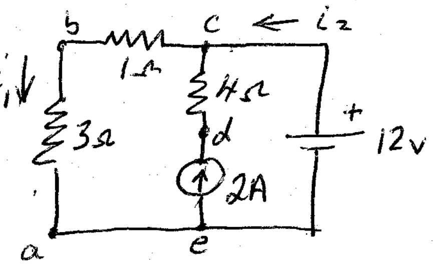
ECE 241 Assignment #1 Model Answers

P2.8, 12, 22, 26, 33, 35, 36, 37, AP2.6, 10

P2.8



(a) Evaluate v_{ba}, v, i_2, v_{de}

(b) Evaluate $P_{3\Omega}$

(c) Evaluate power supplied by the 2A source.

(a) $v_{ca} = 12v \quad \therefore i_1 = \dfrac{12v}{4\Omega} = 3A$ and $v_{ba} = i_1 \times 3\Omega = 9v$

(or $\therefore v_{ba} = \dfrac{3\Omega}{3\Omega + 1\Omega} 12v = 9v$)

$i_1 = i_2 + 2A \quad \therefore i_2 = 3A - 2A = 1A$

Also $v_{ed} = 4\Omega(-2A) = -8v$, so $v_{de} = 12v - v_{cd} = 12 - (-8)v = 20v$

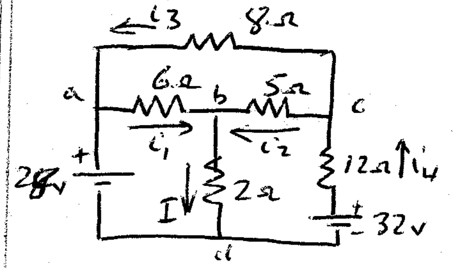
(v undefined)

(b) $P_{3\Omega} = v_{ba} i_1 = 9v \times 3A = 27w$

(or $v_{ba}^2/3\Omega$ or $i_1^2 3\Omega$)

(c) $P_{2A} = v_{de} \times 2A = 20v \times 2A = 40w$ (delivered)

P2.12



Find I by (a) element currents

(b) loop currents

(c) node voltages

Draw a conclusion about the methods

2.12 (a)

	Elim I, i_4	Elim v_b	Elim v_c
(1) $v_a = 28v$			
(2) $28 - v_b = 6i_1$		$\xrightarrow{(9)}$ $28 - 2i_1 - 2i_2 = 6i_1$	$\rightarrow$ (11)
(3) $v_c - v_b = 5i_2$		$\xrightarrow{(9)}$ $v_c - 2i_1 - 2i_2 = 5i_2$	$\xrightarrow{(5)}$ $28 + 8i_3 - 2i_1 = 7i_2$ (12)
(4) $v_b = 2I$	$\xrightarrow{(7)}$ $v_b = 2i_1 + i_2$ (9)		
(5) $v_c - 28 = 8i_3$			
(6) $32 - v_c = 12i_4$	$\xrightarrow{(8)}$ $32 - v_c = 12i_1 + 12i_2$ (10)		$\xrightarrow{(5)}$ $32 - 28 - 8i_3 = 12i_2 + 12i_3$ (13)
(7) $I = i_1 + i_2$			
(8) $i_4 = i_2 + i_3$			

8 eq'ns in 8 unknowns
Elim v_a

3 eq'ns in 3 unknowns
Re-write below

(11) $\rightarrow$ $28 = 8i_1 + 2i_2$ $\quad\therefore i_2 = 14 - 4i_1$, substitute to elim i_2

(12) $\rightarrow$ $28 = 2i_1 + 7i_2 - 8i_3$ $\longrightarrow$ $28 = 2i_1 + 98 - 28i_1 - 8i_3$ (14)

(13) $\quad 4 = 12i_2 + 20i_3$ $\longrightarrow$ $4 = 168 - 48i_1 + 20i_3$ (15)

