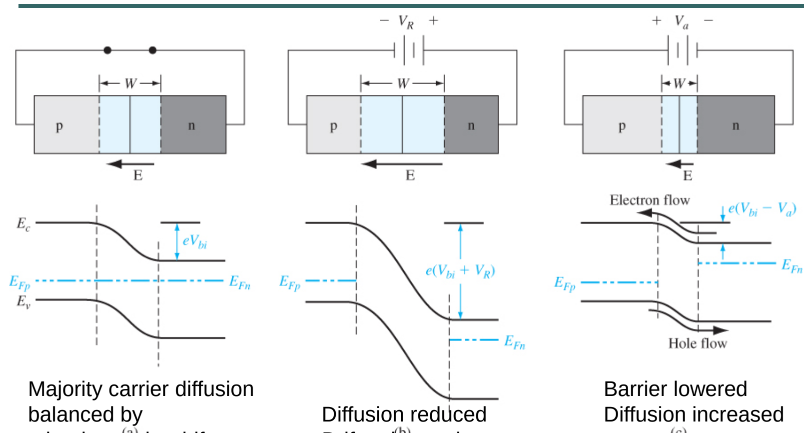
EE415/515 Fundamentals of Semiconductor Devices Fall 2012

Lecture 8: PN Junction Diode (Chapter 8)

Forward & reverse bias



Majority carrier diffusion balanced by minority carrier drift

Diffusion reduced
Drift enhanced

Barrier lowered
Diffusion increased

Figure 8.1 | A pn junction and its associated energy-band diagram for (a) zero bias, (b) reverse bias, and (c) forward bias.

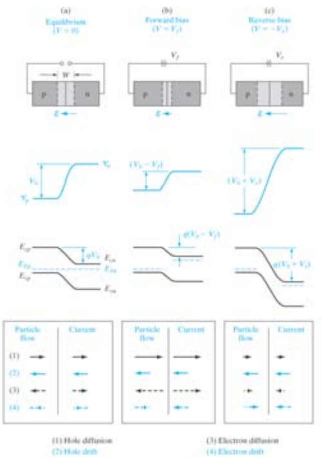


Figure 5.13

Effects of a bias at a p-n junction; transition region width and electric field, electrostatic potential, energy band diagram, and particle flow and current directions within W for (a) equilibrium, (b) forward bias, and (c) reverse bias.

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Terminology/notation

(Note: Complete ionization assumption)

Table 8.1 | Commonly used terms and notation for this chapter

Term	Meaning
N_a	Acceptor concentration in the p region of the pn junction
N_d	Donor concentration in the n region of the pn junction
$n_{n0} = N_d$	Thermal-equilibrium majority carrier electron concentration in the n region
$p_{p0} = N_a$	Thermal-equilibrium majority carrier hole concentration in the p region
$n_{p0} = n_i^2/N_a$	Thermal-equilibrium minority carrier electron concentration in the p region
$p_{n0} = n_i^2/N_d$	Thermal-equilibrium minority carrier hole concentration in the n region
n_p	Total minority carrier electron concentration in the p region
p_n	Total minority carrier hole concentration in the n region
$n_p(-x_p)$	Minority carrier electron concentration in the p region at the space charge edge
$p_n(x_n)$	Minority carrier hole concentration in the n region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the p region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the n region

Built-in potential, carrier densities

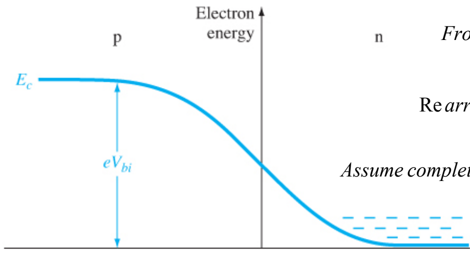

Figure 8.2 | Conduction-band energy through a pn junction.

$$\textit{From before } V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \textit{ where } V_t = kT/e$$

$$\text{Re}\textit{arrange } \frac{n_i^2}{N_a N_d} = \exp -\frac{eV_{bi}}{kT}$$

$$\textit{Assume complete ionization} \rightarrow n_{n0} \approx N_d \text{ so } p_{n0} \approx \frac{n_i^2}{\mathrm{N_d}}$$

$$\textit{and } p_{p0} \approx N_a \text{ so } n_{p0} \approx \frac{n_i^2}{\mathrm{N_a}}$$

$$\textit{Hence } n_{p0} = N_a N_d \exp -\frac{eV_{bi}}{kT} \cdot \frac{1}{N_a} = n_{n0} \exp -\frac{eV_{bi}}{kT}$$

$$\textit{and } p_{n0} = p_{p0} \exp -\frac{eV_{bi}}{kT}$$

Forward bias V_a: $V_{bi} \rightarrow V_{bi}-V_a$

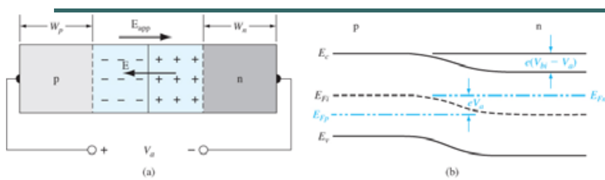

Figure 8.3 | (a) A pn junction with an applied forward-bias voltage showing the directions of the electric field induced by V_a and the space charge electric field. (b) Energy-band diagram of the forward-biased pn junction.

$$n_p = n_{n0} \exp -\frac{e(V_{bi} - V_a)}{kT}$$
$$= n_{n0} \exp -\frac{eV_{bi}}{kT} \exp +\frac{eV_a}{kT}$$
$$= n_{p0} \exp +\frac{eV_a}{kT}$$
$$n_p = n_{p0} \exp \frac{eV_a}{kT} \quad \textit{and} \quad p_{n0} = p_{p0} \exp \frac{eV_a}{kT}$$

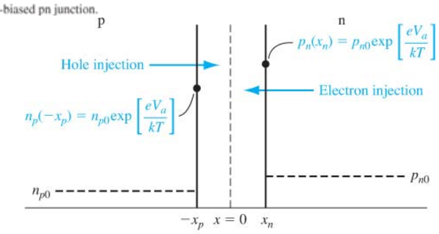

Figure 8.4 | Excess minority carrier concentrations at the space charge edges generated by the forward-bias voltage.

Ex 8.1 Si pn junction at 300 K, doped with N_d=2x10^{16}/cm^3 & N_a=5x10^{16}/cm^3, forward biased with V_a=0.65V. Determine minority carier concentrations at the space charge edges. Does low injection still apply?

$$n_{po} = \frac{n_i^2}{N_a} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^3\ \text{cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{16}} = 1.125\times10^4\ \text{cm}^{-3}$$

$$n_p\left(-x_p\right) = n_{po}\exp\left(\frac{V_a}{V_t}\right) = \left(4.5\times10^3\right)\exp\left(\frac{0.650}{0.0259}\right)$$

$$= 3.57\times10^{14}\ \text{cm}^{-3}$$

$$p_n\left(x_n\right) = p_{no}\exp\left(\frac{V_a}{V_t}\right) = \left(1.125\times10^4\right)\exp\left(\frac{0.650}{0.0259}\right)$$

$$= 8.92\times10^{14}\ \text{cm}^{-3}$$

We have that $n_p\left(-x_p\right) \ll N_a$ and $p_n\left(x_n\right) \ll N_d$ so low injection applies.

