

*EE415/515 Fundamentals
of Semiconductor Devices
Fall 2012*

**Lecture 3:
Density of States, Fermi Level
(Chapter 3.4-3.5/4.1)**

Density of States

- Need to know the density of electrons, n , and holes, p , per unit volume
- To do this, we need to find the density of permitted energy states and then the probability (given by the Fermi function) that these states are occupied by an electron
- See Neamen Sections 3.4 and 3.5 for derivations, and compare these with alternative approaches given below

Density of States

• See figure in “*k*-space.”

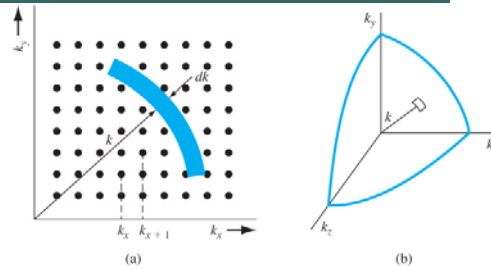


Figure 3.26 (a) A two-dimensional array of allowed quantum states in *k* space. (b) The positive one-eighth of the spherical *k* space.

• Also: $p = \hbar k = (h/2\pi)k$, i.e.

• Direct linear relationship between (wave-vector) *k*-space and (momentum) *p*-space.

• The Neamen derivation (next slide) is based in *k*-space.

• See also a slightly different approach based in *p*-space in following slide.

Density of States

- Consider electron confined to crystal (infinite potential well) of dimensions *a* (volume $V = a^3$)
- It has been shown that $k = n\pi/a$, so $\Delta k = k_{n+1} - k_n = \pi/a$
- Each quantum state occupies volume $(\pi/a)^3$ in *k*-space.
- Number of quantum states in range *k* to *k*+*dk* is $4\pi k^2 \cdot dk$ and the number of electrons in this range *k* to *k*+*dk* is

$$g_T(k)dk = 2(1/8) \cdot 4\pi k^2 dk / (\pi/a)^3 = (k^2 dk / \pi^2) a^3$$
 where “2” comes from spin degeneracy, and “1/8” because $E = (\hbar k)^2 / 2m$, so negative *k* values do not add more energy states, i.e. use $k_x, k_y, k_z > 0$ quadrant only
- Converting *k*-space to energy

Density of States

$E = E_C + p^2/2m_n^*$ in the conduction band, where $E > E_C$

$$\therefore p^2 = \hbar^2 k^2 = 2m_n^*(E - E_C)$$

$$2k \cdot dk = 2m_n^* dE / \hbar^2$$

$$\therefore k^2 dk = (2m_n^*)^{1/2} (E - E_C)^{1/2} m_n^* dE / \hbar^3$$

So writing $g_T(k)dk/V = g_C(E)dE = \left(\frac{k}{\pi}\right)^2 dk \cdot a^3 / a^3 = \frac{1}{\pi^2} (2m_n^*)^{1/2} (E - E_C)^{1/2} m_n^* dE / \hbar^3$

$$= \frac{4\pi}{h^3} (2m_n^*)^{3/2} (E - E_C)^{1/2} dE = 4\pi \left(\frac{2m_n^*}{h^2}\right)^{3/2} (E - E_C)^{1/2} dE$$

is electron density of states/unit energy/unit vol in the conduction band
and electron density/unit energy/unit vol in the conduction band is

$$n(E)dE = f_{FD}(E)g_C(E)dE$$

Density of States (alternate)

Heisenberg says $\Delta p \Delta x = \hbar$,

(but see \hbar in Neamen section 2.1.3 p30, and $\hbar/2$ in Streetman)

and for crystal volume $V=L^3=L_x L_y L_z$

$$\Delta p_x \Delta x = \hbar, \Delta p_y \Delta y = \hbar, \Delta p_z \Delta z = \hbar,$$

and hence for a single electron

$$\Delta p_x \Delta p_y \Delta p_z = \hbar^3 / V \text{ for } \Delta x = L_x, \text{ etc}$$

is the "volume" occupied in p -space by one electron.

Number of allowed conduction electron states $g(p)dp$ from p to $p+dp$

[see Fig 3.26 (slide 3) in k -space with $p = \hbar k$]

is the volume of the p -space "shell" containing all states from p to $p+dp$

$$[4\pi p^2 dp]$$

divided by the p -space volume of each electron, i.e.

Density of states

$$g(p)dp = 2 \frac{(4\pi p^2 dp)}{h^3/V} = \frac{V}{h^3} 2(4\pi p^2 dp) \text{ where the 2 is due to spin degeneracy}$$

To convert to energy density :- $E = E_c + p^2/2m_n^*$ in the conduction band, where $E > E_c$

$$\therefore p^2 = 2m_n^*(E - E_c)$$

$$2p \cdot dp = 2m_n^* dE$$

$$\therefore p^2 dp = (2m_n^*)^{3/2} (E - E_c)^{1/2} m_n^* dE$$

$$\text{So writing } g(p)dp/V = g_c(E)dE = \frac{2m_n^*}{h^3} 4\pi(2m_n^*)^{1/2} (E - E_c)^{1/2} dE$$

$$= \frac{4\pi}{h^3} (2m_n^*)^{3/2} (E - E_c)^{1/2} dE = 4\pi \left(\frac{2m_n^*}{h^2}\right)^{3/2} (E - E_c)^{1/2} dE$$

is electron density of states/unit energy/unit vol in the conduction band

and electron density/unit energy/unit vol in the conduction band is

$$n(E)dE = f_{FD}(E)g_c(E)dE$$

J.E. Morris

ECE415/515 Fall 2012

7

(Compare 1D, 2D systems with 3D)

