EE415/515 Fundamentals of Semiconductor Devices Fall 2012

Lecture 7:
PN Junction
(Chapter 7)

Introduction

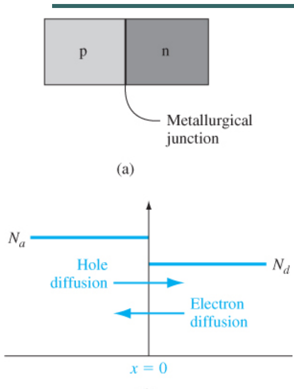


Figure 7.1 | (a) Simplified geometry of a pn junction; (b) doping profile of an ideal uniformly doped pn junction.

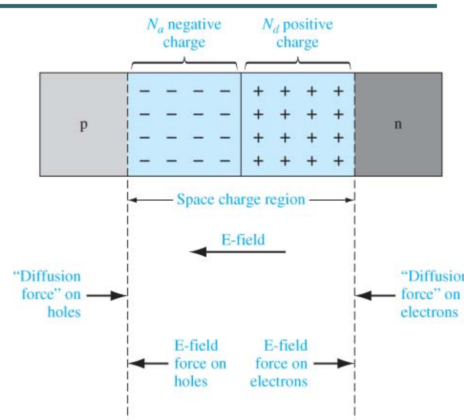


Figure 7.2 | The space charge region, the electric field, and the forces acting on the charged carriers.

Built-in Potential Barrier

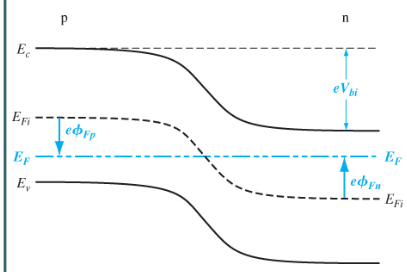


Figure 7.3 | Energy-band diagram of a pn junction in thermal equilibrium.

Define potentials ϕ_{Fn} and ϕ_{Fp} as shown, so

$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

In n - region $n_0 = N_c \exp - \frac{E_c - E_F}{kT} = n_i \exp \frac{E_F - E_{Fi}}{kT}$

so $e\phi_{Fn} = E_{Fi} - E_F = kT \ln \frac{n_i}{n_0} \Rightarrow \phi_{Fn} == -\frac{kT}{e} \ln \frac{N_d}{n_i}$

and similarly

$$p_0 = N_a = n_i \exp \frac{E_{Fi} - E_F}{kT} \Rightarrow \phi_{Fp} == +\frac{kT}{e} \ln \frac{N_a}{n_i}$$

$$\therefore V_{bi} = \frac{kT}{e} \ln \frac{N_d}{n_i} + \frac{kT}{e} \ln \frac{N_a}{n_i} = \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2}$$

Note :

N_d, N_a are NET doping in n-, p - regions respectively

Ex 7.1 (a) Calculate Vbi for a Si pn junction at 300K for: (i) N_a=5x10[15]cm[3], N_d=10[17]cm[3] and (ii) N_a=2x10[16]cm[3], N_d=2x10[15]cm[3] (b) Repeat (a) for GaAs

(a) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

(i) $V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{15})(10^{17})}{(1.5\times10^{10})^2}\right] = 0.736\text{V}$

(ii) $V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{16})(2\times10^{15})}{(1.5\times10^{10})^2}\right] = 0.671\text{V}$

(b)

(i) $V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{15})(10^{17})}{(1.8\times10^{6})^2}\right] = 1.20\text{V}$

(ii) $V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{16})(2\times10^{15})}{(1.8\times10^{6})^2}\right] = 1.14\text{V}$

Space-charge (depletion) region

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon_s} = -\frac{dE(x)}{dx} \quad \text{(Poisson's equation)}$$

$$\rho(x) = -eN_a \quad \text{for } -x_p < x < 0\text{, and}$$

$$\rho(x) = +eN_d \quad \text{for } 0 < x < x_n$$

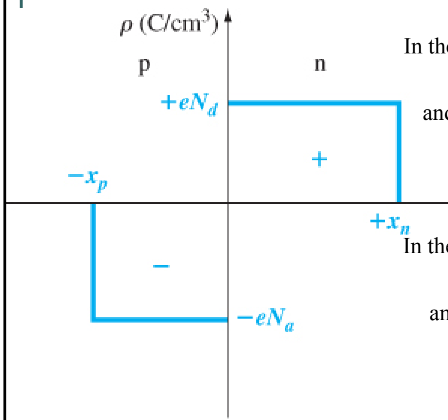


Figure 7.4 | The space charge density in a uniformly doped pn junction assuming the abrupt junction approximation.

