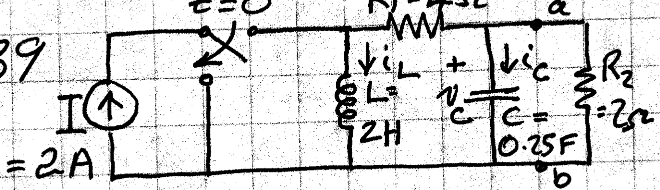
ECE 241 Assignment #4 Model Answers

Problems AP4.9, AP4.10, P4.39, P5.12, P5.38, AP5.8

P4.39



(a) At $t = 0^+$

$v_C(0^+) = 0$

since $i_L(0) = I = 2A$

If $v_C(0^+) = 0$, $v_{R2}(0^+) = 0$ & $i_C(0^+) = -2A = C \frac{dv_C}{dt}\Big|_{t=0^+}$

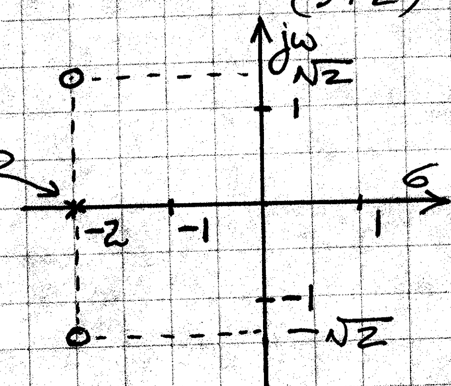
$\therefore \frac{dv_C}{dt}\Big|_{t=0^+} = \frac{-2}{0.25} = -8 \text{ v/s}$

(b) $Y_{ab} = \frac{1}{R_2} + sC + \frac{1}{R_1 + sL} = \frac{1}{2} + 0.25s + \frac{1}{4+2s} = \frac{1 + 2(s+2)(s+2)\frac{1}{4}}{2(s+2)}$

$= \frac{1 + \frac{1}{2}(s+2)^2}{2(s+2)} = 0.25\frac{(s+2)^2 + 2}{(s+2)} = 0.25\frac{(s+2+j\sqrt{2})(s+2-j\sqrt{2})}{(s+2)}$

Pole at $s = -2$

Zeros at $s_1, s_2 = -2 \pm j\sqrt{2}$



(c) Natural response of $v_C(t)$ depends on poles of Z_{ab} or zeros of Y_{ab}

i.e. $s_1, s_2 = -2 \pm j\sqrt{2}$

i.e. $v_C(t) = A e^{-2t} \cos(\sqrt{2}t + \phi)$

At $t = 0^+$ $v_C(0^+) = 0$ $\therefore A\cos\phi = 0$ $\phi = -90°$ for $A \neq 0$

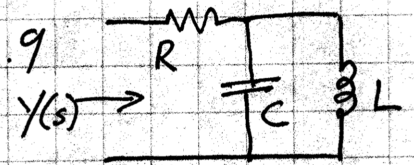
& $v_C(t) = A e^{-2t} \sin\sqrt{2}t$

At $t = 0^+$ $\frac{dv_C}{dt}\Big|_{t=0^+} = -8\text{v/s} = A(e^{-2t}\sqrt{2}\cos\sqrt{2}t + e^{-2t}(-2)\sin\sqrt{2}t)$

$= \sqrt{2}A$ $\therefore A = -8/\sqrt{2} = -5.66$

$\therefore v_C(t) = -5.66\, e^{-2t} \sin\sqrt{2}t$

AP4.9



$$Y(s) = \frac{s^2+36}{s^2+6s+36}$$

$$Z(s) = R + \frac{1}{sC + \frac{1}{sL}} = R + \frac{sL}{1+s^2LC}$$

$$= \frac{sL + R(1+s^2LC)}{1+s^2LC}$$

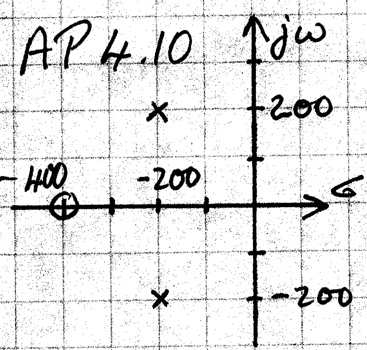
$$\therefore Y(s) = \frac{1+s^2LC}{s^2RLC + sL + R}$$

$$= \frac{s^2 + {}^1\!/_{LC}}{s^2R + s\frac{1}{C} + \frac{R}{LC}} = \frac{s^2+36}{s^2+6s+36}$$

ie. $R = 1$ $\quad C = \frac{1}{6}$ $\quad L = \frac{1}{36C} = \frac{6}{36} = \frac{1}{6}$ $\quad$ Check: $\frac{R}{LC} = .1 \times 36$