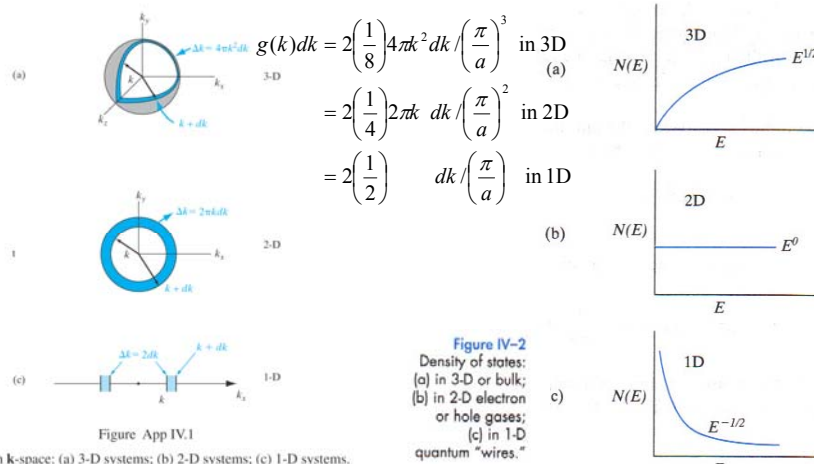


Figure App IV.1
Volume in k-space: (a) 3-D systems; (b) 2-D systems; (c) 1-D systems.

From *Solid State Electronic Devices*, Sixth Edition, by Ben G. Streetman and Sanjay Kumar Banerjee.
ISBN 0-13-149726-X. © 2006 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved.

8

And similarly for holes:-

For electrons in the conduction band, where $E > E_C$:

$$g_C(E) = \frac{4\pi}{h^2} (2m_n^*)^{\frac{3}{2}} \sqrt{E - E_C}$$

For holes in the valence band, where $E < E_V$:

$$g_V(E) = \frac{4\pi}{h^2} (2m_p^*)^{\frac{3}{2}} \sqrt{E_V - E}$$

and $g(E) = 0$ for $E_V < E < E_C$

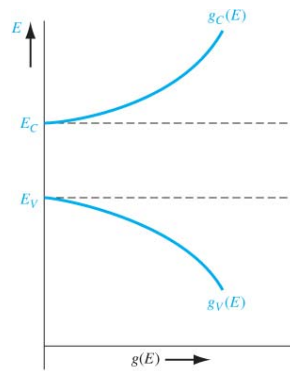


Figure 3.27 | The density of energy states in the conduction band and the density of energy states in the valence band as a function of energy.

Now that we have the densities of states, we need the probabilities that these are filled.

J.E. Morris

ECE415/515 Fall 2012

9

Ex 3.3 Calculate the density of quantum states (/cm³) for a free electron over the energy range of (a) $0 \leq E \leq 2\text{eV}$ & (b) $1 \leq E \leq 2\text{eV}$

$$\begin{aligned} \text{(a)} \quad N &= \frac{4\pi(2m)^{3/2}}{h^3} \int_0^{2\text{eV}} \sqrt{E} \cdot dE \\ &= \frac{4\pi(2m)^{3/2}}{h^3} \cdot \frac{2}{3} \cdot E^{3/2} \Big|_0^{2\text{eV}} \\ &= \frac{4\pi [2(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} \times \frac{2}{3} [2(1.6 \times 10^{-19})]^{3/2} \\ &= 1.28 \times 10^{28} \text{ m}^{-3} = 1.28 \times 10^{22} \text{ cm}^{-3} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad N &= \frac{4\pi(2m)^{3/2}}{h^3} \cdot \frac{2}{3} \cdot E^{3/2} \Big|_{1\text{eV}}^{2\text{eV}} \\ &= 1.06286 \times 10^{28} \times \frac{2}{3} [(2^{3/2} - 1^{3/2})(1.6 \times 10^{-19})^{3/2}] \\ &= 8.29 \times 10^{27} \text{ m}^{-3} = 8.29 \times 10^{21} \text{ cm}^{-3} \end{aligned}$$

J.E. Morris

ECE415/515 Fall 2012

10

Ex 3.3 Calculate the density of quantum states (/cm³) in Si from (E_v-kT) to E_v at T=300 K

$$\begin{aligned}
 N &= \int_{E_v - kT}^{E_v} \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E} \cdot dE \\
 &= \frac{4\pi(2m_p^*)^{3/2}}{h^3} \cdot \left(\frac{-2}{3}\right) \cdot (E_v - E)^{3/2} \Big|_{E_v - kT}^{E_v} \\
 &= \frac{4\pi [2(0.56)(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} \times \left(\frac{-2}{3}\right) (-1) [(0.0259)(1.6 \times 10^{-19})]^{3/2} \\
 &= 7.92 \times 10^{24} \text{ m}^{-3} = 7.92 \times 10^{18} \text{ cm}^{-3}
 \end{aligned}$$

J.E. Morris

ECE415/515 Fall 2012

11

Fermi-Dirac statistics (see also text "derivation")

- First, let's find the **Maxwell-Boltzmann distribution**
- Consider a system of identical particles with random collisions
- Assume 2 particles of initial energies E₁ & E₂ collide with energy transfer δ to give final energies E₃ & E₄
- Conservation of Energy requires that: E₁ + E₂ = E₃ + E₄ = (E₁ + δ) + (E₂ - δ)
- If p(E)=probability that particle has energy E, then:

$$\text{Probability of collision} = C \cdot p(E_1) \cdot p(E_2)$$

- Equilibrium: equal numbers of forward & reverse collisions for constant energy distribution, i.e.