$$\text{In the p-region: } E = \int \frac{\rho(x)}{\varepsilon_s} dx = -\int \frac{eN_a}{\varepsilon_s} dx = -\frac{eN_a}{\varepsilon_s} x + C_1$$

$$\text{and if } E(-x_p) = 0 \text{ then } C_1 = -\frac{eN_a}{\varepsilon_s} x_p$$

$$\& \quad E = -\frac{eN_a}{\varepsilon_s}(x + x_p) \quad \text{for } -x_p < x < 0$$

$$\text{In the n-region: } E = \int \frac{\rho(x)}{\varepsilon_s} dx = \int \frac{eN_d}{\varepsilon_s} dx = \frac{eN_d}{\varepsilon_s} x + C_2$$

$$\text{and if } E(x_n) = 0 \text{ then } C_2 = -\frac{eN_d}{\varepsilon_s} x_n$$

$$\& \quad E = -\frac{eN_d}{\varepsilon_s}(x_n - x) \quad \text{for } 0 < x < x_n$$

$$\text{Charge equality} \Rightarrow N_d x_n = N_a x_p$$

Space-charge (depletion) region

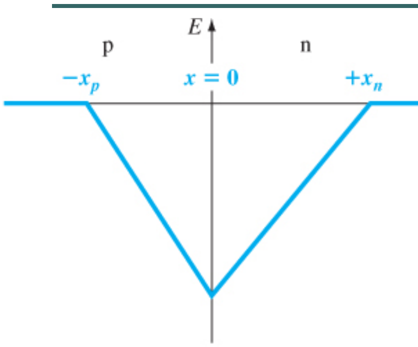


Figure 7.5 | Electric field in the space charge region of a uniformly doped pn junction.

$$E = -\frac{eN_a}{\varepsilon_s}(x + x_p) \quad \text{for } -x_p < x < 0$$

$$E = -\frac{eN_d}{\varepsilon_s}(x_n - x) \quad \text{for } 0 < x < x_n$$

E continuous at x = 0

$$\Rightarrow -\frac{eN_a}{\varepsilon_s} x_p = -\frac{eN_a}{\varepsilon_s} x_n$$

$$\Rightarrow N_a x_p = N_d x_n \quad \text{(charge equality)}$$

$$\text{Max field } E(0) = -\frac{eN_a}{\varepsilon_s} x_p = -\frac{eN_a}{\varepsilon_s} x_n$$

Space-charge (depletion) region

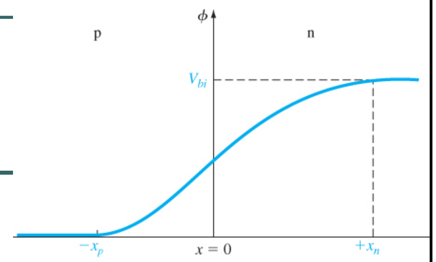


Figure 7.6 | Electric potential through the space charge region of a uniformly doped pn junction.

$$\mathrm{E} = -\frac{eN_a}{\varepsilon_s}(x + x_p) \quad \text{for} -x_p < x < 0$$

$$\mathrm{E} = -\frac{eN_d}{\varepsilon_s}(x_n - x) \quad \text{for } 0 < x < x_n$$

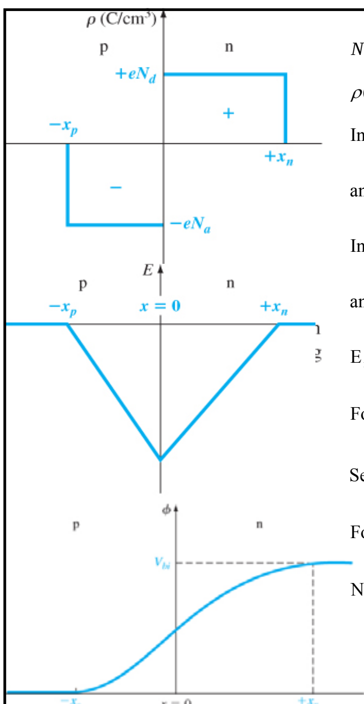
For the p-region: $\phi(\mathrm{x}) = -\int \mathrm{E(x)dx} = \int \frac{eN_a}{\varepsilon_s}(x + x_p)\,dx = \frac{eN_a}{\varepsilon_s}\left(\frac{x^2}{2} + x_p x\right) + C_1'$

Set $\phi(\text{-x}_\text{p}) = 0$, so $C_1' = \frac{eN_a}{\varepsilon_s}.\frac{x_p{}^2}{2}$ & $\phi(\mathrm{x}) = \frac{eN_a}{2\varepsilon_s}(x + x_p)^2$

For the n-region: $\phi(\mathrm{x}) = -\int \mathrm{E(x)dx} = \int \frac{eN_d}{\varepsilon_s}(x_n - x)\,dx = \frac{eN_d}{\varepsilon_s}\left(x_n x - \frac{x^2}{2}\right) + C_2'$

Now $\phi(0_\text{n}) = \phi(0_\text{p})$, so $C_2' = \frac{eN_a}{2\varepsilon_s}x_p{}^2$ & $\phi(\mathrm{x}) = \frac{eN_d}{\varepsilon_s}\left(x_n x - \frac{x^2}{2}\right) + \frac{eN_a}{2\varepsilon_s}x_p{}^2$

Since $\phi(\text{-x}_\text{p}) = 0$, $\mathrm{V_{bi}} = \phi(\mathrm{x_n}) = \frac{eN_d}{2\varepsilon_s}x_n{}^2 + \frac{eN_a}{2\varepsilon_s}x_p{}^2$



