EE415/515 Fundamentals of Semiconductor Devices Fall 2012

Lecture 6:
Excess Carriers
(Chapter 6)

Excess generation/recombination

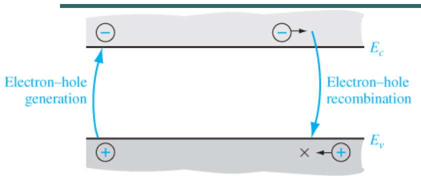


Figure 6.1 | Electron–hole generation and recombination.

At thermal equilibrium, band-to-band EHP generation/recombination …… Carrier concentrations constant, so

$$G_{n0}=G_{p0}=R_{n0}=R_{p0}$$

Excess carriers:

$n=n_0+\delta n,\ p=p_0+\delta p$

Excess generation

$$g_n^{\prime}=g_p^{\prime}$$

Non-equilibrium

$$np \neq n_0p_0=n_i^2$$

Table 6.1 | Relevant notation used in Chapter 6

Symbol	Definition
n_0, p_0	Thermal-equilibrium electron and hole concentrations (independent of time and also usually position)
n, p	Total electron and hole concentrations (may be functions of time and/or position)
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or position)
$g_n^{\prime}$, $g_p^{\prime}$	Excess electron and hole generation rates
$R_n^{\prime}$, $R_p^{\prime}$	Excess electron and hole recombination rates
τ_{n0}, τ_{p0}	Excess minority carrier electron and hole lifetimes

Optical carrier generation & recombination

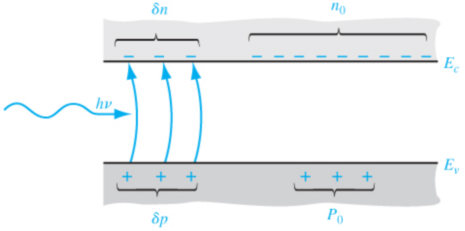
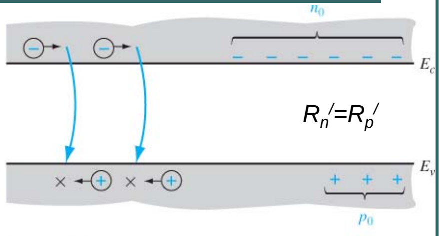


Figure 6.2 | Creation of excess electron and hole densities by photons.



$R_n^{/}=R_p^{/}$

Figure 6.3 | Recombination of excess carriers reestablishing thermal equilibrium.

Electrons in the conduction band recombine with holes in the valence band directly or indirectly.
Direct recombination occurs *spontaneously*, i.e. recombination probability is constant in time.
Electron decay rate at time *t* is proportional to the number of electrons and holes remaining at *t*.
Net rate of change:

$$\frac{dn(t)}{dt} = \alpha_r n_i^2 - \alpha_r n(t)\,p(t)$$

$$\alpha_r n_i^2 = \text{thermal generation rate}$$

Recombination Equations

$n_o p_0 = n_i^2$ where n_0 and p_0 are constant., and $n(t) = n_o + \delta n(t)$ $\quad p(t) = p_0 + \delta p(t)$

$$\frac{d\delta n(t)}{dt} = \alpha_r n_i^{\,2} - \alpha_r [n_o + \delta n(t)][p_0 + \delta p(t)] = -\alpha_r \delta n(t)[(p_0 + n_0) + \delta n(t)] \approx -\alpha_r p_0 \delta n(t)$$

for extrinsic p material => $p_0 >> n_0$ & low level $\delta n(t) << p_0$

with solution $\delta n(t) = \delta n(0)\exp(-\alpha_r p_0 t) = \delta n(0)\exp(-t/\tau_{n0})$

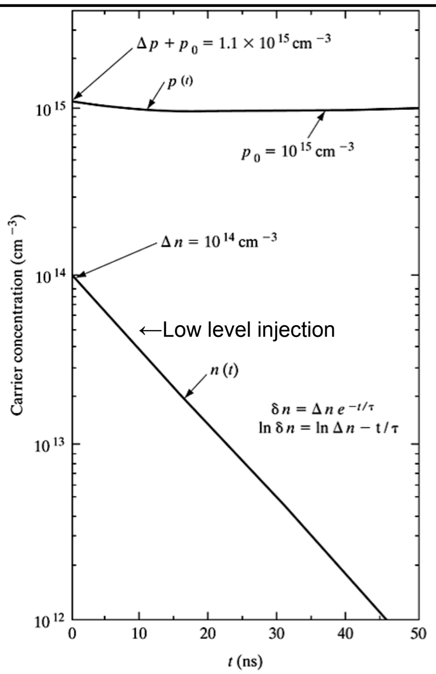
$\tau_{n0} = (\alpha_r p_0)^{-1}$ & $\tau_{p0} = (\alpha_r n_0)^{-1}$ recombination or excess minority carrier lifetimes.

For direct band-to-band recombination $\mathrm{R}_\mathrm{n}^{/} = \mathrm{R}_\mathrm{p}^{/} = \dfrac{-d(\delta n(t))}{dt} = \alpha_r p_0 \delta n(t) = \dfrac{\delta n(t)}{\tau_{n0}}$ in p-type

and $\mathrm{R}_\mathrm{n}^{/} = \mathrm{R}_\mathrm{p}^{/} = \dfrac{-d(\delta p(t))}{dt} = \alpha_r n_0 \delta p(t) = \dfrac{\delta p(t)}{\tau_{p0}}$ in n-type

In general for low level injection: $\tau_{\mathrm{n,p}} = [\alpha_r (n_0 + p_0)]^{-1}$ for $\delta p, \delta n << n_0, p_0$

Decay of excess carriers through recombination



Ex 6.1 Excess carriers generated uniformly to concentration $\delta n(0)=10^{15}/\mathrm{cm}^3$ until generation stopped at t=0.
Calculate the recombination rates for the excess carriers for: (a) t=0, (b) t=1μs (c) t=4μs and (d) t=10 μs, for τ_{n0}=1μs.