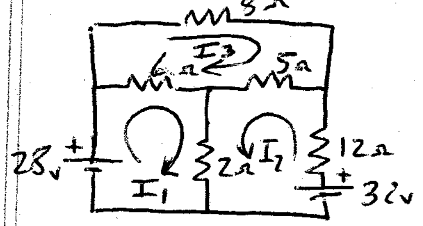
(14) $\rightarrow$ $70 = 26i_1 + 8i_3$ $\xrightarrow{\times 5/2}$ $175 = 65i_1 + 20i_3$

(15) $\rightarrow$ $164 = 48i_1 - 20i_3$ $\longrightarrow$ $164 = 48i_1 - 20i_3$

$\therefore\ 339 = 113i_1$ & $i_1 = 3A$

$\therefore i_2 = 14 - 4i_1 = 2A$ & $I = i_1 + i_2 = 5A$

2.12(b)



Mesh equations

$$28 = 6(I_1 - I_3) + 2(I_1 + I_2)$$

$$32 = 12 I_2 + 5(I_2 + I_3) + 2(I_1 + I_2)$$

$$8 I_3 = 6(I_1 - I_3) + 5(-I_2 - I_3)$$

Rearrange as:

$$8I_1 + 2I_2 - 6I_3 = 28 \qquad (1)$$

$$2I_1 + 19I_2 + 5I_3 = 32 \qquad (2)$$

$$6I_1 - 5I_2 - 19I_3 = 0 \qquad (3)$$

Solve by determinants or elimination

Eliminate I_1 $4\times(2) \rightarrow 8I_1 + 76I_2 + 20I_3 = 128$ $\quad 3\times(2)\ 6I_1 + 57I_2 + 15I_3 = 96$

$4\times(2) - (1) \rightarrow 74I_2 + 26I_3 = 100$ $\quad 3\times(2) - (3) \rightarrow 62I_2 + 34I_3 = 96$

Rearrange to eliminate $I_2 = -\frac{13}{37} I_3 + \frac{50}{37} = \frac{48}{31} - \frac{17I_3}{31}$

$$\therefore (50 - 13I_3)31 = 37(48 - 17I_3) \rightarrow I_3 = \frac{37\times48 - 31\times50}{37\times17 - 13\times31} = \frac{226}{226} = 1A$$

$$\therefore I_2 = \frac{96 - 34I_3}{62} = \frac{62}{62} = 1A$$

$$\therefore I_1 = \frac{32 - 5I_3 - 19I_2}{2} = 4A$$

Hence $I = I_1 + I_2 = 5A$

2.12(c) Nodal analysis: Using the notation from 2.12(a) with $v_a = 28v$

$$\frac{28 - v_b}{6} + \frac{v_c - v_b}{5} = \frac{v_b}{2} \qquad (1)$$

$$\frac{28 - v_c}{8} + \frac{v_b - v_c}{5} + \frac{32 - v_c}{12} = 0 \qquad (2)$$

$(1)\times30 \rightarrow$ $140 = 26v_b - 6v_c$ $\quad \times 49/2 \rightarrow$ & Subtract to elim v_c

$(2)\times120 \rightarrow$ $740 = -24v_b + 49v_c$ $\quad \times 3 \rightarrow$

$13\times49v_b - 3\times49v_c = 49\times70$

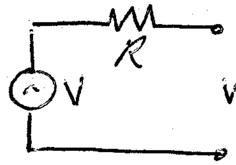
$3\times24v_b - 3\times49v_c = -3\times740$

$$v_b = \frac{3430 + 2220}{565} = 10v \quad \therefore I = 10v/2\Omega = 5A$$

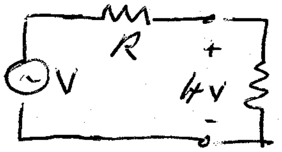
Conclusions: 1. All 3 methods give the same result

2. Mesh and nodal analyses are more efficient than the "brute force" element equations (and nodal is easier here because one begins with 2 unknowns instead of 3)