Minority carrier distribtion

Minority carrier transport equation : holes in N - type

$$D_p\frac{\partial^2(\delta p_n)}{\partial x^2} = -\mu_p E\frac{\partial(\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t}$$

where $\delta p_n = p_n - p_{n0}$

Assuming E = 0, $g' = 0$ outside the depletion region, for $x > x_n$

and steady state, so $\frac{\partial(\delta p_n)}{\partial t} = 0$,

then $D_p\frac{\partial^2(\delta p_n)}{\partial x^2} + \frac{\delta p_n}{\tau_{p0}} = 0$

$$\rightarrow \frac{\partial^2(\delta p_n)}{\partial x^2} + \frac{\delta p_n}{L_p^2} = 0 \text{ for } x > x_n, \text{ where } L_p^2 = D_p\tau_{p0}$$

$$\text{and } \frac{\partial^2(\delta n_p)}{\partial x^2} + \frac{\delta n_p}{L_n^2} = 0 \text{ for } x < -x_p, \text{ where } L_n^2 = D_n\tau_{n0}$$

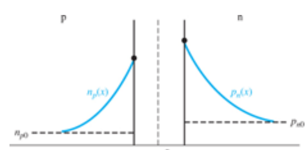

Figure 8.5 | Steady-state minority carrier concentrations in a pn junction under forward bias.

Minority carrier distribtion

General solution for $x \ge x_n$

$$\delta p_n(x) = p_n(x) - p_{n0} = A.\exp\frac{x}{L_p} + B.\exp -\frac{x}{L_p}$$

with boundary conditions

$$p_n(x_n) = p_{n0}\exp\frac{eV_a}{kT} \quad \text{and} \quad p_n(x \to +\infty) = p_{n0}$$

$$So\ A = 0,\ and\ \delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right].\exp -\frac{x - x_n}{L_p}$$

General solution for $x \le x_p$

$$\delta n_p(x) = n_p(x) - n_{p0} = C.\exp\frac{x}{L_n} + D.\exp -\frac{x}{L_n}$$

with boundary conditions

$$n_p(-x_p) = n_{p0}\exp\frac{eV_a}{kT} \quad \text{and} \quad n_p(x \to -\infty) = n_{p0}$$

$$So\ D = 0,\ and\ \delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right].\exp\frac{x + x_p}{L_n}$$

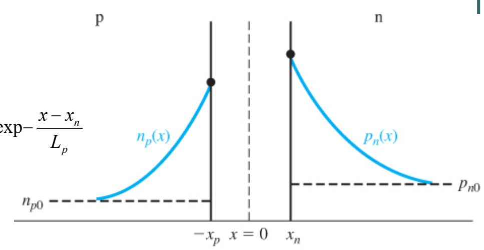

Figure 8.5 | Steady-state minority carrier concentrations in a pn junction under forward bias.

Fwd bias PN: Quasi-Fermi levels

Excess carriers in non - equilibrium

$$p = p_0 + \delta p = n_i\exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

$$n = n_0 + \delta n = n_i\exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

n, p exponential functions of x and of E_{Fp}, E_{Fn}

Hence E_{Fp}, E_{Fn} linear functions of x

Note that at $x \ge x_n$ $E_{Fp} < E_{Fi}$ so $\delta p > n_i$

and for $x >> x_n$ $E_{Fp} > E_{Fi}$ so $\delta p < n_i$

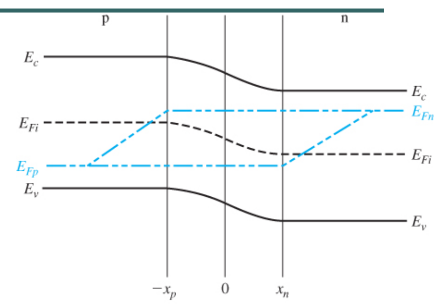

Figure 8.6 | Quasi-Fermi levels through a forward-biased pn junction.

$$np = n_i^2\exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)\exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right) = n_i^2\exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

$$\text{At } x = x_n \quad n_0 p_n(x_n) = n_0 p_{n0}\exp\left(\frac{V_a}{V_t}\right) = n_i^2\exp\left(\frac{V_a}{V_t}\right)$$

PN junction current

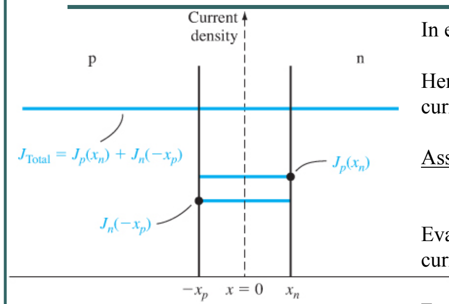

Figure 8.7 | Electron and hole current densities through the space charge region of a pn junction.

In evaluating the junction current:

Total current must be constant $-\infty < x < +\infty$

Hence can evaluate as the sum of electron and hole currents across the depletion region .

<u>Assumption:</u>

Assume currents constant across depletion region

Evaluate electron and hole currents as minority carrier currents at the space charge region edges.

Forward bias:

Majority carriers injected into depletion region

Diffuse across, become minority carriers

Evaluate minority carrier currents at edges

e.g. majority holes in p-type diffuse across the space charge region and become minority holes in n-type.

Evaluate $J_p(x_n) = -eD_p(dp_n(x)/dx)$ at $x = x_n$

PN junction current

$$J_p(x_n) = -eD_p \left.\frac{dp_n(x)}{dx}\right|_{x=x_n} \quad \& \quad J_n(-x_p) = eD_n \left.\frac{dn_p(x)}{dx}\right|_{x=-x_p}$$

$$\text{for } p_n(x) = p_{n0} + p_{n0}\left[\exp\frac{eV_a}{kT} - 1\right]\exp -\frac{x - x_n}{L_p}$$

$$\& \quad n_p(x) = n_{p0} + n_{p0}\left[\exp\frac{eV_a}{kT} - 1\right]\exp\frac{x - (-x_p)}{L_n}$$

$$\text{gives } J_p(x_n) = -eD_p p_{n0}\left[\exp\frac{eV_a}{kT} - 1\right]\exp -\frac{x_n - x_n}{L_p}\cdot\left[-\frac{1}{L_p}\right] = eD_p\frac{p_{n0}}{L_p}\left[\exp\frac{eV_a}{kT} - 1\right]$$

$$\& \quad J_n(-x_p) = eD_n n_{p0}\left[\exp\frac{eV_a}{kT} - 1\right]\exp\frac{-x_p - (-x_p)}{L_n}\cdot\left[\frac{1}{L_n}\right] = eD_n\frac{n_{p0}}{L_n}\left[\exp\frac{eV_a}{kT} - 1\right]$$

$$\therefore \; J = J_p(x_n) + J_n(-x_p) = e\left(D_p\frac{p_{n0}}{L_p} + D_n\frac{n_{p0}}{L_n}\right)\left[\exp\frac{eV_a}{kT} - 1\right] = J_s\left[\exp\frac{eV_a}{kT} - 1\right]$$