$$R'_n = \frac{\delta n}{\tau_{n0}} = \frac{10^{15} e^{-t/10^{-6}}}{10^{-6}}\ \mathrm{cm}^{-3}\,\mathrm{s}^{-1}$$

$$\text{For } t = 0, \quad R'_n = \frac{10^{15}}{10^{-6}} = 10^{21}\ \mathrm{cm}^{-3}\,\mathrm{s}^{-1}$$

$$t = 1\ \mu\mathrm{s}, \ R'_n = \frac{10^{15} e^{-1/1}}{10^{-6}} = 3.68 \times 10^{20}\ \mathrm{cm}^{-3}\,\mathrm{s}^{-1}$$

$$t = 4\ \mu\mathrm{s}, \ R'_n = \frac{10^{15} e^{-4/1}}{10^{-6}} = 1.83 \times 10^{19}\ \mathrm{cm}^{-3}\,\mathrm{s}^{-1}$$

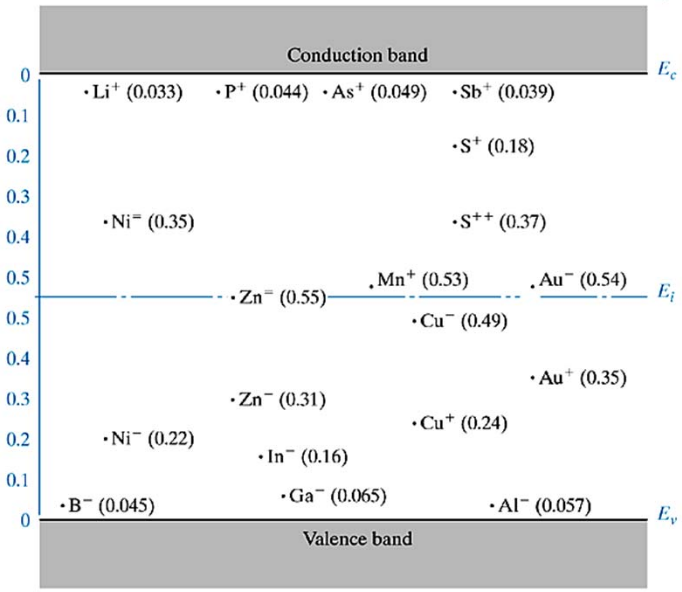
$$t = 10\ \mu\mathrm{s}, \ R'_n = \frac{10^{15} e^{-10/1}}{10^{-6}} = 4.54 \times 10^{16}\ \mathrm{cm}^{-3}\,\mathrm{s}^{-1}$$

Indirect Recombination, trapping

- For Group IV semiconductors the probability of direct recombination is small. (Why?)
- Most recombination occurs through recombination levels in the band gap. Energy dissipated as heat (aka phonons or lattice vibrations) not photons.
- Any impurity or defect which can trap one type of charge and then the next can serve as a *recombination* center.

Impurity energy levels

- Donor levels indicated by a + sign.
- Indicates whether it is positive (donor) or negative (acceptor) when ionized.
- Note that some impurities introduce multiple levels, e.g. Zn, which can accept two electrons.



Indirect Recombination, Trapping

- Example: assume recombination level E_r is below E_F and is filled at equilibrium. Then
  - Hole can be captured. (electron falls from E_r to E_v)
  - Then electron can be captured (electron falls from E_c to E_r..
  - One EHP lost. Excess energy given up as heat (phonons.)
- What if probability of capture is not equal for electrons and holes?
- As before, the first captured carrier can return to its band before being annihilated.
  - Delays process.
  - Called *temporary trapping.*
  - Defect or impurity called a *trapping center* if this event is more likely.
- If it is more likely that the annihilation will occur *before* re-excitation of the original trapped carrier
  - Called a *recombination center.*
- Deep levels are usually slower (more energy required.)

Continuity equations

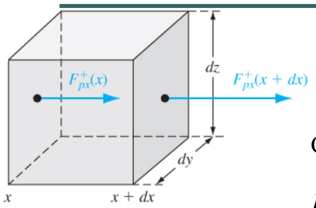


Figure 6.4 | Differential volume showing *x* component of the hole-particle flux.

Consider holes flowing into and out of cube

$$F^+_{px}(x+dx) = F^+_{px}(x) + \frac{\partial F^+_{px}(x)}{\partial x}dx \text{ in number/cm}^2 - s$$

$$\text{So } \frac{\partial p}{\partial t}dxdydz = \left[F^+_{px}(x) - F^+_{px}(x+dx)\right]dydz = -\frac{\partial F^+_{px}(x)}{\partial x}dxdydz$$

$$\Rightarrow -\frac{\partial F^+_{px}(x)}{\partial x}dxdydz + g_p dxdydz - \frac{p}{\tau_{pt}}dxdydz \text{ with gen/recomb}$$

$$\text{So } \frac{\partial p}{\partial t} = -\frac{\partial F^+_{px}(x)}{\partial x} + g_p - \frac{p}{\tau_{pt}} \text{ and } \frac{\partial n}{\partial t} = -\frac{\partial F^-_{nx}(x)}{\partial x} + g_n - \frac{n}{\tau_{nt}}$$

Time dependent diffusion

$$J_p = e\mu_p pE - eD_p \frac{\partial p}{\partial x} \text{ and } J_n = e\mu_n nE + eD_n \frac{\partial n}{\partial x}$$

$$\text{Hole flux } F_p^+ = \frac{J_p}{+e} = \mu_p pE - D_p \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}} = -\mu_p \frac{\partial(pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$= D_p \frac{\partial^2 p}{\partial x^2} - \mu_p \left[p\frac{\partial E}{\partial x} + E\frac{\partial p}{\partial x} \right] + g_p - \frac{p}{\tau_{pt}}$$

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2 (\delta p)}{\partial x^2} - \mu_p \left[p\frac{\partial E}{\partial x} + E\frac{\partial(\delta p)}{\partial x} \right] + g_p - \frac{p}{\tau_{pt}}$$

$$\text{Similarly } \frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n \left[n\frac{\partial E}{\partial x} + E\frac{\partial(\delta n)}{\partial x} \right] + g_n - \frac{n}{\tau_{nt}}$$

Ambipolar transport

Applied field E_{app} tends to separate charges. But charge separation creates restoring field E_{int}, which tends to hold pos/neg charges together. The electrons and holes may then drift/diffuse together with a single effective mobility/diffusion constant → ambipolar diffusion/transport.

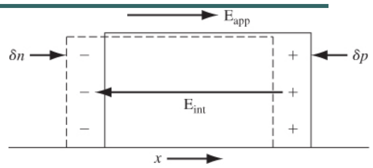


Figure 6.5 | The creation of an internal electric field as excess electrons and holes *tend* to separate.

