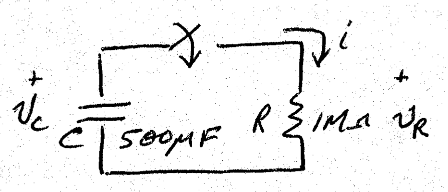
ECE 241 Assignment #2 Model Answers

Problems: P 3.7, 3.11, 3.13, 16.3, 16.6, 16.23, 16.29 AP16.1

3.7



$v_c(0) = 100\text{v}$

Following Example 1

Current $i = I_0 e^{-t/\tau}$ and so $v = v_c = v_R = I_0 R e^{-t/\tau}$

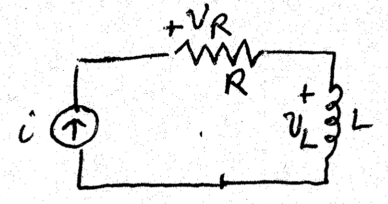
But clearly $v(0) = I_0 R = 100\text{v}$ (given)

so $v(t) = 100\, e^{-t/\tau}$ where $\tau = RC = 500\times10^{-6}\times10^{6}$
$= 500\,\text{s}.$

If $v(T) = 14\text{v}$ at $t = T$ $\quad 14 = 100\, e^{-T/500\text{s}}$

$T = 500\text{s} \ln(100/14) = 983\text{s}$

3.11



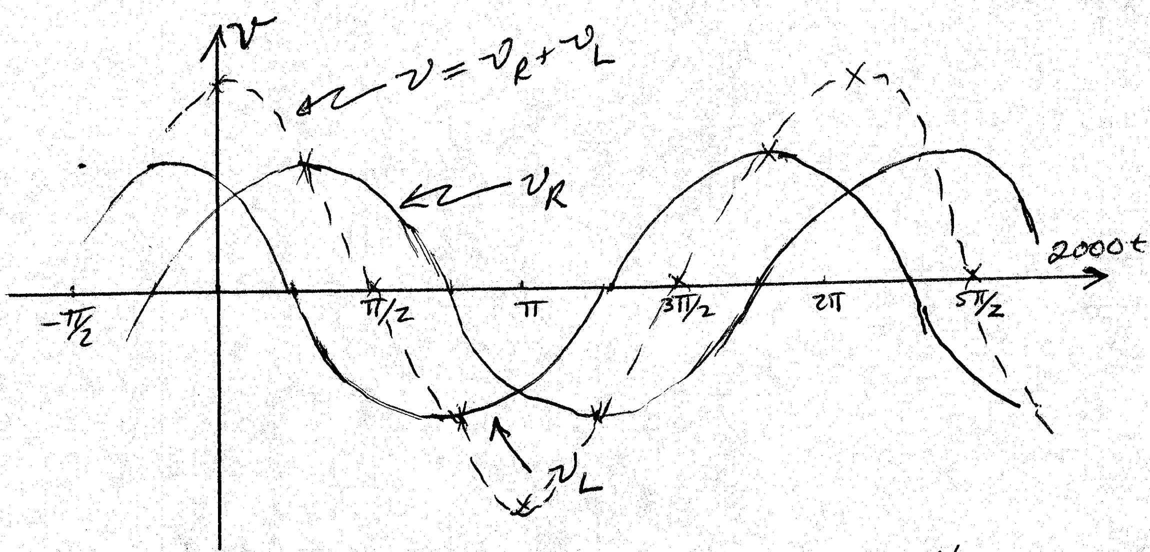
$i = 2\cos(2000t - \pi/4)$

$R = 20\Omega \qquad L = 10\text{mH}$

$v_R = iR = 40\cos(2000t - \pi/4)$

$v_L = L\frac{di}{dt} = 2\times10^{-2}(-\sin 2000t - \pi/4)\,2000$

$= -40\sin(2000t - \pi/4)$

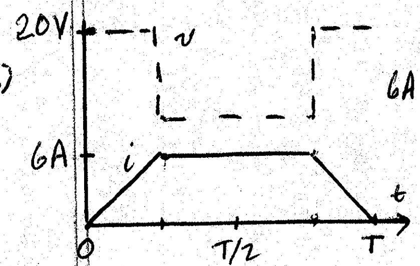


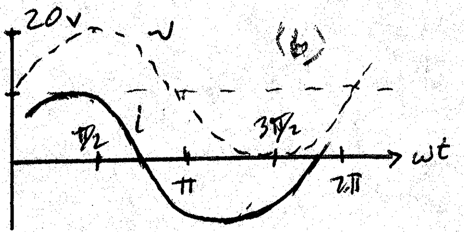
Amplitude of $v \approx \frac{3.2}{2}40 = 64\text{v}$ (actually $\sim 57\text{v}$ so this plot not very good!)

Phase of $v = 0°$ (sine wave)

3.13 Calculate average and effective values for each waveform

(a)





(a) $v_{AV} = \dfrac{20\frac{T}{4} + 10\frac{T}{2} + 20\frac{T}{4}}{T}$

$= 15v$

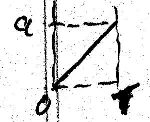
