Excess generation/recombination

At thermal equilibrium, band-to-band EHP generation/recombination …… Carrier concentrations constant, so \( G_{n0} = G_{p0} = R_{n0} = R_{p0} \)

**Excess carriers:**
\[ n = n_0 + \delta n, \quad p = p_0 + \delta p \]

**Excess generation**
\[ g_n = g_p \]

**Non-equilibrium**
\[ np \neq n_p p_0 = n_i^2 \]

Table 6.1 | Relevant notation used in Chapter 6

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_0, p_0 )</td>
<td>Thermal-equilibrium electron and hole concentrations (independent of time and also usually position)</td>
</tr>
<tr>
<td>( n, p )</td>
<td>Total electron and hole concentrations (may be functions of time and/or position)</td>
</tr>
<tr>
<td>( b_n = n - n_i )</td>
<td>Excess electron concentrations (may be functions of time and/or position)</td>
</tr>
<tr>
<td>( b_p = p - p_i )</td>
<td>Excess hole concentrations (may be functions of time and/or position)</td>
</tr>
<tr>
<td>( g_n, g_p )</td>
<td>Excess electron and hole generation rates</td>
</tr>
<tr>
<td>( R_n, R_p )</td>
<td>Excess electron and hole recombination rates</td>
</tr>
<tr>
<td>( \tau_{e0}, \tau_{p0} )</td>
<td>Excess minority carrier electron and hole lifetimes</td>
</tr>
</tbody>
</table>
Optical carrier generation & recombination

Electrons in the conduction band recombine with holes in the valence band directly or indirectly. Direct recombination occurs spontaneously, i.e. recombination probability is constant in time. Electron decay rate at time \( t \) is proportional to the number of electrons and holes remaining at \( t \).

Net rate of change:

\[
\frac{dn(t)}{dt} = \alpha, n_i^2 - \alpha, n(t) p(t) \\
\alpha, n_i^2 = \text{thermal generation rate}
\]

Recombination Equations

\[n, p_0 = n_i^2 \quad \text{where} \quad n_0 \quad \text{and} \quad p_0 \quad \text{are constant, and} \quad n(t) = n_0 + \delta n(t) \quad p(t) = p_0 + \delta p(t)\]

\[
\frac{d\delta n(t)}{dt} = \alpha, n_i^2 - \alpha, [n_0 + \delta n(t)][p_0 + \delta p(t)] = -\alpha, n_0 \delta n(t) + \delta n(t) = -\alpha, p_0 \delta n(t)
\]

for extrinsic p material \( \Rightarrow p_0 >> n_0 \) & low level \( \delta n(t) << p_0 \) with solution \( \delta n(t) = \delta n(0) \exp(-\alpha, p_0 t) \)

\[\tau_{p0} = (\alpha, p_0)^{-1} \quad \text{&} \quad \tau_{n0} = (\alpha, n_0)^{-1} \quad \text{recombination or excess minority carrier lifetimes.}\]

For direct band - to - band recombination \( R_n = R_p = -\frac{d(\delta n(t))}{dt} = \alpha, p_0 \delta n(t) = \frac{\delta n(t)}{\tau_{p0}} \quad \text{in p - type}\]

\[\text{and} \quad R_n' = R_p' = -\frac{d(\delta p(t))}{dt} = \alpha, n_0 \delta p(t) = \frac{\delta p(t)}{\tau_{n0}} \quad \text{in n - type}\]

In general for low level injection : \( \tau_{np} = \left[\alpha, (n_0 + p_0)\right]^{-1} \quad \text{for} \quad \delta p, \delta n << n_0, p_0 \)
Ex 6.1 Excess carriers generated uniformly to concentration $\delta n(0)=10^{15}$ cm$^{-3}$ until generation stopped at $t=0$. Calculate the recombination rates for the excess carriers for: (a) $t=0$, (b) $t=1\mu s$ (c) $t=4\mu s$ and (d) $t=10\mu s$, for $\tau_{n0}=1\mu s$.

$$R_n = \frac{\delta n}{\tau_{n0}} = \frac{10^{15} e^{-t/\tau_{n0}}}{10^{-4}} \text{ cm}^{-3} \text{ s}^{-1}$$

For $t=0$, $R_n = 10^{15} \times 10^{-4} = 10^{11}$ cm$^{-3}$ s$^{-1}$

$t=1\mu s$, $R_n = 10^{15} e^{-1/1} \times 10^{-6} = 3.68 \times 10^{10}$ cm$^{-3}$ s$^{-1}$

$t=4\mu s$, $R_n = 10^{15} e^{-4/1} \times 10^{-6} = 1.83 \times 10^{10}$ cm$^{-3}$ s$^{-1}$

$t=10\mu s$, $R_n = 10^{15} e^{-10/1} \times 10^{-6} = 4.54 \times 10^{10}$ cm$^{-3}$ s$^{-1}$
Indirect Recombination, trapping

- For Group IV semiconductors the probability of direct recombination is small. (Why?)
- Most recombination occurs through recombination levels in the band gap. Energy dissipated as heat (aka phonons or lattice vibrations) not photons.
- Any impurity or defect which can trap one type of charge and then the next can serve as a recombination center.