$E = E_{app} + E_{int}$

$$\text{Poisson's equation : } \nabla . E_{int} = \frac{\partial E_{int}}{\partial x} = \frac{e(\delta p - \delta n)}{\varepsilon_s}$$

$$\text{Assume } |E_{int}| << |E_{app}|$$

$$\text{and } \delta p \approx \delta n \text{ (very small} |\delta p - \delta n| \to E_{int})$$

$$\text{Generation } g = g_n = g_p$$

$$\text{and recombination } R = R_n = \frac{n}{\tau_{nt}} = R_p = \frac{p}{\tau_{pt}}$$

$$\text{So setting } \delta p = \delta n : D_p \frac{\partial^2 (\delta n)}{\partial x^2} - \mu_p \left(E\frac{\partial(\delta n)}{\partial x} + p\frac{\partial E}{\partial x} \right) + g - R = \frac{\partial(\delta n)}{\partial t}$$

$$\text{and } D_n \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n \left(E\frac{\partial(\delta n)}{\partial x} + n\frac{\partial E}{\partial x} \right) + g - R = \frac{\partial(\delta n)}{\partial t}$$

$$D_p\mu_n n. \frac{\partial^2 (\delta n)}{\partial x^2} - \mu_p\mu_n n. \left(E\frac{\partial(\delta n)}{\partial x} + p\frac{\partial E}{\partial x} \right) + \mu_n n.(g - R) = \mu_n n. \frac{\partial(\delta n)}{\partial t}$$

$$D_n\mu_p p. \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n\mu_p p. \left(E\frac{\partial(\delta n)}{\partial x} + n\frac{\partial E}{\partial x} \right) + \mu_p p.(g - R) = \mu_p p. \frac{\partial(\delta n)}{\partial t}$$

Ambipolar transport (cont’d)

Add equations to eliminate $\frac{\partial E}{\partial \mathrm{x}}$

$$\left(D_p\mu_n n + D_n\mu_p p\right)\frac{\partial^2(\delta n)}{\partial x^2} + \mu_p\mu_n(p-n)E\frac{\partial(\delta n)}{\partial x} + \left(\mu_n n + \mu_p p\right)(g-R) = \left(\mu_n n + \mu_p p\right)\frac{\partial(\delta n)}{\partial t}$$

$$D'\frac{\partial^2(\delta n)}{\partial x^2} + \mu' E\frac{\partial(\delta n)}{\partial x} + (g-R) = \frac{\partial(\delta n)}{\partial t} \quad \text{where } \mu' = \frac{\mu_p\mu_n(p-n)}{\mu_n n + \mu_p p}$$

$$\text{and } \mathrm{D}' = \frac{D_p\mu_n n + D_n\mu_p p}{\mu_n n + \mu_p p} = \frac{D_p D_n(e/kT)n + D_n D_p(e/kT)p}{D_n(e/kT)n + D_p(e/kT)p} = \frac{D_p D_n(n+p)}{D_n n + D_p p}$$

D' = ambipolar diffusion coefficient, and μ' = ambipolar mobility

Ambipolar transport: extrinsic, low level injection

For extrinsic p-type and low level injection: $\mathrm{p}_0 >> n_0$ and $\delta\mathrm{n} << \mathrm{p}_0$

$$\mathrm{D}' = \frac{D_p D_n[(n_0+\delta n)+(p_0+\delta p)]}{D_n(n_0+\delta n)+D_p(p_0+\delta p)} \Rightarrow \frac{D_p D_n[p_0]}{D_p(p_0)} \Rightarrow D_n$$

$$\mu' = \frac{\mu_p\mu_n(p-n)}{\mu_n n + \mu_p p} \Rightarrow \frac{\mu_p\mu_n(p)}{\mu_p p} \Rightarrow \mu_{\mathrm{n}}$$

$$g - R = g_n - R_n = (G_{n0} + g'_n) - (R_{n0} + R'_n) = g'_n - R'_n \quad (\text{since } G_{n0} = R_{n0})$$

$$\mathrm{g - R} = g'_n - \frac{\delta n}{\tau_n} \text{ for electrons, and } \mathrm{g - R} = g'_p - \frac{\delta p}{\tau_p} \text{ for holes}$$

$$D_n\frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E\frac{\partial(\delta n)}{\partial x} + (g' - \frac{\delta n}{\tau_{n0}}) = \frac{\partial(\delta n)}{\partial t} \quad \text{for p-type}$$

$$D_p\frac{\partial^2(\delta p)}{\partial x^2} + \mu_p E\frac{\partial(\delta p)}{\partial x} + (g' - \frac{\delta p}{\tau_{p0}}) = \frac{\partial(\delta p)}{\partial t} \quad \text{for n-type}$$

Ambipolar transport equations

Table 6.2 | Common ambipolar transport equation simplifications

Specification	**Effect**
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate)	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

Ex 6.2 N-type GaAs N_d=10^{16}/cm^3, with 10^{14} EHPs/cm^3 uniformly created at t=0. Find times at which the minority carrier hole concentration reaches (a) 1/e and (b) 10% of the initial value, if τ_{p0}=50ns.

See Example 6.2 :

For n - type: $D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t}$

For uniform excess hole distribution : $\frac{\partial(\delta p)}{\partial x} = 0 = \frac{\partial^2(\delta p)}{\partial x^2}$

and $g' = 0$ for $t > 0$, so $\frac{\partial(\delta p)}{\partial t} = -\frac{\delta p}{\tau_{p0}} \Rightarrow \delta p(t) = \delta p(0)\exp - \frac{t}{\tau_{p0}}$

(a) $10^{14} e^{-t/50} = 10^{14} e^{-1}$

$\Rightarrow t = 50\text{ ns}$

(b) $10^{14} e^{-t/50} = 10^{13}$

$t = (50 \times 10^{-9}) \ln\left(\frac{10^{14}}{10^{13}}\right) = 1.15 \times 10^{-7}\text{ s}$

or $t = 115\text{ ns}$

Ex 6.3 For n-type Si at T=300K with N_d=5x10^{16}/cm^3. Assume uniform g$^{/}$=5x10^{21}/cm^{3}s^{-1} for t≥0 and τ_{p0}=10^{-7}s.
(a) Find δp(t) at (i) t=0, (ii) t=10^{-7}s, (iii) t=5x10^{-7}s, and (iv) t⟶∞.
(b) Is low level injection maintained?