P2-22



$V_{oc} = 5v$

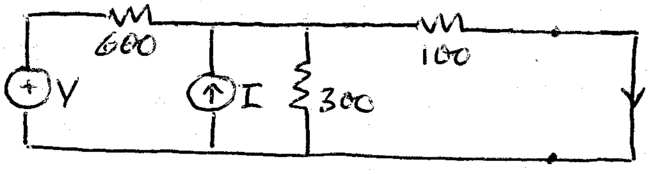


$V = V_{oc} = 5v$

$$4v = \frac{8000}{8000+R} V = \frac{8000 \times 5}{8000+R}$$

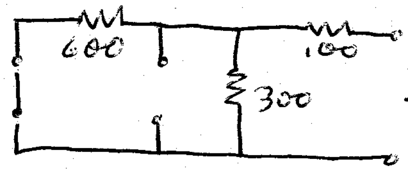
$$\therefore R = \frac{40{,}000 - 8000 \times 4}{4} = 2000\,\Omega$$

P2-26



$60\,mA = I_{sc}$

No dependent sources, so can find $R_{thevenin}$ (ie R_T) by zeroing sources

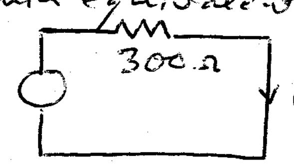


$$\leftarrow R_T = 100 + 300 \| 600 = 100 + \frac{300 \times 600}{300+600} = 100 + 200 = 300\,\Omega$$

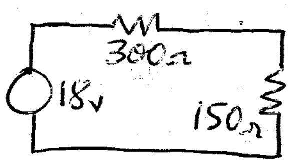
[OR find R_T from V_{oc}, but to do this requires finding some relationship between V, I (unknown) and $I_{sc} = 60mA$] (see below)

Thevenin equivalent:

V_T

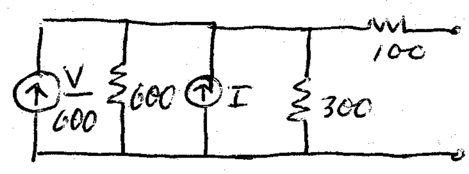


$60mA$ $\therefore V_T = 18v$



$$I = \frac{18v}{300+150\,\Omega} = \frac{18v}{0.45K\Omega} = 40mA$$

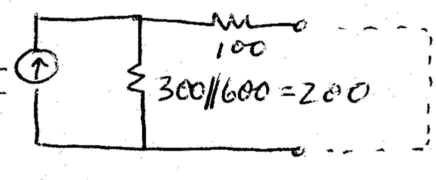
By $R_T = V_{oc}/I_{sc}$, convert source to



$$\therefore V_{oc} = \left(I + \frac{V}{600}\right) 200\,\Omega$$

$$I_{sc} = \left(I + \frac{V}{600}\right) \frac{200}{200+100} = 60mA$$

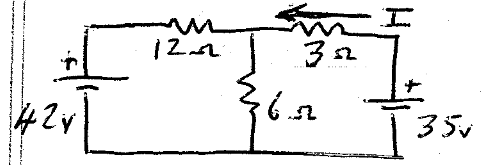
=



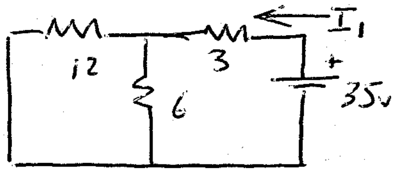
$$\therefore R_T = V_{oc}/I_{sc} = \frac{200}{200/300} = 300\,\Omega \text{ as before}$$

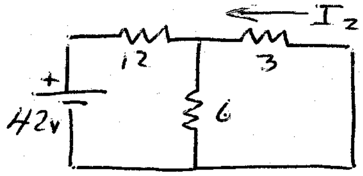
(Note also, can get $I + \frac{V}{600} = 60mA \times \frac{3}{2} = 90mA$)