Impurity energy levels

- Donor levels indicated by a + sign.
- Indicates whether it is positive (donor) or negative (acceptor) when ionized.
- Note that some impurities introduce multiple levels, e.g. Zn, which can accept two electrons.
Indirect Recombination, Trapping

- Example: assume recombination level $E_r$ is below $E_F$ and is filled at equilibrium. Then
  - Hole can be captured. (electron falls from $E_r$ to $E_V$)
  - Then electron can be captured (electron falls from $E_C$ to $E_r$)
  - One EHP lost. Excess energy given up as heat (phonons.)
- What if probability of capture is not equal for electrons and holes?
- As before, the first captured carrier can return to its band before being annihilated.
  - Delays process.
  - Called temporary trapping.
  - Defect or impurity called a trapping center if this event is more likely.
- If it is more likely that the annihilation will occur before re-excitation of the original trapped carrier
  - Called a recombination center.
- Deep levels are usually slower (more energy required.)

Continuity equations

Consider holes flowing into and out of cube

$$F_{p_0}^+(x + dx) = F_{p_0}^+(x) + \frac{\partial F_{p_0}^+(x)}{\partial x} \ dx \ \text{in number/cm}^3 = x$$

So

$$\frac{\partial p}{\partial t} \ dx dy dz = \left[ F_{p_0}^+(x) - F_{p_0}^+(x + dx) \right] dy dz = -\frac{\partial F_{p_0}^+(x)}{\partial x} \ dx dy dz$$

$$\Rightarrow -\frac{\partial F_{p_0}^+(x)}{\partial x} \ dx dy dz + g_p \ dx dy dz = -\frac{p}{\tau_{p_0}} \ dx dy dz \ \text{with gen/recomb}$$

So

$$\frac{\partial p}{\partial t} = -\frac{\partial F_{p_0}^+(x)}{\partial x} + g_p \ -\frac{p}{\tau_{p_0}} \ \text{and} \ \frac{\partial n}{\partial t} = -\frac{\partial F_{n_0}^-(x)}{\partial x} + g_n - \frac{n}{\tau_n}$$
Time dependent diffusion

\[ J_p = e\mu_p p E - eD_p \frac{\partial p}{\partial x} \]
\[ J_n = e\mu_n n E + eD_n \frac{\partial n}{\partial x} \]

Hole flux \( F_p^+ = \frac{J_p}{e} = \mu_p p E - D_p \frac{\partial p}{\partial x} \)

\[ \frac{\partial p}{\partial t} + g_p = \frac{\partial F_p^+}{\partial x} = \frac{\partial (pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_p} \]

Similarly

\[ \frac{\partial (\phi_p)}{\partial t} = D_p \frac{\partial^2 (\phi_p)}{\partial x^2} - \mu_p \left[ \nu \frac{\partial \phi_p}{\partial x} + \frac{\partial^2 (\phi_p)}{\partial x^2} \right] + g_p - \frac{p}{\tau_p} \]

Ambipolar transport

Applied field \( E_{app} \) tends to separate charges. But charge separation creates restoring field \( E_{int} \), which tends to hold pos/neg charges together. The electrons and holes may then drift/diffuse together with a single effective mobility/diffusion constant \( \rightarrow \) ambipolar diffusion/transport.