Diode characteristic

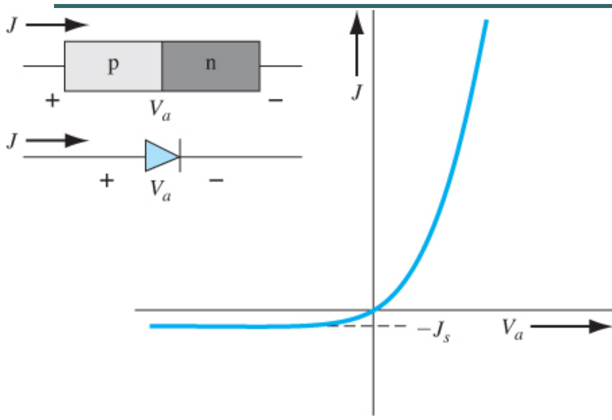

Figure 8.8 | Ideal *I–V* characteristic of a pn junction diode.

Ex 8.2 Find the ideal reverse saturation current density for a GaAs pn junction at 300 K with N_d=2x10^{16}/cm^3, N_a=8x10^{15}/cm^3, D_n=210cm^2/s, τ_{n0}=10^{-7}s, & τ_{p0}=5x10^{-8}s.

$$J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right]$$

$$=\left(1.6\times10^{-19}\right)\left(1.8\times10^{6}\right)^2$$

$$\times\left[\frac{1}{8\times10^{15}}\sqrt{\frac{210}{10^{-7}}}+\frac{1}{2\times10^{16}}\sqrt{\frac{8}{5\times10^{-8}}}\right]$$

$$J_s = 3.30\times10^{-18}\ \text{A/cm}^2$$

Forward bias approximation

$J = J_s(\exp[eV_a/kT]-1)$

$\approx J_s \exp[eV_a/kT]$ for $eV_a >> kT$

$\ln(J) = eV_a/kT$

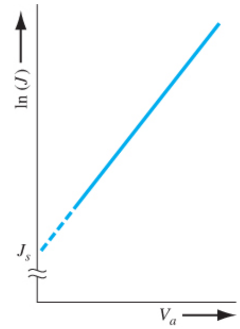

Figure 8.9 | Ideal I–V characteristic of a pn junction diode with the current plotted on a log scale.

Ex 8.3 **Determine the electron and hole current densities at the space charge region edges, and determine the total current density in the diode for a GaAs pn junction at 300 K and forward bias 1.05V with $N_d=2x10^{16}/cm^3$, $N_a=8x10^{15}/cm^3$, $D_n=210cm^2/s$, $\tau_{n0}=10^{-7}s$, & $\tau_{p0}=5x10^{-8}s$.**

We find $L_n = \sqrt{D_n \tau_{no}} = \sqrt{(210)(10^{-7})} = 4.583\times10^{-3}$ cm

and $L_p = \sqrt{D_p \tau_{po}} = \sqrt{(8)(5\times10^{-8})} = 6.325\times10^{-4}$ cm

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.8\times10^6)^2}{8\times10^{15}} = 4.05\times10^{-4}\ \text{cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.8\times10^6)^2}{2\times10^{16}} = 1.62\times10^{-4}\ \text{cm}^{-3}$$

$$J_n(-x_p) = \frac{eD_n n_{po}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right] = \frac{(1.6\times10^{-19})(210)(4.05\times10^{-4})}{4.583\times10^{-3}}\times\left[\exp\left(\frac{1.05}{0.0259}\right)-1\right]$$

$J_n(-x_p) = 1.20\ \text{A/cm}^2$

$$J_p(x_n) = \frac{eD_p p_{no}}{L_p}\left[\exp\left[\frac{eV_a}{kT}\right]-1\right] = \frac{(1.6\times10^{-19})(8)(1.62\times10^{-4})}{6.325\times10^{-4}}\times\left[\exp\left(\frac{1.05}{0.0259}\right)-1\right]$$

$J_p(x_n) = 0.1325\ \text{A/cm}^2$

The total current density is: $J_T = J_n(-x_p) + J_p(x_n) = 1.20 + 0.1325$

$J_T = 1.33\ \text{A/cm}^2$

Complete currents

$$J_p(x_n) = -eD_p \left.\frac{dp_n(x)}{dx}\right|_{x=x_n} \quad \text{for } p_n(x) = p_{n0} + p_{n0}\left[\exp\frac{eV_a}{kT} - 1\right]\exp - \frac{x - x_n}{L_p}$$

$$\text{gives } J_p(x_n) = eD_p \frac{p_{n0}}{L_p}\left[\exp\frac{eV_a}{kT} - 1\right]\exp - \frac{x - x_n}{L_p} \quad \text{for } x > x_n$$

$$\& \quad J_n(-x_p) = eD_n \frac{n_{p0}}{L_n}\left[\exp\frac{eV_a}{kT} - 1\right]\exp\frac{x - (-x_p)}{L_n} \quad \text{for } x < -x_p$$

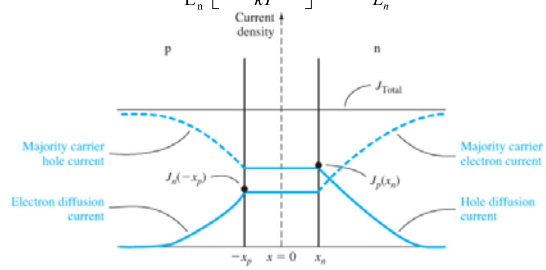

Figure 8.10 | Ideal electron and hole current components through a pn junction under forward bias.

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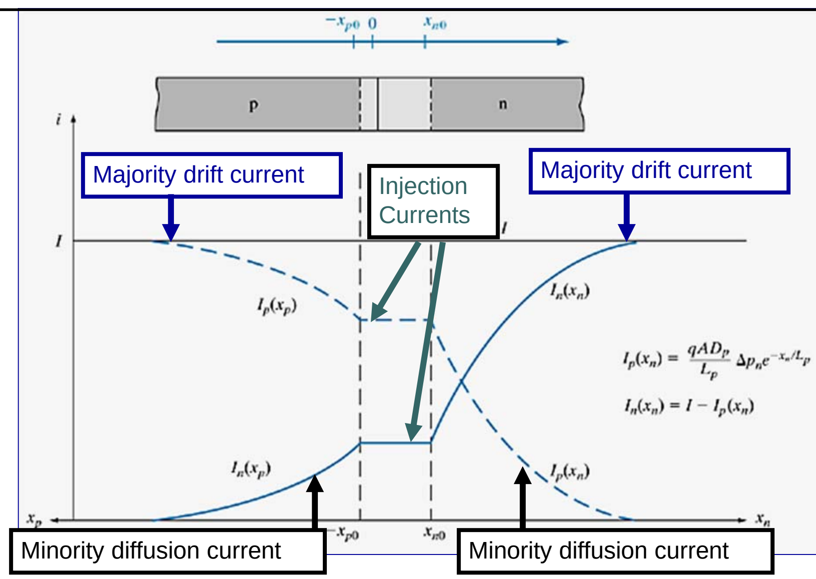

Figure 5—17[1] Electron and hole components of current in a forward-biased p-n junction. Why is there a difference between the two currents?