See Example 6.3 :

$$\text{For n-type: } D_p\frac{\partial^2(\delta p)}{\partial x^2}-\mu_p E\frac{\partial(\delta p)}{\partial x}+g^{/}-\frac{\delta p}{\tau_{p0}}=\frac{\partial(\delta p)}{\partial t}$$

$$\text{For uniform excess hole distribution : } \frac{\partial(\delta p)}{\partial x}=0=\frac{\partial^2(\delta p)}{\partial x^2}$$

$$\text{so } \frac{\partial(\delta p)}{\partial t}=\text{g}^{/}-\frac{\delta p}{\tau_{p0}}\Rightarrow \delta p(t)=g^{/}\tau_{p0}(1-\exp-\frac{t}{\tau_{p0}})$$

(a) $\delta p(t)=g'\tau_{p0}\left(1-e^{-t/\tau_{p0}}\right)=\left(5\times10^{21}\right)\left(10^{-7}\right)\left(1-e^{-t/\tau_{p0}}\right)=5\times10^{14}\left(1-e^{-t/\tau_{p0}}\right)$

(i) $\delta p(0)=5\times10^{14}\left(1-e^{-0}\right)=0$

(ii) $\delta p\left(10^{-7}\right)=5\times10^{14}\left(1-e^{-1/1}\right)=3.16\times10^{14}\text{ cm}^{-3}$

(iii) $\delta p\left(5\times10^{-7}\right)=5\times10^{14}\left(1-e^{-5/1}\right)=4.966\times10^{14}\text{ cm}^{-3}$

(iv) $\delta p(\infty)=5\times10^{14}\left(1-e^{-\infty}\right)=5\times10^{14}\text{ cm}^{-3}$

(b) $\delta p(\max)=5\times10^{14}=(0.01)N_d$

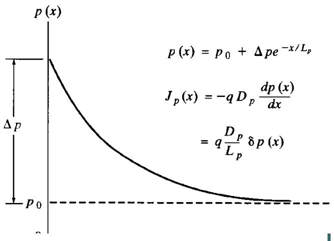
Yes, low-injection condition is met.

Steady State Carrier Injection; Diffusion Length

- In steady state the concentrations of excess carriers do not change.
- We should recognize this as exponential decay.

$$\frac{d^2\delta n}{dx^2}=\frac{\delta n}{D_n\tau_n}\equiv\frac{\delta n}{L_n^2} \quad \text{Electron diffusion length } L_n\equiv\sqrt{D_n\tau_n}$$

$$\frac{d^2\delta p}{dx^2}=\frac{\delta p}{D_p\tau_p}\equiv\frac{\delta p}{L_p^2} \quad \text{Hole diffusion length } L_p\equiv\sqrt{D_p\tau_p}$$



$$p(x)=p_0+\Delta p e^{-x/L_p}$$

$$J_p(x)=-qD_p\frac{dp(x)}{dx}$$

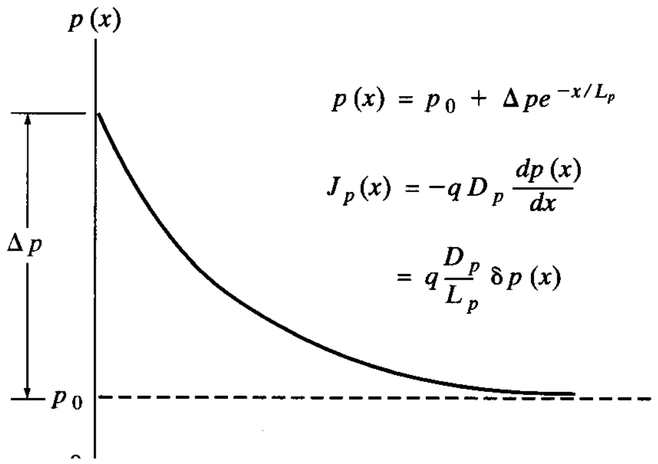
$$=q\frac{D_p}{L_p}\delta p(x)$$

Assume injection of excess holes N - type at x = 0

$\delta p(x)=\delta p(0)\ \exp(-x/L_p)$

At x = L_p holes are reduced to 1/e. Average distance before recombining.

Diffusion Length



$$p\,(x) = p_0 + \Delta p e^{-x/L_p}$$

$$J_p(x) = -q D_p \frac{dp\,(x)}{dx}$$

$$= q \frac{D_p}{L_p} \delta p\,(x)$$

Ex 6.4 P-type Si, T=300K, N_a=5x10^{16}/cm^3, τ_{n0}=5x10^{-7}s, D_n=25cm^2/s, with electrons being generated at x=0, so δn(0)=10^{15}/cm^3.
(a) Calculate diffusion length L_n. (b) Determine δn at
(i) x= 0, (ii) x= +30µm (iii) x= -50µm (iv) x= +85µm & (v) x= 120µm.

See Example 6.4 :

$$\text{For p-type: } D_n \frac{\partial^2(\delta n)}{\partial x^2} - \mu_n E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$$

$$\text{For steady state } \frac{\partial(\delta n)}{\partial t} = 0,\ E = 0,\ g' = 0 \text{ for } x \neq 0, \text{ we have}$$

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} - \frac{\delta n}{\tau_{n0}} = 0 \ . \Rightarrow \frac{\partial^2(\delta n)}{\partial x^2} - \frac{\delta n}{D_n \tau_{n0}} = 0 \Rightarrow \frac{\partial^2(\delta n)}{\partial x^2} - \frac{\delta n}{L_n{}^2} = 0$$

$$\text{with solution } \delta n(x) = A\exp - \frac{x}{L_n} + B\exp \frac{x}{L_n}$$

$$\text{For } x > 0, B = 0 \text{ for } x \to \infty, \text{ so } \delta n(x) = \delta n(0)\exp - \frac{x}{L_n}$$

$$\text{For } x < 0, A = 0 \text{ for } x \to -\infty, \text{ so } \delta n(x) = \delta n(0)\exp + \frac{x}{L_n}$$

Ex 6.4 P-type Si, T=300K, $N_a=5\text{x}10^{16}/\text{cm}^3$, $\tau_{n0}=5\text{x}10^{-7}\text{s}$, $D_n=25\text{cm}^2/\text{s}$, with electrons being generated at x=0, so $\delta n(0)=10^{15}/\text{cm}^3$. (a) Calculate diffusion length L_n. (b) Determine δn at (i) x= 0, (ii) x= +30μm (iii) x= -50μm (iv) x= +85μm & (v) x= 120μm.