2.33

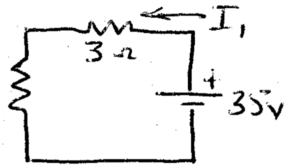


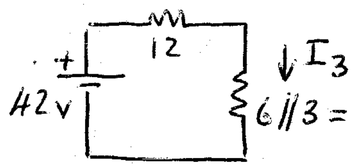
Find I by superposition





$6 \| 12 = \frac{6 \times 12}{18} = 4\Omega$

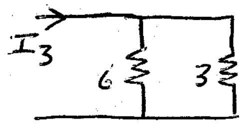




$6 \| 3 = \frac{6 \times 3}{6+3} = 2\Omega$

$$I_1 = \frac{35v}{3 + 4\Omega} = 5A$$

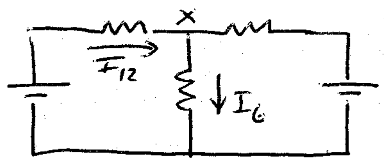
$$I_3 = 42/(12+2) = 3A$$



$$I_2 = -\frac{6}{3+6} I_3 = -\frac{2}{3}(3A) = -2A$$

$$\therefore I = I_1 + I_2 = 5 - 2 = 3A$$

(Check:

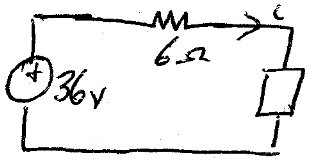


$V_x = 35 - 3\Omega \times 3A = 26v$

$\therefore I_6 = \frac{26}{6} = 4\tfrac{1}{3}A \quad I_{12} = \frac{42-26}{12} = 1\tfrac{1}{3}$

$\therefore I_6 = I + I_{12} = 3 + 1\tfrac{1}{3} = 4\tfrac{1}{3}A$ Check.)

P2.35



(a) $36 = 6i + (6i + 4i^2)$

$4i^2 + 12i - 36 = 0$

$i = \dfrac{-12 \pm \sqrt{12^2 + 4 \times 4 \times 36}}{8}$

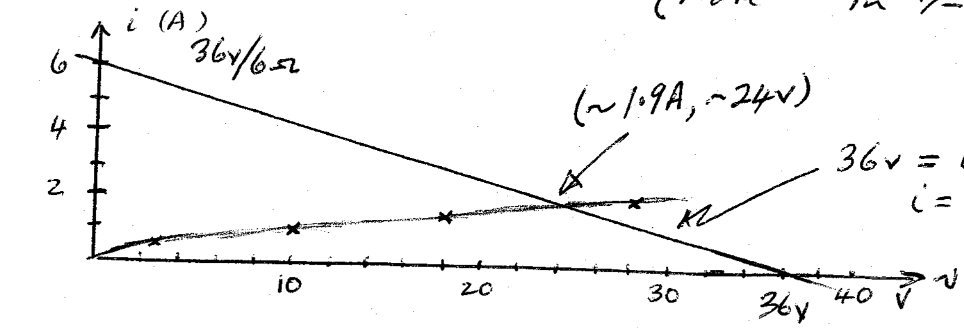
$= \dfrac{-12 \pm 12\sqrt{1+4}}{8}$

(Note – in +/– not possible)

