EE415/515 Fundamentals of Semiconductor Devices Fall 2012

Lecture 5:
Drift & Diffusion
(Chapter 5)

Drift in Electric Field

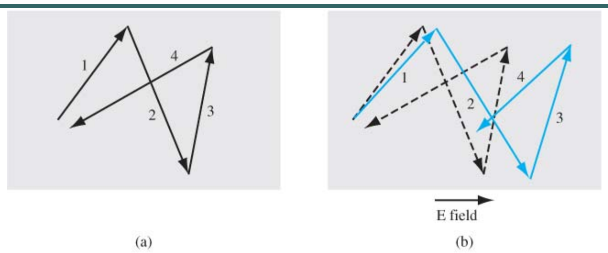
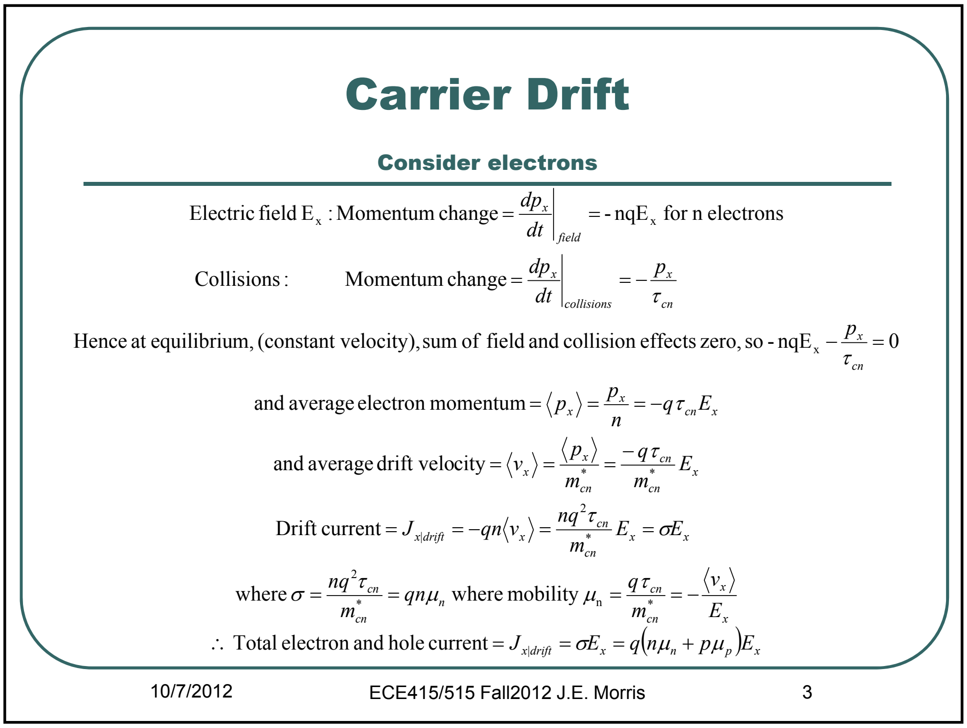


Figure 5.1 | Typical random behavior of a hole in a semiconductor (a) without an electric field and (b) with an electric field.

- Electrons have thermal energy of $kT/2$.
- Mean square thermal velocity $\frac{1}{2}m_n^* v_{th}^2 = 3kT/2$

  $m_n^* = 0.26m_0$; m_0 = free electron mass.

  $v_{th} = 2.3\text{x}10^7$ cm/s at T = 300K
- Under an applied electric field E_x the electron acquires a *drift velocity* v_d.



Carrier Drift

Consider electrons

Electric field E_x : Momentum change $= \left.\frac{dp_x}{dt}\right|_{field} = -nqE_x$ for n electrons

Collisions: Momentum change $= \left.\frac{dp_x}{dt}\right|_{collisions} = -\frac{p_x}{\tau_{cn}}$

Hence at equilibrium, (constant velocity), sum of field and collision effects zero, so $-nqE_x - \frac{p_x}{\tau_{cn}} = 0$

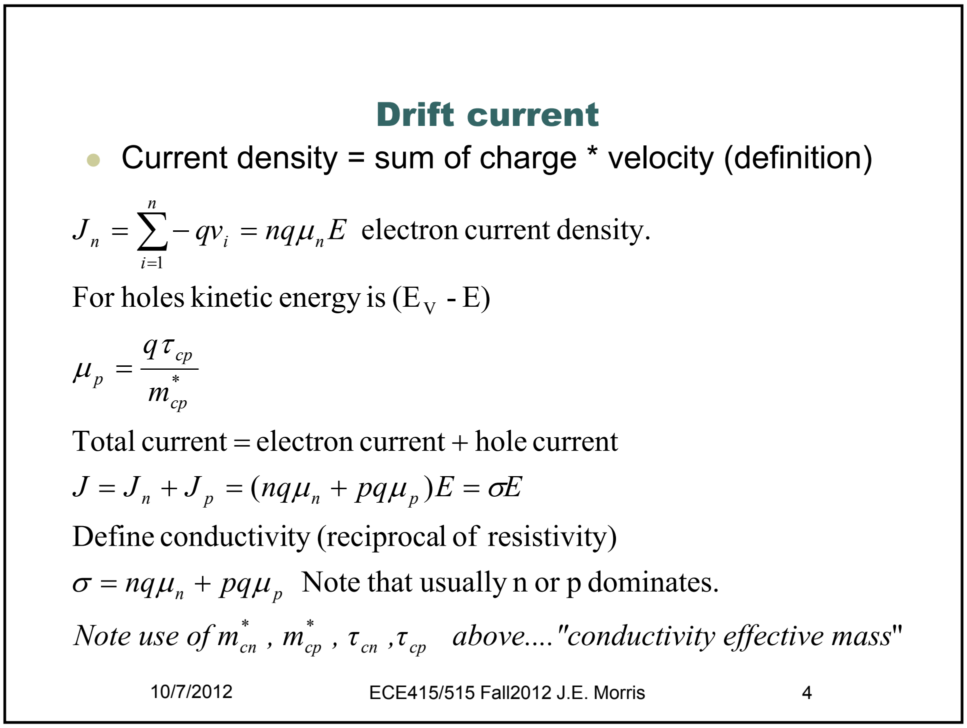
and average electron momentum $= \langle p_x \rangle = \frac{p_x}{n} = -q\tau_{cn}E_x$