$i_{AV} = \dfrac{\frac{1}{2}6\frac{T}{4} + 6\frac{T}{2} + \frac{1}{2}6\frac{T}{4}}{T}$

$= 4.5A$

$v_{eff} = \sqrt{\left(20^2\frac{T}{4} + 10^2\frac{T}{2} + 20^2\frac{T}{4}\right)/T}$

$= \left(\frac{400}{4} + \frac{100}{2} + \frac{400}{4}\right)^{1/2} = \sqrt{250}$

$= 15.81v$



$y = \frac{a}{T}t$

$y^2 = \frac{a^2}{T^2}t^2$

$\int_0^T y^2 dt = \frac{a^2}{T^2}\frac{t^3}{3}\Big]_0^T = \frac{a^2}{3}$

$i_{eff} = \sqrt{\left(\frac{1}{3}6^2\frac{T}{4} + 6^2\frac{T}{2} + \frac{1}{3}6^2\frac{T}{4}\right)/T}$

$= \sqrt{24} = 4.90A$

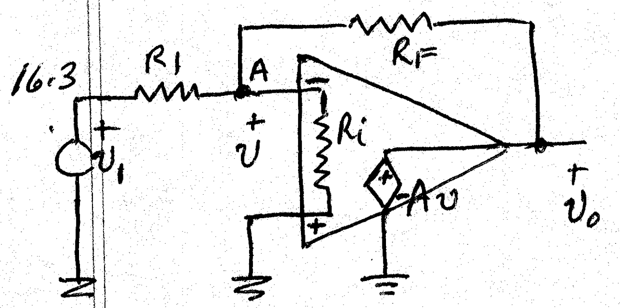
(b) $v_{AV} = 10v$ (DC component) $\quad i_{AV} = 0A$ (DC component)

$v_{eff} = \left[\frac{1}{T}\int_0^T (10 + 10\sin\omega t)^2 dt\right]^{1/2} = \left[\frac{100}{T}\int_0^T (1 + \sin\omega t)^2 dt\right]^{1/2} = 10\left[\frac{1}{T}\int_0^T (1 + 2\sin\omega t + \frac{1}{2} - \frac{1}{2}\cos 2\omega t)\, dt\right]^{1/2}$

$= 10\left(\frac{1}{T}\,\frac{3}{2}T\right)^{1/2} = 10\sqrt{1.5} = 12.25v$

$i_{eff} = i_{rms} = \frac{6}{\sqrt{2}} = 4.24A$ (for sine wave, even if phase shifted)

16.3



At A

(a) $$\frac{v_1 - v}{R_1} + \frac{v_0 - v}{R_F} - \frac{v}{R_i} = 0$$

and $v_0 = -Av$

$$\therefore \quad \frac{v_1}{R_1} = v\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_F}\right) + \frac{Av}{R_F}$$

