# EE415/515 Fundamentals of Semiconductor Devices Fall 2012

## Lecture 14: Bipolar Junction Transistor
## (Chapter 12.4-12.8)

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## Base-width modulation (Early Effect)

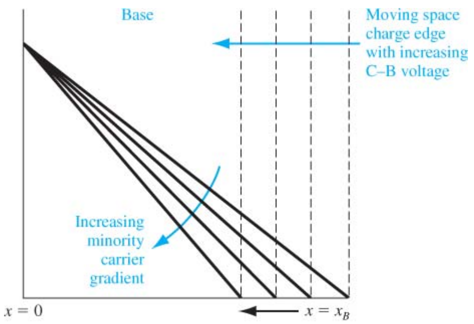


**Figure 12.21** | The change in the base width and the change in the minority carrier gradient as the B–C space charge width changes.

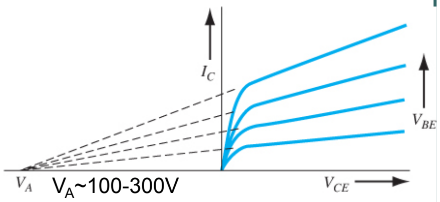


**Figure 12.22** | The collector current versus collector–emitter voltage showing the Early effect and Early voltage.

$$g_o \equiv \frac{dI_C}{dV_{CE}} = \frac{I_C}{V_{CE}+V_A} = \frac{1}{r_o}$$

$$I_C = g_o\left(V_{CE}+V_A\right)$$

Base-width modulation (narrowing) with CB reverse bias due to depletion width

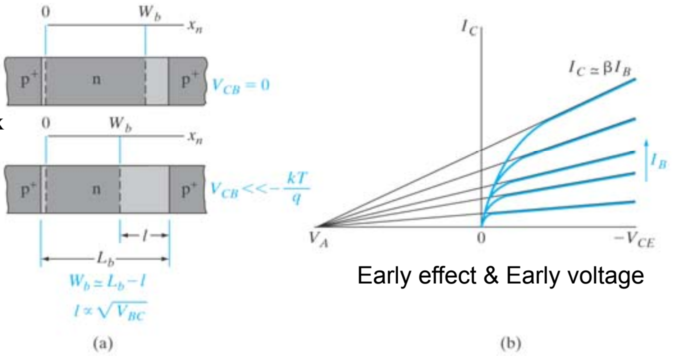
For p$^+$n CB:

depletion width approx

$$l = \sqrt{\frac{2\varepsilon V_{BC}}{qN_d}}$$

and $W_b \approx L_b - l$

β increases with $V_{BC}$



Early effect & Early voltage

Figure 7.18

The effects of base narrowing on the characteristics of a p$^+$-n-p$^+$ transistor: (a) decrease in the effective base width as the reverse bias on the collector junction is increased; (b) common-emitter characteristics showing the increase in $I_C$ with increased collector voltage. The black lines in (b) indicate the extrapolation of the curves to the Early voltage $V_A$.

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**Example 12.8 For uniformly doped Si NPN BJT with $N_B$=5x10$^{16}$/cc, $N_C$=2x10$^{15}$/cc, $x_{B0}$=0.70μm<<$L_B$, $D_B$=25cm$^2$/s, & $V_{BE}$=0.60V. Calculate the change in $J_C$ for $V_{CB}$=2V to 10V and determine $V_A$.**