and average drift velocity $= \langle v_x \rangle = \frac{\langle p_x \rangle}{m^*_{cn}} = \frac{-q\tau_{cn}}{m^*_{cn}}E_x$

Drift current $= J_{x|drift} = -qn\langle v_x \rangle = \frac{nq^2\tau_{cn}}{m^*_{cn}}E_x = \sigma E_x$

where $\sigma = \frac{nq^2\tau_{cn}}{m^*_{cn}} = qn\mu_n$ where mobility $\mu_n = \frac{q\tau_{cn}}{m^*_{cn}} = -\frac{\langle v_x \rangle}{E_x}$

$\therefore$ Total electron and hole current $= J_{x|drift} = \sigma E_x = q(n\mu_n + p\mu_p)E_x$



Drift current

- Current density = sum of charge * velocity (definition)

$J_n = \sum_{i=1}^{n} -qv_i = nq\mu_n E$ electron current density.

For holes kinetic energy is $(E_V - E)$

$$\mu_p = \frac{q\tau_{cp}}{m^*_{cp}}$$

Total current = electron current + hole current

$J = J_n + J_p = (nq\mu_n + pq\mu_p)E = \sigma E$

Define conductivity (reciprocal of resistivity)

$\sigma = nq\mu_n + pq\mu_p$ Note that usually n or p dominates.

Note use of m^*_{cn}, m^*_{cp}, τ_{cn}, τ_{cp} *above...."conductivity effective mass"*

Drift (summary)

$$-qE\tau_{cn} = m^*_{cn} v_d \quad \text{Impulse (force x time) = momentum gain}$$

$$v_d = -\frac{qE\tau_{cn}}{m^*_{cn}} \quad \text{electron drift velocity proportional to field}$$

$$\mu_n = \frac{q\tau_{cn}}{m^*_{cn}} \quad \text{depends on mean scattering time \& eff. mass}$$

$$v_d = \mu_n E$$

Table 5.1 | Typical mobility values at $T = 300$ K and low doping concentrations

	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

Ex 5.1 A drift current density J_{drift}=75A/cm² is required in a p-type Si device for field E=120V/cm. Determine the required impurity doping concentration, assuming electron and hole mobilities in Table 5.1.

$$J_{drift} = e\mu_p pE \approx e\mu_p N_a E$$

$$75 = 1.6x10^{-19} x480xN_a x120$$

$$\text{which gives } \mathrm{N_a} = 8.14x10^{15} / cm^3$$

Conductivity/resistivity

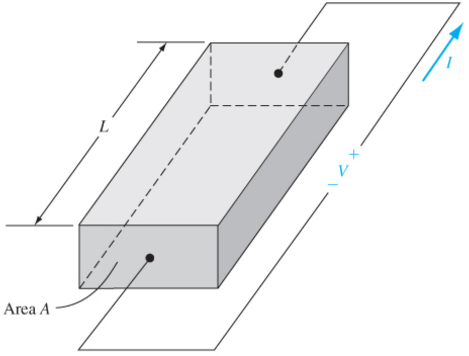


Figure 5.5 | Bar of semiconductor material as a resistor.

$$J = I / A \text{ and } E = V / L$$

$$J_{drift} = \sigma E \rightarrow \frac{\mathrm{I}}{\mathrm{A}} = \sigma \frac{V}{L} \rightarrow V = \left(\frac{L}{\sigma A}\right) I$$

$$V = \left(\frac{\rho L}{A}\right) I = IR$$

$$J_{drift} = e(n\mu_n + p\mu_p)E = \sigma E$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e(n\mu_n + p\mu_p)}$$

$$\sigma = e(n\mu_n + p\mu_p) \approx en\mu_n \text{ if } p << n$$

$$\approx eN_d\mu_n \text{ if } n \approx N_d$$

Mobility Values

- Note that mobility depends on *total* dopant concentration.
- For compensated semiconductors
  - Scattering depends on (N_a+ N_d) *Remember this!*
  - Carrier density depends on (N_a- N_d)
- Compensated semiconductors can have much lower mobility than uncompensated material of the same carrier density.