$$C \cdot p(E_1) \cdot p(E_2) = C \cdot p(E_3) \cdot p(E_4)$$

$$p(E_1) \cdot p(E_2) = p(E_1 + \delta) \cdot p(E_2 - \delta)$$

$$\text{i.e. } \frac{p(E_1 + \delta)}{p(E_1)} = \frac{p(E_2 - \delta)}{p(E_2)} = \lambda \text{ (constant)}$$

$$\text{i.e. } p(E_i + \delta) = \lambda \cdot p(E_i)$$

J.E. Morris

ECE415/515 Fall 2012

12

MB continued & FD intro

Result $p(E_i + \delta) = \lambda \cdot p(E_i)$ suggests an exponential distribution,

$$\begin{aligned} \text{i.e. } p_{MB}(E) &= A \cdot \exp(-\alpha E) \\ &= A \cdot \exp(-E/kT) \end{aligned}$$

- Check: $A \exp(-\alpha(E_i + \delta)) = \lambda \cdot A \exp(-\alpha E_i)$ if $\lambda = \exp(-\alpha \delta)$
- For Fermi-Dirac statistics, repeat as for MB, but with the Pauli Exclusion Principle included, i.e.
- Can only get transfer of energy $E_1, E_2 \rightarrow E_3, E_4$ if E_1 & E_2 are occupied AND if E_3 & E_4 are unoccupied

J.E. Morris

ECE415/515 Fall 2012

13

Fermi function

So the collision probability is $C \cdot p(E_1) \cdot p(E_2) \cdot [1 - p(E_3)] \cdot [1 - p(E_4)]$
and applying Equilibrium and Energy Conservation again:

$$C \cdot p(E_1) \cdot p(E_2) \cdot [1 - p(E_1 + \delta)] \cdot [1 - p(E_2 - \delta)] = C \cdot p(E_1 + \delta) \cdot p(E_2 - \delta) \cdot [1 - p(E_1)] \cdot [1 - p(E_2)]$$

$$\text{i.e. } \frac{1 - p(E_1)}{p(E_1)} \cdot \frac{p(E_1 + \delta)}{1 - p(E_1 + \delta)} = \frac{1 - p(E_2 - \delta)}{p(E_2 - \delta)} \cdot \frac{p(E_2)}{1 - p(E_2)}$$

$$\text{and } \frac{1 - p(E_i)}{p(E_i)} \cdot \frac{p(E_i + \delta)}{1 - p(E_i + \delta)} = \lambda \text{ (constant)} \rightarrow \text{an exponential solution again,}$$

$$\text{i.e. } \frac{1 - p(E_i)}{p(E_i)} = A \cdot \exp(+\beta E) \text{ (where } \lambda = A^2 \exp(2\beta E))$$

$$\text{i.e. } p(E) = \frac{1}{1 + A \exp(+\beta E)} \rightarrow \frac{1}{A} \exp(-\beta E) \text{ for } \beta \text{ large,}$$

$$\rightarrow \beta = \frac{1}{kT},$$

$$\text{and writing } \frac{1}{A} = \exp\left(\frac{E_F}{kT}\right), f_{FD} = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

J.E. Morris

ECE415/515 Fall 2012

14

MB/FD/(BE) comparisons

Maxwell-Boltzmann	Bose-Einstein	Fermi-Dirac
Particles distinguishable	Particles indistinguishable (wave functions overlap – cannot find individual particle)	
Any number of particles can be in the same energy state		Only one particle in any energy state (Pauli) but note spin degeneracy
Particles are statistically independent		
$\sum_{i=1}^M N_i = N$	$\sum_{i=1}^M N_i E_i = E$	
Atoms, molecules, etc	Bosons Photons, phonons, ⁴ He nuclei (integral spins)	Fermions Electrons, protons, neutrons, ³ He nuclei ($\frac{1}{2}$ -odd-integral spins)