**First calculate base widths for $V_{CB}$=2V and 10V**

The space charge width extending into the base region is

$$x_{dB} = \left\{ \frac{2\in_s (V_{bi} + V_{BC})}{e} \left[ \frac{N_C}{N_B} \cdot \frac{1}{(N_B + N_C)} \right] \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85\times10^{-14})(V_{bi} + V_{CB})}{1.6\times10^{-19}} \times \left[ \frac{2\times10^{15}}{5\times10^{16}} \cdot \frac{1}{(5\times10^{16} + 2\times10^{15})} \right] \right\}^{1/2}$$

$$= \left\{ (9.956\times10^{-12})(V_{bi} + V_{CB}) \right\}^{1/2}$$

Now $V_{bi} = V_t \ln\left( \frac{N_B N_C}{n_i^2} \right)$

$$= (0.0259)\ln\left[ \frac{(5\times10^{16})(2\times10^{15})}{(1.5\times10^{10})^2} \right]$$

$$= 0.6946\,\mathrm{V}$$

For $V_{CB} = 2\,\mathrm{V}$,

$$x_{dB} = \left\{ (9.956\times10^{-12})(0.6946 + 2) \right\}^{1/2}$$

$$= 5.18\times10^{-6}\,\mathrm{cm} = 0.0518\,\mu\mathrm{m}$$

For $V_{CB} = 10\,\mathrm{V}$,

$$x_{dB} = \left\{ (9.956\times10^{-12})(0.6946 + 10) \right\}^{1/2}$$

$$= 1.03\times10^{-5}\,\mathrm{cm} = 0.103\,\mu\mathrm{m}$$

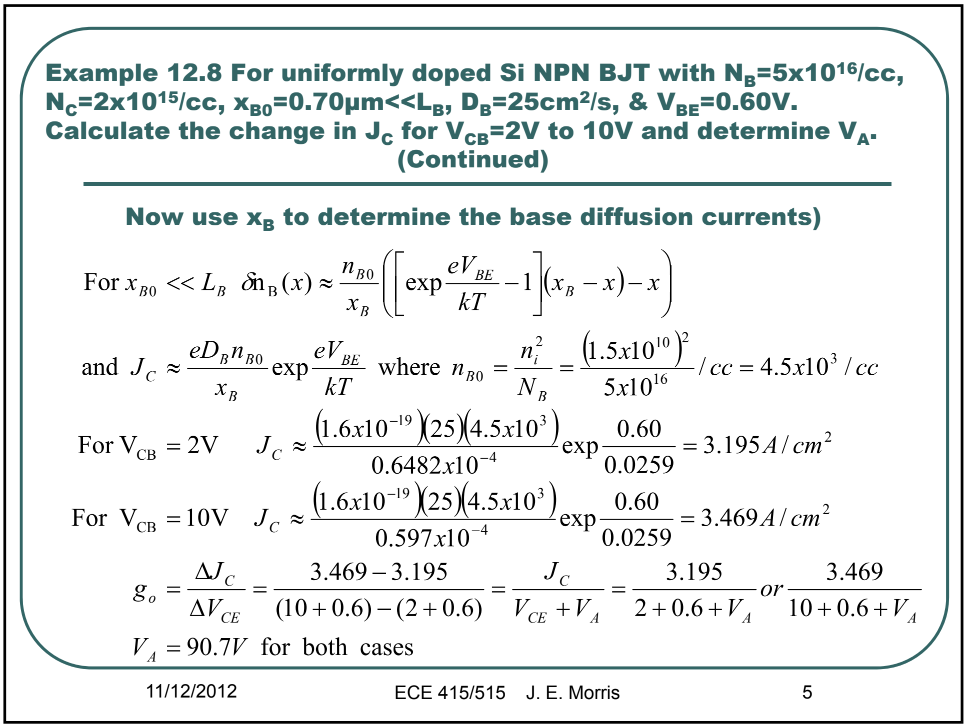
Neglecting the B-E space charge width, we find the neutral base width to be:

$V_{CB} = 2\,\mathrm{V}$,

$$x_B = x_{B0} - x_{dB} = 0.70 - 0.0518 = 0.6482\,\mu\mathrm{m}$$

$V_{CB} = 10\,\mathrm{V}$,

$$x_B = 0.70 - 0.103 = 0.597\,\mu\mathrm{m}$$



**Example 12.8 For uniformly doped Si NPN BJT with $N_B$=5x$10^{16}$/cc, $N_C$=2x$10^{15}$/cc, $x_{B0}$=0.70μm<<$L_B$, $D_B$=25cm$^2$/s, & $V_{BE}$=0.60V. Calculate the change in $J_C$ for $V_{CB}$=2V to 10V and determine $V_A$. (Continued)**