Scattering and mobility

- Electric field accelerates electrons.
- For steady state acceleration is balanced by scattering.
- Scattering decreases carrier lifetimes τ_{cn}, τ_{cp}
- With increasing temperature
  - Lattice scattering increases
  - Impurity scattering decreases
- Mobility “adds” reciprocally.

$$\mu_n = \frac{q\tau_{cn}}{m^*_{cn}} \qquad v_d = \mu_n E$$

$$J_n = \sum_{i=1}^{n} -qv_i = nq\mu_n E$$

$$\mu_p = \frac{q\tau_{cp}}{m^*_{cp}}$$

$$J = J_n + J_p = (nq\mu_n + pq\mu_p)E = \sigma E$$

$$\sigma = nq\mu_n + pq\mu_p$$

$$\mu = \frac{q\tau_c}{m^*} \qquad \frac{1}{\mu} = \sum_i \frac{1}{\mu_i}$$

Lattice Scattering

- A perfectly periodic lattice would not scatter electrons.
- Vibrations of atoms due to temp. disturb periodicity.
- “Phonons” have an energy *h*ν. Lowest levels have energy 0.063 eV.
- As temperature decreases vibration decreases and scattering decreases.
  - Varies as T^{-n} where $1.66 < n < 3$
  - Compare text $\mu_L \propto T^{-1.5}$
- Mobility increases as temperature **decreases** with lattice scattering.

Impurity Scattering

- Dopant impurities cause local distortions in the crystal lattice.
  - As temperature decreases electron velocity decreases.
  - Electrons remain near impurities a longer time => larger scattering.
- Mobility increases as temperature **increases** with impurity scattering.
  - About $T^{3/2}$
  - Compare text $\mu_I \propto T^{+1.5}/N_I$, and note $N_I = N_a + N_d$
- Other impurities and crystal defects have similar effects. (e.g. contaminents, surface effects, grain boundaries in poly-crystalline material.)

Combining scattering processes

- May have two or more scattering processes.
- Probability of scattering in time dt by process i is

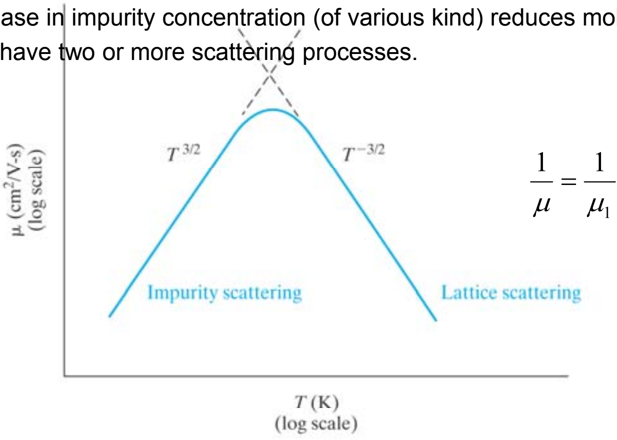
$\frac{dt}{\tau_i}$ where τ_i is the average time between scattering events

Total probability of scattering is sum over individual probabilities

$\frac{dt}{\tau_c} = \sum_i \frac{dt}{\tau_i}$ dominated by shortest scattering time

$\mu = \frac{q\tau_c}{m^*}$ $\quad \frac{1}{\mu} = \sum_i \frac{1}{\mu_i}$ Mobility adds reciprocally.

- For lattice scattering mobility increases as temperature **decreases**
- For impurity scattering mobility increases as temperature **increases**
- Increase in impurity concentration (of various kind) reduces mobility
- May have two or more scattering processes.



$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \ldots.$$

Figure 3.22

Approximate temperature dependence of mobility with both lattice and impurity scattering.

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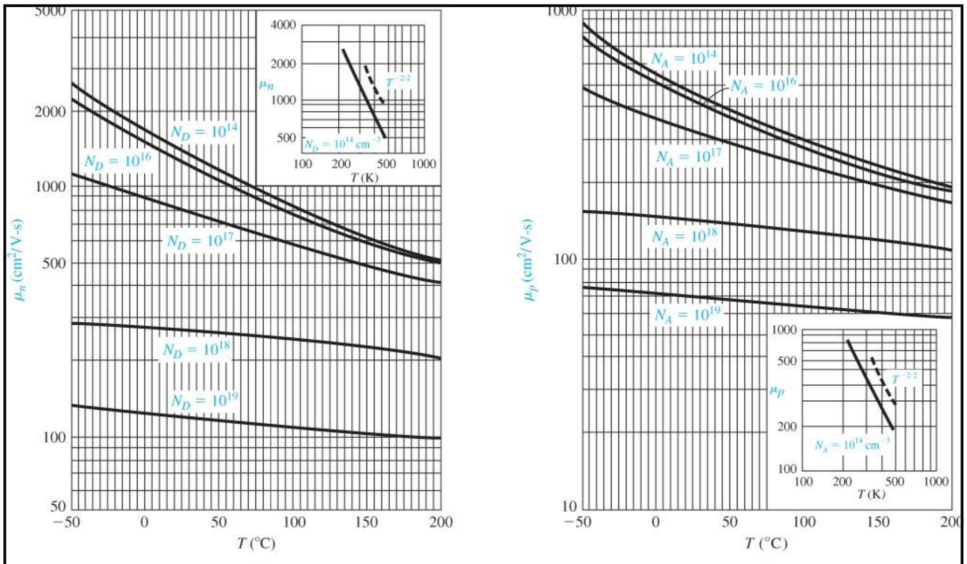


(a)

(b)

igure 5.2 | (a) Electron and (b) hole mobilities in silicon versus temperature for various doping concentrations. Inserts show emperature dependence for "almost" intrinsic silicon.
From Pierret [8].)