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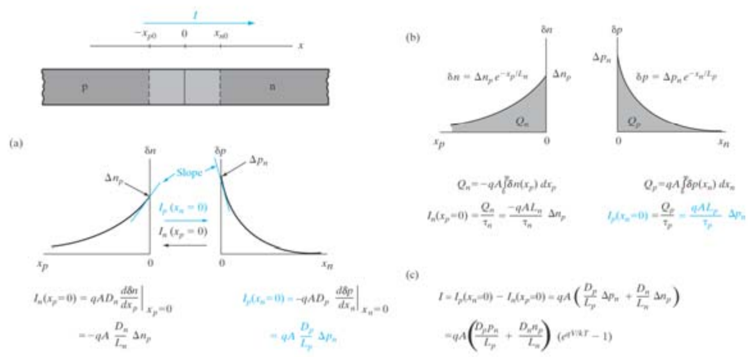

Figure 5.16

Two methods for calculating junction current from the excess minority carrier distributions: (a) diffusion currents at the edges of the transition region; (b) charge in the distributions divided by the minority carrier lifetimes; (c) the diode equation.

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Ex 8.4 Determine the electric field in the neutral n region and neutral p region for the GaAs pn junction at 300 K and forward bias 1.05V with N_d=2x10^{16}/cm^3, N_a=8x10^{15}/cm^3, D_n=210cm^2/s, τ_{n0}=10^{-7}s, & τ_{p0}=5x10^{-8}s.

In the n-region, for $N_d = 2\times10^{16}\ \text{cm}^{-3}$, $\mu_n \cong 6000\ \text{cm}^2/\text{V-s}$

$$J = e\mu_n N_d \mathrm{E}_n$$

or $$\mathrm{E}_n = \frac{J}{e\mu_n N_d} = \frac{1.3325}{(1.6\times10^{-19})(6000)(2\times10^{16})} = 0.0694\ \text{V/cm}$$

In the p-region, for $N_a = 8\times10^{15}\ \text{cm}^{-3}$, $\mu_p \cong 320\ \text{cm}^2/\text{V-cm}$

$$J = e\mu_p N_a \mathrm{E}_p$$

or $$\mathrm{E}_p = \frac{J}{e\mu_p N_a} = \frac{1.3325}{(1.6\times10^{-19})(320)|8\times10^{15}|} = 3.25\ \text{V/cm}$$

Ex 8.5 Consider a GaAs pn junction initially biased at V_a=1.05V at T=300 K, and calculate the change in bias voltage required to maintain the same current when T increases to 310 K.

From Example 8.5, we have

$$\frac{E_g - eV_{a2}}{kT_2} = \frac{E_g - eV_{a1}}{kT_1}$$

Let $T_2 = 310\text{ K},\ T_1 = 300\text{ K},\ E_g = 1.42\text{ eV},$

and $V_{a1} = 1.050\text{ V}.$

Then

$$\frac{1.42 - V_{a2}}{310} = \frac{1.42 - 1.050}{300}$$

which yields $V_{a2} = 1.0377\text{ V}$

so $\Delta V = 1.0377 - 1.050 = -0.0123\text{ V}$

or $\Delta V = -12.3\text{ mV}$ per $10°$ C increase in T.

Short diode (Implicit assumption above that x→ ± ∞)

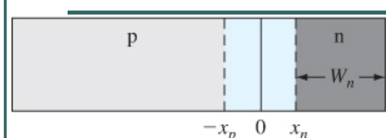

Figure 8.11 | Geometry of a "short" diode.

Assume "long" p-region and "short" n-type

As before, for n - region $x_n \le x \le x_n + W_n$

$$\delta p_n(x) = p_n(x) - p_{n0} = A.\exp\frac{x}{L_p} + B.\exp-\frac{x}{L_p}$$

but with boundary conditions

$$p_n(x_n) = p_{n0}\exp\frac{eV_a}{kT} \text{ (as before) and } p_n(x_n + W_n) = p_{n0}$$

$$\textit{giving } \delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\frac{\sinh\frac{x_n + W_n - x}{L_p}}{\sinh\frac{W_n}{L_p}}$$

$$\text{(Check : } \delta p_n(x_n) = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \ \& \ \delta p_n(x_n + W_n) = 0\text{)}$$

Short diode

$$For\ \delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]\frac{\sinh\frac{x_n+W_n-x}{L_p}}{\sinh\frac{W_n}{L_p}}$$

$$and\ \ J_p = -eD_p\frac{d\left(\delta p_n(x)\right)}{dx} = e\frac{D_p}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]\frac{\cosh\frac{x_n+W_n-x}{L_p}}{\sinh\frac{W_n}{L_p}}$$

$$\text{Now if } W_n << L_p\,,\ \sinh\frac{W_n}{L_p} \approx \frac{W_n}{L_p}\,,\ \sinh\frac{x_n+W_n-x}{L_p} \approx \frac{x_n+W_n-x}{L_p}\,\text{, so } \delta p_n(x) \approx p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]\frac{x_n+W_n-x}{W_n}$$

$$\text{and } \cosh\frac{x_n+W_n-x}{L_p} \approx 1,\ \text{so } J_p \approx e\frac{D_p}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]\left(\frac{L_p}{W_n}\right) = e\frac{D_p}{W_n}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]$$

$$and\ \ J = J_p + J_n = e\left(\frac{D_p}{W_n}+\frac{D_n}{L_n}\right)\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]$$

Note that $W_n << L_p$ implies no hole recombination in the N - region, and that $J_p(x)$ is constsnt

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Generation & Recombination Currents:
Reverse bias: Space charge region EHP generation

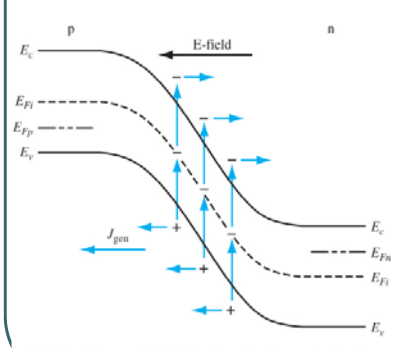

Figure 8.12 | Generation process in a reverse-biased pn junction.