J.E. Morris

ECE415/515 Fall 2012

15

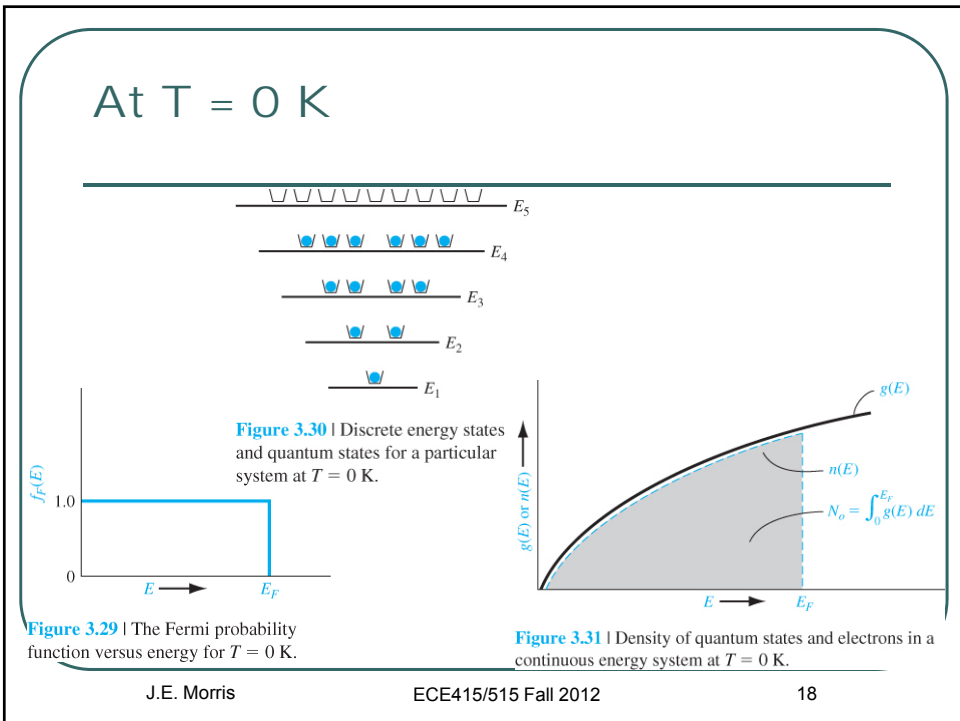
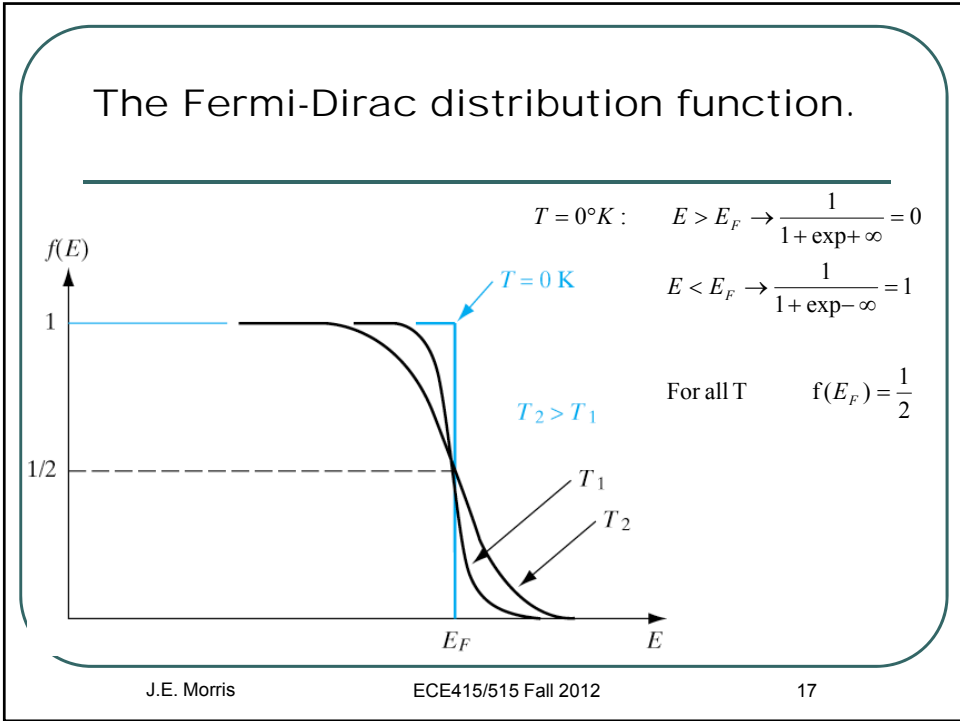
Fermi distributions

- Electrons in solids obey Fermi-Dirac statistics due to the Pauli exclusion principle (each state can have only one electron – but remember spin!)
- Probability density function gives the ratio of filled to total allowed states at a given energy.
- Using statistical mechanics to count states we find the Fermi-Dirac distribution function:
 - $f(E) = \{1 + \exp[(E-E_f)/kT]\}^{-1}$
 - k is Boltzmann's constant
 - = 8.62×10^{-5} eV/K
 - = 1.38×10^{-23} J/K
 - **E_f is the Fermi level**, [where $f(E_f) = \frac{1}{2}$]

J.E. Morris

ECE415/515 Fall 2012

16



At $T > 0 \text{ K}$

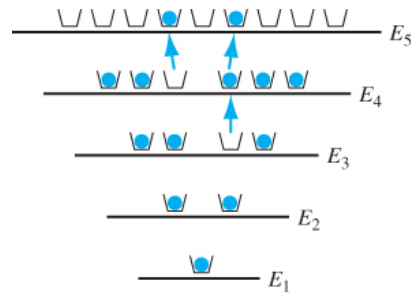


Figure 3.32 | Discrete energy states and quantum states for the same system shown in Figure 3.30 for $T > 0 \text{ K}$.

J.E. Morris

ECE415/515 Fall 2012

19

Fermi levels

- The Fermi distribution gives the *probability* that an *available* state will be occupied.
 - Within the band gap there are no available states.
 - Even with non-zero $f(E)$ there is no occupancy in the gap. (An energy state at the Fermi level would have a probability of $\frac{1}{2}$ of being occupied, if there was one.)
- At 0K every available state up to the Fermi level is occupied and every state above the Fermi level is empty.
- Fermi function is “symmetric” about the Fermi level
- For holes:

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{\exp\left(\frac{E - E_F}{kT}\right)}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_F - E}{kT}\right)}$$

J.E. Morris

ECE415/515 Fall 2012

20

Fermi function symmetry: electrons and holes

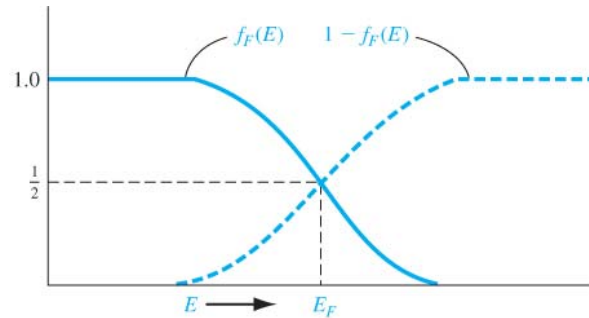


Figure 3.34 | The probability of a state being occupied, $f_F(E)$, and the probability of a state being empty, $1 - f_F(E)$.