$\therefore R = 1\Omega$ $\quad C = 0.17F$ $\quad L = 0.17H$

AP4.10



Pole-zero diagram for $Z(s) = K \frac{s+400}{(s+200-200j) \times (s+200+200j)}$

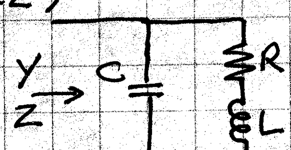
$$= K \frac{s+400}{(s+200)^2 + 200^2}$$

$$= K \frac{s+400}{s^2 + 400s + 8\times10^4}$$

Need a circuit with $Z(0) = \frac{K}{8\times10^4}$ ie. no series C or parallel L

" " " " $Z(\infty) = 0$ ie. parallel C, not series L

Looks like must be C // (R in series with L)



$$Y(s) = sC + \frac{1}{R+sL}$$

$$= \frac{1+sC(R+sL)}{R+sL}$$

$$Z(s) = \frac{L}{LC} \frac{s + R/L}{s^2 + s\frac{RC}{LC} + \frac{1}{LC}}$$

$$= \frac{1}{C} \frac{s + R/L}{s^2 + sR/L + 1/LC}$$

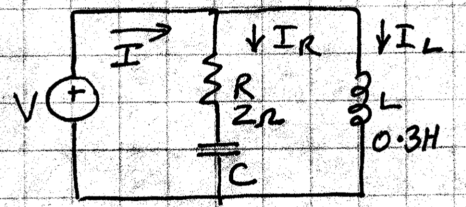
Hence $R/L = 400$

$C = 1$

$\frac{1}{LC} = 8\times10^4$ $\therefore L = \frac{1}{8\times10^4} = 12.5\mu H$

$R = 400 \times 12.5\times10^{-6} = 5m\Omega$

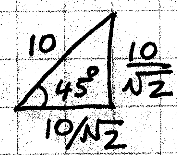
P5.17



$i_R = 10\sqrt{2}\cos(10t + 45°)$ A

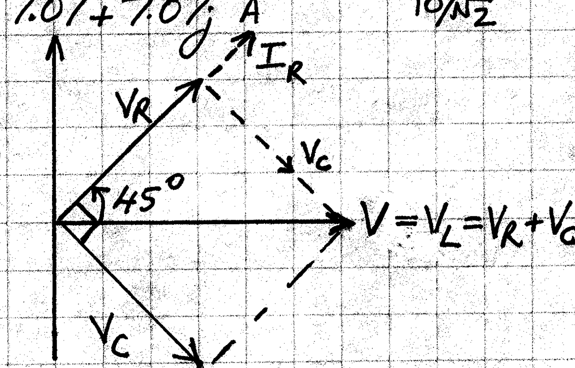
(a) $\tilde{I}_R = 10\angle 45°$

$= 7.07 + 7.07j$ A



$\therefore \tilde{V}_R = \tilde{I}_R R = 14.14 + 14.14j$ V

(b)