**Now use $x_B$ to determine the base diffusion currents)**

$$\text{For } x_{B0} << L_B \quad \delta n_B(x) \approx \frac{n_{B0}}{x_B}\left(\left[\exp\frac{eV_{BE}}{kT}-1\right](x_B - x) - x\right)$$

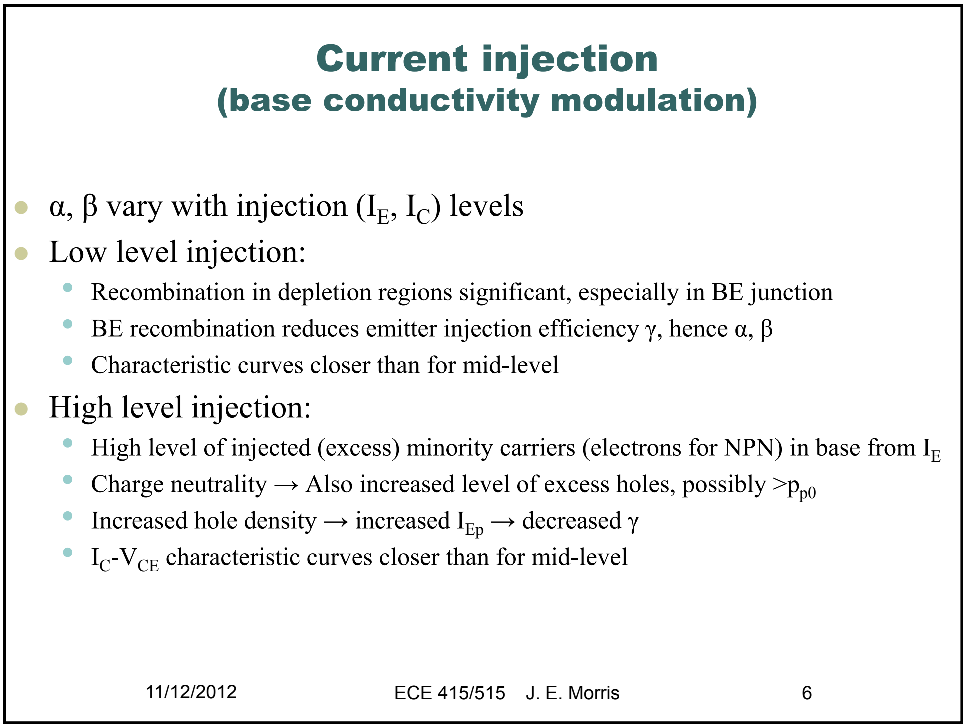
$$\text{and } J_C \approx \frac{eD_B n_{B0}}{x_B}\exp\frac{eV_{BE}}{kT} \text{ where } n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{5x10^{16}}/cc = 4.5x10^3/cc$$

$$\text{For } V_{CB} = 2V \quad J_C \approx \frac{(1.6x10^{-19})(25)(4.5x10^3)}{0.6482x10^{-4}}\exp\frac{0.60}{0.0259} = 3.195A/cm^2$$

$$\text{For } V_{CB} = 10V \quad J_C \approx \frac{(1.6x10^{-19})(25)(4.5x10^3)}{0.597x10^{-4}}\exp\frac{0.60}{0.0259} = 3.469A/cm^2$$

$$g_o = \frac{\Delta J_C}{\Delta V_{CE}} = \frac{3.469-3.195}{(10+0.6)-(2+0.6)} = \frac{J_C}{V_{CE}+V_A} = \frac{3.195}{2+0.6+V_A} \; or \; \frac{3.469}{10+0.6+V_A}$$