$N_a x_p = N_d x_n$ (charge equality) & $\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon_s} = -\frac{dE(x)}{dx}$

$\rho(x) = -eN_a$ for $-x_p < x < 0$, and $\rho(x) = +eN_d$ for $0 < x < x_n$

In the p-region: $\mathrm{E} = \int \frac{\rho(x)}{\varepsilon_s}dx = -\int \frac{eN_a}{\varepsilon_s}dx = -\frac{eN_a}{\varepsilon_s}x + C_1$

and $\mathrm{E(-x_p)} = 0 \Rightarrow \mathrm{C_1} = -\frac{eN_a}{\varepsilon_s}x_p$ & $\mathrm{E} = -\frac{eN_a}{\varepsilon_s}(x + x_p)$ for $-x_p < x < 0$

In the n-region: $\mathrm{E} = \int \frac{\rho(x)}{\varepsilon_s}dx = \int \frac{eN_d}{\varepsilon_s}dx = \frac{eN_d}{\varepsilon_s}x + C_2$

and $\mathrm{E(x_n)} = 0 \Rightarrow \mathrm{C_2} = -\frac{eN_d}{\varepsilon_s}x_n$ & $\mathrm{E} = -\frac{eN_d}{\varepsilon_s}(x_n - x)$ for $0 < x < x_n$

$\mathrm{E_{max}} = -\frac{eN_a}{\varepsilon_s}x_p = -\frac{eN_d}{\varepsilon_s}x_n$ at x = 0

For the p-region: $\phi(\mathrm{x}) = -\int \mathrm{E(x)dx} = \int \frac{eN_a}{\varepsilon_s}(x + x_p)\,dx = \frac{eN_a}{\varepsilon_s}\left(\frac{x^2}{2} + x_p x\right) + C_1'$

Set $\phi(\text{-x}_\text{p}) = 0$, so $C_1' = \frac{eN_a}{\varepsilon_s}.\frac{x_p{}^2}{2}$ & $\phi(\mathrm{x}) = \frac{eN_a}{2\varepsilon_s}(x + x_p)^2$

For the n-region: $\phi(\mathrm{x}) = -\int \mathrm{E(x)dx} = \int \frac{eN_d}{\varepsilon_s}(x_n - x)\,dx = \frac{eN_d}{\varepsilon_s}\left(x_n x - \frac{x^2}{2} +\right) + C_2'$

Now $\phi(0_\text{n}) = \phi(0_\text{p})$, so $C_2' = \frac{eN_a}{2\varepsilon_s}x_p{}^2$ & $\phi(\mathrm{x}) = \frac{eN_d}{\varepsilon_s}\left(x_n x - \frac{x^2}{2}\right) + \frac{eN_a}{2\varepsilon_s}x_p{}^2$

Since $\phi(\text{-x}_\text{p}) = 0$, $\mathrm{V_{bi}} = \phi(\mathrm{x_n}) = \frac{eN_d}{2\varepsilon_s}x_n{}^2 + \frac{eN_a}{2\varepsilon_s}x_p{}^2$

Space-charge region width

$$\mathrm{V_{bi}} = \phi(\mathrm{x_n}) = \frac{eN_d}{2\varepsilon_s} x_n^{\,2} + \frac{eN_a}{2\varepsilon_s} x_p^{\,2} \quad \text{and} \quad \mathrm{N_a} x_p = N_d x_n$$

$$\text{Substitute } x_p = \frac{N_d x_n}{N_a} \Rightarrow \mathrm{V_{bi}} = \frac{eN_d}{2\varepsilon_s} x_n^{\,2} + \frac{eN_a}{2\varepsilon_s}\left(\frac{N_d x_n}{N_a}\right)^2 = \frac{e}{2\varepsilon_s}\left[N_d + \frac{N_d^{\,2}}{N_a}\right] x_n^{\,2} = \frac{e}{2\varepsilon_s} \cdot \frac{N_d}{N_a}\left[N_a + N_d\right] x_n^{\,2}$$

$$\text{so } \mathrm{x_n} = \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \cdot \frac{N_a}{N_d} \cdot \frac{1}{[N_a + N_d]}}$$

$$\text{Substitute } x_n = \frac{N_a x_p}{N_d} \Rightarrow \mathrm{V_{bi}} = \frac{eN_a}{2\varepsilon_s} x_p^{\,2} + \frac{eN_d}{2\varepsilon_s}\left(\frac{N_a x_p}{N_d}\right)^2 = \frac{e}{2\varepsilon_s}\left[N_a + \frac{N_a^{\,2}}{N_d}\right] x_p^{\,2} = \frac{e}{2\varepsilon_s} \cdot \frac{N_a}{N_d}\left[N_a + N_d\right] x_p^{\,2}$$

$$\text{so } \mathrm{x_p} = \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \cdot \frac{N_d}{N_a} \cdot \frac{1}{[N_a + N_d]}}$$

$$\text{and } \mathrm{W} = \mathrm{x_n} + x_p \text{ so } W = \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \cdot \frac{1}{[N_a + N_d]}}\left[\sqrt{\frac{N_d}{N_a}} + \sqrt{+\frac{N_a}{N_d}}\right] = \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \cdot \frac{1}{[N_a + N_d]}} \frac{N_a + N_d}{\sqrt{N_a N_d}}$$

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \cdot \frac{(N_a + N_d)}{N_a N_d}}$$

Ex 7.2 Si pn junction at 300K with zero applied bias has N_d=5x10^{16}/cm^3 and N_a=5x10^{15}/cm^3. Find x_n, x_p, W, $|E_{max}|$.