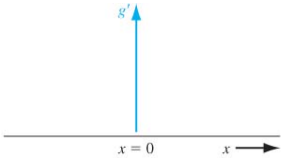


Figure 6.6 | Steady-state generation rate at $x = 0$.

(a) $L_n = \sqrt{D_n \tau_{n0}} = \left[(25)(5\times10^{-7})\right]^{1/2} = 3.536\times10^{-3}\,\text{cm}$
or $L = 35.36\,\mu\text{m}$

(b) $\delta n = 10^{15} e^{-x/L_n}\ (x \ge 0)$ or $\delta n = 10^{15} e^{+x/L_n}\ (x \le 0)$
(i) $\delta n = 10^{15} e^{0} = 10^{15}\,\text{cm}^{-3}$
(ii) $\delta n = 10^{15} e^{-30/35.36} = 4.28\times10^{14}\,\text{cm}^{-3}$
(iii) $\delta n = 10^{15} e^{-50/35.36} = 2.43\times10^{14}\,\text{cm}^{-3}$
(iv) $\delta n = 10^{15} e^{-85/35.36} = 9.04\times10^{13}\,\text{cm}^{-3}$
(v) $\delta n = 10^{15} e^{-120/35.36} = 3.36\times10^{13}\,\text{cm}^{-3}$

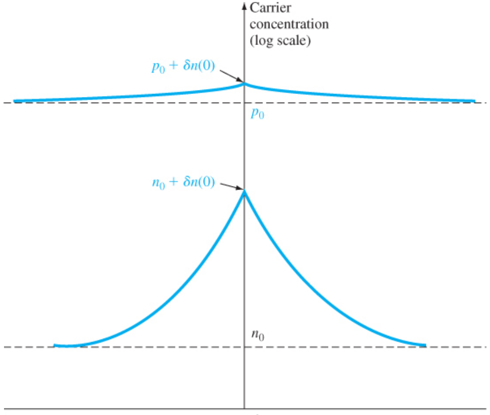


Figure 6.7 | Steady-state electron and hole concentrations for the case when excess electrons and holes are generated at $x = 0$.

Ex 6.5 N-type semiconductor with constant applied field E_0 in +x direction. Finite number of EHPs generated at x=0 at t=0, and then $g'=0$ for t>0. $D_p=10\text{cm}^2/\text{s}$, $\tau_{p0}=10^{-7}\text{s}$, $\mu_p=400\text{cm}^2/\text{V-s}$, & $E_0=100\text{V/cm}$. Find δp for (a) $t=10^{-7}\text{s}$ at (i) x=20μm, x=40μm, & (iii) x=60μm., and (b) x=40μm at (i) $t=5\text{x}10^{-8}\text{s}$, (ii) $t=10^{-7}\text{s}$, and (iii) $t=2\text{x}10^{-7}\text{s}$.

See Example 6.5 :

For n - type: $D_p \dfrac{\partial^2(\delta p)}{\partial x^2} - \mu_p E_0 \dfrac{\partial(\delta p)}{\partial x} - \dfrac{\delta p}{\tau_{p0}} = \dfrac{\partial(\delta p)}{\partial t}$

with solution of the form $\delta\text{p}(\text{x,t}) = \text{p}'(x,t)\exp-\dfrac{t}{\tau_{p0}}$

Substitute $\left[D_p \dfrac{\partial^2 \text{p}'(x,t)}{\partial x^2} - \mu_p E_0 \dfrac{\partial \text{p}'(x,t)}{\partial x} - \dfrac{\text{p}'(x,t)}{\tau_{p0}}\right]\exp-\dfrac{t}{\tau_{p0}} = \left[\dfrac{\partial \text{p}'(x,t)}{\partial x} - \dfrac{\text{p}'(x,t)}{\tau_{p0}}\right]\exp-\dfrac{t}{\tau_{p0}}$

$$D_p \frac{\partial^2 \text{p}'(x,t)}{\partial x^2} - \mu_p E_0 \frac{\partial \text{p}'(x,t)}{\partial x} = \frac{\partial \text{p}'(x,t)}{\partial x}$$

has solution $\text{p}'(\text{x,t}) = \dfrac{1}{\sqrt{4\pi D_p t}}\exp-\dfrac{(x-\mu_p E_0 t)^2}{4D_p t}$

$$\delta p(x,t) = \frac{e^{-\frac{t}{\tau_{p0}}}}{\sqrt{4\pi D_p t}}\exp-\frac{(x-\mu_p E_0 t)^2}{4D_p t}$$

Ex 6.5 N-type semiconductor with constant applied field E_0 in +x direction. Finite number of EHPs generated at x=0 at t=0, and then $g'=0$ for t>0. $D_p=10\text{cm}^2/\text{s}$, $\tau_{p0}=10^{-7}\text{s}$, $\mu_p=400\text{cm}^2/\text{V-s}$, & $E_0=100\text{V/cm}$. Find δp for (a) $t=10^{-7}$s at (i) x=20μm, x=40μm, & (iii) x=60μm., and (b) x=40μm at (i) $t=5\text{x}10^{-8}$s, (ii) $t=10^{-7}$s, and (iii) $t=2\text{x}10^{-7}$s.

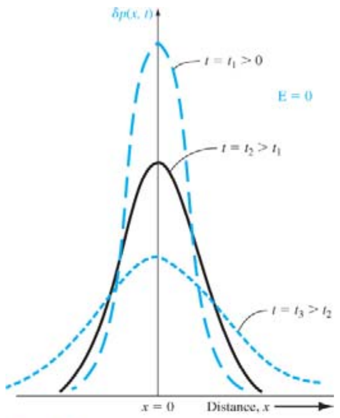


Figure 6.8 | Excess hole concentration versus distance at various times for zero applied electric field.

