EE415/515 Fundamentals of Semiconductor Devices Fall 2012

Lecture 4:
Extrinsic Semiconductors
(Chapter 4.2-4.6)

Extrinsic: N-type (P) doping

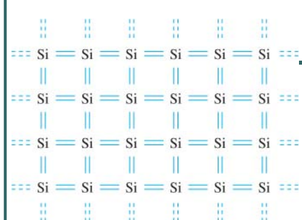


Figure 4.3 | Two-dimensional representation of the intrinsic silicon lattice.

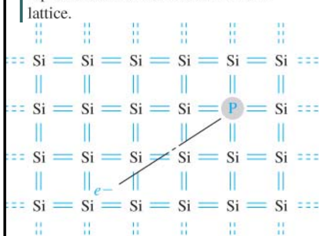


Figure 4.4 | Two-dimensional representation of the silicon lattice doped with a phosphorus atom.

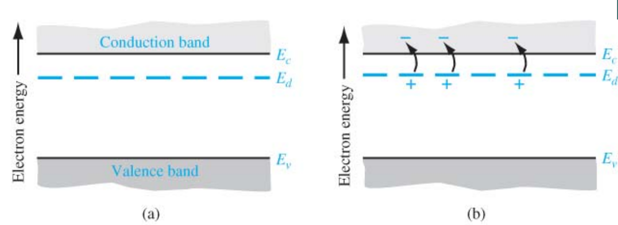


Figure 4.5 | The energy-band diagram showing (a) the discrete donor energy state and (b) the effect of a donor state being ionized.

Donor level $E_d \leq E_C$

Extrinsic: P-type (B) doping

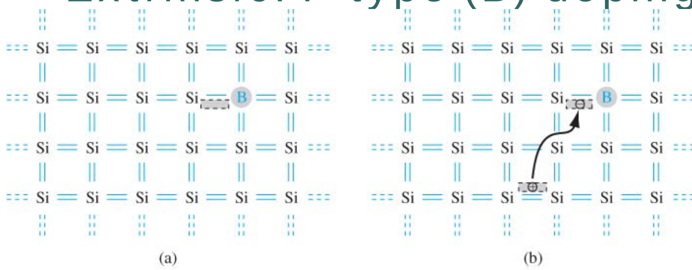
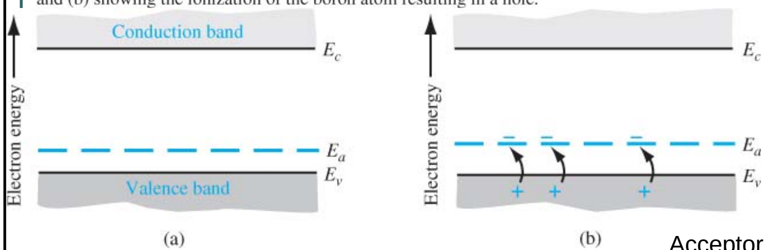


Figure 4.6 | Two-dimensional representation of a silicon lattice (a) doped with a boron atom and (b) showing the ionization of the boron atom resulting in a hole.



Acceptor level $E_a \geq E_V$

Figure 4.7 | The energy-band diagram showing (a) the discrete acceptor energy state and (b) the effect of an acceptor state being ionized.

Electron orbit around donor impurity

$\frac{e^2}{4\pi\varepsilon}\cdot\frac{1}{r_n^2} = \frac{m^* v^2}{r_n}$ and quantized angular momentum $m^* v r_n = n\hbar$

gives $r_n = n^2\hbar^2 \frac{4\pi\varepsilon_r\varepsilon_0}{me^2} = \mathrm{n}^2\varepsilon_r\left(\frac{m_0}{m^*}\right)a_0$

where Bohr radius $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_0 e^2} = 0.053nm$

For Si: $m^* = 0.26m_0$ & $\varepsilon_r = 11.8$

For $n = 1$: $r_1 = 2.4nm \approx 6a_0 \approx$ 4 x Si lattice constants

i.e. ~ 6^3 x 8 atoms/unit cell = 1728 atoms enclosed in orbit

i.e. essentially free due to (ε_r) electron screening

i.e. orbit encompasses many lattice atoms → essentially free due to shielding effects and lattice vibrations (phonons)