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(5\times10^{16})(5\times10^{15})}{(1.5\times10^{10})^2}\right] = 0.7184\ \mathrm{V}$$

Then

$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right\}^{1/2} = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184)}{(1.6\times10^{-19})} \times \left(\frac{5\times10^{15}}{5\times10^{16}}\right)\left(\frac{1}{5\times10^{15} + 5\times10^{16}}\right)\right\}^{1/2}$$

$$\Rightarrow x_n = 4.11\times10^{-6}\ \mathrm{cm}$$

$$x_p = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184)}{(1.6\times10^{-19})} \times \left(\frac{5\times10^{16}}{5\times10^{15}}\right)\left(\frac{1}{5\times10^{15} + 5\times10^{16}}\right)\right\}^{1/2}$$

$$\Rightarrow x_p = 4.11\times10^{-5}\ \mathrm{cm}$$

Now $W = x_n + x_p = 4.11\times10^{-6} + 4.11\times10^{-5} = 4.52\times10^{-5}\ \mathrm{cm}$

Now $|E_{max}| = \dfrac{eN_d x_n}{\epsilon_s} = \dfrac{(1.6\times10^{-19})(5\times10^{16})(4.11\times10^{-6})}{(11.7)(8.85\times10^{-14})} = 3.18\times10^{4}\ \mathrm{V/cm}$

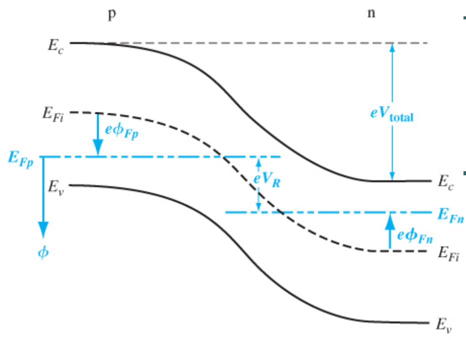


Figure 7.7 | Energy-band diagram of a pn junction under reverse bias.

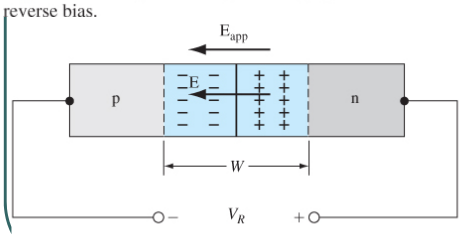


Figure 7.8 | A pn junction, with an applied reverse-biased voltage, showing the directions of the electric field induced by V_R and the space charge electric field.

Reverse bias

$$V_{total} = |\phi_{Fn}| + |\phi_{Fp}| + V_R = V_{bi} + V_R$$

$$W \Rightarrow \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \cdot \frac{(N_a + N_d)}{N_a N_d}}$$

$$E_{\max} = -\frac{eN_d x_n}{\varepsilon_s} = -\frac{eN_a x_p}{\varepsilon_s}$$

$$= -\sqrt{\frac{2e(V_{bi} + V_R)}{\varepsilon_s} \cdot \frac{N_a N_d}{(N_a + N_d)}}$$

$$= -2\frac{V_{bi} + V_R}{W}$$

Ex 7.3 Si pn junction at 300K has $N_d = 5\times10^{16}/cm^3$ and $N_a = 5\times10^{15}/cm^3$. Find V_{bi}, x_n, x_p, and W for reverse bias (a) $V_R = 4V$ & (b) $V_R = 8V$.

(a) $$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(5\times10^{15})(5\times10^{16})}{(1.5\times10^{10})^2}\right] = 0.718\text{ V}$$

$$x_n = \left\{\frac{2\epsilon_s (V_{bi} + V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184+4)}{1.6\times10^{-19}} \times \left(\frac{5\times10^{15}}{5\times10^{16}}\right)\left(\frac{1}{5\times10^{15} + 5\times10^{16}}\right)\right\}^{1/2} = 1.054\times10^{-5}\text{ cm}$$

or $x_n = 0.1054\ \mu\text{m}$

$$x_p = \left\{\frac{2\epsilon_s (V_{bi} + V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184+4)}{1.6\times10^{-19}} \times \left(\frac{5\times10^{16}}{5\times10^{15}}\right)\left(\frac{1}{5\times10^{15} + 5\times10^{16}}\right)\right\}^{1/2} = 1.054\times10^{-4}\text{ cm}$$

or $x_p = 1.054\ \mu\text{m}$

Ex 7.3 Si pn junction at 300K has $N_d=5\times10^{16}/cm^3$ and $N_a=5\times10^{15}/cm^3$. Find V_{bi}, x_n, x_p, and W for reverse bias (a) $V_R=4V$ & (b) $V_R=8V$. (cont'd)