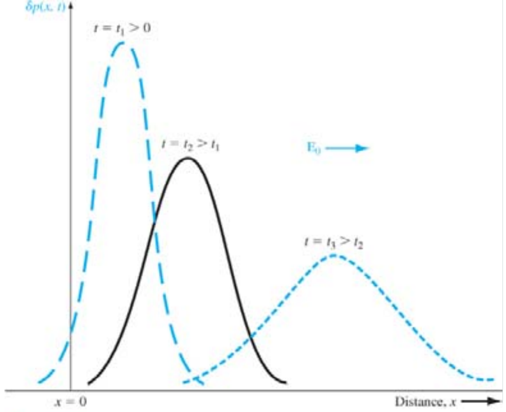


Figure 6.9 | Excess hole concentration versus distance at various times for a constant applied electric field.

Ex 6.5 N-type semiconductor with constant applied field E_0 in +x direction. Finite number of EHPs generated at x=0 at t=0, and then $g'=0$ for t>0. $D_p=10\text{cm}^2/\text{s}$, $\tau_{p0}=10^{-7}\text{s}$, $\mu_p=400\text{cm}^2/\text{V-s}$, & $E_0=100\text{V/cm}$. Find δp for (a) $t=10^{-7}$s at (i) x=20μm, x=40μm, & (iii) x=60μm., and (b) x=40μm at (i) $t=5\text{x}10^{-8}$s, (ii) $t=10^{-7}$s, and (iii) $t=2\text{x}10^{-7}$s.

$$\delta p(x,t)=\frac{e^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}}\exp\left[\frac{-(x-\mu_p E_0 t)^2}{4D_p t}\right]$$

(a) $\mu_p E_0 t=(400)(100)(10^{-7})=4\times10^{-3}\,\text{cm}\ =40\,\mu\text{m}$

(i) $x=20\,\mu\text{m},\quad \delta p=\frac{0.36788}{3.545\times10^{-3}}\exp\left[\frac{-(-2\times10^{-3})^2}{4\times10^{-6}}\right]=38.18$

(ii) $x=40\,\mu\text{m},\quad \delta p=\frac{0.36788}{3.545\times10^{-3}}\exp[0]\ =103.8$

(iii) $x=60\,\mu\text{m},\quad \delta p=\frac{0.36788}{3.545\times10^{-3}}\exp\left[\frac{-(2\times10^{-3})^2}{4\times10^{-6}}\right]\quad \delta p=38.18$

(b) $x=40\,\mu\text{m}$

(i) $t=5\times10^{-8}\,\text{s},\quad \delta p=\frac{0.60653}{2.50663\times10^{-3}}\exp\left[\frac{-(2\times10^{-3})^2}{2\times10^{-6}}\right]=32.75$

(ii) $t=10^{-7}\,\text{s},\quad \delta p=\frac{0.36788}{3.545\times10^{-3}}\exp[0]\ =103.8$

(iii) $t=2\times10^{-7}\,\text{s},\quad \delta p=\frac{0.1353}{5.013\times10^{-3}}\exp\left[\frac{-(-4\times10^{-3})^2}{8\times10^{-6}}\right]=3.65$

Dielectric relaxation time constant

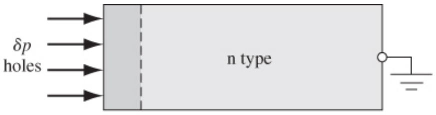


Figure 6.10 | The injection of a concentration of holes into a small region at the surface of an n-type semiconductor.

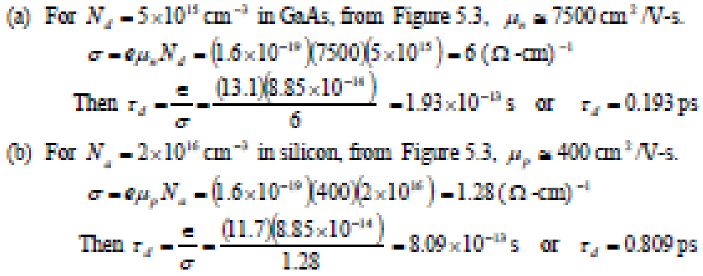
$$Poisson:\ \nabla \bullet E = \frac{\rho}{\varepsilon}\quad Ohm's:\ J = \sigma E\quad \text{Continuity}:\ \nabla \bullet J = -\frac{\partial \rho}{\partial t}$$

$$\nabla \bullet J = \sigma \nabla \bullet E = \frac{\sigma\rho}{\varepsilon} = -\frac{\partial \rho}{\partial t} = -\frac{d\rho}{dt},\ \text{i.e.}\ \frac{d\rho}{dt} + \frac{\sigma}{\varepsilon}\rho = 0$$

$$\text{with solution}\ \ \rho(t) = \rho(0)\exp -\frac{t}{\tau_d}\ \ \text{where}\ \ \tau_d = \frac{\varepsilon}{\sigma} = \varepsilon\rho$$

$$\tau_d = \text{dielectric relaxation time constant}$$

Ex 6.6 Find the dielectric relaxation time for (a) N-type GaAs with N_d=5x10^{15}/cm^3 & (b) P=type Si with N_a=2x10^{16}/cm^3.



(a) For $N_d = 5\times10^{15}\ \text{cm}^{-3}$ in GaAs, from Figure 5.3, $\mu_n \cong 7500\ \text{cm}^2/\text{V-s}$.

$$\sigma = e\mu_n N_d = (1.6\times10^{-19})(7500)(5\times10^{15}) = 6\ (\Omega\text{-cm})^{-1}$$

$$\text{Then}\ \tau_d = \frac{\varepsilon}{\sigma} = \frac{(13.1)(8.85\times10^{-14})}{6} = 1.93\times10^{-13}\ \text{s}\quad \text{or}\quad \tau_d = 0.193\ \text{ps}$$

(b) For $N_a = 2\times10^{16}\ \text{cm}^{-3}$ in silicon, from Figure 5.3, $\mu_p \cong 400\ \text{cm}^2/\text{V-s}$.

$$\sigma = e\mu_p N_a = (1.6\times10^{-19})(400)(2\times10^{16}) = 1.28\ (\Omega\text{-cm})^{-1}$$

$$\text{Then}\ \tau_d = \frac{\varepsilon}{\sigma} = \frac{(11.7)(8.85\times10^{-14})}{1.28} = 8.09\times10^{-13}\ \text{s}\quad \text{or}\quad \tau_d = 0.809\ \text{ps}$$

Haynes-Shockley experiment

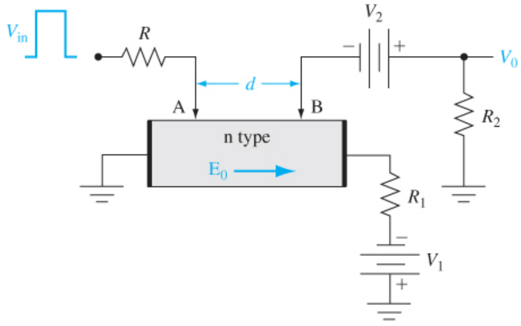


Figure 6.11 | The basic Haynes–Shockley experimental arrangement.