$$V_A = 90.7V \text{ for both cases}$$



## Current injection
## (base conductivity modulation)

- α, β vary with injection ($I_E$, $I_C$) levels
- Low level injection:
  - Recombination in depletion regions significant, especially in BE junction
  - BE recombination reduces emitter injection efficiency γ, hence α, β
  - Characteristic curves closer than for mid-level
- High level injection:
  - High level of injected (excess) minority carriers (electrons for NPN) in base from $I_E$
  - Charge neutrality → Also increased level of excess holes, possibly >$p_{p0}$
  - Increased hole density → increased $I_{Ep}$ → decreased γ
  - $I_C$-$V_{CE}$ characteristic curves closer than for mid-level

# High injection:

## Injected minority carriers in base ≥ majority

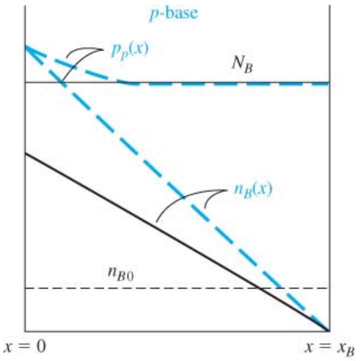


**Figure 12.23** | Minority and majority carrier concentrations in the base under low and high injection (solid line: low injection; dashed line: high injection).

1. High minority electron injection into B (NPN) increases majority hole density at BE junction, and increases $J_{pE}$, decreasing $\alpha, \beta$
2. Low injection:

$$p_p(0) = p_{p0} = N_a \quad \text{and} \quad n_p(0) = n_{p0} \exp\frac{eV_{BE}}{kT}$$

so $p_p(0) n_p(0) = p_{p0} n_{p0} \exp\frac{eV_{BE}}{kT}$

High injection:

$p_p(0) n_p(0) = p_{p0} n_{p0} \exp\frac{eV_{BE}}{kT}$ still,

but $p_p(0) \approx n_p(0)$ so $n_p(0) = n_{p0} \exp\frac{eV_{BE}}{2kT}$

$\therefore$ $n_p(0)$, $J_{nE}$ increase more slowly with $V_{BE}$

# High/low-level injection effects

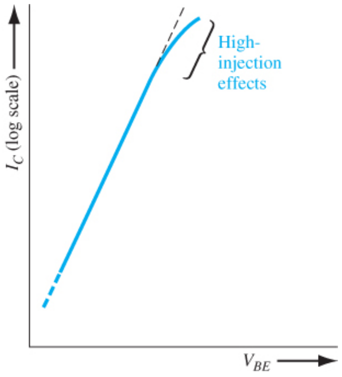


**Figure 12.25** | Collector current versus base–emitter voltage showing high-injection effects.

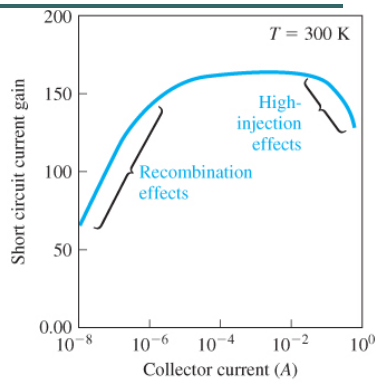


**Figure 12.24** | Common-emitter current gain versus collector current. *(From Shur [14].)*

## Thermal effects

- Joule heating by $I_C V_{BC}$
- Dominant effect:
  - Carrier lifetimes $\tau_n$, $\tau_p$ increase → increase $\beta$
  - Thermal runaway
- Also:
  - Mobilities $\mu_n$, $\mu_p$ decrease (as $T^{-3/2}$) → $D_n$, $D_p$ decrease
  - Increase base transit time $\tau_t$ → decrease $\beta$

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## Emitter band-gap narrowing