Ex 5.2 Use Fig 5.2 above to find the hole mobility in Si for:
(a) T=25°C for (i) $N_a=10^{16}/cm^3$ and (ii) $N_a=10^{18}/cm^3$, and
(b) $N_a=10^{14}/cm^3$ for (i) T=0°C and (ii) T=100°C

Using Figure 5.2:

(a) $T = 25°C$,

(i) $N_a = 10^{16}\ cm^{-3}$, $\Rightarrow \mu_p \cong 410\ cm^2/V\text{-}s$

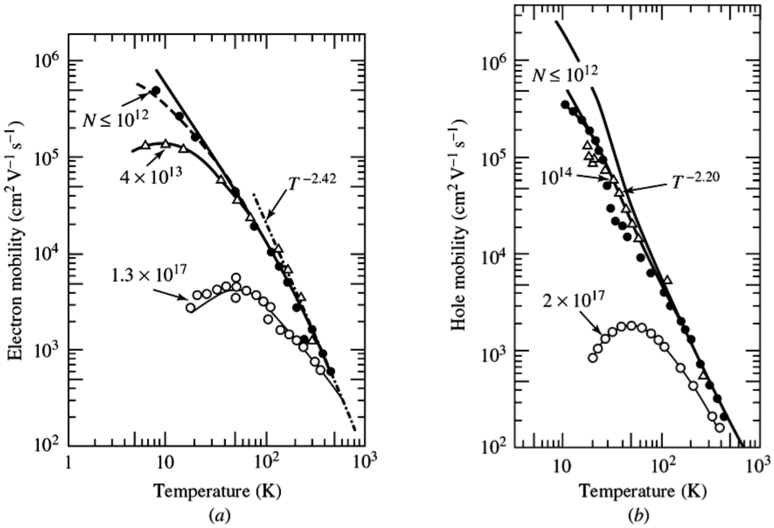
(ii) $N_a = 10^{18}\ cm^{-3}$, $\Rightarrow \mu_p \cong 130\ cm^2/V\text{-}s$

(b) $N_a = 10^{14}\ cm^{-3}$,

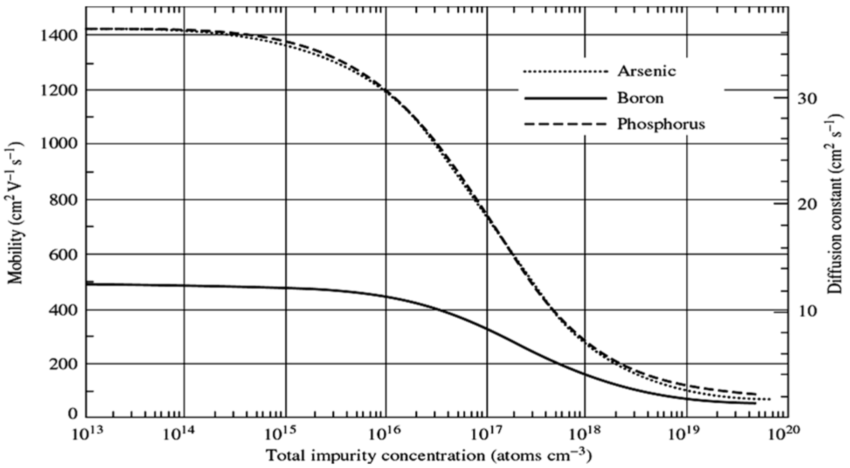
(i) $T = 0°C$, $\Rightarrow \mu_p \cong 550\ cm^2/V\text{-}s$

(ii) $T = 100°C$, $\Rightarrow \mu_p \cong 300\ cm^2/V\text{-}s$

Low-field mobility in silicon as a function of temperature for electrons and holes. The solid lines represent the theoretical predictions for pure lattice scattering.



Electron and hole mobilities in silicon at 300 K as functions of the total dopant concentration. The values plotted are the results of curve fitting measurements from several sources.



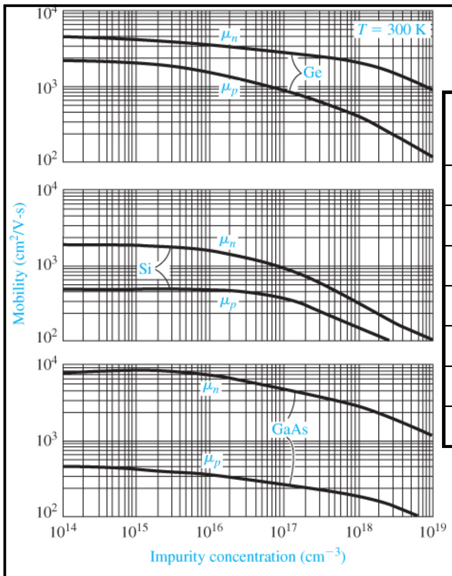


Figure 5.3 | Electron and hole mobilities versus impurity concentrations for germanium, silicon, and gallium arsenide at $T = 300$ K.
(From Sze [14].)

Doping	μ(As) (cm^2/Vs)	μ(P) (cm^2/Vs)	μ(B) (cm^2/Vs)
1E13	1423	1424	486
1E14	1413	1416	485
1E15	1367	1374	478
1E16	1184	1194	444
1E17	731	727	328
1E18	285	279	157
1E19	108	115	72

ρ vs $N_I=N_a+N_d$

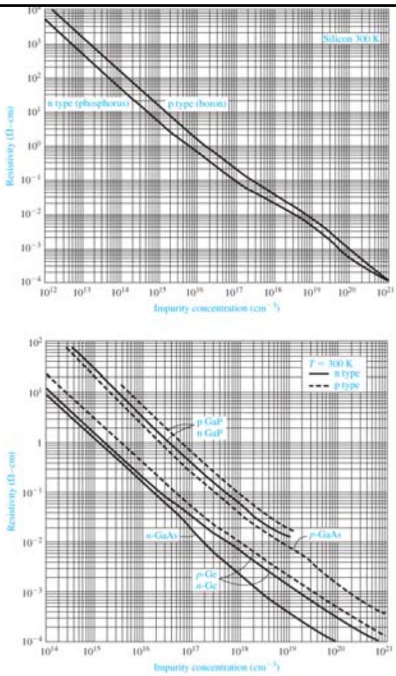


Figure 5.4 | Resistivity versus impurity concentration at T = 300 K in (a) silicon and (b) germanium, gallium arsenide, and gallium phosphide.
(From Sze [14].)