Poisson's equation: \( \nabla \cdot E_{int} = \frac{\partial E_{int}}{\partial x} = \frac{\varepsilon_0}{\epsilon_p} (\dot{\phi} - \dot{n}) \)

Assume \( \left| E_{int} \right| \ll \left| E_{app} \right| \)

and \( \dot{\phi} = \dot{n} \) (very small \( \dot{\phi} - \dot{n} \) \( \rightarrow \) \( E_{int} \))

Generation \( g = g_p = \dot{p} \)

and recombination \( R = R_n = \frac{n}{\tau_n} = R_p = \frac{p}{\tau_p} \)

So setting \( \dot{\phi} = \dot{n} \):

\[ D_p \frac{\partial^2 (\dot{\phi})}{\partial x^2} = \mu_p \left[ \nu \frac{\partial \dot{\phi}}{\partial x} + \frac{\partial^2 (\dot{\phi})}{\partial x^2} \right] + g - R = \frac{\partial \dot{n}}{\partial x} \]

and

\[ D_n \frac{\partial^2 (\dot{n})}{\partial x^2} + \mu_n \left[ \nu \frac{\partial \dot{n}}{\partial x} + \frac{\partial^2 (\dot{n})}{\partial x^2} \right] + g - R = \frac{\partial \dot{\phi}}{\partial x} \]

\[ D_{n,p,n} \frac{\partial^2 (\dot{n})}{\partial x^2} - \mu_n \mu_p \nu \left( \frac{\partial \dot{n}}{\partial x} + \frac{\partial^2 (\dot{n})}{\partial x^2} \right) + \mu_n n (g - R) = \mu_n \frac{\partial \dot{n}}{\partial x} \]

\[ D_{n,p,n} \frac{\partial^2 (\dot{n})}{\partial x^2} + \mu_n \mu_p \nu \left( \frac{\partial \dot{n}}{\partial x} + \frac{\partial^2 (\dot{n})}{\partial x^2} \right) + \mu_p n (g - R) = \mu_p \frac{\partial \dot{\phi}}{\partial x} \]

Figure 6.5 | The creation of an internal electric field as excess electrons and holes tend to separate.

\[ E = E_{app} + E_{int} \]
Ambipolar transport (cont’d)

Add equations to eliminate \( \frac{\partial E}{\partial x} \)

\[
(D_n \mu_n n + D_p \mu_p p) \left( \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n \mu_p (p - n) \right) \frac{\partial \bar{c}(\delta n)}{\partial x} + \left( \mu_n n + \mu_p p \right) (g - R) = \left( \mu_n n + \mu_p p \right) \frac{\partial \bar{c}(\delta n)}{\partial t}
\]

\[
D_p \frac{\partial^2 (\delta \pi)}{\partial x^2} + \mu' E \frac{\partial \bar{c}(\delta \pi)}{\partial x} + (g - R) = \frac{\partial \bar{c}(\delta \pi)}{\partial t}
\]

where \( \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} \)

and \( D' = \frac{D_n \mu_n n + D_p \mu_p p}{\mu_n n + \mu_p p} = \frac{D_n D_p (e / kT) n + D_p D_p (e / kT) p}{D_n (e / kT) n + D_p (e / kT) p} = \frac{D_n D_p (n + p)}{D_n n + D_p p} \)

\( D' \) = ambipolar diffusion coefficient, and \( \mu' \) = ambipolar mobility

Ambipolar transport: extrinsic, low level injection

For extrinsic p - type and low level injection: \( p_0 >> n_0 \) and \( \delta n << p_0 \)

\[
D' = \frac{D_n D_p [(n_0 + \delta n) + (p_0 + \delta p)]}{D_n (n_0 + \delta n) + D_p (p_0 + \delta p)} \Rightarrow \frac{D_n D_p (p_0)}{D_p (p_0)} \Rightarrow D_v
\]

\[
\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} = \frac{\mu_n \mu_p (p)}{\mu_p p} = \mu_n
\]

\( g - R = g_x - R_x = (G_{x_0} + g_x') - (R_{x_0} + R_x') = g_x' - R_x' \) (since \( G_{x_0} = R_{x_0} \))

\( g - R = g_x' - \frac{\delta \pi}{\tau_x} \) for electrons, and \( g - R = g_x' - \frac{\delta \pi}{\tau_p} \) for holes

\[
D_n \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n E \frac{\partial \bar{c}(\delta \pi)}{\partial x} + \left( g' - \frac{\delta \pi}{\tau_{x_0}} \right) = \frac{\partial \bar{c}(\delta n)}{\partial t}
\]

for p - type

\[
D_p \frac{\partial^2 (\delta \pi)}{\partial x^2} + \mu_p E \frac{\partial \bar{c}(\delta \pi)}{\partial x} + \left( g' - \frac{\delta \pi}{\tau_{p_0}} \right) = \frac{\partial \bar{c}(\delta \pi)}{\partial t}
\]