$\therefore i = \frac{3}{2}(5^{1/2} - 1)$

$\approx \frac{3}{2}\, 1.24 = 1.92A$

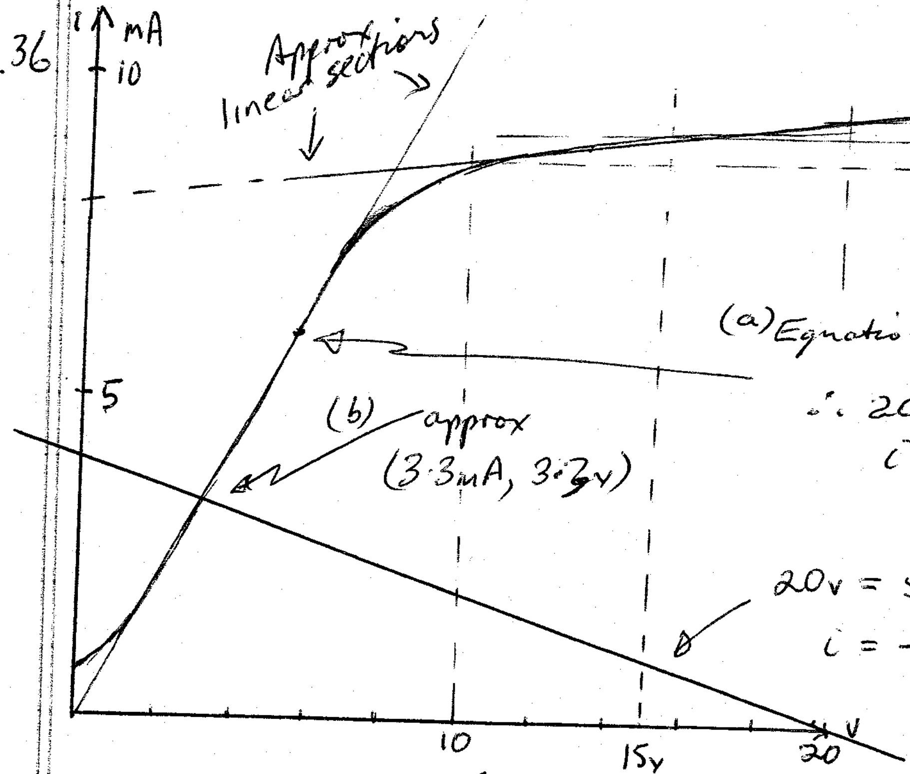
(b)

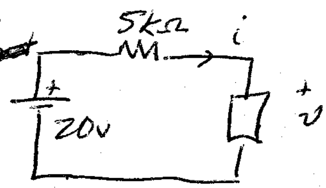


$36v = 6i + v$

$i = -\frac{v}{6} + 6$

P2.36





(a) Equation $i \approx \frac{6mA}{6V} v = 10^{-3} v$

$\therefore 20 = i\,(5000 + 1000)$

$i = 20v/6K = 3\frac{1}{3}mA$

$20v = 5k\,i + v$

$i = -\frac{v}{5K} + \frac{20v}{5K}$ ← 4mA

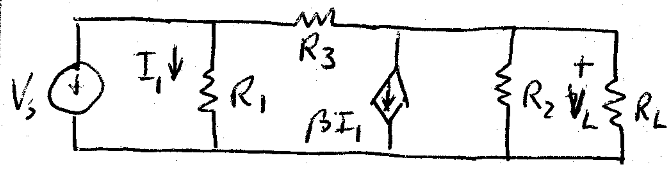
P2.37 (a) done

(b) use upper section $i = 8mA + \frac{1mA}{13.5_V} v$

$\therefore i = 8mA + \frac{15}{13.5} mA + \frac{5}{13.5} \cos\omega t \; mA$

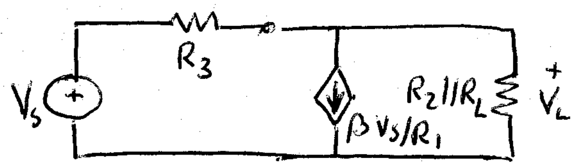
$\approx 9.1 mA + 0.37 mA \cos\omega t$

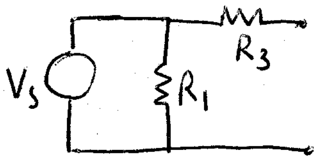
AP2.6



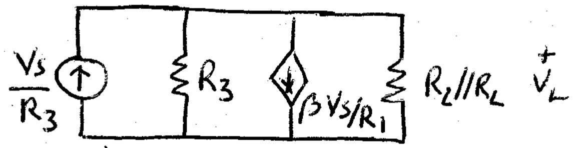
$R_1 = 10K \quad R_2 = 75K \quad R_3 = 150K$
$\beta = 100 \quad R_L = 20K$