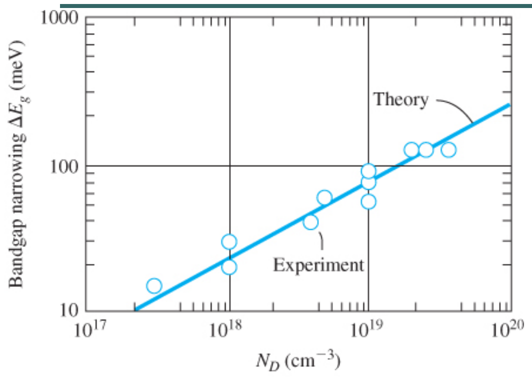


**Figure 12.26** | Bandgap narrowing factor versus donor impurity concentration in silicon.
*(From Sze [19].)*

For N - type emitter .....
As $N_E$ increases, donors closer together
Interaction splits/broadens $E_D$ level,
which reaches $E_C$. $E_G$ decreased.

$$n_{iE}^2 = N_c N_v \exp\frac{-E_G}{kT}$$

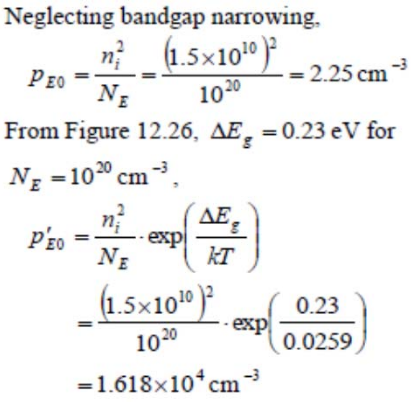
$$= N_c N_v \exp\frac{-(E_{G0} - \Delta E_G)}{kT}$$

$$= n_i^2 \exp\frac{\Delta E_G}{kT}$$

$$p'_{E0} = \frac{n_{iE}^2}{N_E} = \frac{n_i^2}{N_E}\exp\frac{\Delta E_G}{kT}$$

decreases less with $N_E$ increases,
(may increase.)
Emitter injection doesn't increase as fast
with $N_E$ increases, (may decrease.)

**Ex 12.9 Determine the thermal equilibrium minority carrier concentration for an emitter doping concentration $N_E$=$10^{20}$/cc, taking band-gap narrowing into account.**



Neglecting bandgap narrowing,

$$p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{10^{20}} = 2.25\,\mathrm{cm}^{-3}$$

From Figure 12.26, $\Delta E_g = 0.23$ eV for

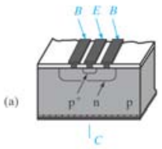
$N_E = 10^{20}\,\mathrm{cm}^{-3}$,

$$p'_{E0} = \frac{n_i^2}{N_E}\cdot\exp\left(\frac{\Delta E_g}{kT}\right)$$

$$= \frac{(1.5\times10^{10})^2}{10^{20}}\cdot\exp\left(\frac{0.23}{0.0259}\right)$$

$$= 1.618\times10^{4}\,\mathrm{cm}^{-3}$$

E, C areas differ → different $\alpha_N$ and $\alpha_I$



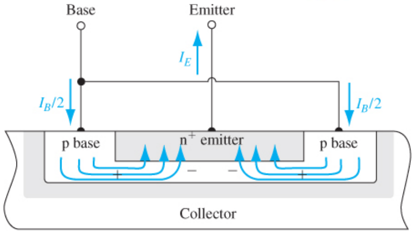
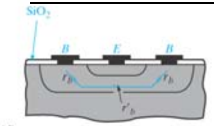
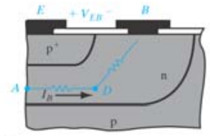


**Figure 12.27** | Cross section of an npn bipolar transistor showing the base current distribution and the lateral potential drop in the base region.