for n - type
### Ambipolar transport equations

#### Table 6.2 | Common ambipolar transport equation simplifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state</td>
<td>$\frac{\partial (\delta n)}{\partial t} = 0$, $\frac{\partial (\delta p)}{\partial t} = 0$</td>
</tr>
<tr>
<td>Uniform distribution of excess carriers (uniform generation rate)</td>
<td>$D_n \frac{\partial^2 (\delta n)}{\partial x^2} = 0$, $D_p \frac{\partial^2 (\delta n)}{\partial x^2} = 0$</td>
</tr>
<tr>
<td>Zero electric field</td>
<td>$E \frac{\partial (\delta n)}{\partial x} = 0$, $E \frac{\partial (\delta p)}{\partial x} = 0$</td>
</tr>
<tr>
<td>No excess carrier generation</td>
<td>$g' = 0$</td>
</tr>
<tr>
<td>No excess carrier recombination (infinite lifetime)</td>
<td>$\frac{\delta n}{\tau_{n0}} = 0$, $\frac{\delta p}{\tau_{p0}} = 0$</td>
</tr>
</tbody>
</table>

---

**Ex 6.2** N-type GaAs $N_d=10^{16}/\text{cm}^3$, with $10^{14}$ EHPs/\text{cm}^3 uniformly created at $t=0$. Find times at which the minority carrier hole concentration reaches (a) $1/e$ and (b) $10\%$ of the initial value, if $\tau_{p0}=50\text{ns}$.

See Example 6.2:

For n-type: $D_n \frac{\partial^2 (\delta p)}{\partial x^2} - \mu_n E \frac{\partial (\delta p)}{\partial x} + g' = \frac{\delta p}{\tau_{p0}} = \frac{\partial (\delta p)}{\partial t}$

For uniform excess hole distribution: $\frac{\partial (\delta p)}{\partial t} = 0 = \frac{\delta p}{\tau_{p0}}$

and $g' = 0$ for $t > 0$, so $\frac{\partial (\delta p)}{\partial t} = -\frac{\delta p}{\tau_{p0}} \Rightarrow \delta p(t) = \delta p(0) \exp\left(-\frac{t}{\tau_{p0}}\right)$

(a) $10^{14} e^{-t/50} = 10^{14} e^{-1}$

$\Rightarrow t = 50 \text{ ns}$

(b) $10^{14} e^{-t/10} = 10^{13}$

$$t = \left(\frac{50 \times 10^{-3}}{10^{13}}\right) \ln\left(\frac{10^{14}}{10^{13}}\right) = 1.15 \times 10^{-3} \text{ s}$$

or $t = 115 \text{ ns}$
Ex 6.3  For n-type Si at T=300K with N_d=5x10^{16}/cm^3. Assume uniform g'=5x10^{21}/cm^3s^{-1} for t\geq0 and \tau_{p0}=10^{-7}s.
(a) Find \delta p(t) at (i) t=0, (ii) t=10^{-7}s, (iii) t=5x10^{-7}s, and (iv) t\rightarrow\infty. (b) Is low level injection maintained?

See Example 6.3:

For n-type : \( \frac{\partial^2 \delta(p)}{\partial x^2} - g' - \frac{\delta(p)}{\tau_{p0}} = 0 \)

For uniform excess distribution : \( \frac{\partial^2 \delta(p)}{\partial x^2} = 0 \)

\( \frac{\partial \delta(p)}{\partial t} = g' \frac{\partial \delta(p)}{\partial x} = \frac{\partial \delta(p)}{\partial x}(1 - \exp^{-t/\tau_{p0}}) \)

\( \phi(t) = g' \tau_{p0} \int_0^t (1 - \exp^{-\tau/\tau_{p0}}) \)

(i) \( \phi(0) = 5x10^{-4}(1 - \exp^{-0}) = 0 \)
(ii) \( \phi(10^{-7}) = 5x10^{-4}(1 - \exp^{-1}) = 3.156x10^{-4} \)
(iii) \( \phi(5x10^{-7}) = 5x10^{-4}(1 - \exp^{-5}) = 4.966x10^{-5} \)
(iv) \( \phi(\infty) = 5x10^{-4}(1 - \exp^{-\infty}) = 5x10^{-4} \)

(b) \( \phi(\text{max}) = 5x10^{-4} \)
Yes, low injection condition is met.

Steady State Carrier Injection; Diffusion Length

- In steady state the concentrations of excess carriers do not change.
- We should recognize this as exponential decay.

\[ \frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} = \frac{\delta n}{L_n^2} \]
Electron diffusion length \( L_n = \sqrt{D_n \tau_n} \)

\[ \frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} = \frac{\delta p}{L_p^2} \]
Hole diffusion length \( L_p = \sqrt{D_p \tau_p} \)

Assume injection of excess holes N-type at \( x = 0 \)
\( \delta p(x) = \delta p(0) \exp(-x/L_p) \)
At \( x = L_p \) holes are reduced to 1/e. Average distance before recombining.
### Diffusion Length

For p-type Si, $T=300K$, $N_a=5 \times 10^{16}/\text{cm}^3$, $\tau_{n0}=5 \times 10^{-7}\text{s}$, $D_n=25\text{cm}^2/\text{s}$, with electrons being generated at $x=0$, so $\delta n(0)=10^{15}/\text{cm}^3$.