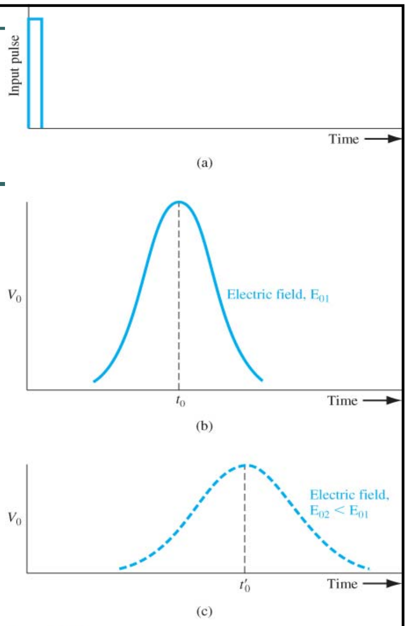


Figure 6.12 | (a) The idealized excess carrier pulse at terminal A at $t = 0$. (b) The excess carrier pulse versus time at terminal B for a given applied electric field. (c) The excess carrier pulse versus time at terminal B for a smaller applied electric field.

The Haynes-Schockley Experiment (1951 Bell Labs)

- Allows independent measurement of mobility and diffusion coefficient for minority carriers.
- Pulse of holes in n-type bar. Drifts in field and diffuses.
- Monitor concentration. Time to reach point gives mobility. Spreading gives diffusion coefficient.

$$v_d = \frac{L}{t_d} \qquad \mu_p = \frac{v_d}{\mathbf{E}}$$

For negligible recombination $\frac{\partial \delta \mathrm{p(x,t)}}{\partial \mathrm{t}} = D_p \frac{\partial^2 \delta p(x,t)}{\partial x^2}$

$$\delta p(x,t) = \left[\frac{\Delta P}{2\sqrt{\pi D_p t}}\right] \exp\left(\frac{-x^2}{4D_p t}\right) \quad \text{Gaussian}$$

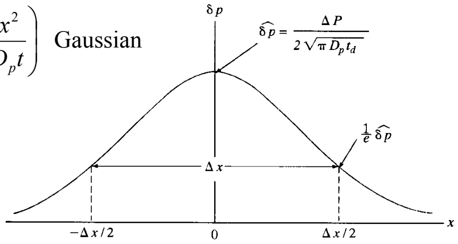


Figure 4–19
Calculation of D_p from the shape of the δp distribution after time t_d. No drift or recombination is included.

Calculating D_p, μ_p

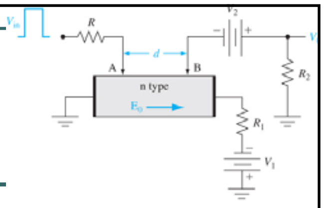


Figure 6.11 | The basic Haynes–Shockley experimental arrangement.

Pulse arrives at x = d when $\mu_p E_0 = d/t_0 \Rightarrow \mu_p = d/E_0 t_0$

Pulse shape at detector as shown: At t_1 & t_2, magnitude = max/e

If $t_2 - t_1$ small, $t \approx t_1, t_2$ so $\left[\dfrac{\exp -\dfrac{t}{\tau_{p0}}}{(4\pi D_p t)^{1/2}}\right]$ approx constant, so

$$(d - \mu_p E_0 t_1)^2 = 4D_p t_1 \text{ and } (d - \mu_p E_0 t_2)^2 = 4D_p t_2$$

Adding gives: $D_p = \dfrac{(\mu_p E_0)^2 (t_2 - t_1)^2}{8(t_2 + t_1)}$

Also: Area under curve $= S = K\exp -\dfrac{t_0}{\tau_{p0}} = K\exp -\dfrac{d}{\mu_p E_0 \tau_{p0}}$

plotting lnS vs $1/E_0$ gives slope $\dfrac{-d}{\mu_p \tau_{p0}}$ and hence find τ_{p0}

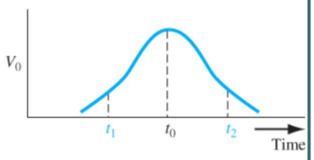


Figure 6.13 | The output excess carrier pulse versus time to determine the diffusion coefficient.

Quasi-Fermi levels

- True Fermi level E_F only meaningful at equilibrium.
- Define E_{Fn} and E_{Fp} as *quasi-Fermi levels,* steady state analogs of the equilibrium E_F.
- Deviation of E_{Fn} and E_{Fp} from E_F shows how far concentrations are from equilibrium values.

In thermal equilibrium: $n_0 = n_i \exp\left(\dfrac{E_F - E_{Fi}}{kT}\right)$ & $p_0 = n_i \exp\left(\dfrac{E_{Fi} - E_F}{kT}\right)$

In non-equilibrium: $n = n_0 + \delta n = n_i \exp\left(\dfrac{E_{Fn} - E_{Fi}}{kT}\right)$ & $p = p_0 + \delta p = n_i \exp\left(\dfrac{E_{Fi} - E_{Fp}}{kT}\right)$

Ex 6.7 Si at 300K with N_d=3x10^{15}/cm^3, N_a=10^{16}/cm^3. Excess carriers generated such that steady state δn=δp=4x10^{14}/cm^3. (a) Find thermal equilibrium Fermi level wrt E_{Fi}. (b) Find E_{Fn} & E_{Fp} wrt E_{Fi}.

$$p_o = N_a - N_d = 10^{16} - 3\times10^{15} = 7\times10^{15}\ \text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{7\times10^{15}} = 3.214\times10^{4}\ \text{cm}^{-3}$$

(a) In thermal equilibrium,

$$E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right) = (0.0259)\ln\left(\frac{7\times10^{15}}{1.5\times10^{10}}\right) = 0.33808\ \text{eV}$$