$$W=\left\{\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right\}^{1/2}$$

$$=\left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184+4)}{1.6\times10^{-19}}\times\left[\frac{5\times10^{15}+5\times10^{16}}{(5\times10^{15})(5\times10^{16})}\right]\right\}^{1/2}=1.159\times10^{-4}\text{ cm}$$

or $W=1.159\ \mu m$

(b) $V_{bi}=0.718\text{ V}$

$$x_n=\left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184+8)}{1.6\times10\times10^{-19}}\times\left(\frac{5\times10^{15}}{5\times10^{16}}\right)\left(\frac{1}{5\times10^{15}+5\times10^{16}}\right)\right\}^{1/2}=1.432\times10^{-5}\text{ cm}$$

or $x_n=0.1432\ \mu m$

$$x_p=\left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184+8)}{1.6\times10^{-19}}\times\left(\frac{5\times10^{16}}{5\times10^{15}}\right)\left(\frac{1}{5\times10^{15}+5\times10^{16}}\right)\right\}^{1/2}=1.432\times10^{-4}\text{ cm}$$

or $x_p=1.432\ \mu m$

$$W=\left\{\frac{2(11.7)(8.85\times10^{-14})(0.7184+8)}{1.6\times10^{-19}}\times\left[\frac{5\times10^{15}+5\times10^{16}}{(5\times10^{15})(5\times10^{16})}\right]\right\}^{1/2}=1.576\times10^{-4}\text{ cm}$$

or $W=1.576\ \mu m$

Ex 7.4 $|E_{max}|$ in a GaAs pn junction at 300K is limited to $7.2\times10^4 V/cm$ with $N_d=5\times10^{15}/cm^3$ & $N_a=3\times10^{16}/cm^3$. Find max V_R that can be applied.

$$V_{bi}=V_t\ln\left(\frac{N_aN_d}{n_i^2}\right)=(0.0259)\ln\left[\frac{(5\times10^{15})(3\times10^{16})}{(1.8\times10^{6})^2}\right]=1.173\text{ V}$$

$$|E_{max}|=\left\{\frac{2e(V_{bi}+V_R)}{\epsilon_s}\left(\frac{N_aN_d}{N_a+N_d}\right)\right\}^{1/2}$$

$$\text{Now }(7.2\times10^4)^2=\left\{\frac{2(1.6\times10^{-19})(V_{bi}+V_R)}{(13.1)(8.85\times10^{-14})}\times\left[\frac{(5\times10^{15})(3\times10^{16})}{5\times10^{15}+3\times10^{16}}\right]\right\}$$

$$5.184\times10^{9}=1.1829\times10^{9}(V_{bi}+V_R)$$

$$V_{bi}+V_R=1.173+V_R=4.382$$

$$\text{Then }V_R=3.21\text{ V}$$

Junction Capacitance: Depletion layer capacitance

(also known as Transition Capacitance)

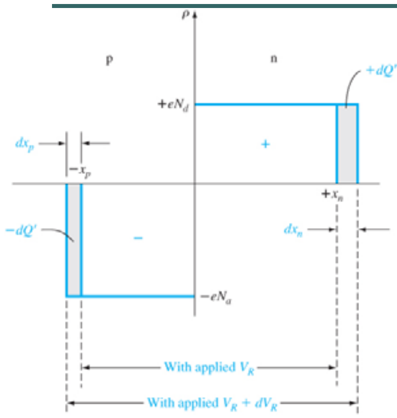


Figure 7.9 | Differential change in the space charge width with a differential change in reverse-biased voltage for a uniformly doped pn junction.

$$C' = \frac{dQ'}{dV_R}$$

where $dQ' = eN_d dx_n = eN_a dx_p$

$$x_n = \sqrt{\frac{2\varepsilon_s}{e}\cdot\frac{N_a}{N_d}\cdot\frac{1}{[N_a+N_d]}}\cdot(V_{bi}+V_R)^{\frac{1}{2}}$$

so $C' = eN_d \dfrac{dx_n}{dV_R}$

$$= \frac{1}{2}eN_d\sqrt{\frac{2\varepsilon_s}{e}\cdot\frac{N_a}{N_d}\cdot\frac{1}{[N_a+N_d]}}(V_{bi}+V_R)^{-\frac{1}{2}}$$

$$= \sqrt{\frac{e\varepsilon_s}{2}\cdot\frac{N_aN_d}{[N_a+N_d]}}(V_{bi}+V_R)^{-\frac{1}{2}}$$

$$= \frac{\varepsilon_s}{W}$$

Using $x_p \Rightarrow$ the same result

Ex 7.5 GaAs pn junction with $N_a=5\times10^{15}/cm^3$ & $N_d=2\times10^{16}/cm^3$ at 300K. (a) Calculate V_{bi}. (b) Find C' for $V_R=4V$. (c) Find C' for $V_R=8V$.