$$\text{Recombination rate R} = \frac{C_nC_pN_t(np-n_i^2)}{C_n(n+n')+C_p(p+p')}$$

$$\text{i.e. generation } G \approx \frac{C_nC_pN_tn_i^2}{C_nn'+C_pp'} \text{ for n = p} \approx 0 \text{ in the space charge region}$$

$$\text{Write } G = \frac{n_i}{\tau_{p0}+\tau_{n0}} = \frac{n_i}{2\tau_0}\,,\ \text{where } \tau_{p0}+\tau_{n0} = \frac{1}{N_tC_p}+\frac{1}{N_tC_n}\,,$$

$$\text{assuming } n' = p' \approx n_i \text{ if traps at } E_{Fi}$$

$$\text{Then } J_{gen} = \int_0^W eGdx = \frac{en_iW}{2\tau_0} \approx \frac{n_i}{\tau_0}\sqrt{\frac{e}{2}\left[\frac{1}{N_a}+\frac{1}{N_d}\right]\varepsilon_sV_R}\,,$$

and reverse current $J_R = J_s + J_{gen}$ does not saturate with reverse bias

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Ex 8.6 Consider a GaAs pn junction at 300 K with N_d=8x10^{16}/cm^3, N_a=2x10^{15}/cm^3, D_p=9.80cm^2/s, $\tau_0=\tau_{n0}=\tau_{p0}$=5x10^{-8}s. (a) Calculate the ideal reverse bias saturation current density (b) Find the reverse bias current density at V_R=5V (c) Determine the ratio of J_{gen} to J_s

(a) $J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right] = (1.6\times10^{-19})(1.8\times10^6)^2\times\left[\frac{1}{2\times10^{15}}\sqrt{\frac{207}{5\times10^{-8}}}+\frac{1}{8\times10^{16}}\sqrt{\frac{9.8}{5\times10^{-8}}}\right]$

or $J_s = 1.677\times10^{-17}$ A/cm^2

(b) $V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(8\times10^{16})}{(1.8\times10^6)^2}\right] = 1.174$ V

$$W = \left\{\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right\}^{1/2} = \left\{\frac{2(13.1)(8.85\times10^{-14})(1.174+5)}{1.6\times10^{-19}}\times\left[\frac{2\times10^{15}+8\times10^{16}}{(2\times10^{15})(8\times10^{16})}\right]\right\}^{1/2}$$

or $W = 2.141\times10^{-4}$ cm

$$J_{gen} = \frac{en_iW}{2\tau_0} = \frac{(1.6\times10^{-19})(1.8\times10^6)(2.141\times10^{-4})}{2(5\times10^{-8})},\ \text{i.e. } J_{gen} = 6.166\times10^{-10}\ \text{A/cm}^2$$

(c) $\frac{J_{gen}}{J_s} = \frac{6.166\times10^{-10}}{1.677\times10^{-17}} = 3.68\times10^7$

Generation & Recombination Currents:

Forward bias: Space charge region recombination

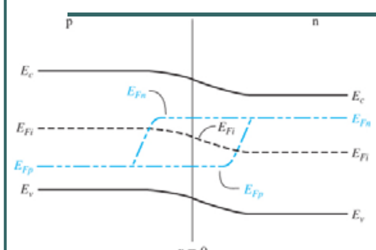

Figure 8.13 | Energy-band diagram of a forward-biased pn junction including quasi-Fermi levels.

$(E_{Fn}-E_{Fi})+(E_{Fi}-E_{Fp}) = eV_a$

and at the center of the space charge region where maximum recombination occurs

$E_{Fn}-E_{Fi} = E_{Fi}-E_{Fp} = \frac{1}{2}eV_a$

For recombination rate $R = \frac{C_nC_pN_t(np-n_i^2)}{C_n(n+n')+C_p(p+p')}$ again,

and $n = n_i\exp\frac{E_{Fn}-E_{Fi}}{kT}$ & $p = n_i\exp\frac{E_{Fi}-E_{Fp}}{kT}$

we find that R peaks at the middle of the space charge region, as shown

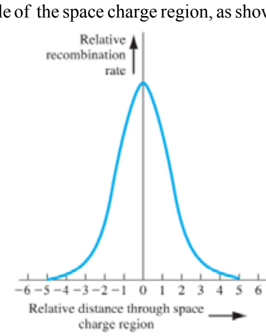

Figure 8.14 | Relative magnitude of the recombination rate through the space charge region of a forward-biased pn junction.

Recombination current

$$\text{Recombination rate R} = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n+n') + C_p(p+p')} = \frac{np - n_i^2}{\tau_{p0}(n+n') + \tau_{n0}(p+p')}$$

$$= \frac{np - n_i^2}{2\tau_0[(n+n_i)+(p+n_i)]}$$ if traps at E_{Fi} again, and at the center of the space charge region

Using $n = n_i \exp\frac{E_{Fn} - E_{Fi}}{kT} = n_i \exp\frac{eV_a}{2kT}$ & $p = n_i \exp\frac{E_{Fi} - E_{Fp}}{kT} n_i \exp\frac{eV_a}{2kT}$

$$R_{\max} = \frac{1}{2\tau_0} \cdot \frac{n_i^2\left[\exp\frac{eV_a}{kT} - 1\right]}{n_i\left[\exp\frac{eV_a}{2kT} + 1\right]} = \frac{n_i}{2\tau_0} \cdot \frac{\left[\exp\frac{eV_a}{kT} - 1\right]}{\left[\exp\frac{eV_a}{2kT} + 1\right]} \rightarrow \frac{n_i}{2\tau_0}\exp\frac{eV_a}{2kT} \text{ if } V_a >> kT/e$$

Then $J_{rec} = \int_0^W eR\,dx = \frac{eWn_i}{2\tau_0}\exp\frac{eV_a}{2kT} \rightarrow J_{r0}\exp\frac{eV_a}{2kT}$

Recombination adds current

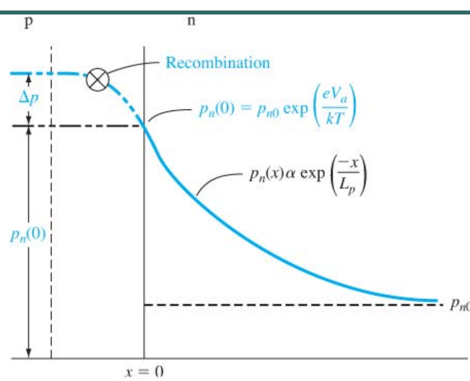

Figure 8.15 | Because of recombination, additional holes from the p region must be injected into the space charge region to establish the minority carrier hole concentration in the n region.