J.E. Morris

ECE415/515 Fall 2012

21

Approximate FD \rightarrow MB for $(E - E_F) \gg kT$

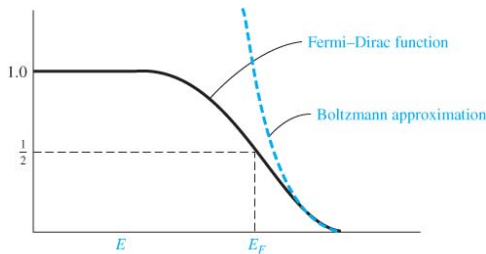


Figure 3.35 | The Fermi-Dirac probability function and the Maxwell-Boltzmann approximation.

See Example 3.6:
 $f(E_F + 3kT) \approx 5\%$ for 300K
 So $f(E)$ varies quickly

Example 3.8:
 $(f_{MB} - f_{FD}) / f_{FD} = 5\%$
 at $E - E_F = kT \ln(1/0.05) \approx 3kT$

Note $f(E) = \{1 + \exp[(E - E_f)/kT]\}^{-1}$
 $\approx \exp[-(E - E_f)/kT]$
 for
 “ $(E - E_F) \gg kT$ ” condition
 where
 $\exp[(E - E_f)/kT] \gg 1$

J.E. Morris

ECE415/515 Fall 2012

22

Ex 3.6 Assume $E_F = E_C - 0.3\text{eV}$ and $T = 300\text{ K}$.
Determine probability of state occupied at
(a) $E = E_C + kT/4$ & (b) $E = E_C + kT$

$$\begin{aligned}
 \text{(a) } f_F(E) &\equiv \exp\left[\frac{-(E - E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(E_C + kT/4 - E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(0.30 + 0.0259/4)}{0.0259}\right] \\
 &= 7.26 \times 10^{-6} \\
 \text{(b) } f_F(E) &\equiv \exp\left[\frac{-(E - E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(0.30 + 0.0259)}{0.0259}\right] \\
 &= 3.43 \times 10^{-6}
 \end{aligned}$$

J.E. Morris

ECE415/515 Fall 2012

23

Ex 3.7 Assume $E_F = E_C - 0.3\text{eV}$. Determine T at
which the probability of state at
 $E = E_C + 0.025\text{eV}$ is occupied is 8×10^{-6} .

$$\begin{aligned}
 f_F(E) &= \exp\left[\frac{-(E - E_F)}{kT}\right] \\
 8 \times 10^{-6} &= \exp\left[\frac{-(0.30 + 0.025)}{kT}\right] \\
 \exp\left[\frac{+0.325}{kT}\right] &= 1.25 \times 10^5 \\
 \frac{0.325}{kT} &= \ln(1.25 \times 10^5) = 11.736 \\
 kT &= \frac{0.325}{11.736} = 0.02769 = (0.0259) \left(\frac{T}{300}\right)
 \end{aligned}$$

so

$$T = 321\text{ K}$$

J.E. Morris

ECE415/515 Fall 2012

24

Ex.3.8 Calculate the energy, in terms of kT and E_F , at which the difference between the FD and MB distributions is 2% of the FD value.

$$\frac{\exp\left[\frac{-(E-E_F)}{kT}\right] - \frac{1}{1 + \exp\left[\frac{E-E_F}{kT}\right]}}{\frac{1}{1 + \exp\left[\frac{E-E_F}{kT}\right]}} = 0.02$$

$$\exp\left[\frac{-(E-E_F)}{kT}\right] \left\{ 1 + \exp\left[\frac{E-E_F}{kT}\right] \right\} - 1 = 0.02$$

Then

$$\exp\left[\frac{-(E-E_F)}{kT}\right] = 0.02$$

$$E - E_F = kT \ln\left(\frac{1}{0.02}\right) = 3.9kT$$

Electron & hole densities

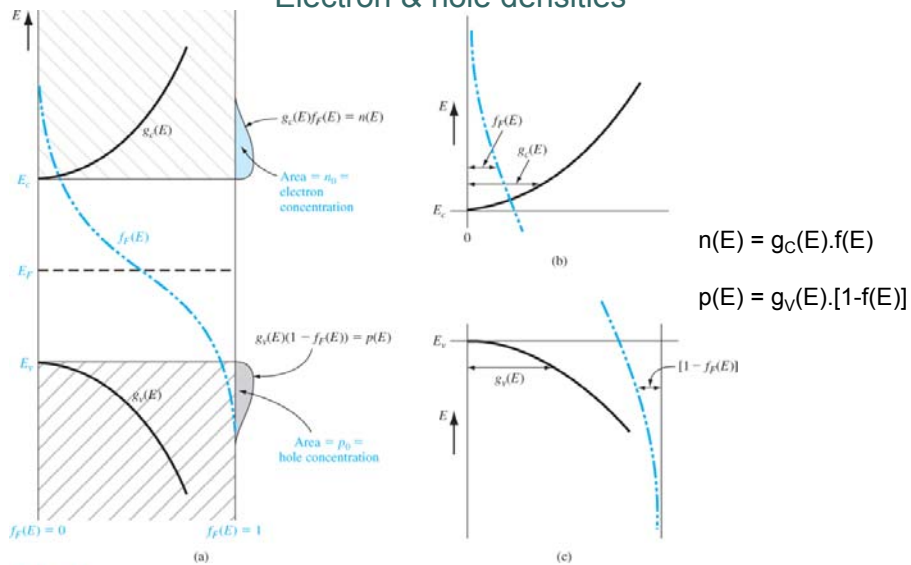


Figure 4.1 (a) Density of states functions, Fermi-Dirac probability function, and areas representing electron and hole concentrations for the case when E_f is near the midgap energy; (b) expanded view near the conduction-band energy; and (c) expanded view near the valence-band energy.