Ionization Energy

Table 4.3 | Impurity ionization energies in silicon and germanium

Impurity	Ionization energy (eV) Si	Ge
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

Table 4.4 | Impurity ionization energies in gallium arsenide

Impurity	Ionization energy (eV)
Donors	
Selenium	0.0059
Tellurium	0.0058
Silicon	0.0058
Germanium	0.0061
Acceptors	
Beryllium	0.028
Zinc	0.0307
Cadmium	0.0347
Silicon	0.0345
Germanium	0.0404

Extrinsic carrier concentrations & Law of Mass Action

In general $n_0 = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$ & $p_0 = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$

Write $n_0 = N_C \exp\left[\frac{-(E_C - E_{Fi}) + (E_F - E_{Fi})}{kT}\right] = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right]\exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$

$$= n_i \exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$$

and $p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$

So $n_0 p_0 = N_C N_V \exp\left[\frac{-(E_C - E_F)}{kT}\right]\exp\left[\frac{-(E_F - E_V)}{kT}\right] = N_C N_V \exp\left[\frac{-(E_C - E_V)}{kT}\right]$

$$= N_C N_V \exp\left[\frac{-E_G}{kT}\right] = n_i^2$$

or $n_0 p_0 = n_i \exp\left[\frac{(E_F - E_{Fi})}{kT}\right] n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right] = n_i^2$

$n_0 p_0 = n_i^2$ is the Law of Mass Action

Ex 4.5 Determine the thermal equilibrium concentrations of electrons and holes in Si at T=300 K if $E_F=E_V+0.215$eV.

$$p_o = N_\upsilon \exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$$

$$= \left(1.04\times10^{19}\right)\exp\left[\frac{-0.215}{0.0259}\right]$$

$$= 2.58\times10^{15}\,\text{cm}^{-3}$$

We find $E_c - E_F = 1.12 - 0.215 = 0.905$ eV

$$n_o = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= \left(2.8\times10^{19}\right)\exp\left[\frac{-0.905}{0.0259}\right]$$

$$= 1.87\times10^{4}\,\text{cm}^{-3}$$

Minority carrier suppression

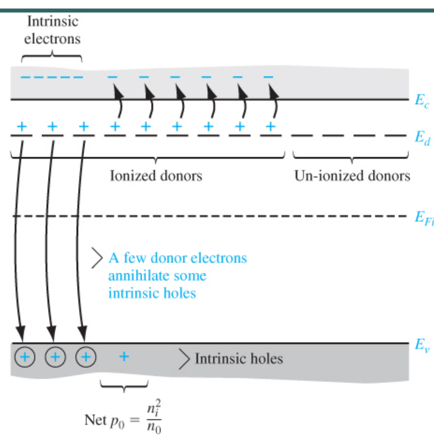


Figure 4.15 | Energy-band diagram showing the redistribution of electrons when donors are added.

Majority & Minority Cariers

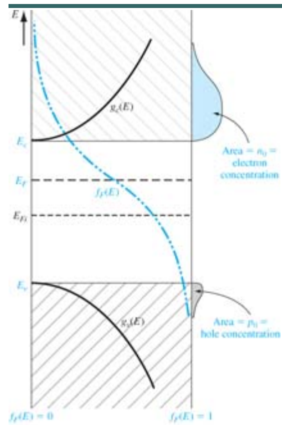


Figure 4.8 | Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is above the intrinsic Fermi energy.

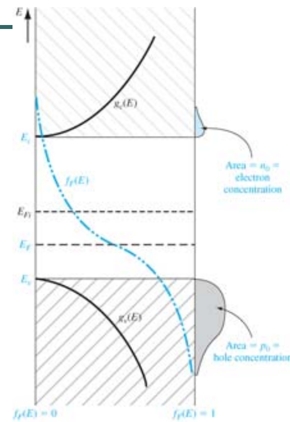


Figure 4.9 | Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is below the intrinsic Fermi energy.

