}$$
$$= 12.5 + 2.5s + \frac{10}{s}$$
$$= \frac{2.5s^2 + 12.5s + 10}{s}$$
$$= 2.5\,\frac{s^2 + 5s + 4}{s} = \frac{2.5}{s}(s+4)(s+1)$$

Forced response $s = j5$

$$Z(j5) = 2.5\,\frac{-25 + 25j + 4}{5j} = 2.5\left(-\tfrac{21}{5}/j + 5\right) = 12.5 + j10.5$$
$$= 16.325 \angle 40^\circ$$

$$\therefore\ i_F = \frac{20}{16.325}\angle -40^\circ = 1.225\cos(5t - 40^\circ)$$

Natural response : Zeros of $Z(s)$ $s_1, s_2 = -1, -4$

$\therefore$ Natural response $i_N = Ae^{-t} + Be^{-4t}$

Total response : $i = 1.225\cos(5t - 40^\circ) + Ae^{-t} + Be^{-4t}$

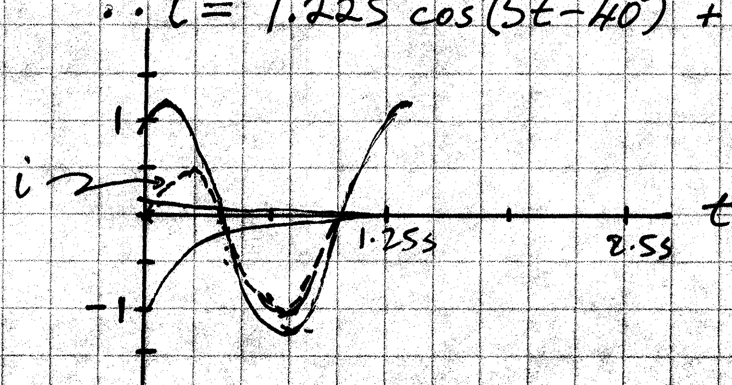
At $t = 0^+$ $i = 0$ $\therefore\ A + B = -1.225\cos -40^\circ = -0.9384$

$$V = 20V = L\left.\frac{di}{dt}\right|_{t=0} = 2.5\left[-1.225 \cdot 5\sin(5t-40) - Ae^{-t} - 4Be^{-4t}\right]_{t=0}$$
$$= -2.5\left[A + 4B - 3.937\right]$$

$\therefore\ A + 4B - 3.937 = -8$

$$\left.\begin{aligned} A + 4B &= -4.063 \\ A + B &= -0.9384 \end{aligned}\right\} \begin{aligned} 3B &= -3.1246 \\ B &= -1.0415 \end{aligned}$$
$$A = 1.0415 - 0.9384 = +0.1031$$

$$\therefore\ i = 1.225\cos(5t - 40^\circ) + 0.1031e^{-t} - 1.0415e^{-4t}$$



$\omega = 5 = 2\pi f$

$\therefore\ T = \frac{1}{f} \approx \frac{1}{.8} \approx 1.25s$

P7.3. (a) Current coil $i = 10\sqrt{2}\cos(\omega t + 45°)$
Voltage coil $v = 220\sqrt{2}\cos(\omega t - 15°)$

$I = 10\angle 45°$ $V = 220\angle -15°$ $P = IV\cos\theta,\ \theta = 45° - (-15°)$

$\therefore P = 10 \times 220 \cos 60° = 1100\,W$

(b) $i = 5\cos(\omega t - 120°)$ $v = 20\cos(\omega t - 75°)$

$I = \frac{5}{\sqrt{2}}\angle -120°$ $V = \frac{20}{\sqrt{2}}\angle -75°$ $\theta = -120° - (-75°) = -45°$

$\therefore P = \frac{5}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \cos -45° = 50\frac{1}{\sqrt{2}} = 35.4\,W$

P7.5 200V 60Hz 10KW at leading pf = 0.55

(a) Current & reactive power : Av power $P = 10^4 W = VI\cos\theta = 200 I (0.55)$