(a) $I_1 = V_s/R_1$, simplify circuit





$V_{oc} = V_s$
$I_{sc} = V_s/R_3$



$$V_L = \left(-\beta \frac{V_s}{R_1} + \frac{V_s}{R_3}\right)\left(R_3 // (R_2 // R_L)\right)$$

$$V_L/V_s = \left(-\frac{\beta}{R_1} + \frac{1}{R_3}\right) \frac{R_3 R_2 R_L}{R_3 R_2 + R_2 R_L + R_L R_3}$$

$$= -\frac{\beta}{R_1} \frac{R_2 R_L \quad R_3}{(R_L + R_2) R_3 + R_2 R_L} + \frac{R_2 R_L}{R_3 (R_2 + R_L) + R_2 R_L}$$

$$= -\frac{\beta}{R_1} \frac{R_2 R_L}{R_2 + R_L + \left(\frac{R_2 R_L}{R_3}\right)} + \frac{1}{R_3 \frac{(R_2 + R_L)}{R_2 R_L} + 1}$$

$$= -\frac{\beta}{R_1} \frac{(R_2 // R_L)}{1 + \frac{(R_2 // R_L)}{R_3}} + \frac{1}{1 + R_3/(R_2 // R_L)}$$

$$\longrightarrow -\frac{\beta}{R_1}(R_2 // R_L) \quad \text{for } R_3 \gg (R_2 // R_L)$$

(b) $$V_L/V_s = -\frac{100}{10K} \frac{300K}{19} \frac{1}{1 + \frac{300}{19 \times 150}} + \frac{1}{1 + \frac{150}{300/19}}$$

$$\boxed{\begin{aligned} &R_2 // R_L \\ &= \frac{75 \times 20}{95} K. \\ &= 300/19 \ K \end{aligned}}$$

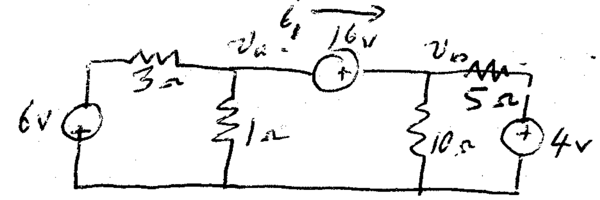
$$= -\frac{3000}{19} \cdot \frac{1}{1 + 2/19} + \frac{1}{1 + 19/2}$$

$$= -157.9/1.105 + 1/10.5 = -142.9 \quad + .095$$

$$\approx -142.8$$

Approximation ($R_3 \to \infty$) gives -157.9

AP2.10



(a) when try to apply nodal analysis at v_a, v_b, cannot write the usual current equations through the sources because no defined resistance.

(b) Hence:

$$\frac{-6-v_a}{3} - \frac{v_a}{1} - i_1 = 0$$

$$\frac{4-v_b}{5} - \frac{v_b}{10} + i_1 = 0$$

and (!) $\quad v_b - v_a = 16\text{v}$

Solve $-6 - 4v_a = 3i_1$

$8 - 3v_b = -10i_1$

$v_a = \dfrac{-6-3i_1}{4} \qquad v_b = \dfrac{8+10i_1}{3}$

So $v_b - v_a = 16 = \dfrac{+6+3i_1}{4} + \dfrac{8+10i_1}{3}$

$12 \times 16 = 32 + 40i_1 + 18 + 9i_1$

$49i_1 = 192 - 18 - 32 = 142$

$i_1 = 142/49 = 2\frac{44}{49}$

$\therefore v_a = -\frac{3}{2} - \dfrac{3 \times 142}{4 \times 49}$

$= -3\frac{33}{49}$

$v_b = \dfrac{8 + \frac{1420}{49}}{3} = 12\frac{16}{49}$

Check: At a $\quad \xleftarrow{33/49} \quad \xrightarrow{2\frac{44}{49}} \quad \uparrow 3\frac{33}{49}$

At b $\quad \xrightarrow{2\frac{44}{49}} \quad \xrightarrow{1\frac{31}{49}} \quad \downarrow \frac{18}{49}$