Effective density of states & Fermi integral

Electron density in the conduction band

$$n_0 = \int_{E_C}^{E_C'} n(E) dE = \int_{E_C}^{E_C'} f_{FD}(E) g_C(E) dE \text{ where } E_C' \text{ is the top of the conduction band}$$

$$\approx \int_{E_C}^{\infty} 4\pi \left(\frac{2m_n^*}{h^2} \right)^{\frac{3}{2}} \frac{(E - E_C)^{\frac{1}{2}}}{1 + \exp \frac{E - E_F}{kT}} dE \text{ for } E_C' \rightarrow \infty$$

$$= 4\pi \left(\frac{2m_n^*}{h^2} \right)^{\frac{3}{2}} F_{1/2}(\eta_F),$$

$$\text{where } F_{1/2}(\eta_F) = \int_0^{\infty} \frac{\eta^{\frac{1}{2}} d\eta}{1 + \exp(\eta - \eta_F)}, \eta = \frac{E - E_C}{kT}, \eta_F = \frac{E_F - E_C}{kT}$$

This is the (tabulated) Fermi Integral

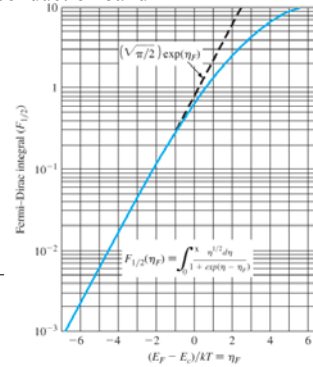


Figure 4.10 The Fermi-Dirac integral $F_{1/2}$ as a function of the Fermi energy. (From See [14].)

A similar treatment gives a similar result for $g_v(E)$ and p_0 for holes in the valence band

Effective density of states

For conduction electrons with $E - E_F \gg kT$ (usually so)

$$f_{FD}(E) \approx f_{MB}(E) = \exp - \frac{E - E_F}{kT}$$

$$\text{so } n_0 \approx \int_{E_C}^{\infty} 4\pi \left(\frac{2m_n^*}{h^2} \right)^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}} \exp - \frac{E - E_F}{kT} dE = 4\pi \left(\frac{2m_n^*}{h^2} \right)^{\frac{3}{2}} \exp - \frac{E_C - E_F}{kT} \int_0^{\infty} (kT)^{\frac{1}{2}} x^{\frac{1}{2}} e^{-x} kT dx$$

$$[\text{writing } x = \frac{E - E_C}{kT}, dE = kT dx, \exp - \frac{E - E_F}{kT} = \exp - \frac{E_C - E_F}{kT} \exp - \frac{E - E_C}{kT}, \text{ and using } \int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx = \frac{\sqrt{\pi}}{2}]$$

$$= 4\pi \left(\frac{2m_n^* kT}{h^2} \right)^{\frac{3}{2}} \exp - \frac{E_C - E_F}{kT} \cdot \frac{\pi^{\frac{1}{2}}}{2} = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{\frac{3}{2}} \exp - \frac{E_C - E_F}{kT} = N_C \exp - \frac{E_C - E_F}{kT}$$

$$\text{where } N_C = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{\frac{3}{2}}$$

N_C is the effective density of states as if all electrons at conduction band edge E_C . Only limiting assumption is that $E_C - E_F \gg kT$; if so, result holds for intrinsic and extrinsic.

(Note: $E_F \rightarrow E_C$ for N-type, but approx usually still OK.)

Electron Concentrations

- Simplify: represent states throughout the conduction band by an *effective density of states* N_c located at E_c .

$$n_0 = N_c f(E_c)$$

- If $E_c - E_F > 3kT$ then

$$f(E_c) \sim e^{-(E_c - E_F)/kT} \quad (\text{Maxwell-Boltzmann})$$

- $kT = 0.026$ eV at 300K, so usually OK

$$n_0 = N_c e^{-(E_c - E_F)/kT}; N_c = 2[2\pi m_n^* kT/h^2]^{3/2}$$

- Termed *non-degenerate*.
- If E_F within $3kT$ of either band edge or inside band → **degenerate** and we need to use the full Fermi function.

Hole Concentrations

- Concentration of holes in valence band

$$p_0 = N_v [1 - f(E_v)]$$

- If $E_F - E_v > 3kT$ then Fermi function reduces to Maxwell Boltzmann and

$$p_0 = N_v \exp[-(E_F - E_v)/kT]$$

$$N_v = [2\pi m_p^* kT/h^2]^{3/2}$$

- $p_0 = \int g_v(E)[1-f(E)]dE$

- $1-f(E) = 1 - [1 + \exp((E - E_F)/kT)]^{-1}$

$$= [\exp(E - E_F)/kT] / [1 + \exp((E - E_F)/kT)]$$

$$= 1 / [1 + \exp((E_F - E)/kT)] \approx \exp(-(E_F - E)/kT)$$

Ex 4.1 Determine the probability that a quantum state at $E=E_C+kT$ is occupied by an electron, and calculate the electron concentration in GaAs at $T=300$ K if $E_F=E_C-0.25$ eV.