(a) $V_{bi} = V_t \ln\left(\dfrac{N_aN_d}{n_i^2}\right) = (0.0259)\ln\left[\dfrac{(2\times10^{16})(5\times10^{15})}{(1.8\times10^{6})^2}\right] = 1.16V$

(b) $C' = \left\{\dfrac{e\varepsilon_s N_aN_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2} = \left\{\dfrac{(1.6\times10^{-19})(13.1)(8.85\times10^{-14})}{2(1.162+4)}\times\dfrac{(5\times10^{15})(2\times10^{16})}{(5\times10^{15}+2\times10^{16})}\right\}^{1/2}$

$C' = 8.48\times10^{-9}\ F/cm^2$

(c) $C' = \left\{\dfrac{(1.6\times10^{-19})(13.1)(8.85\times10^{-14})}{2(1.162+8)}\times\dfrac{(5\times10^{15})(2\times10^{16})}{(5\times10^{15}+2\times10^{16})}\right\}^{1/2}$

$C' = 6.36\times10^{-9}\ F/cm^2$

Single-sided junctions

(p^+n for $N_a >> N_d$ (below), or pn^+ for $N_a << N_d$)

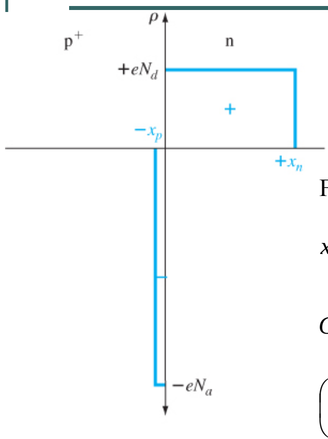


Figure 7.10 | Space charge density of a one-sided p⁻n junction.

For $N_a >> N_d$:

$$x_p << x_n \quad \& \quad W \approx x_n = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{eN_d}}$$

$$C' = \frac{\varepsilon_s}{W} \approx \sqrt{\frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)}}$$

$$\left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi} + V_R)}{e\varepsilon_s N_d}$$ which gives N_d & V_{bi}

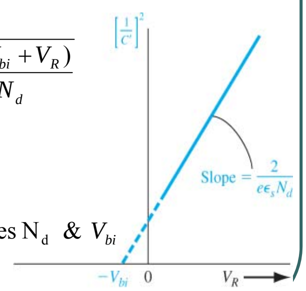


Figure 7.11 | $(1/C')^2$ versus V_R of a uniformly doped pn junction.

Ex 7.6 C' of a Si pn+ junction with c/s area 10^{-5}cm² is 0.105pF at 300K and V_R=3V, and V_{bi}=0.765V. Find N_a & N_d.

For a one-sided junction $C' = \left\{\frac{e\,\epsilon_s N_a}{2(V_{bi} + V_R)}\right\}^{1/2}$

$$C = A \cdot C' = (10^{-5})C' = 0.105\times10^{-12} = (10^{-5})\left\{\frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})N_a}{2(3+0.765)}\right\}^{1/2}$$

$$(0.105\times10^{-12})^2 = (10^{-5})^2(2.20\times10^{-32})N_a$$

So $N_a = 5.01\times10^{15}\,\text{cm}^{-3}$

We have $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

Then $N_d = \frac{n_i^2}{N_a}\exp\left(\frac{V_{bi}}{V_t}\right) = \frac{(1.5\times10^{10})^2}{5.01\times10^{15}}\exp\left(\frac{0.765}{0.0259}\right)$

$N_d = 3.02\times10^{17}\,\text{cm}^{-3}$

(Reverse bias) Junction Breakdown

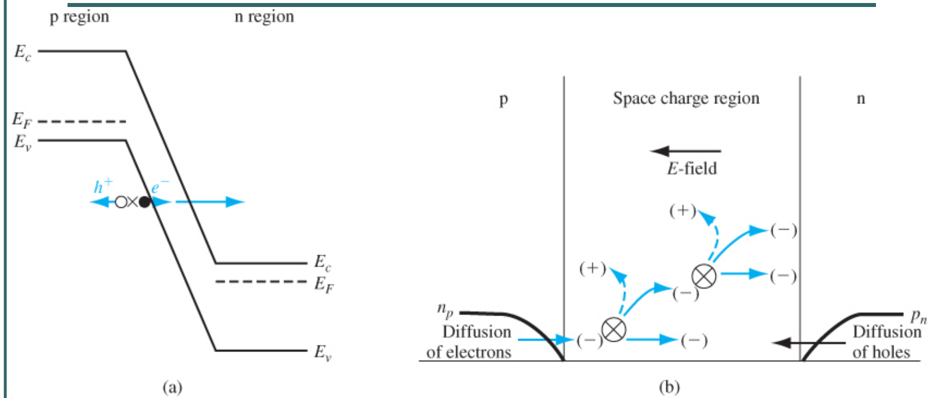


Figure 7.12 | (a) Zener breakdown mechanism in a reverse-biased pn junction; (b) avalanche breakdown process in a reverse-biased pn junction.