Temperature effects

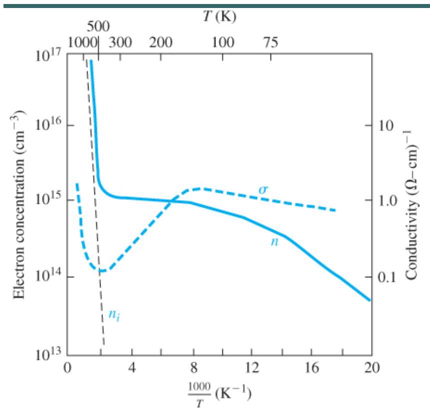


Figure 5.6 | Electron concentration and conductivity versus inverse temperature for silicon.
(After Sze [14].)

σ(T) doesn't follow n(T) exactly because of μ(T)

Ex 5.3 Compensated p-type Si at 300K has $N_a=2.8\text{x}10^{17}/\text{cm}^3$ and $N_d=8\text{x}10^{16}/\text{cm}^3$. Find (a) hole mobility, (b) conductivity, and (c) resistivity.

$$\text{(a) For } N_I = N_a + N_d = 2.8\times10^{17} + 8\times10^{16}$$

$$= 3.6\times10^{17}\ \text{cm}^{-3},$$

$$\Rightarrow \mu_p = 200\ \text{cm}^2/\text{V-s}$$

$$\text{(b) } \sigma = e\mu_p(N_a - N_d)$$

$$= \left(1.6\times10^{-19}\right)(200)\left(2\times10^{17}\right)$$

$$\sigma = 6.4\ (\Omega\text{-cm})^{-1}$$

$$\text{(a)}\quad \rho = \frac{1}{\sigma} = \frac{1}{6.4} = 0.156\ \Omega\text{-cm}$$

Ex 5.4 A p-type Si bar (Fig 5.5 above) has c/s area $A=10^{-6}$ cm^2 and length $L=1.2\text{x}10^{-3}$cm. A current of 2mA is required for 5V applied. What is the required (a) resistance, (b) resistivity, and (c) impurity doping concentration? (d) What is the resulting hole mobility?

$$\text{(a)}\quad R = \frac{V}{I} = \frac{5}{2\times10^{-3}} = 2500\ \Omega$$

$$\text{(b)}\quad \rho = \frac{RA}{L} = \frac{(2500)\left(10^{-6}\right)}{1.2\times10^{-3}} = 2.083\ \Omega\text{-cm}$$

$$\text{(c)}\quad \sigma = \frac{1}{\rho} = \frac{1}{2.083} = 0.480\ (\Omega\text{-cm})^{-1}$$

$$= e\mu_p N_a$$

$$\text{Then}\quad \mu_p N_a = \frac{0.48}{1.6\times10^{-19}} = 3.00\times10^{18}$$

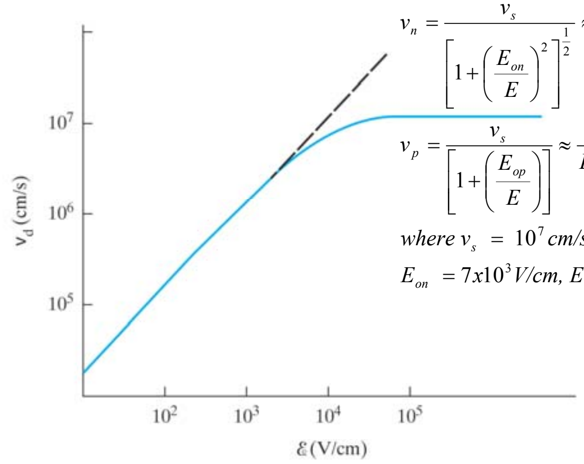
Using Figure 5.3 and trial and error,

$$N_a \approx 7.3\times10^{15}\ \text{cm}^{-3}$$

$$\text{(d)}\quad \mu_p \approx 410\ \text{cm}^2/\text{V-s}$$

High Field Effects

- We assumed Ohm’s law was valid I=V/R
  - Drift current proportional to electric field, conductivity constant.
- For large electric fields (>10^3 V/cm) current shows *sublinear* dependence on field. Ohm’s law invalid.
- *Hot carrier effect:* drift velocity ≥ thermal velocity (~10^7 cm/s)
  - Effective temperature T_e due to increasing kinetic energy.
  - Begins scattering with “optical” phonons, which transfers energy to the lattice effectively.
- At high fields Ohm’s law doesn’t hold and current is lower than expected.
  - “Hot carriers” have velocities exceeding thermal velocity.
  - Velocity saturates.



$$v_n = \frac{v_s}{\left[1+\left(\frac{E_{on}}{E}\right)^2\right]^{\frac{1}{2}}} \approx \frac{E}{E_{on}} v_s \quad \textit{for small } E$$

$$v_p = \frac{v_s}{\left[1+\left(\frac{E_{op}}{E}\right)\right]} \approx \frac{E}{E_{op}} v_s \quad \textit{for small } E$$

where $v_s = 10^7$ *cm/s at* 300*K*,

$E_{on} = 7x10^3$ *V/cm*, $E_{op} = 2x10^4$ *V/cm*

Figure 3.24

Saturation of electron drift velocity at high electric fields for Si.