$\therefore I = \frac{50}{0.55} = 90.9A$

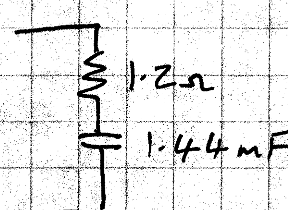
Leading pf $\therefore \theta = \cos^{-1}(.55) = -56.6°$

& $P_x = VI\sin\theta = 200 \times 90.9 \sin -56.6°$
$= -15{,}177.5 \approx -15.2\,KVAR$

(b) Series elements for load : Leading pf $\therefore$ capacitive

$R = \frac{P}{I^2} = \frac{10^4 W}{(90.9)^2 A^2} = 1.21\,\Omega$

$X_c = \frac{-15.2 \times 10^3}{(90.9)^2} = -1.84\,\Omega = -\frac{1}{\omega C}$ $\therefore C = \frac{1}{1.84\ 2\pi\ 60} = 1.44\,mF$

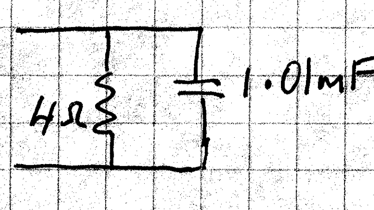


(c) Parallel elements for load :

$P = V^2/R$ $R = \frac{V^2}{P} = \frac{200^2}{10^4} = 4\,\Omega$

$X_c = V^2/P_x = 200^2/-15.2\times10^3 = -2.63\,\Omega$

so $C = \frac{1}{2.63\ 2\pi\ 60} = 1.008\,mF$



P7.9 Load: 100 KVA at 0.7 pf lagging
20 KW at 0 pf

Lagging pf ⇒ inductive load $\cos\theta = 0.7 \rightarrow \theta = 45.6°$

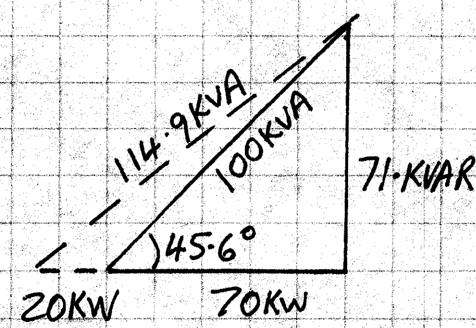
$P = VI\cos\theta = 100(0.7) = 70\,KW$
$P_X = VI\sin\theta = 100\sin 45.6° = 71.4\,KVARs$

∴ Apparent power

$P_A = \left[(P_X)_{tot}^2 + P_{tot}^2\right]^{1/2}$

$= \left[71.4^2 + (70+20)^2\right]^{1/2}$

$= 114.88\,KVA$

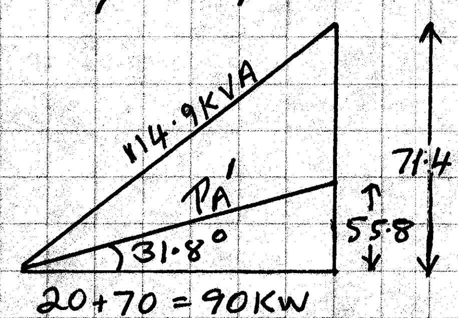


P7.10 Specify reactive KVA for capacitor to correct pf of plant above (in P7.9) to 0.85 lagging.

$\theta = \cos^{-1} 0.85 = 31.79°$

∴ New $P_X' = 90\tan 31.79° = 55.78\,KVARs$

∴ New Apparent Power $P_A' = (90^2 + 55.8^2)^{1/2}$
$= 105.9\,KVA$



& reactive KVARs of Capacitor $= 71.4 - 55.8$
$= 15.6\,KVAR$

$\dfrac{\text{New Power loss}}{\text{Old Power loss}} = \dfrac{I'^2 R}{I^2 R}$

$= \dfrac{(P_A'/V)^2}{(P_A/V)^2} = \left(\dfrac{P_A'}{P_A}\right)^2 = \left(\dfrac{105.9}{114.9}\right)^2 = 0.85$