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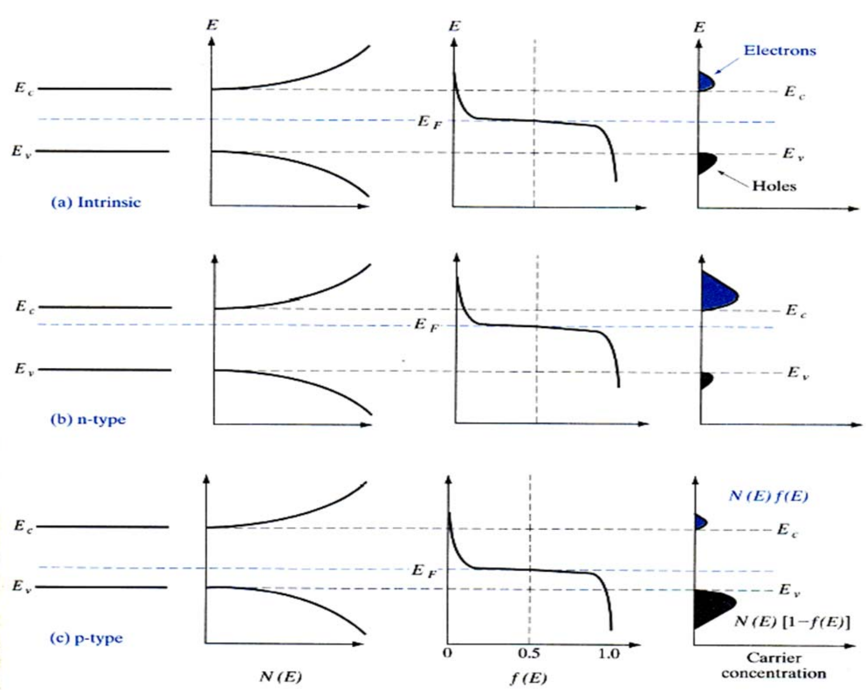

Donor & acceptor levels (& E_F)

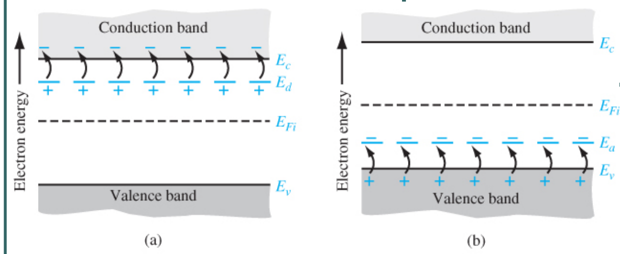

Figure 4.12 | Energy-band diagrams showing complete ionization of (a) donor states and (b) acceptor states.

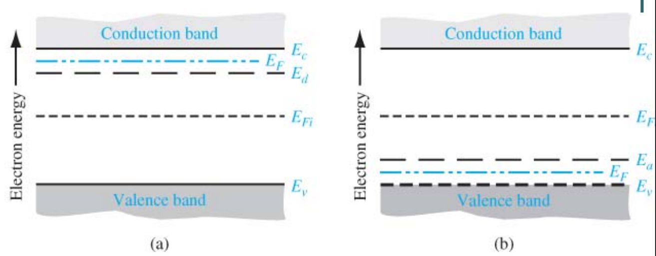

Figure 4.13 | Energy-band diagram at $T = 0$ K for (a) n-type and (b) p-type semiconductors.

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Extrinsic Fermi Level

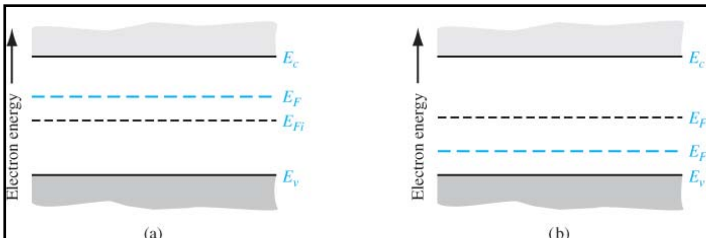

Figure 4.17 | Position of Fermi level for an (a) n-type ($N_d > N_a$) and (b) p-type ($N_a > N_d$) semiconductor.