Base resistance: physical PNP structure



($r_b$ resistance to contacts)
$r_b{}'$ base spreading resistance (thin base)
$V_{EB}$ increased near B contacts



$V_{EA} = V_{EB} - I_B(R_{AD}+R_{DB})$ & $V_{ED} = V_{EB} - I_BR_{DB}$
Forward EB bias greatest at E edge ($V_{ED}$)

Figure 7.20

Effects of a base resistance: (a) cross section of an implanted transistor; (b) and (c) top view, showing emitter and base areas and metallized contacts; (d) illustration of base resistance; (e) expanded view of distributed resistance in the active part of the base region.

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# Emitter current crowding

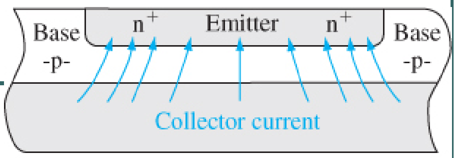

Figure 12.28 | Cross section of an npn bipolar transistor showing the emitter current crowding effect.

Forward BE bias greatest at E edge
Hence hole injection greatest at E edge
Possible high injection effects here

Minimize crowding
→ minimize $r_{bb}$ effects

→ (a) long thin E contact
or (b) inter-digitated structure

(Power BJTs)

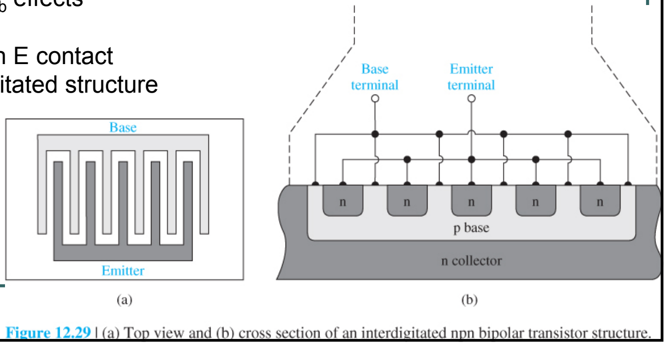

Figure 12.29 | (a) Top view and (b) cross section of an interdigitated npn bipolar transistor structure.

# Non-uniform base doping:

## Leads to electric field in base (assumed zero)

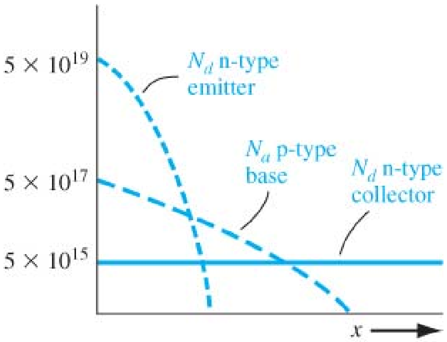

Figure 12.31 | Impurity concentration profiles of a double-diffused npn bipolar transistor.

$$J_p = e\mu_p N_a E - eD_p \frac{dN_a}{dx} = 0$$

in equilibrium,

$$\text{so } E = +\frac{kT}{e} \cdot \frac{1}{N_a} \cdot \frac{dN_a}{dx}$$

$\frac{dN_a}{dx} < 0$ here, so $E_x < 0$,

accelerates electrons to collector,

i.e. drift current + diffusion current

Diffusion doping leads to donor concentration gradient $N_d(x)$ in the base
($N_d$ previously assumed to be constant)
Hence $N_d(0) > N_d(W_b)$ leading to electric field drift assisting base transport of holes
Reduces base transit time $\tau_t$

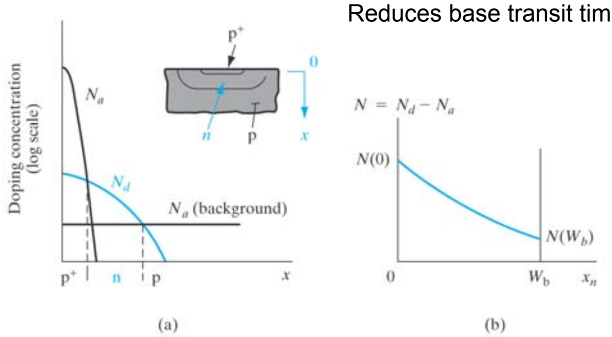

Figure 7.17

Graded doping in the base region of a p-n-p transistor: (a) typical doping profile on a semilog plot; (b) approximate exponential distribution of the net donor concentration in the base region on a linear plot.