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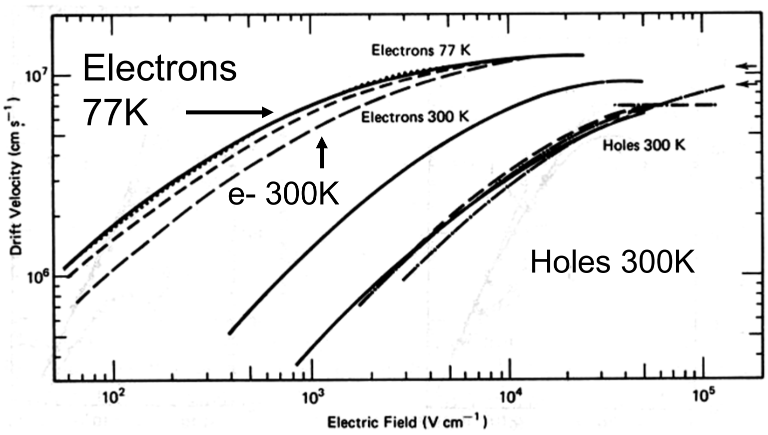
ISBN 0-13-149726-X.



Electrons 77K

e- 300K

Holes 300K

Figure 1.17 Drift velocities of electrons (at 77 K and 300 K) and holes (at 300 K) in silicon as functions of the applied field showing velocity saturation at high fields. The presence of several curves indicates the variation in reported data. An empirical "best fit" to these curves is given in Equation 1.2.12 and Table 1.2.[5]

High field effects

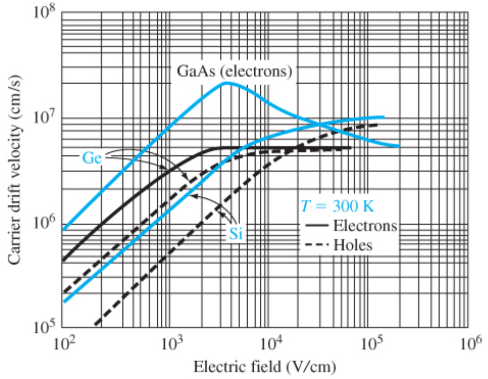


Figure 5.7 | Carrier drift velocity versus electric field for high-purity silicon, germanium, and gallium arsenide. (*From Sze [14].*)

- *Scattering limited velocity* gives fairly constant current at high fields.
  - (Si, Ge and others)
  - Some electrons can exceed that velocity.
- Can have a decrease in velocity at high fields (GaAs and others) causing negative conductivity and current instabilities.

GaAs negative differential resistance

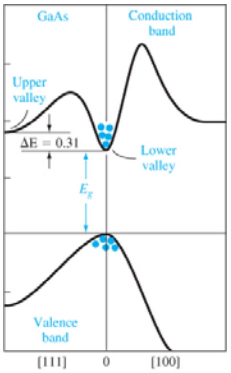


Figure 5.8 | Energy-band structure for gallium arsenide showing the upper valley and lower valley in the conduction band.
(From Sze [15].)

Electrons in conduction band:

Lower valley: m_n^* small, high E gives high v
Scatter into higher valley, higher m_n^*
Hence lower mobility, μ, and current

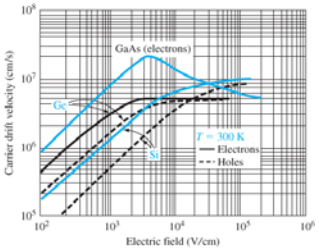


Figure 5.7 | Carrier drift velocity versus electric field for high-purity silicon, germanium, and gallium arsenide.
(From Sze [14].)

Diffusion currents

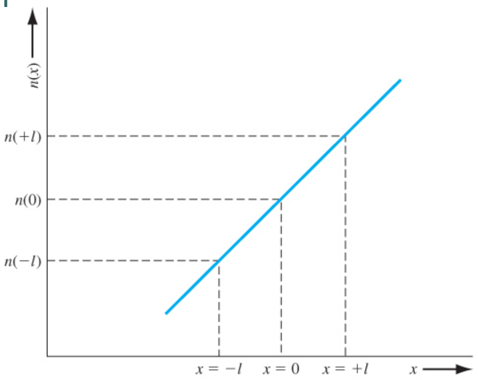


Figure 5.10 | Electron concentration versus distance.

For electron concentration gradient shown,
find net diffusion current at $x = 0$

At x: *half electrons move* $\rightarrow +x$, *half move* $\rightarrow -x$ *with thermal velocity* v_{th}

$$\textit{Electron flux at } x = 0 \quad F_n = \frac{n(-l)}{2} v_{th} - \frac{n(+l)}{2} v_{th}$$

$$= [n(-l) - n(+l)] \frac{v_{th}}{2}$$

$$= \{[n(0) - l\frac{dn}{dx}] - [n(0) + l\frac{dn}{dx}]\} \frac{v_{th}}{2}$$

for the linear variation shown, or if non – linear by keeping the first 2 *terms of a Taylor expansion*

$$\text{i.e. } F_n = -v_{th} l \frac{dn}{dx}$$

$$\textit{and current } J_{nx|dif} = -eF_n = +e\, v_{th} l \frac{dn}{dx} = eD_n \frac{dn}{dx}$$

Total current

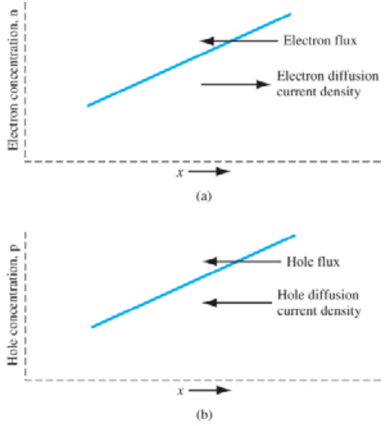


Figure 5.11 | (a) Diffusion of electrons due to a density gradient. (b) Diffusion of holes due to a density gradient.