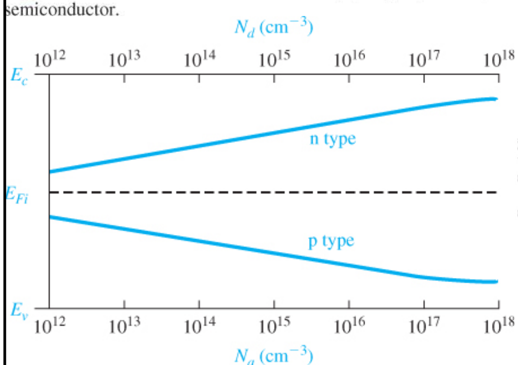

Figure 4.18 | Position of Fermi level as a function of donor concentration (n type) and acceptor concentration (p type).

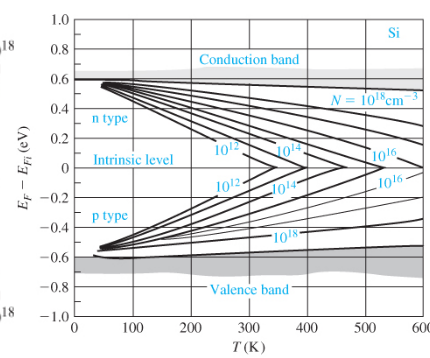

Figure 4.19 | Position of Fermi level as a function of temperature for various doping concentrations. *(From Sze [14].)*

Position of extrinsic Fermi level

Assuming the MB approximation :-

$$n_0 = N_c \exp - \frac{E_c - E_F}{kT} \rightarrow E_c - E_F = kT.\ln\left(\frac{N_c}{n_0}\right)$$

$$and\ \ for\ N_d >> n_i\ ,\ n_0 \approx N_d \rightarrow E_c - E_F = kT.\ln\left(\frac{N_c}{N_d}\right)$$

$$Or,\ n_0 = n_i \exp\frac{E_F - E_{Fi}}{kT} \rightarrow E_F - E_{Fi} = kT.\ln\left(\frac{n_0}{n_i}\right)$$

$$and\ similarly\ \ E_F - E_V = kT.\ln\left(\frac{N_V}{p_0}\right) = kT.\ln\left(\frac{N_V}{N_a}\right),\ E_{Fi} - E_F = kT.\ln\left(\frac{p_0}{n_i}\right)$$

Ex 4.12 For Si at T=300 K, N_d=8x10^{15}/cm^3, N_a=5x10^{15}/cm^3, find E_F wrt E_c.

$$n_o = N_d - N_a = 8\times10^{15} - 5\times10^{15}$$

$$n_o = 3\times10^{15}\ \mathrm{cm}^{-3}$$

$$E_c - E_F = kT\ \ln\left(\frac{N_c}{n_o}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{3\times10^{15}}\right)$$

$$E_c - E_F = 0.2368\ \mathrm{eV}$$

Ex 4.13 For n-type Si at T=300 K, doped with As, find maximum doping N_d for which the MB approximation is still valid, i.e. when E_d-E_F=3kT.