$$\begin{aligned}
 f_F &\cong \exp\left[\frac{-(E-E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(E_C+kT-E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(0.25+0.0259)}{0.0259}\right] \\
 f_F &= 2.36 \times 10^{-5} \\
 n_o &= N_c \exp\left[\frac{-(E_C-E_F)}{kT}\right] \\
 &= (4.7 \times 10^{17}) \exp\left[\frac{-0.25}{0.0259}\right] \quad (N_c \text{ from Table 4.1}) \\
 n_o &= 3.02 \times 10^{13} \text{ cm}^{-3}
 \end{aligned}$$

J.E. Morris

ECE415/515 Fall 2012

31

Ex 4.2 (a) Calculate the equilibrium hole concentration in Si at $T=250$ K if $E_F=E_V+0.27$ eV. [$N_V(300\text{K})=1.04 \times 10^{19}/\text{cm}^3$]
 (b) What is $p_o(250\text{K})/p_o(400\text{K})$? (See Example 4.2)

$$\begin{aligned}
 \text{(a) } N_v &= (1.04 \times 10^{19}) \left(\frac{250}{300}\right)^{3/2} \\
 &= 7.9115 \times 10^{18} \text{ cm}^{-3} \\
 kT &= (0.0259) \left(\frac{250}{300}\right) = 0.021583 \text{ eV} \\
 p_o &= N_v \exp\left[\frac{-(E_F-E_V)}{kT}\right] \\
 &= (7.9115 \times 10^{18}) \exp\left[\frac{-0.27}{0.021583}\right] \\
 p_o &= 2.919 \times 10^{13} \text{ cm}^{-3} \\
 \text{(b) Ratio} &= \frac{2.919 \times 10^{13}}{6.43 \times 10^{15}} = 4.54 \times 10^{-3}
 \end{aligned}$$

J.E. Morris

ECE415/515 Fall 2012

32

Intrinsic Concentrations

- $E_F = E_{Fi}$ for intrinsic material

$$n_0 = n_i = N_c \exp[-(E_c - E_{Fi})/kT] \quad p_i = N_v \exp[-(E_{Fi} - E_v)/kT]$$
- $n_0 p_0 = \text{constant}$.
- Plug in above $\Rightarrow n_0 p_0 = n_i^2 = [N_c N_v] \exp[-(E_c - E_v)/kT]$

$$n_i = [N_c N_v]^{1/2} \exp[-E_g/2kT]$$

$$= 2[2\pi kT/h^2]^{3/2} (m_n m_p)^{3/2} \exp[-E_g/2kT]$$
- Remember $n_0 p_0 = n_i^2$ and $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ for Si
- You'll use both often!
- Also for Si: 6 different conduction band valleys (2 each x,y,z)

$$(m_n^*)^{\frac{3}{2}} = 6(m_x m_y m_z)^{\frac{1}{2}}$$
- Note: m_n^* varies in different sources
- See Appendix F for more detail (complications) on effective mass

J.E. Morris

ECE415/515 Fall 2012

33

Ex 4.3 (a) Calculate the intrinsic carrier concentration in GaAs at $T=400 \text{ K}$ and $T=250 \text{ K}$, assuming $E_g=1.42 \text{ eV}$. (b) What is $n_i(400\text{K})/n_i(250\text{K})$?

(a) For $T = 400 \text{ K}$,

$$n_i^2 = (4.7 \times 10^{17}) (7 \times 10^{18}) \left(\frac{400}{300}\right)^3 \times \exp\left[\frac{-1.42}{(0.0259)(400/300)}\right]$$

$$= 1.081 \times 10^{19}$$

$$n_i = 3.29 \times 10^9 \text{ cm}^{-3}$$

For $T = 250 \text{ K}$,

$$n_i^2 = (4.7 \times 10^{17}) (7 \times 10^{18}) \left(\frac{250}{300}\right)^3 \times \exp\left[\frac{-1.42}{(0.0259)(250/300)}\right]$$

$$= 5.09 \times 10^7$$

$$n_i = 7.13 \times 10^3 \text{ cm}^{-3}$$

(b)

$$\frac{n_i(400)}{n_i(250)} = \frac{3.288 \times 10^9}{7.135 \times 10^3} = 4.61 \times 10^5$$