For electron concentration gradient shown, find net diffusion current at $x = 0$

$$\text{Electron current } J_{nx|diff} = eD_n \frac{dn}{dx}$$

$$\text{Similarly hole current } J_{px|diff} = -eD_p \frac{dp}{dx}$$

$$\text{and } J_x = [\mathrm{en}\mu_n E_x + e\mu_p E_x] + [eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}]$$

$$\rightarrow J = [\mathrm{en}\mu_n E + e\mu_p E] + [eD_n \nabla n - eD_p \nabla p]$$

$E \rightarrow +x$ electrons ← electron current →

holes → hole current →

dn/dx > 0 electrons ← electron current →

dp/dx > 0 holes ← hole current ←

Ex 5.5 Hole density is given by $p(x)=10^{16}\exp{-x/L_p}$ ($x \geq 0$) in Si where $L_p=2\times10^{-4}$cm and $D_p=8$cm^2/s. Determine the hole diffusion current density at (a) x=0, (b) x=2x10^{-4}cm and (c) x=10^{-3}cm.

$$J_p = -eD_p \frac{dp}{dx}$$

$$= -eD_p \frac{d}{dx}\left[10^{16} e^{-x/L_p}\right]$$

$$= -eD_p \left(10^{16}\right)\left(\frac{-1}{L_p}\right) e^{-x/L_p}$$

$$= \frac{+eD_p \left(10^{16}\right)}{L_p} e^{-x/L_p}$$

$$= \frac{\left(1.6\times10^{-19}\right)(8)\left(10^{16}\right)}{2\times10^{-4}} e^{-x/L_p}$$

$$J_p = 64\exp\left(\frac{-x}{L_p}\right)$$

(a) For $x = 0$,

$$J_p = 64 \text{ A/cm}^2$$

(b) For $x = 2\times10^{-4}$ cm,

$$J_p = 64\exp\left(\frac{-2\times10^{-4}}{2\times10^{-4}}\right) = 23.54 \text{ A/cm}^2$$

(c) For $x = 10^{-3}$ cm,

$$J_p = 64\exp\left(\frac{-10^{-3}}{2\times10^{-4}}\right) = 0.431 \text{ A/cm}^2$$

Graded impurity distribution induces electric field

For varied donor doping $\frac{dN_d(x)}{dx} < 0,$

electrons will diffuse left to right → $E_x > 0$

$$E_x = -\frac{d\phi}{dx} = -\frac{d}{dx}\left[\frac{1}{e}(E_F - E_{Fi})\right] = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

For $n_0 = n_i \exp\frac{E_F - E_{Fi}}{kT} \approx N_d(x)$

$$E_F - E_{Fi} = kT.\ln\frac{N_d(x)}{n_i} = kT\left[\ln N_d(x) - \ln n_i\right]$$

$$-\frac{dE_{Fi}}{dx} = \frac{kT}{N_d(x)}\frac{dN_d(x)}{dx}$$

$$E_x = -\frac{kT}{e}.\frac{1}{N_d(x)}.\frac{dN_d(x)}{dx}$$

Einstein Relation

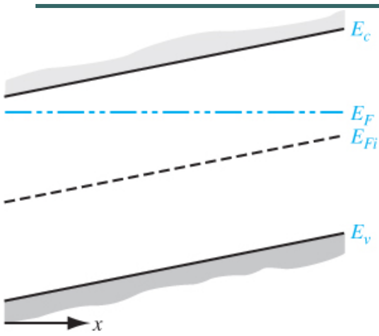


Figure 5.12 | Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration.

For semiconductor shown in isolation (no connections) electron and hole currents are zero

$$J_n = en\mu_n E_x + eD_n\frac{dn}{dx} = 0$$

and for $n(x) \approx N_d(x)$

$$eN_d(x)\mu_n E_x + eD_n\frac{dN_d(x)}{dx} = 0$$

$$-eN_d(x)\mu_n\left[\frac{kT}{e}.\frac{1}{N_d(x)}.\frac{dN_d(x)}{dx}\right] + eD_n\frac{dN_d(x)}{dx} = 0$$

i.e. $\frac{D_n}{\mu_n} = \frac{kT}{e}$ and similarly $\frac{D_p}{\mu_p} = \frac{kT}{e}$

Einstein Relation $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$

Mobilities and diffusion constants

Table 5.2 | Typical mobility and diffusion coefficient values at $T = 300$ K ($\mu = \text{cm}^2/\text{V-s}$ and $D = \text{cm}^2/\text{s}$)

	μ_m	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

Ex 5.6 Assume $N_d(x)=10^{16}\exp{-x/L}$ at 300K in an N-type semiconductor, where L=0.02cm. Determine the induced electric field at (a) x=0 and (b) $x=10^{-4}$cm.

$$\frac{dN_d(x)}{dx} = -\frac{\left(10^{16}\right)}{L}e^{-x/L}$$

So

$$E_x = -\left(\frac{kT}{e}\right)\frac{\left(\frac{-10^{16}}{L}\right)e^{-x/L}}{10^{16}e^{-x/L}}$$

$$= \left(\frac{kT}{e}\right)\left(\frac{1}{L}\right) = \frac{0.0259}{2\times10^{-2}}$$

or $E_x = 1.295$ V/cm

Ex 5.7 Assume D_n=215cm^2/s at 300K and determine μ_n.

$$\mu_n = \frac{D_n}{\left(\frac{kT}{e}\right)} = \frac{215}{0.0259}$$

$$\mu_n = 8301\ \text{cm}^2/\text{V-s}$$

Hall Effect

For currents in the +ve x-direction as shown:
 Forces F=qvxB on holes/electrons as shown

V_H +ve for holes
V_H -ve for electrons

Hall Effect:
Magnetic field shifts the hole/electron distribution in the -y direction.