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right)$$

We have $E_c - E_F = (E_c - E_d) + (E_d - E_F)$

$= 0.05 + 3kT = 0.05 + 3(0.0259)$

$= 0.1277\ \text{eV}$

(E_c-E_d=0.05eV from Table 4.3)

So $0.1277 = (0.0259)\ln\left(\frac{N_c}{n_o}\right)$

Or $\frac{N_c}{n_o} = \exp(4.9305) = 138.45$

Then $n_o = \frac{N_c}{138.45} = \frac{2.8\times10^{19}}{138.45}$

$n_o = 2.02\times10^{17}\ \text{cm}^{-3}$

Compensation and Space Charge Neutrality

- Consider n-type semiconductor doped with both donors and acceptors. $N_d > N_a$
- E_F is well above E_A, so all states filled.
  - Mostly filled with donated conduction band electrons.
  - Unless N_d is close to N_a then $n_0 = N_d - N_a$; called *compensation*

Charge neutrality requires:

$$p_0 + N_d^+ = n_0 + N_a^-$$

$$\therefore n_o = p_0 + (N_d^+ - N_a^-)$$

Electron & hole concentrations

Charge neutrality requires :

$$p_0 + N_d^+ = n_0 + N_a^-$$

$$\therefore n_o = p_0 + (N_d^+ - N_a^-)$$

$$\therefore n_o = \frac{n_i^2}{n_0} + (N_d^+ - N_a^-)$$

$$\therefore n_o^2 = n_i^2 + (N_d^+ - N_a^-)n_0$$

$$\therefore n_0 = \left(\frac{N_d^+ - N_a^-}{2}\right) + \sqrt{\left(\frac{N_d^+ - N_a^-}{2}\right)^2 + n_i^2}$$

$$\& \qquad p_0 = \left(\frac{N_a^- - N_d^+}{2}\right) + \sqrt{\left(\frac{N_a^- - N_d^+}{2}\right)^2 + n_i^2}$$

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Compensation, etc

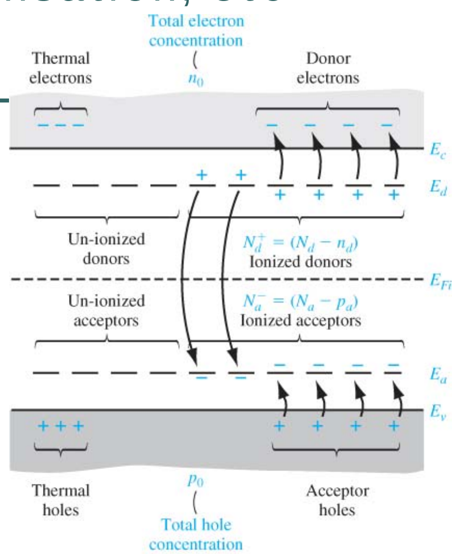

Figure 4.14 | Energy-band diagram of a compensated semiconductor showing ionized and un-ionized donors and acceptors.

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Ex 4.9 Find thermal equilibrium electron and hole densities in Si with doping $N_d=7\text{x}10^{15}/\text{cm}^3$ and $N_a=3\text{x}10^{15}/\text{cm}^3$ for (a) T=250 K & (b) T=400 K.

From Example 4.3, $n_i(250) = 7.0\times10^7\ \text{cm}^{-3}$

$n_i(400) = 2.38\times10^{12}\ \text{cm}^{-3}$, then

(a) $T = 250\text{K}$

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$= \frac{7\times10^{15} - 3\times10^{15}}{2} + \sqrt{\left(\frac{7\times10^{15} - 3\times10^{15}}{2}\right)^2 + \left(7\times10^7\right)^2}$$

$$n_o = 4\times10^{15}\ \text{cm}^{-3} \text{ and } p_o = \frac{n_i^2}{n_o} = \frac{\left(7\times10^7\right)^2}{4\times10^{15}} = 1.225\ \text{cm}^{-3}$$

(b) $T = 400\text{K}$

$$n_o = \frac{7\times10^{15} - 3\times10^{15}}{2} + \sqrt{\left(\frac{7\times10^{15} - 3\times10^{15}}{2}\right)^2 + \left(2.38\times10^{12}\right)^2}$$

$$n_o \cong 4\times10^{15}\ \text{cm}^{-3} \text{ and } p_o = \frac{\left(2.38\times10^{12}\right)^2}{4\times10^{15}} = 1.416\times10^9\ \text{cm}^{-3}$$

Ex 4.10 Find thermal equilibrium electron and hole densities in Ge ($n_i=2.4\text{x}10^{13}/\text{cm}^3$) with doping $N_d=2\text{x}10^{14}/\text{cm}^3$ and $N_a=0$ for (a) T=250 K & (b) T=350 K. (c) What happens with very low-doped materials as T increases?