(a) Calculate diffusion length $L_n$.  
(b) Determine $\delta n$ at
(i) $x=0$, (ii) $x=+30 \mu\text{m}$ (iii) $x=-50 \mu\text{m}$ (iv) $x=+85 \mu\text{m}$ & (v) $x=120 \mu\text{m}$.

Ex 6.4  

See Example 6.4:

For p-type:  
$$D_n \frac{\partial^2 (\delta n)}{\partial x^2} - \mu_L E \frac{\partial (\delta n)}{\partial x} + \frac{g_i}{\tau_{n0}} \frac{\partial \delta n}{\partial t} = \frac{\partial n}{\partial t}$$

For steady state $\frac{\partial n}{\partial t} = 0$, $E = 0$, $g_i = 0$ for $x \neq 0$, we have

$$D_n \frac{\partial^2 (\delta n)}{\partial x^2} = \frac{\partial \delta n}{\partial x}$$

with solution $\delta n(x) = A \exp \frac{x}{L_n} + B \exp -\frac{x}{L_n}$

For $x > 0$, $B = 0$ for $x \to \infty$, so $\delta n(x) = \delta n(0) \exp -\frac{x}{L_n}$

For $x < 0$, $A = 0$ for $x \to -\infty$, so $\delta n(x) = \delta n(0) \exp +\frac{x}{L_n}$
Ex 6.4  P-type Si, T=300K, \( N_n = 5 \times 10^{16}/\text{cm}^3 \), \( T_n = 5 \times 10^{-7} \text{s} \), \( D_n = 25 \text{cm}^2/\text{s} \), with electrons being generated at \( x=0 \), so \( \delta n(0) = 10^{15}/\text{cm}^3 \).

(a) Calculate diffusion length \( L_n \).  
(b) Determine \( \delta n \) at

(i) \( x=0 \),

(ii) \( x=+30 \mu\text{m} \)

(iii) \( x=-50 \mu\text{m} \)

(iv) \( x=+85 \mu\text{m} \)

(v) \( x=120 \mu\text{m} \).

Ex 6.5  N-type semiconductor with constant applied field \( E_0 \) in +x direction. Finite number of EHPs generated at \( x=0 \) at \( t=0 \), and then \( g'=0 \) for \( t>0 \). \( D_p=10 \text{cm}^2/\text{s} \), \( \tau_p=10^{-7} \text{s} \), \( \mu_p=400 \text{cm}^2/\text{V-s} \), \( E_0=100 \text{V/cm} \).

Find \( \delta p \) for (a) \( t=10^{-7} \text{s} \) at (i) \( x=20 \mu\text{m} \), (ii) \( x=40 \mu\text{m} \), (iii) \( x=60 \mu\text{m} \), and (b) \( x=40 \mu\text{m} \) at (i) \( t=5 \times 10^{-8} \text{s} \), (ii) \( t=10^{-7} \text{s} \), and (iii) \( t=2 \times 10^{-7} \text{s} \).
Ex 6.5 N-type semiconductor with constant applied field $E_0$ in +x direction. Finite number of EHPs generated at $x=0$ at $t=0$, and then $g'=0$ for $t>0$. $D_p=10\text{cm}^2/\text{s}$, $\tau_{p0}=10^{-7}\text{s}$, $\mu_p=400\text{cm}^2/\text{V-s}$, & $E_0=100\text{V/cm}$. Find $\delta p$ for (a) $t=10^{-7}\text{s}$ at (i) $x=20\mu\text{m}$, $x=40\mu\text{m}$, & (iii) $x=60\mu\text{m}$, and (b) $x=40\mu\text{m}$ at (i) $t=5\times10^{-8}\text{s}$, (ii) $t=10^{-7}\text{s}$, and (iii) $t=2\times10^{-7}\text{s}$.

---

Figure 6.8 Excess hole concentration versus distance at various times for zero applied electric field.

---

Figure 6.9 Excess hole concentration versus distance at various times for a constant applied electric field.