J.E. Morris

ECE415/515 Fall 2012

34

Intrinsic concentrations

- In general, if $E_F \neq E_{Fi}$, then re-write these as

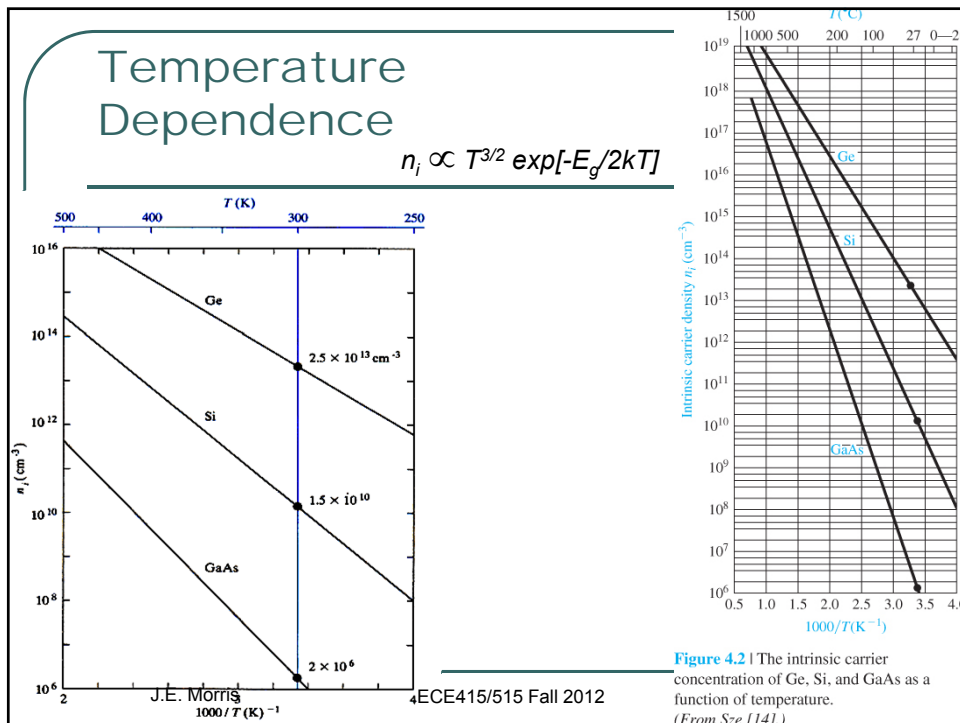
$$n_o = n_i \exp[(E_F - E_{Fi})/kT]$$

$$p_o = p_i \exp[(E_{Fi} - E_F)/kT]$$
- Also used frequently and helpful for visualization
- With each problem consider what information you have and what you are looking for.
- Law of Mass Action:
 - $n_o p_o = n_i^2$
 - Independent of E_F , (i.e. of doping)
 - So applies to both intrinsic and extrinsic at equilibrium

J.E. Morris

ECE415/515 Fall 2012

35



J.E. Morris

ECE415/515 Fall 2012

Physical constants (@300 K unless noted)

Boltzmann's const (k)	$1.38 * 10^{-23}$ J/K $8.62 * 10^{-5}$ eV/K	Planck's constant (h)	$6.63 * 10^{-34}$ J-s $4.14 * 10^{-15}$ eV-s
Room temp value of kT	0.0259 eV	Electron rest mass (m_0)	$9.11 * 10^{-31}$ kg
Electronic charge (q)	$1.60 * 10^{-19}$ C	Permittivity of free space (ϵ_0)	$8.85 * 10^{-14}$ F/cm
Density of states Nc Nv	$2.8 * 10^{19}$ cm ⁻³ $1.04 * 10^{19}$ cm ⁻³	Relative permittivity of silicon (ϵ_r)	11.9
Speed of light (c)	$3 * 10^{10}$ cm/s	Electron volt (eV)	$1.6 * 10^{-19}$ J
Band Gap Si E _g	1.11 eV	Band gap Ge E _g	0.67 eV

J.E. Morris

ECE415/515 Fall 2012

37

Effective density of states & masses

Table 4.1 | Effective density of states function and density of states effective mass values

	N_c (cm ⁻³)	N_v (cm ⁻³)	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

See Appendix B for m^* values

Table 4.2 | Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm ⁻³
Gallium arsenide	$n_i = 1.8 \times 10^6$ cm ⁻³
Germanium	$n_i = 2.4 \times 10^{13}$ cm ⁻³

J.E. Morris

ECE415/515 Fall 2012

38

Intrinsic E_F position

$$\begin{aligned}
 n_0 &= p_0 = n_i = p_i \\
 N_C \exp\left[-\frac{(E_C - E_{Fi})}{kT}\right] &= N_V \exp\left[-\frac{(E_{Fi} - E_V)}{kT}\right] \\
 \therefore \ln(N_C) + \frac{E_{Fi} - E_C}{kT} &= \ln(N_V) + \frac{E_V - E_{Fi}}{kT} \\
 E_{Fi} &= \frac{1}{2}(E_C + E_V) + \frac{1}{2}kT \cdot \ln\left(\frac{N_V}{N_C}\right) \\
 &= \frac{1}{2}(E_C + E_V) + \frac{3}{4}kT \cdot \ln\left(\frac{m_p^*}{m_n^*}\right) \\
 \text{i.e. } E_{Fi} &= E_{midgap} + \frac{3}{4}kT \cdot \ln\left(\frac{m_p^*}{m_n^*}\right) \rightarrow E_{midgap} \text{ if } m_p^* = m_n^*
 \end{aligned}$$

J.E. Morris

ECE415/515 Fall 2012

39

Ex 4.4 Find the position of E_{Fi} at $T=300$ K wrt the band-gap center for (a) GaAs, and (b) Ge.

(a) GaAs

$$\begin{aligned}
 E_{Fi} - E_{midgap} &= \frac{3}{4}kT \ln\left(\frac{m_p^*}{m_n^*}\right) \\
 &= \frac{3}{4}(0.0259)\ln\left(\frac{0.48}{0.067}\right) \\
 &\Rightarrow +38.25 \text{ meV}
 \end{aligned}$$

(b) Ge

$$\begin{aligned}
 E_{Fi} - E_{midgap} &= \frac{3}{4}(0.0259)\ln\left(\frac{0.37}{0.55}\right) \\
 &\Rightarrow -7.70 \text{ meV}
 \end{aligned}$$

J.E. Morris

ECE415/515 Fall 2012

40

Summary: Fermi function

- Fermi function describes probability of a given state being occupied at temperature T.

$$f(E) = \{1 + \exp[(E-E_F)/kT]\}^{-1}$$

- For intrinsic semiconductors E_F is mid-gap.
- If $E - E_F > 3kT$ (non-degenerate, with E_F in the forbidden gap) then

$$f(E_c) \sim e^{-(E-E_F)/kT}$$

- Use *effective density of states* N_c located at E_c or N_v located at E_v . (still non-degenerate)

$$n_0 = N_c e^{-(E_c-E_F)/kT}; N_c = 2[2\pi m_n^* kT/h^2]^{3/2}$$

$$p_0 = N_v e^{-(E_v-E_F)/kT}; N_v = [2\pi m_p^* kT/h^2]^{3/2}$$