Note $v_0 = A(v^+ - v^-) = -Av$

$$\therefore \quad v_0 = -Av = -A\frac{v_1}{R_1}\,\frac{1}{\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_F} + \frac{A}{R_F}} = -\frac{R_F}{R_1}v_1\,\frac{1}{1 + \left(\frac{R_F}{R_1} + \frac{R_F}{R_i} + 1\right)/A}$$

(b) $R_1 = 20K \quad R_F = 1M \quad R_i = 10M \quad A = 5\times10^4$

Standard formula $$\frac{v_0}{v_1} = -\frac{R_F}{R_1} = \frac{-10^6}{2\times10^4} = -50$$

For $$\frac{v_0}{v_1} = -\frac{R_F}{R_1}\,\frac{1}{1 + \left(\frac{R_F}{R_1} + \frac{R_F}{R_i} + 1\right)/A} = \frac{-50}{1 + \frac{50 + 0.1 + 1}{5\times10^4}} = \frac{-50}{1 + \frac{51.1}{5}10^{-4}}$$

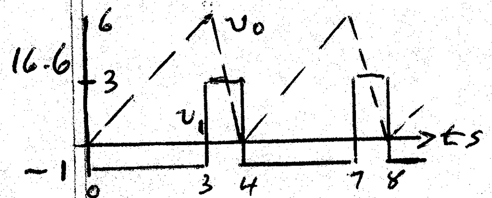
$$= -50/1.001022 = -49.95$$

(c) $R_i = 2M\Omega \quad A = 2\times10^4$

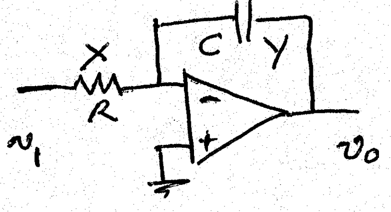
$$\frac{v_0}{v_1} = -50/\left(1 + \frac{50 + .5 + 1}{2\times10^4}\right) = -50/1.002575 = -49.87$$

In the approximation, A is more important than R_i, since large A forces a high effective R_i by negative feedback. Also, approximation reasonable for realistic A, R_i values.

16.6



v_0 is $-\int v_1\,dt$



$$v_0 = -\frac{1}{RC}\int_0^t v_1\,dt + IC = 0 \text{ at } t=0$$

$$-\frac{1}{RC}\cdot\underline{-1v \times 3s} = 6v \quad \therefore \frac{1}{RC} = 2$$

Select any reasonable values

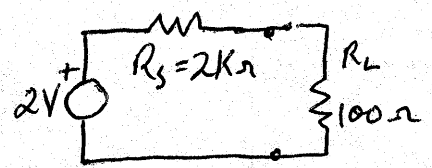
e.g. $R = 1K\Omega \quad C = 500\mu F$

$R = 10K\Omega \quad C = 50\mu F$

$R = 100K\Omega \quad C = 5\mu F$

Probably best choice → $R = 1M\Omega \quad C = 0.5\mu F$

16.23

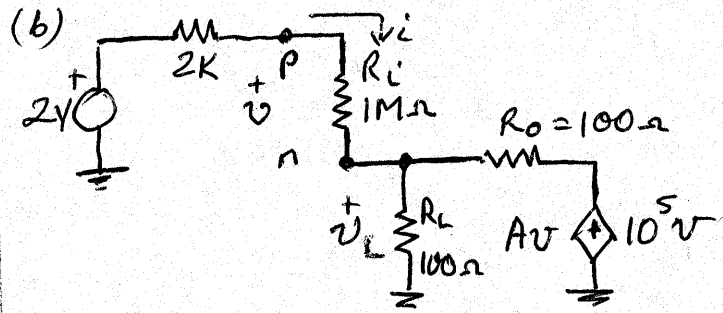


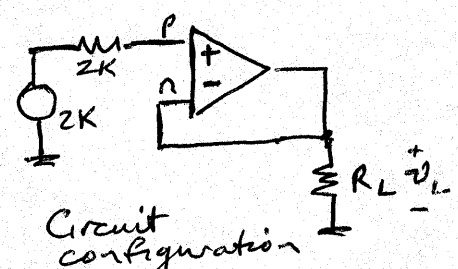
(a)

$$v_L = \frac{100}{2100} 2\text{v} = 0.0952\ \text{v}$$

$$P_L = v_L^2 / R_L = 90.7\ \mu\text{W}$$

(b)