---

\[ \phi(x,t) = \frac{\exp\left(\frac{-x-\mu_p E_0 t}{4D_p t}\right)}{4D_p t} \]

(i) $x=20\mu\text{m}$, $\phi = \frac{0.35783}{3.545\times10^{-6}} \exp\left[\frac{-1.8\times10^{-8}}{4\times10^{-6}}\right] \approx 38.18$

(ii) $x=40\mu\text{m}$, $\phi = \frac{0.35783}{3.545\times10^{-6}} \exp[0] = 38.18$

(iii) $x=60\mu\text{m}$, $\phi = \frac{0.35783}{3.545\times10^{-6}} \exp[0] \approx 38.18$

(j) $x=40\mu\text{m}$

(i) $t=5\times10^{-8}\text{s}$, $\phi = \frac{0.60652}{1.50663\times10^{-5}} \exp\left[\frac{-3.2\times10^{-8}}{2\times10^{-5}}\right] \approx 32.75$

(ii) $t=10^{-7}\text{s}$, $\phi = \frac{0.35783}{3.545\times10^{-6}} \exp[0] = 38.18$

(iii) $t=2\times10^{-7}\text{s}$, $\phi = \frac{0.35783}{3.545\times10^{-6}} \exp[0] \approx 38.18$
**Dielectric relaxation time constant**

![Diagram](image)

Figure 6.10 The injection of a concentration of holes into a small region at the surface of an n-type semiconductor.

\[ \text{Poisson: } \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \text{ Ohm's: } J = \sigma \vec{E} \]

Continuity: \( \nabla \cdot J = -\frac{\partial \rho}{\partial t} \)

\[ \nabla \cdot J = \sigma \nabla \cdot \vec{E} = \frac{\sigma \rho}{\varepsilon} = -\frac{\partial \rho}{\partial t}, \text{ i.e. } \frac{\partial \rho}{\partial t} + \frac{\sigma}{\varepsilon} \rho = 0 \]

with solution \( \rho(t) = \rho(0) \exp\left( \frac{-t}{\tau_d} \right) \) where \( \tau_d = \frac{\varepsilon}{\sigma} \equiv \tau_d \) is the dielectric relaxation time constant

---

**Ex 6.6 Find the dielectric relaxation time for**
(a) N-type GaAs with \( N_d = 5 \times 10^{15} / \text{cm}^3 \) &
(b) P-type Si with \( N_a = 2 \times 10^{16} / \text{cm}^3 \).

---

(a) For \( N_d = 5 \times 10^{15} / \text{cm}^3 \) in GaAs, from Figure 5.3, \( \mu_e = 7500 \text{ cm}^2 / \text{V-s.} \)
\[ \sigma = \varepsilon \mu_e N_d = \left( 1.6 \times 10^{-19} \right) \left( 7500 \times 5 \times 10^{15} \right) = 6 (\Omega \cdot \text{cm})^{-1} \]
Then \( \tau_d = \frac{\varepsilon}{\sigma} = \frac{(13.1)(8.85 \times 10^{-12})}{6} = 1.93 \times 10^{-12} \text{ s or } \tau_d = 0.193 \text{ ps} \)

(b) For \( N_a = 2 \times 10^{16} / \text{cm}^3 \) in silicon, from Figure 5.3, \( \mu_p = 400 \text{ cm}^2 / \text{V-s.} \)
\[ \sigma = \varepsilon \mu_p N_a = \left( 6.6 \times 10^{-19} \right) \left( 400 \times 2 \times 10^{16} \right) = 1.28 (\Omega \cdot \text{cm})^{-1} \]
Then \( \tau_d = \frac{\varepsilon}{\sigma} = \frac{(11.7)(8.85 \times 10^{-12})}{1.28} = 8.09 \times 10^{-12} \text{ s or } \tau_d = 0.809 \text{ ps} \)
The Haynes-Schockley Experiment (1951 Bell Labs)

- Allows independent measurement of mobility and diffusion coefficient for minority carriers.
- Pulse of holes in n-type bar. Drifts in field and diffuses.
- Monitor concentration. Time to reach point gives mobility. Spreading gives diffusion coefficient.

\[ v_p = \frac{L}{t_p} \quad \mu_p = \frac{v_j}{E} \]

For negligible recombination

\[ \frac{\partial^2 \delta p(x, t)}{\partial t^2} = D_p \frac{\partial^2 \delta p(x, t)}{\partial x^2} \]

\[ \delta p(x, t) = \left[ \frac{\Delta P}{2 \sqrt{\pi D_p t}} \right] \exp \left( \frac{-x^2}{4D_p t} \right) \text{ Gaussian} \]

\[ \delta p = \frac{\Delta P}{2 \sqrt{\pi \tau D_p}} \]

Figure 6.12 (a) The idealized excess minority carrier pulse at terminal A at \( t = 0 \). (b) The excess minority carrier pulse versus time at terminal B for a given applied electric field. (c) The excess minority carrier pulse versus time at terminal B for a smaller electric field.