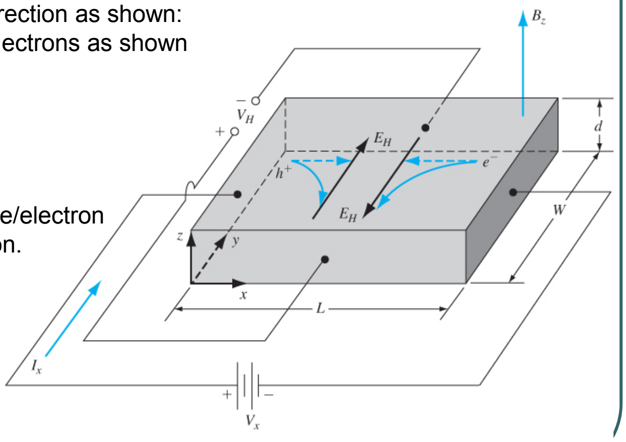


Figure 5.13 | Geometry for measuring the Hall effect.

Hall Effect: Consider P-type

If magnetic field is applied perpendicular to current direction, holes are deflected.

F=q(E+v x B)

i.e. here $F_y = q(E_y - v_x B_z)$

For steady state current must be constant so forces must balance.

$E_y = v_x B_z = (J_x B_z)/(qp_0) = R_H J_x B_z$

$V_{AB} = E_y w$ = Hall voltage, measured for applied J_x *&* B_z

w=width of bar in y direction

$R_H = (qp_0)^{-1}$

= *Hall coefficient*

= $E_y/J_x B_z$

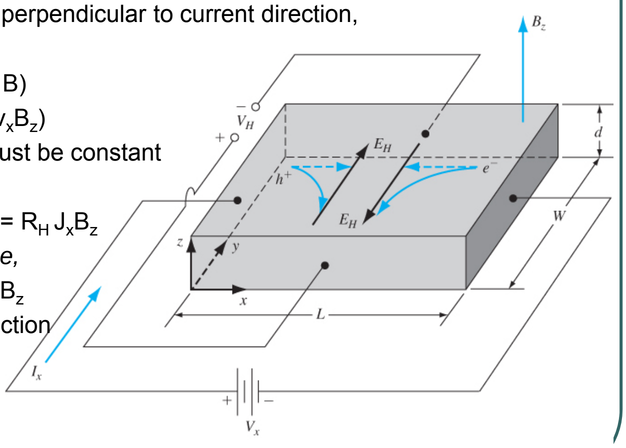


Figure 5.13 | Geometry for measuring the Hall effect.

Hall effect: Calculating mobility

- Can measure Hall voltage for applied current & magnetic field to calculate carrier concentration
- Can measure resistance to calculate resistivity

$$\rho(\Omega - cm) = \frac{Rwt}{L} = \frac{V_{CD}/I_x}{L/wt} = \frac{1}{\sigma} = \frac{1}{qp_0\mu_p}$$

L = length (x); t = thickness (z)

$$\mu_p = \frac{\sigma}{qp_0} = \frac{1/\rho}{q(1/qR_H)} = \frac{R_H}{\rho}$$

Measuring Hall coefficient and resistivity over a range of temperatures gives carrier concentration and mobility vs. temperature.

Electrons: Similar effect, opposite Hall voltage sign

Ex 5.8 P-type Si as in Fig 5.13 above. $L=0.2\text{cm}$, $W=0.01\text{cm}$, $d=8\times10^{-4}\text{cm}$, $p=10^{16}/\text{cm}^3$, $\mu_p=320\text{cm}^2/\text{V-s}$. For $V_x=10\text{V}$, $B=500\text{gauss}=0.05\text{tesla}$, find I_x and V_H.

From Equation (5.59),

$$I_x = \frac{(\mu_p)(epV_xWd)}{L}$$

$$= \frac{(320)(1.6\times10^{-19})(10^{16})(10)(10^{-2})(8\times10^{-4})}{0.2}$$

$$I_x = 2.048\times10^{-4}\text{ A}$$

or $I_x = 0.2048\text{ mA}$

From Equation (5.53),

$$V_H = \frac{I_xB_z}{epd} = \frac{(2.048\times10^{-4})(5\times10^{-2})}{(1.6\times10^{-19})(10^{22})(8\times10^{-6})}$$

$$= 8\times10^{-4}\text{V}$$

or $V_H = 0.80\text{ mV}$

A precision, linear-output, Hall-sensor chip. The integrated circuit contains bias elements, temperature-compensation circuitry, and on-chip amplification. The chip area is 1.12 by 1.98 mm^2 and the Hall element (large pattern on the). lower right-hand side) measures 230 by 335 μm^2. (Courtesy: G. B. Hocker, Honeywell Corporation