Total forward current

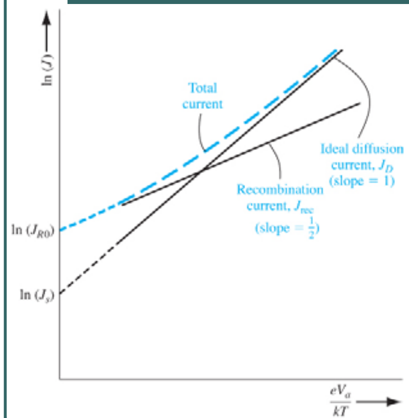

Figure 8.16 | Ideal diffusion, recombination, and total current in a forward-biased pn junction.

$$J = J_{rec} + J_D = J_{ro}\exp\frac{eV_a}{2kT} + J_s\exp\frac{eV_a}{kT}$$

$$\rightarrow I = I_s\left[\exp\frac{eV_a}{nkT} - 1\right]$$

LowV_a $\rightarrow$ diffusion dominant, n $\rightarrow$ 1

LowV_a $\rightarrow$ recombination dominant, n $\rightarrow$ 2

$$\ln J_{rec} = \ln J_{ro} + \frac{eV_a}{2kT} \quad \text{and} \quad \ln J_D = \ln J_s + \frac{eV_a}{kT}$$

so discontinuity in lnI vs V_a

High level injection

At high forward V_a low injection approximation

(minority δn_p << majority p_0

or minority δp_n << majority n_0)

$$np = n_i^2\exp\frac{eV_a}{kT},$$

and $n = n_0 + \delta n$, $p = p_0 + \delta p$

$$\text{so } (n_0 + \delta n)(p_0 + \delta p) = n_i^2\exp\frac{eV_a}{kT}$$

$\approx \delta n.\delta p$ if $\delta n > n_0$, $\delta p > p_0$

Hence, since $\delta n. = \delta p$,

$$\delta n. = \delta p = n_i\exp\frac{eV_a}{2kT}, \text{ and}$$

High injection current $\propto$ excess current :- $I \propto \exp\frac{eV_a}{2kT}$

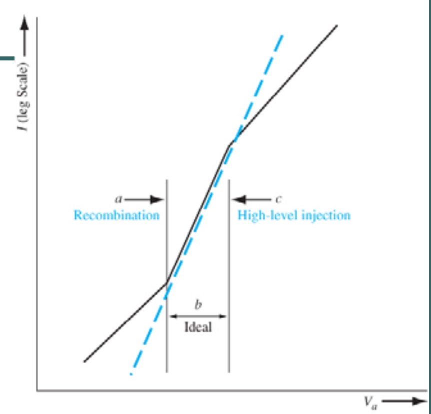

Figure 8.17 | Forward-bias current versus voltage from low forward bias to high forward bias.

Small signal: Diffusion resistance

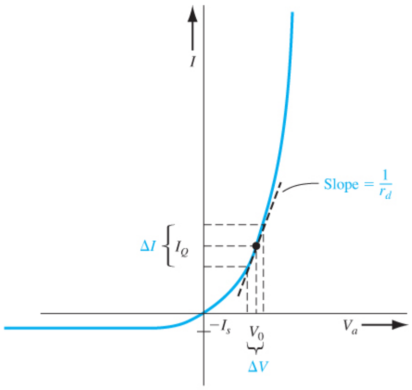

Figure 8.18 | Curve showing the concept of the small-signal diffusion resistance.

$$I_D = I_s\left[\exp\frac{eV_a}{kT} - 1\right]$$

$$g_D = \left.\frac{dI_D}{dV_a}\right|_{V_a=V_D} = I_s\frac{e}{kT}\exp\frac{eV_D}{kT}$$

$$= \frac{e}{kT}\left(I_D + I_s\right)$$

$$\approx \frac{I_{DQ}}{kT/e}$$

$$r_d = \frac{kT/e}{I_{DQ}}$$

Diffusion capacitance

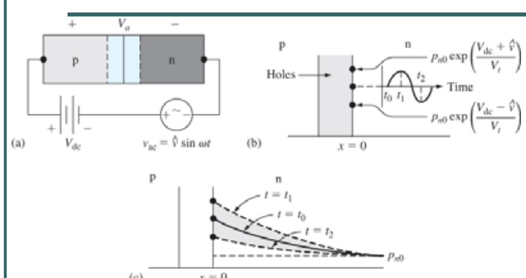

Figure 8.19 | (a) A pn junction with an ac voltage superimposed on a forward-biased dc value; (b) the hole concentration versus time at the space charge edge; (c) the hole concentration versus distance in the n region at three different times.

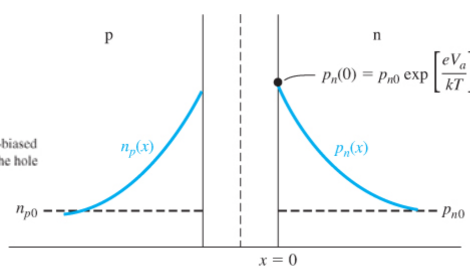

Figure 8.20 | The dc characteristics of a forward-biased pn junction used in the small-signal admittance calculations.

Incremental charge changes ΔQ

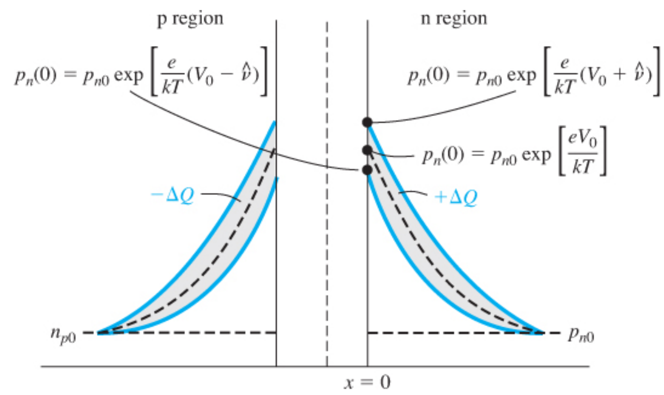

Figure 8.21 | Minority carrier concentration changes with changing forward-bias voltage.

Diffusion capacitance

If $V_a = V_0 + v_1(t)$ then $p_n(x=0,t) = p_{n0}\exp\frac{e[V_0 + v_1(t)]}{\text{kT}}$

i.e. $p_n(0,t) = p_{dc}\exp\frac{ev_1(t)}{\text{kT}}$, and for $v_1(t) << kT/e$, then $p_n(0,t) \approx p_{dc}\left[1+\frac{v_1(t)}{kT}\right] = p_{dc}\left[1+\frac{\hat{V}_1}{kT}\exp.j\omega t\right]$

For n - region (x > 0), $\delta p_n(x,t) = \delta p_0(x) + p_1(x)\exp.j\omega t$

Substituting in $D_p\frac{\partial^2 \delta p_n}{\partial x^2} - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial \delta p_n}{\partial t}$ gives $D_p\left[\frac{\partial^2 \delta p_0(x)}{\partial x^2} + \frac{\partial^2 p_1(x)}{\partial x^2}e^{j\omega t}\right] - \frac{\delta p_0(x) + p_1(x)e^{j\omega t}}{\tau_{p0}} = j\omega p_1(x)e^{j\omega t}$

and clustering time independent and time dependent terms gives $D_p\frac{\partial^2 p_1(x)}{\partial x^2} - \frac{p_1(x)}{\tau_{p0}} - j\omega p_1(x) = 0$

i.e. $\frac{\partial^2 p_1(x)}{\partial x^2} - C_p^{\ 2} p_1(x) = 0$ where $L_p^{\ 2} = D_p\tau_{p0}$ and $C_p^{\ 2} = \frac{(1 + j\omega\tau_{p0})}{L_p^{\ 2}}$,

with general solution $p_1(x) = K_1 e^{-C_p x} + K_2 e^{+C_p x}$

Diffusion capacitance

For general solution $p_1(x) = K_1 e^{-C_p x} + K_2 e^{+C_p x}$

Boundary conditions require $p_1(x=\infty) = 0, K_2 = 0$, and $p_1(0) = K_1 = p_{dc}\frac{\hat{V}_1}{kT}$,

so $p_1(x) = p_{dc}\frac{\hat{V}_1}{kT}\exp - \frac{\sqrt{1+j\omega\tau_{p0}}}{L_p}x$