For $T = 250\text{K}$, $n_i^2 = \left(1.04\times10^{19}\right)\left(6\times10^{18}\right)\left(\frac{250}{300}\right)^3 \times \exp\left[\frac{-0.66}{(0.0259)(250/300)}\right]$

$= 1.894\times10^{24}$ or $n_i = 1.376\times10^{12}\ \text{cm}^{-3}$

For $T = 350\text{K}$, $n_i^2 = \left(1.04\times10^{19}\right)\left(6\times10^{18}\right)\left(\frac{350}{300}\right)^3 \times \exp\left[\frac{-0.66}{(0.0259)350/300}\right]$

$= 3.236\times10^{28}$ or $n_i = 1.80\times10^{14}\ \text{cm}^{-3}$

(a) $T = 250\text{K}$

$$n_o = \frac{2\times10^{14}}{2} \times \sqrt{\left(\frac{2\times10^{14}}{2}\right)^2 + \left(1.376\times10^{12}\right)^2}$$

$$n_o \cong 2\times10^{14}\ \text{cm}^{-3} \quad p_o = \frac{n_i^2}{n_o} = \frac{\left(1.376\times10^{12}\right)^2}{2\times10^{14}} = 9.47\times10^9\ \text{cm}^{-3}$$

(b) $T = 350\text{K}$

$$n_o = \frac{2\times10^{14}}{2} \times \sqrt{\left(\frac{2\times10^{14}}{2}\right)^2 + \left(1.80\times10^{14}\right)^2}$$

$$n_o = 3.059\times10^{14}\ \text{cm}^{-3} \quad p_o = \frac{\left(1.80\times10^{14}\right)^2}{3.059\times10^{14}} = 1.059\times10^{14}\ \text{cm}^{-3}$$

(c) Tend towards intrinsic with n=p

Degenerate semiconductor: high doping

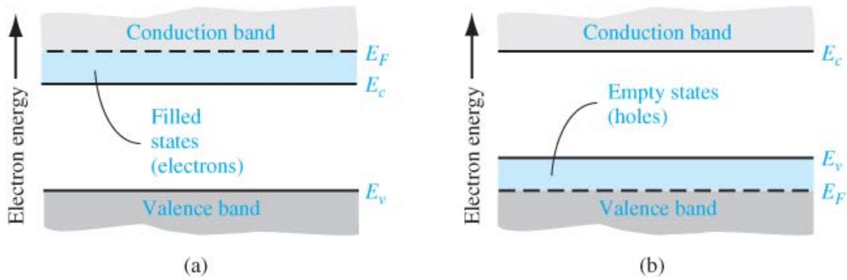

Figure 4.11 | Simplified energy-band diagrams for degenerately doped (a) n-type and (b) p-type semiconductors.

Donor (acceptor) ionization/freeze-out

Modified FD function for donor/acceptor levels

$$\textit{Density of electrons at the donor level } n_d = N_d - N_d^+ = \frac{N_d}{1+\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$\textit{and } \mathrm{p}_a = N_a - N_a^- = \frac{N_a}{1+\frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)} \quad \textit{where } g \sim 4$$

$$\textit{If } kT << E_d - E_F \textit{ then } \mathrm{n_d} \approx 2N_d \exp\frac{-(E_d - E_F)}{\mathrm{kT}} \;\&\; \mathrm{n_0} \approx N_c \exp\frac{-(E_c - E_F)}{\mathrm{kT}}$$

$$\textit{so } \frac{n_d}{n_d + n_0} = \frac{1}{1+\frac{N_c}{2N_d}\exp\frac{-(E_c - E_F)}{kT}} \to 1, \; i.e.\, n_0 \to 0 \;\; as \;\; T \to 0$$