Figure 6–19 Calculation of \( D_p \) from the shape of the \( \delta p \) distribution after time \( t_p \). No drift or recombination is included.
Calculating $D_p$, $\mu_p$

Pulse arrives at $x = d$ when $\mu_p E_0 = d / t_0 \Rightarrow \mu_p = d / E_0$

Pulse shape at detector as shown: At $t_1$, & $t_2$, magnitude = max/c

If $t_2 - t_1$ small, $t \approx t_1, t_2$ so $\left[ \exp \left( \frac{-t}{\tau_{ph}} \right) \right]$ approx constant, so

$$(d - \mu_p E_p t_1)^2 = 4D_p t_1$$

and $(d - \mu_p E_p t_2)^2 = 4D_p t_2$

Adding gives: $D_p = \frac{(\mu_p E_p)^2 (t_2 - t_1)^2}{8(t_2 + t_1)}$

Also: Area under curve $= S = \text{Kexp} - \frac{t_1}{\tau_{ph}} = \text{Kexp} - \frac{d}{\mu_p E_p \tau_{ph}}$

plotting $\ln S vs 1/E_0$ gives slope $-\frac{d}{\mu_p \tau_{ph}}$ and hence find $\tau_{ph}$

---

Quasi-Fermi levels

- True Fermi level $E_F$ only meaningful at equilibrium.
- Define $E_{Fn}$ and $E_{FP}$ as quasi-Fermi levels, steady state analogs of the equilibrium $E_F$.
- Deviation of $E_{Fn}$ and $E_{FP}$ from $E_F$ shows how far concentrations are from equilibrium values.

In thermal equilibrium: $n_0 = n_i \exp \left( \frac{E_F - E_{Fn}}{kT} \right)$ & $p_0 = n_i \exp \left( \frac{E_{Fn} - E_F}{kT} \right)$

In non-equilibrium: $n = n_0 + \delta n = n_i \exp \left( \frac{E_{Fn} - E_F}{kT} \right)$ & $p = p_0 + \delta p = n_i \exp \left( \frac{E_{FP} - E_F}{kT} \right)$
Ex 6.7 Si at 300K with \( N_d = 3 \times 10^{15} / \text{cm}^3 \), \( N_a = 10^{16} / \text{cm}^3 \). Excess carriers generated such that steady state \( \delta n = \delta p = 4 \times 10^{14} / \text{cm}^3 \). (a) Find thermal equilibrium Fermi level wrt \( E_{Fn} \). (b) Find \( E_{Fn} \) & \( E_{fp} \) wrt \( E_{Ff} \).

\[
\begin{align*}
\delta n &= N_d - N_a - 10^{-15} = 7 \times 10^{14} \text{ cm}^{-3} \\
\delta p &= \frac{n^n}{p^p} (1.5 \times 10^{16})  = 2.214 \times 10^4 \text{ cm}^{-3} \\
&
\end{align*}
\]

(a) In thermal equilibrium,
\[
E_{Fp} - E_F = kT \ln \left( \frac{N_d}{\delta n} \right) = -0.0259 \ln \left( \frac{7 \times 10^{14}}{1.5 \times 10^{16}} \right) = -0.33808 \text{ eV}
\]

(b) Quasi-Fermi levels,
\[
\begin{align*}
E_{Fp} - E_F &= kT \ln \left( \frac{N_d + \delta n}{\delta p} \right) = -0.0259 \ln \left( \frac{7 \times 10^{14} + 4 \times 10^{14}}{1.5 \times 10^{16}} \right) = -0.33952 \text{ eV} \\
E_{Fp} - E_F &= kT \ln \left( \frac{\delta p + \delta p}{\delta n} \right) = -0.0259 \ln \left( \frac{3.214 \times 10^4 + 4 \times 10^4}{1.5 \times 10^{16}} \right) = 0.26395 \text{ eV}
\end{align*}
\]

Gradients in the quasi-Fermi Levels

- Drift and diffusion imply gradient in the steady state quasi-Fermi level.
- Currents proportional to gradient.

\[
\begin{align*}
J_n (x) &= \mu_n n(x) \frac{dE_{Fn}}{dx} \quad \text{or} \quad J_n (x) = \frac{\sigma_n}{q} \frac{dE_{Fn}}{dx} \\
J_p (x) &= \mu_p p(x) \frac{dE_{Fp}}{dx} \quad \text{or} \quad J_p (x) = \frac{\sigma_p}{q} \frac{dE_{Fp}}{dx}
\end{align*}
\]
Excess carrier recombination lifetime: Shockley-Read-Hall theory (Recombination centers)

Figure 6.16 | The four basic trapping and emission processes for the case of an acceptor-type trap.