$j_p(t) = \hat{J}_p e^{j\omega t} = -eD_p \frac{\partial p_1(x)}{\partial x} e^{j\omega t}\Big|_{x=0}$ and hence

$$\hat{I}_p = \frac{eAD_p p_{n0}}{L_p}\exp\frac{eV_0}{kT}\sqrt{1+j\omega\tau_{p0}}\frac{\hat{V}_1}{kT}, \text{ and } \hat{I}_n = \frac{eAD_n n_{p0}}{L_n}\exp\frac{eV_0}{kT}\sqrt{1+j\omega\tau_{n0}}\frac{\hat{V}_1}{kT}$$

$$\therefore Y = \frac{\hat{I}_p + \hat{I}_n}{\hat{V}_1} = \left[I_{p0}\sqrt{1+j\omega\tau_{p0}} + I_{n0}\sqrt{1+j\omega\tau_{n0}}\right]/kT,$$

and for low frequency $\omega\tau_{p0}$, $\omega\tau_{n0} << 1$, so $\sqrt{1+j\omega\tau_0} \approx 1 + j\omega\tau_0/2$

$$Y = \left[I_{p0}(1+j\omega\tau_{p0}/2) + I_{n0}(1+j\omega\tau_{n0}/2)\right]/kT = \frac{I_{p0}+I_{n0}}{kT} + j\omega\frac{I_{p0}\tau_{p0}+I_{n0}\tau_{n0}}{2kT}$$

$$= g_d + j\omega C_d, \text{ so } C_d = \frac{I_{p0}\tau_{p0}+I_{n0}\tau_{n0}}{2kT}$$

Ex 8.7 Si pn junction at 300 K with N_d=8x10^{16}/cm^3, N_a=2x10^{15}/cm^3, D_n=25cm^2/s, D_p=10cm^2/s, τ_{n0}=5x10^{-7}s, τ_{p0}=10^{-7}s, and c/s area A=10^{-3}cm^2. Find the diffusion resistance and diffusion capacitance if the diode is forward biased at (a) V_a=0.550V and (b) V_a=0.610V.

$$I_{Sn} = A\cdot\frac{en_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} = (10^{-3})\cdot\frac{(1.6\times10^{-19})(1.5\times10^{10})^2}{2\times10^{15}}\sqrt{\frac{25}{5\times10^{-7}}} \text{ or } I_{Sn} = 1.273\times10^{-13}\text{ A}$$

$$I_{Sp} = A\cdot\frac{en_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{po}}} = (10^{-3})\cdot\frac{(1.6\times10^{-19})(1.5\times10^{10})^2}{8\times10^{16}}\sqrt{\frac{10}{10^{-7}}} \text{ or } I_{Sp} = 4.5\times10^{-15}\text{ A}$$

Then $I_S = I_{Sn} + I_{Sp} = 1.318\times10^{-13}$ A

(a) $I_D = I_S\exp\left(\frac{V_a}{V_t}\right) = (1.318\times10^{-13})\exp\left(\frac{0.550}{0.0259}\right) = 2.2\times10^{-4}$ A, $r_d = \frac{V_t}{I_D} = \frac{0.0259}{2.2\times10^{-4}} = 118\,\Omega$

(b) $I_D = (1.318\times10^{-13})\exp\left(\frac{0.610}{0.0259}\right) = 2.23\times10^{-3}$ A, and $r_d = \frac{0.0259}{2.23\times10^{-3}} = 11.6\,\Omega$

We find $I_{pO} = I_{Sp}\exp\left(\frac{V_a}{V_t}\right)$, $I_{nO} = I_{Sn}\exp\left(\frac{V_a}{V_t}\right)$, and then

(a) $I_{pO} = 7.511\times10^{-6}$ A; $I_{nO} = 2.125\times10^{-4}$ A (b) $I_{pO} = 7.617\times10^{-5}$ A; $I_{nO} = 2.155\times10^{-3}$ A

We find $C_d = \left(\frac{1}{2V_t}\right)\left[I_{pO}\tau_{pO} + I_{nO}\tau_{nO}\right]$, so

(a) $C_d = \frac{1}{2(0.0259)}\left[(7.511\times10^{-6})(10^{-7}) + (2.125\times10^{-4})(5\times10^{-7})\right]$ or $C_d = 2.07\times10^{-9}\text{ F} = 2.07\text{ nF}$

(b) $C_d = \frac{1}{2(0.0259)}\left[(7.617\times10^{-5})(10^{-7}) + (2.155\times10^{-3})(5\times10^{-7})\right]$ or $C_d = 2.09\times10^{-8}\text{ F} = 20.9\text{ nF}$

Equivalent circuit (note r_s effect)

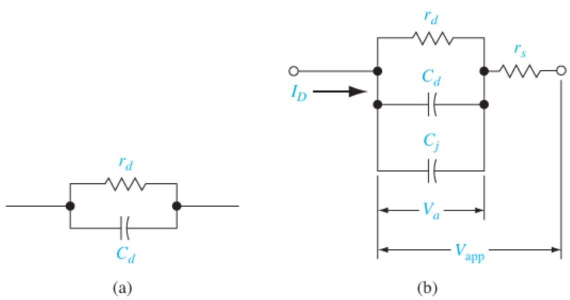

Figure 8.22 | (a) Small-signal equivalent circuit of ideal forward-biased pn junction diode; (b) complete small-signal equivalent circuit of pn junction.

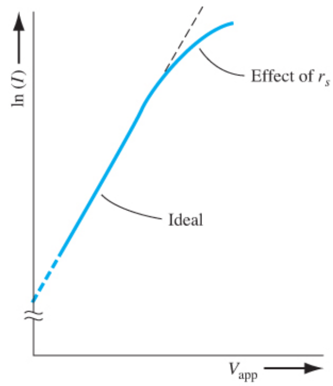

Figure 8.23 | Forward-biased I–V characteristics of a pn junction diode showing the effect of series resistance.

Overall DC characteristic

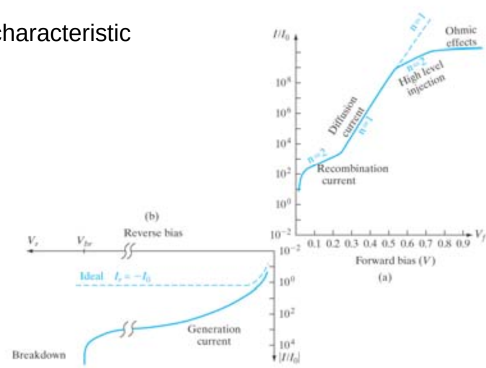

Figure 5.37

Forward and reverse current-voltage characteristics plotted on semilog scales, with current normalized with respect to saturation current, I_0; (a) the ideal forward characteristic is an exponential with an ideality factor **n** = 1 (dashed straight line on log-linear plot). The actual forward characteristics of a typical diode (solid line) have four regimes of operation; (b) ideal reverse characteristic (dashed line) is a voltage-independent current = $-I_0$. Actual leakage characteristics (solid line) are higher due to generation in the depletion region, and also show breakdown at high voltages.