Carrier freeze-out

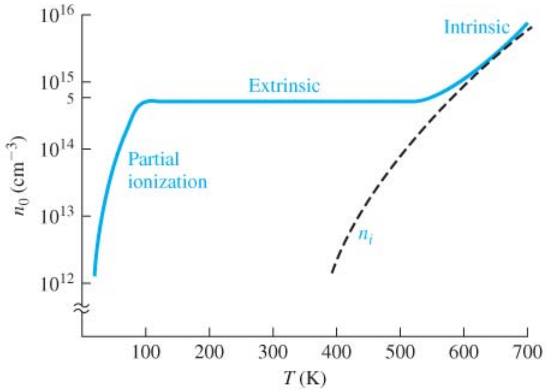

Figure 4.16 | Electron concentration versus temperature showing the three regions: partial ionization, extrinsic, and intrinsic.

Arrhenius plot: semi-log vs 1/T

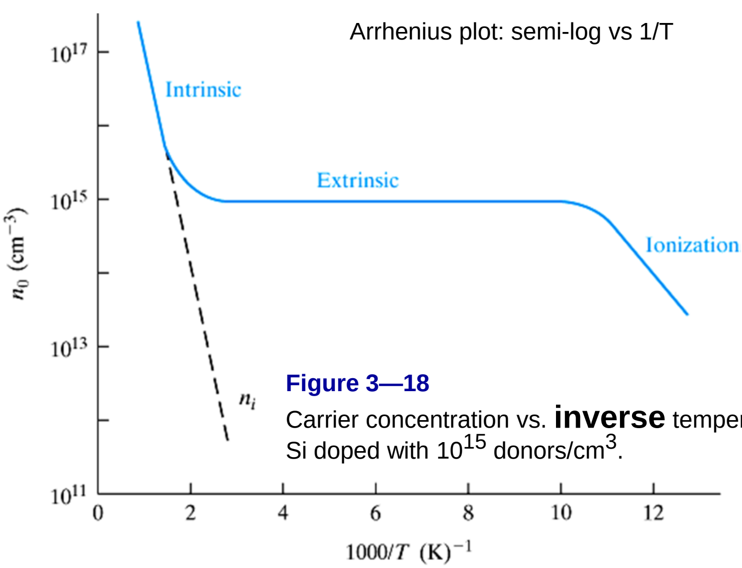

Figure 3—18
Carrier concentration vs. **inverse** temperature for Si doped with 10^{15} donors/cm^3.

- High temp *intrinsic* dominates.
- Above ~100K
  extrinsic
  $n_0 \sim N_d$
- Low temp *ionization region,* electrons donated to conduction band

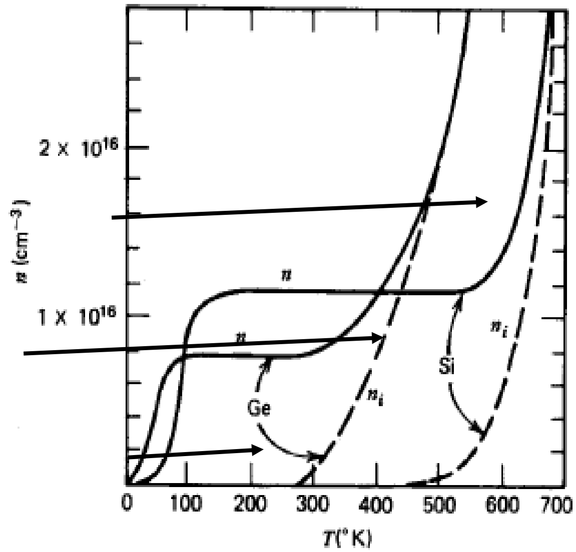

Figure 1.9 Electron concentration versus temperature for two doped semiconductors: (*a*) Silicon doped with 1.15×10^{16} arsenic atoms cm^{-3},[1] (*b*) Germanium doped with 7.5×10^{15} arsenic atoms cm^{-3}.[2]

Ex 4.7 Find fraction of electrons still in donor states at (a) 250 K and (b) 200 K, for P doping $N_d=10^{16}/\mathrm{cm}^3$. (c) How does the fraction vary as T decreases?