Avalanche

$I_n(0) = I_{n0}$ & $I_n(W) = M_n I_{n0}$

(M_n electron multiplication factor)

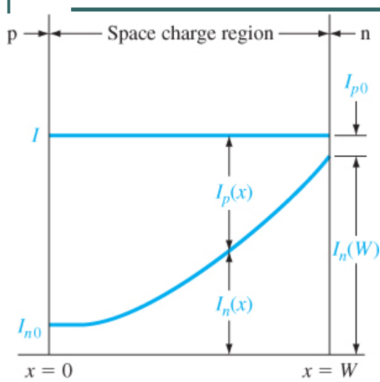


Figure 7.13 | Electron and hole current components through the space charge region during avalanche multiplication.

At point x, $dI_n(x) = [\alpha_n I_n(x) + \alpha_p I_p(x)]dx$, where

$\alpha_n,\ \alpha_p$ = EHP ionization rates for electrons, holes

Total current $I = I_n(x) + I_p(x) \Rightarrow I_p(x) = I - I_n(x)$

$dI_n(x) = [\alpha_n I_n(x) + \alpha_p (I - I_n(x))]dx$

$= \alpha_p I + (\alpha_n - \alpha_p) I_n(x)$

Simplify with $\alpha_p \approx \alpha_n = \alpha \Rightarrow dI_n(x) = \alpha I$

$$\int_0^W dI_n(x) = \int_0^W \alpha I.dx = I\int_0^W \alpha.dx$$

$$= I_n(W) - I_n(0) = M_n I_{n0} - I_{n0} \approx I - I_{n0}$$

so $\dfrac{I - I_{n0}}{I} = \int_0^W \alpha.dx = 1 - \dfrac{1}{M_n}$

Avalanche when $M_n \to \infty$, i.e. when $\int_0^W \alpha.dx = 1$

Avalanche & Zener breakdowns

For $E_{max} = \dfrac{eN_d x_n}{\varepsilon_s}$

$$\approx \frac{eN_d}{\varepsilon_s}\sqrt{\frac{2\varepsilon_s}{eN_d}V_R}$$

$$= \sqrt{\frac{2eN_d}{\varepsilon_s}V_R}$$

$$\Rightarrow V_B = \frac{\varepsilon_s E_{crit}^{\;\;2}}{2eN_B}$$

(N_B lower of N_a, N_d)

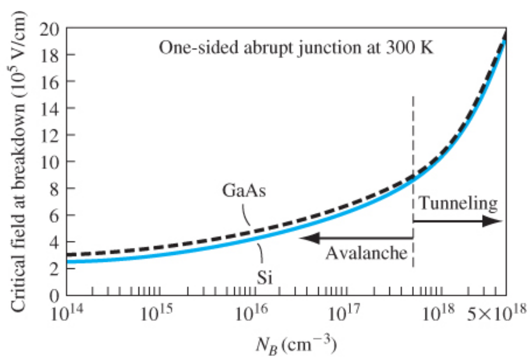


Figure 7.14 | Critical electric field at breakdown in a one-sided junction as a function of impurity doping concentrations. *(From Sze and Ng [14].)*

Ex 7.7 Design single-sided, planar, uniformly doped Si planar diode for reverse breakdown V_B=60V. What is max doping concentration on low-doped side to meet spec?

$V_B = \varepsilon_s E_{crit}^2/2eN$

$N = \varepsilon_s E_{crit}^2/2eV_B$

$= 11.7 \times 8.85\times10^{-12}(2\times10^7)^2/2\times1.6\times10^{-19}\times60$ (from Fig 7.14 for light doping)

$= 2.16\times10^{21}/m^3$

$= 2.16\times10^{15}/cm^3$

Iteration:
$E_{crit} \approx 3.5\times10^7$V/m from Fig 7.14, so incr N & repeat

Compare Fig 7.15 $N = 9\times10^{15}/cm^3$

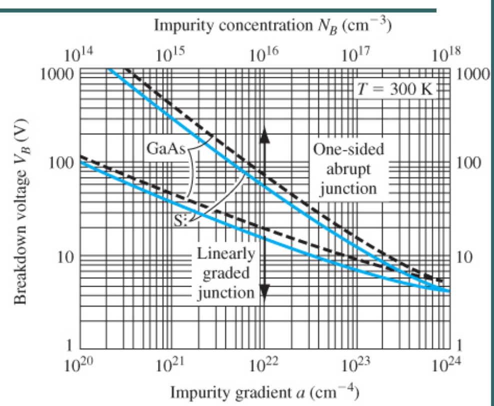


Figure 7.15 | Breakdown voltage versus impurity concentration in uniformly doped and linearly graded junctions. *(From Sze [14].)*

Linearly graded junction

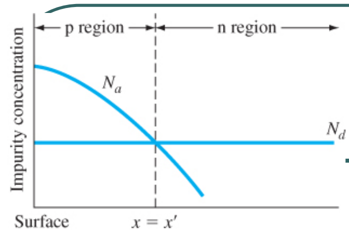


Figure 7.16 | Impurity concentrations of a pn junction with a nonuniformly doped p region.

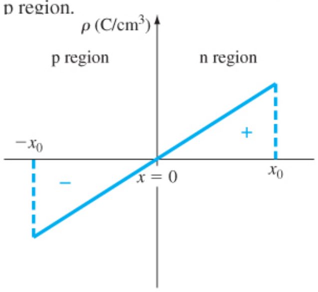


Figure 7.17 | Space charge density in a linearly graded pn junction.