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Switching transient: On → off

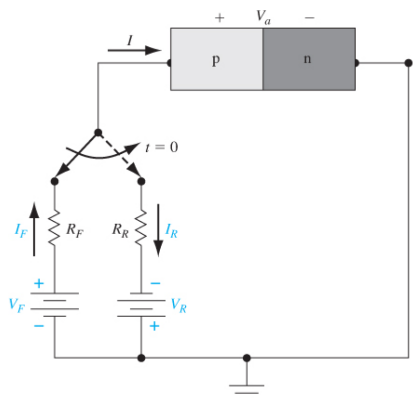


Figure 8.24 | Simple circuit for switching a diode from forward to reverse bias.

Initially forward current $I = I_F = \dfrac{V_F - V_a}{R_F}$

For t > 0, $I = -I_R \approx -\dfrac{V_R}{R_R}$ if diode voltage << V_R,

and note that diode voltage cannot change instantaneously at $t = 0$

due to junction/diffusion capacitance, i.e diode still forward biased at $t = 0^+$

On → off

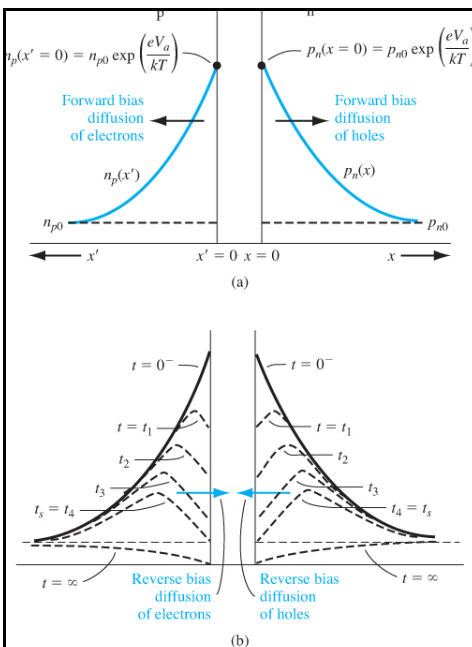
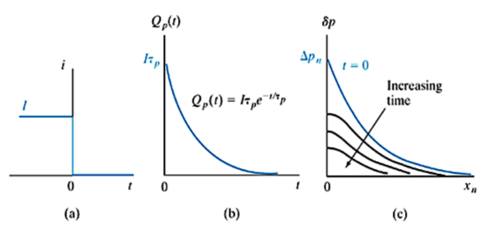


Figure 8.25 | (a) Steady-state forward-bias minority carrier concentrations; (b) minority carrier concentrations at various times during switching.



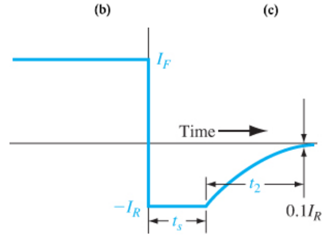


Figure 8.26 | Current characteristic versus time during diode switching.

Switching transient: On → off

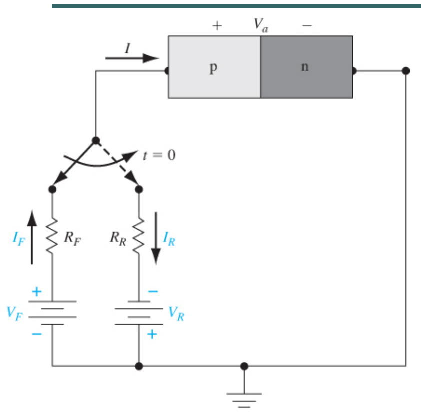


Figure 8.24 | Simple circuit for switching a diode from forward to reverse bias.

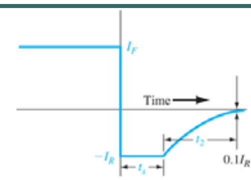


Figure 8.26 | Current characteristic versus time during diode switching.

Storage time : -

$$t_s = \tau_{p0}\left(erf^{-1}\left[\frac{I_F}{I_F + I_R}\right]^2\right) \approx \tau_{p0}\ln\left[1 + \frac{I_F}{I_R}\right]$$

Recovery time : –

$$\text{erf}\sqrt{\frac{t_2}{\tau_{p0}}} + \frac{\exp-\frac{t_2}{\tau_{p0}}}{\sqrt{\frac{\pi t_2}{\tau_{p0}}}} = 1 + \frac{I_R}{10 I_F}$$

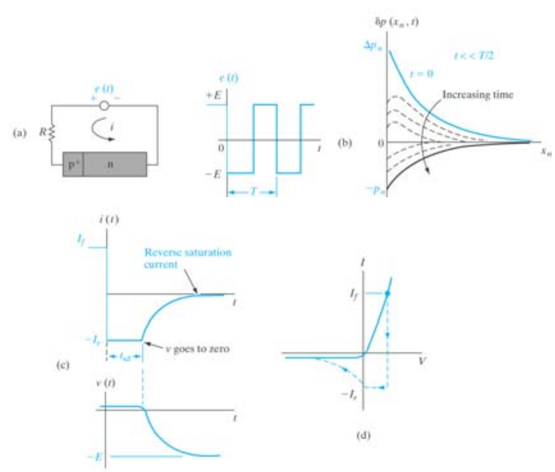


Figure 5.28

Storage delay time in a p+-n diode: (a) circuit and input square wave; (b) hole distribution in the n-region as a function of time during the transient; (c) variation of current and voltage with time; (d) sketch of transient current and voltage on the device I–V characteristic.

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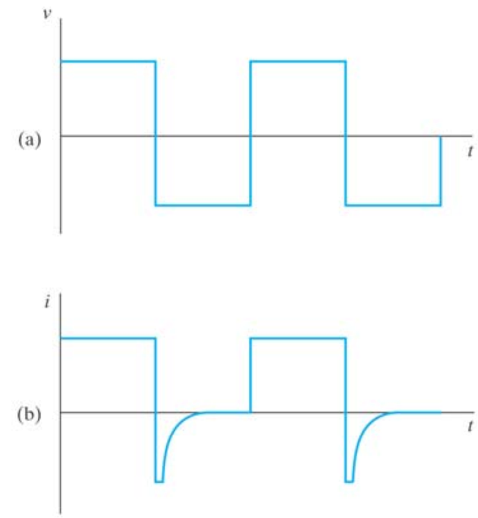

Figure 5.29

Effects of storage delay time on switching signal: (a) switching voltage; (b) diode current.

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Assignment #4

7.5	8.8
7.25	8.16
7.39	8.23
7.45	8.29