Surface states

Figure 6.17 | Distribution of surface states within the forbidden bandgap.

In bulk: \[ R = \frac{\dot{\phi}_p}{\tau_{p0}} = \frac{\dot{\phi}_n}{\tau_{p0}} \]

At the surface: \[ R_s = \frac{\dot{\phi}_p}{\tau_{p0S}} \]

\[ \tau_{p0S} < \tau_{p0} \] so \[ \dot{\phi}_s < \dot{\phi}_n \]

Figure 6.18 | Steady-state excess hole concentration versus distance from a semiconductor surface.
**Ex 6.9** See Example 6.9, where $x=0$ at the N-type semiconductor surface. (a) Find the steady-state excess carrier concentration as a function of $x$, the distance from the surface, for $\delta p_B = 10^{14}/\text{cm}^3$, $\tau_{p^0} = 10^{-6}\text{s}$, $\tau_{pS} = 0\text{s}$, $D_p = 10\text{cm}^2/\text{s}$, with no applied field. (b) What is the recombination rate for carriers at the surface?

(a) For $\tau_{p^0} = 0 \Rightarrow \phi_p = -\phi_{pS} \left( \frac{\tau_{p^0}}{\tau_{p^0}} \right) = 0$

From Equation (6.109), $\phi(x) = g \tau_{p^0} + A e^{-x/\tau_{p^0}}$

As $x \to \infty$, $\phi = g \tau_{p^0} = 10^{4}\text{ cm}^{-1} \Rightarrow A = 0$

As $x \to 0$, $\phi = 0 \Rightarrow B = -g \tau_{p^0}$

Then $\phi(x) = g \tau_{p^0} \left( 1 - e^{-x/\tau_{p^0}} \right)$

(b) $\phi(x) = 0$

(c) $R = \frac{\phi(0)}{\tau_{p^0}} = \frac{\phi(0)}{0} = R = \infty$

Note: $\phi(0) = 0$ is a result of $R = \infty$.

---

**Surface recombination velocity**

Excess carrier concentration gradient at the surface, hence diffusion towards surface for recombination

$$-D_p \left[ n_0 \cdot \frac{d(\phi_p)}{dx} \right]_{\text{surf}} = s \phi_p \Bigg|_{\text{surf}}$$

$$\phi_{\text{surf}} = \phi_p(0) = g \tau_{p^0} + B$$

$$d(\phi_p) \Bigg|_{\text{surf}} = \frac{d(\phi_p)}{dt} \Bigg|_{t=0} = -\frac{B}{L_p}$$

$$-\frac{B}{L_p} \frac{D_p}{L_p} = s(g \tau_{p^0} + B) \Rightarrow B = \frac{-sg \tau_{p^0}}{s + D_p / L_p}$$

and $\phi_p(x) = g \tau_{p^0} \left( 1 - \frac{s}{s + D_p / L_p} \exp \left( -\frac{x}{L_p} \right) \right)$
Ex 6.10 (a) Find \( \delta p(x) \) for (i) \( s=\infty \) and (ii) \( s=0 \).
(b) What does (i) an infinite surface recombination velocity \( (s=\infty) \)
and (ii) a zero surface recombination velocity \( (s=0) \) imply?

\[
\delta p(x) = g' \tau_{p0} \left( 1 - \frac{se^{-x/L_p}}{D_pL_p} \right)
\]

(i) For \( s \to \infty \), \( \delta p(x) = g' \tau_{p0} (1 - e^{-x/L_p}) \)
(ii) For \( s \to 0 \), \( \delta p(x) = g' \tau_{p0} \)

(b) 
(i) For \( s \to \infty \), \( \delta p(0) = 0 \)
(ii) For \( s \to 0 \), \( \delta p(0) = g' \tau_{p0} \to \delta p(x) = \text{constant} \)

Assignment #3

<table>
<thead>
<tr>
<th>5.7</th>
<th>6.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.21</td>
<td>6.14</td>
</tr>
<tr>
<td>5.35</td>
<td>6.15</td>
</tr>
<tr>
<td>5.49</td>
<td>6.31</td>
</tr>
</tbody>
</table>