(a) $T = 250\,\mathrm{K}$

$$N_c = (2.8\times10^{19})\left(\frac{250}{300}\right)^{3/2} = 2.13\times10^{19}\ \mathrm{cm}^{-3}$$

$$kT = (0.0259)\left(\frac{250}{300}\right) = 0.021583\,\mathrm{eV}$$

$$\frac{n_d}{n_o+n_d} = \frac{1}{1+\dfrac{N_c}{2N_d}\exp\left[\dfrac{-(E_c-E_d)}{kT}\right]}$$

$$= \frac{1}{1+\dfrac{(2.13\times10^{19})}{2(10^{16})}\exp\left[\dfrac{-0.045}{0.02158}\right]}$$

$$= 7.50\times10^{-3}$$

(b) $T = 200\,\mathrm{K}$

$$N_c = (2.8\times10^{19})\left(\frac{200}{300}\right)^{3/2} = 1.524\times10^{19}\ \mathrm{cm}^{-3}$$

$$kT = (0.0259)\left(\frac{200}{300}\right) = 0.017267\ \mathrm{eV}$$

$$\frac{n_d}{n_o+n_d} = \frac{1}{1+\dfrac{(1.524\times10^{19})}{2(10^{16})}\exp\left[\dfrac{-0.045}{0.017267}\right]}$$

$$= 1.75\times10^{-2}$$

(c) Fraction increases as temperature decreases.

Summary: Temperature effects on carrier concentration

- At low temperatures dopants are not completely ionized.
- Ionization increases with temperature.
- Above ~100K donors are completely ionized and n_0=N_d constant.
- At high temps n_i dominates (exponential).

Summary

- $E_F = E_{Fi}$ for intrinsic material.

  $n_i = N_c exp[-(E_c-E_{Fi})/kT]$

  $p_i = N_v exp[-(E_{Fi}-E_v)/kT]$
- Combine, knowing $n_0 p_0$ = constant=n_i^2.

  $n_i = [N_c N_v]^{1/2} exp[-E_g/2kT]$
- Rewrite previous equations.

  $n_0 = n_i \, exp[(E_F - E_{Fi})/kT]$

  $p_0 = p_i \, exp[(E_{Fi} - E_F)/kT]$

  in general at equilibrium

Review: Find the majority and minority carrier concentrations

- If we dope Silicon with 10^{17} cm^{-3} of Boron what are concentrations
  - at room temperature?
  - just above 0 K?
  - 700 K?
- Silicon with 10^{17} cm^{-3} of phosphorous and 10^{16} cm^{-3} of boron at room temp?
- You should be able to do the room temp calculations (approximately) in your head and understand how they will vary with temperature.

Now put different materials together (Fermi levels align)

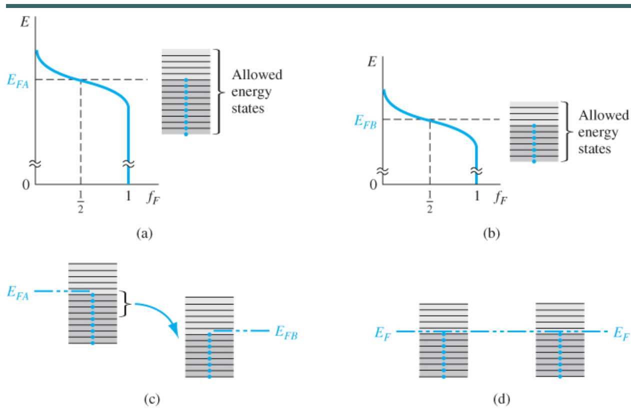

Figure 4.20 | The Fermi energy of (a) material A in thermal equilibrium, (b) material B in thermal equilibrium, (c) materials A and B at the instant they are placed in contact, and (d) materials A and B in contact at thermal equilibrium.

Assignment #2

3.29	4.37
3.40	4.46
4.6	4.50
4.19	4.61