(b) Quasi-Fermi levels,

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_o+\delta p}{n_i}\right) = (0.0259)\ln\left(\frac{7\times10^{15}+4\times10^{14}}{1.5\times10^{10}}\right) = 0.33952\ \text{eV}$$

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_o+\delta n}{n_i}\right) = (0.0259)\ln\left(\frac{3.214\times10^{4}+4\times10^{14}}{1.5\times10^{10}}\right) = 0.26395\ \text{eV}$$

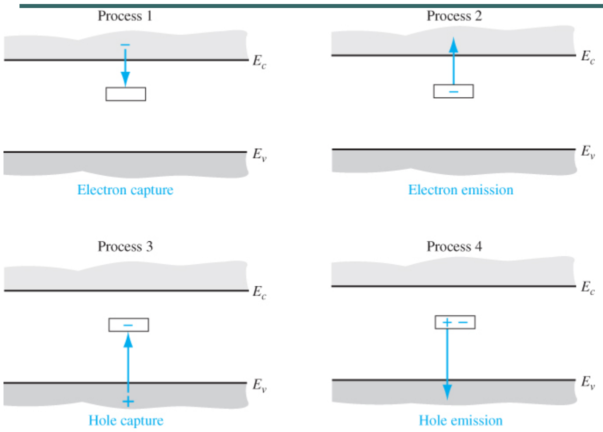
Gradients in the quasi-Fermi Levels

- Drift and diffusion imply gradient in the steady state quasi-Fermi level.
- Currents proportional to gradient.

$$J_n(x) = \mu_n n(x)\frac{dE_{Fn}}{dx} \quad \text{or } J_n(x) = \frac{\sigma_n}{q}\frac{dE_{Fn}}{dx}$$

$$J_p(x) = \mu_p p(x)\frac{dE_{Fp}}{dx} \quad \text{or } J_p(x) = \frac{\sigma_p}{q}\frac{dE_{Fp}}{dx}$$

Excess carrier recombination lifetime: Shockley-Read-Hall theory (Recombination centers)



Read Section 6.5

Figure 6.16 | The four basic trapping and emission processes for the case of an acceptor-type trap.

Surface states

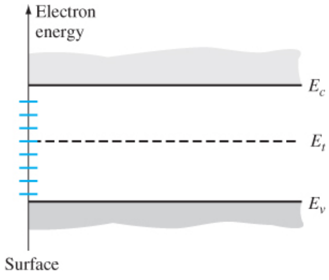


Figure 6.17 | Distribution of surface states within the forbidden bandgap.

In bulk : $R = \frac{\delta p}{\tau_{p0}} = \frac{\delta p_B}{\tau_{p0}}$

At the surface : $R_S = \frac{\delta p_S}{\tau_{p0S}}$

$\tau_{p0S} < \tau_{p0}$ so $\delta p_S < \delta p_B$

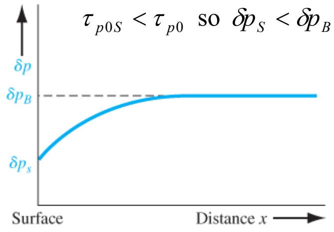


Figure 6.18 | Steady-state excess hole concentration versus distance from a semiconductor surface.

Ex 6.9 See Example 6.9, where x=0 at the N-type semiconductor surface. (a) Find the steady-state excess carrier concentration as a function of x, the distance from the surface, for $\delta p_B=10^{14}/cm^3$, $\tau_{p0}=10^{-6}s$, $\tau_{p0S}=0s$, $D_p=10cm^2/s$, with no applied field. (b) What is the recombination rate for carriers at the surface?

(a) For $\tau_{p0s}=0 \Rightarrow \delta p_s=\delta p_B\left(\frac{\tau_{p0s}}{\tau_{p0}}\right)=0$

From Equation (6.109), $\delta p(x)=g'\tau_{p0}+Ae^{+x/L_p}+Be^{-x/L_p}$

As $x\to\infty$, $\delta p=g'\tau_{p0}=10^{14}\text{ cm}^{-3} \Rightarrow A=0$

As $x\to 0$, $\delta p=0 \Rightarrow B=-g'\tau_{p0}$

Then $\delta p(x)=g'\tau_{p0}\left(1-e^{-x/L_p}\right)$

(b) $\delta p(x=0)=0$

(c) $R'=\frac{\delta p(0)}{\tau_{p0s}}=\frac{\delta p(0)}{0}\Rightarrow R'=\infty$

Note: $\delta p(0)=0$ is a result of $R'=\infty$.

Surface recombination velocity

Excess carrier concentration gradient at the surface, hence diffusion towards surface for recombination

$$-D_p\left[\hat{n}\bullet\frac{d(\delta p)}{dx}\right]_{surf}=s\,\delta p\,|_{surf}$$

$$\delta p_{surf}=\delta p(0)=g'\tau_{p0}+B$$

$$\left.\frac{d(\delta p)}{dx}\right|_{surf}=\left.\frac{d(\delta p)}{dx}\right|_{x=0}=-\frac{B}{L_p}$$

$$-B\frac{D_p}{L_p}=s\left(g'\tau_{p0}+B\right)\Rightarrow B=\frac{-sg'\tau_{p0}}{s+D_p/L_p}$$

and $\delta p(x)=g'\tau_{p0}\left(1-\frac{s}{s+D_p/L_p}\exp-\frac{x}{L_p}\right)$

Ex 6.10 (a) Find δp(x) for (i) s=∞ and (ii) s=0.
(b) What does (i) an infinite surface recombination velocity (s=∞) and (ii) a zero surface recombination velocity (s=0) imply?

(a) $\delta p(x) = g'\tau_{p0}\left(1 - \frac{s e^{-x/L_p}}{(D_p/L_p)+s}\right)$

(i) For $s \to \infty$, $\delta p(x) = g'\tau_{p0}\left(1 - e^{-x/L_p}\right)$

(ii) For $s \to 0$, $\delta p(x) = g'\tau_{p0}$

(b)

(i) For $s \to \infty$, $\delta p(0) = 0$

(ii) For $s \to 0$, $\delta p(0) = g'\tau_{p0}$, $\Rightarrow \delta p(x) =$ constant

Assignment #3

5.7
5.21
5.35
5.49
6.3
6.14
6.15
6.31