Circuit configuration

$$2\text{v} = 2000i + 10^6 i + v_L$$

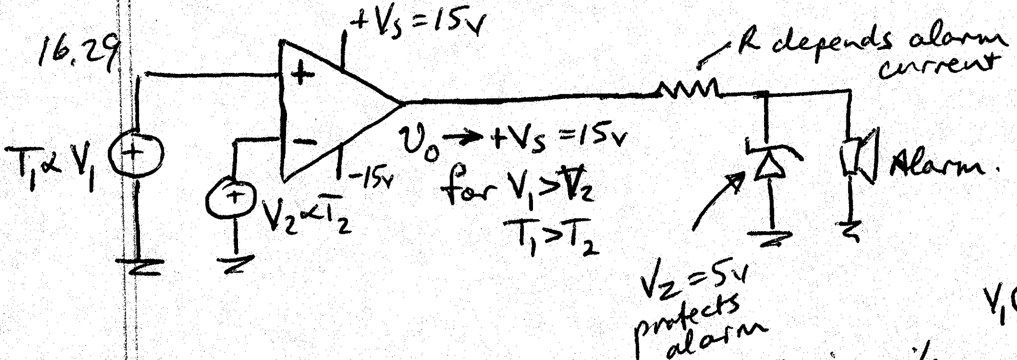
$$\left.\begin{aligned} i &= \frac{v_L}{100} + \frac{v_L - 10^5 v}{100} \\ v &= 10^6 i \end{aligned}\right\} \quad \left.\begin{aligned} i &= \frac{v_L}{50} - 10^9 i \\ \therefore i &= \frac{v_L/50}{1+10^9} \end{aligned}\right\} \quad 2 = \frac{1002 \times 10^3 \times v_L}{50(1+10^9)} + v_L$$

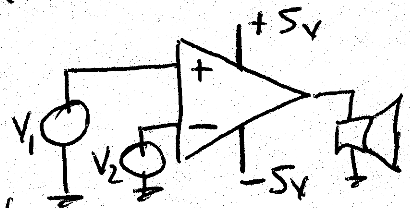
$$v_L = \frac{2}{1 + \frac{1.002 \times 10^6}{50 \times (1+10^9)}} \approx \frac{2}{1 + \frac{10^6}{5 \times 10^{10}}} = \frac{2}{1 + 2\times 10^{-5}} \approx 2(1 - 2\times10^{-5})\text{v}$$

$$\approx 2\text{v}$$

& $P_L \approx 2^2/100 = 40\ \text{mW}$ Buffer is effective with practical op amp parameters

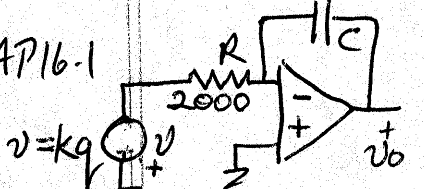
16.29





Other possible output circuits e.g. simplest

AP16.1



$K = 20\text{mV}\cdot\text{s/cm}^3$

$$\int q\,dt = \pm 200\,\text{cm}^3 \rightarrow V_o = \pm 10\text{v}$$

$$v_o = -\frac{1}{RC}\int -v\,dt = +\frac{K}{RC}\int q\,dt = \frac{20\times10^{-3}(200)}{2000\,C} = 10\text{v}$$

$$\therefore C = \frac{4}{2\times10^4}\text{F} = 200\,\mu\text{F}$$ using the sensor R_{out} as R

OR increase R to say $200K - 2K = 198K$, $C = 2\mu F$ etc.