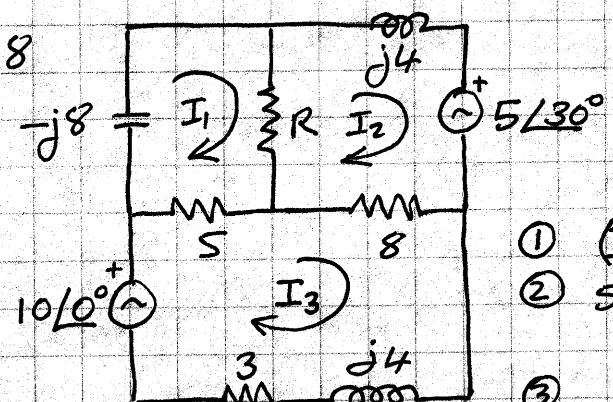


(c) $V = \dfrac{V_R}{\cos 45°} = \dfrac{10 \times 2}{1/\sqrt{2}}$

$= 20 \times 1.414 = 28.3$V

& $v(t) = \dfrac{20}{\sqrt{2}}\sqrt{2}\cos 10t = 20\cos 10t$ V

P5.38



$5\angle 30° = 5\cos 30° + j\sin 30°$

$= 5\sqrt{3}/2 + j\,5/2$

① $(I_1 - I_2)R = -(-j8)I_1 + 5(I_3 - I_1)$

② $\dfrac{5\sqrt{3}}{2} + j\dfrac{5}{2} = -j4I_2 + R(I_1 - I_2) + 8(I_3 - I_2)$

③ $10 = 5(I_3 - I_1) + 8(I_3 - I_2) + I_3(3 + j4)$

$$\begin{aligned}
&I_1(R + 5 - j8) + I_2(-R) + I_3(-5) = 0\\
&I_1(R) + I_2(-R - j4 - 8) + I_3(8) = 5\sqrt{3}/2 + j\,5/2\\
&I_1(-5) + I_2(-8) + I_3(5 + 8 + 3 + j4) = 10
\end{aligned}$$

By determinants, terms in R do not disappear ∴ use $R = 5\Omega$

$$\begin{aligned}
&I_1(10 - j8) + I_2(-5) + I_3(-5) = 0\\
&I_1(5) + I_2(-13 - j4) + I_3(8) = 5\sqrt{3}/2 + 5/2\,j\\
&I_1(-5) + I_2(-8) + I_3(16 + j4) = 10
\end{aligned}$$

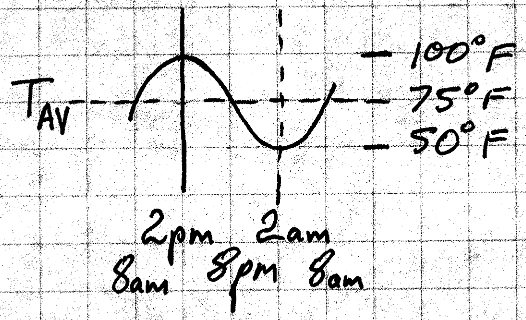
Solving by determinants gives $I = \dfrac{1000 + 100\sqrt{3} + 300j + 50\sqrt{3}j}{1851 + j1224}$

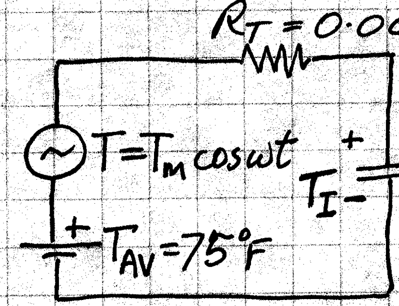
$= \dfrac{1173.2 + 386.6j}{1851 + 1224j} = \dfrac{1235.26\angle 18.24°}{2219.09\angle 33.48°}$

$= 0.56\angle -15.24°$ A

AP 5.8 Assume the outside temperature varies sinusoidally

$T_{max} = 100°F$ at 2pm
$T_{min} = 50°F$ at 2am
$\therefore T_{AV} = 75°F$, $T_M = 25°F$





$t = 0$ at 2pm
Period $T = 24$hr
$\therefore \omega = \frac{2\pi}{T}$

Consider DC $\left.T_I\right)_{DC} = 75°F$ (interior temperature)

Consider AC interior temperature $\left.T_I\right)_{AC}$

$$\left.T_I\right)_{AC} = \frac{Z_c}{Z_c + Z_R} T = \frac{1/j\omega C_p}{R_T + 1/j\omega C_p} T$$

$$= \frac{1}{1 + j\omega R_T C_p} T = \frac{1 - j\omega R_T C_p}{1 + (\omega R_T C_p)^2} T$$

$\omega R_T C_p$

$= \frac{2\pi}{24} 10^{-3} \, 6\times10^3 \; \frac{\text{rad}}{\text{hr}} \; \frac{°\text{F hr}}{\text{Btu}} \; \frac{\text{Btu}}{°\text{F}}$

$= \frac{\pi}{2}$ radians

$$= \frac{1 - j\frac{\pi}{2}}{1 + (\pi/2)^2} \, 25°F \angle 0°$$

$$= \frac{25°F}{\sqrt{1 + (\pi/2)^2}} \angle 0° + \tan^{-1} - \frac{\pi}{2}$$

$$= \frac{25°F}{1.86} \angle -57.52°$$

$$= 13.44° \angle -57.52°$$

So complete $T_I = 75°F + 13.44° \cos(\omega t - 57.52°)$

Minimum $T_I = 75 - 13.44 = 61.56°F$

occurring at $\omega t - 57.52° = 180°$

i.e. $t = \frac{237.52}{360°}$ 24hrs after 2pm

$= 15.835$ hrs (or 15hr 50.1min) after 2pm

i.e. at 5:50.1 am (3hrs 50min after the external min at 2am)