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_s} = \frac{eax}{\varepsilon_s} \quad \text{where } a \text{ defines gradient}$$

$$\therefore \mathrm{E}(x) = \int \frac{eax}{\varepsilon_s}dx = \frac{eax^2}{2\varepsilon_s} + A \rightarrow 0 \text{ at } x = \pm x_0 \,,$$

$$\text{so } \mathrm{E}(x) = \frac{ea(x^2 - x_0^2)}{2\varepsilon_s}$$

$$E_{max} \text{ when } \frac{d\mathrm{E}(x)}{dx} = 0 \text{ at } x = 0,\ E_{max} = -\frac{eax_0^2}{2\varepsilon_s}$$

$$\phi(x) = -\int Edx = -\frac{ea(x^3/3 - x_0^2 x)}{2\varepsilon_s} + B$$

$$\text{Define } \phi(\text{-}x_0) = 0,\ B = +\frac{eax_0^3}{3\varepsilon_s} \Rightarrow \phi(x) = -\int Edx$$

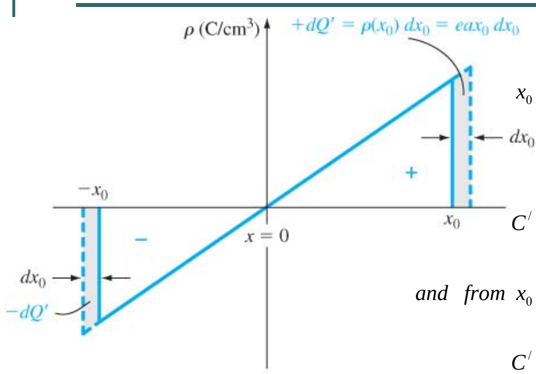
$$= -\frac{ea(x^3 - 3x_0^2 x - 2x_0^3)}{6\varepsilon_s} \Rightarrow V_{bi} = \phi(+x_0) = \frac{2}{3}\frac{eax_0^3}{\varepsilon_s}$$

$$\text{so } x_0 = \left[\frac{3}{2}\frac{\varepsilon_s}{ea}(V_{bi} + V_R)\right]^{\frac{1}{3}}$$

$$\text{As before } V_{bi} = \frac{kT}{e}\ln\left[\frac{N_d(x_0)N_a(-x_0)}{n_i^2}\right]$$

$$\text{where } N_d(x_0) = ax_0 = N_a(-x_0),\ \text{so } V_{bi} = \frac{kT}{e}\ln[ax_0/n_i]^2$$

Linear junction capacitance



$$x_0 = \left[\frac{3}{2}\frac{\varepsilon_s}{ea}(V_{bi} + V_R)\right]^{\frac{1}{3}}$$

$$C' = \frac{dQ'}{dV_R} = (eax_0)\frac{dx_0}{dV_R}$$

$$\text{and from } x_0 = \left[\frac{3\varepsilon_s}{2ea}(V_{bi} + V_R)\right]^{1/3}$$

$$C' = (ea)\left[\frac{3\varepsilon_s}{2ea}(V_{bi} + V_R)\right]^{1/3}\left[\frac{3\varepsilon_s}{2ea}\right]^{1/3}\frac{1}{3}(V_{bi} + V_R)^{-2/3}$$

$$= \left[\frac{ea\varepsilon_s^2}{12(V_{bi} + V_R)}\right]^{1/3}$$

Figure 7.18 | Differential change in space charge width with a differential change in reverse-biased voltage for a linearly graded pn junction.

Hyperabrupt junctions

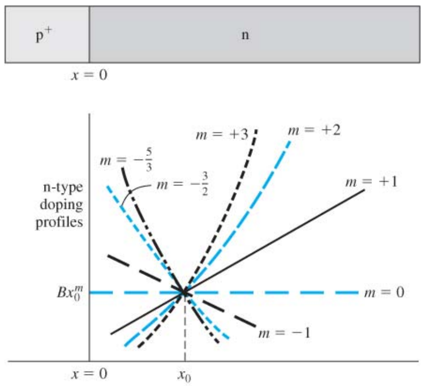


Figure 7.19 | Generalized doping profiles of a one-sided p^+n junction.
(From Sze [14].)

Doping for x > 0: $N = Bx^m$

("Hyperabrupt" if m < 0)

It can be shown that

$$C' = \left[\frac{eB\varepsilon_s^{\,m+1}}{(m+2)(V_{bi}+V_R)}\right]^{\frac{1}{m+2}}$$

$$= C_0 (V_{bi}+V_R)^{-\frac{1}{m+2}}$$

If used for tuning $f_r = \left(2\pi\sqrt{LC}\right)^{-1}$

For $f_r \propto V_R$

need $f_r \propto C^{-\frac{1}{2}} \propto V_R^{\left(-\frac{1}{m+2}\right)\left(-\frac{1}{2}\right)} \propto V_R$

i.e. $\frac{1}{m+2}\cdot\frac{1}{2} = 1$, $m = -\frac{3}{2}$