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# Base field

For balanced electron drift and diffusion at equilibrium, and assuming $n_p(x_n) \approx N_d(x_n)$

$$I_n(x_n) = qA\mu_n N_d(x_n)\xi(x_n) + qaD_n \frac{dN_d(x_n)}{dx_n} = 0 \text{ gives}$$

$$\xi(x_n) = -\frac{D_n}{\mu_n}\frac{1}{N_d(x_n)}\frac{dN_d(x_n)}{dx_n} = -\frac{kT}{q}\frac{1}{N_d(x_n)}\frac{dN_d(x_n)}{dx_n}$$

For an exponential distribution $N_d(x_n) = N_d(0)\exp - ax_n/W_b$

where $N_d(W_b) = N_d(0)\exp - aW_b/W_b$ so $a = \ln[N(0)/N(W_b)]$

$$\xi(x_n) = -\frac{kT}{q}\frac{1}{N_d(x_n)}\frac{dN_d(x_n)}{dx_n}$$

$$= -\frac{kT}{q}\frac{1}{N_d(0)\exp - ax_n/W_b}(-a/W_b)N_d(0)\exp - ax_n/W_b$$

$$= \frac{kT}{q}\frac{a}{W_b}$$

# Breakdown voltage: Punch-through

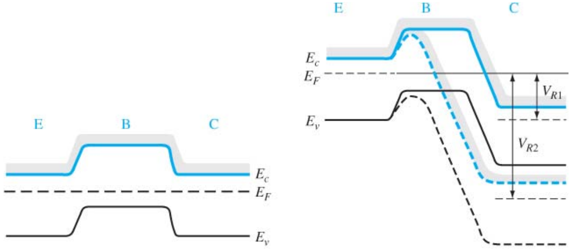


**Figure 12.32** | Energy-band diagram of an npn bipolar transistor (a) in thermal equilibrium, and (b) with a reverse-biased B–C voltage before punch-through, $V_{R1}$, and after punch-through, $V_{R2}$.

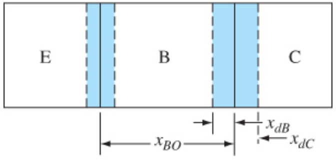


**Figure 12.33** | Geometry of a bipolar transistor to calculate the punch-through voltage.

Punch - through when $x_{dB} = x_{B0}$, i.e. when $x_{dB} = x_{B0} = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_{pt})}{e} \cdot \frac{N_C}{N_B} \cdot \frac{1}{N_C + N_B}}$

i.e. when $V_{pt} = \frac{e x_{B0}^2}{2\varepsilon_s} \cdot \frac{N_B}{N_C} (N_B + N_C)$ for $V_{pt} >> V_{bi}$

# Breakdown voltage: Avalanche

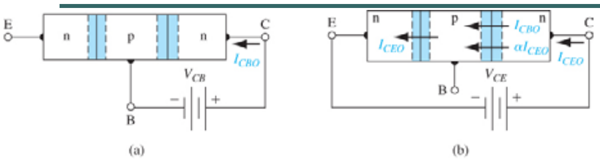


**Figure 12.34** | (a) Open-emitter configuration with saturation current $I_{CBO}$; (b) Open-base configuration with saturation current $I_{CEO}$.

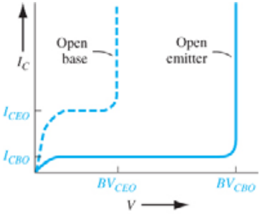


**Figure 12.35** | Relative breakdown voltages and saturation currents of the open-base and open-emitter configurations.

$I_{CBO}$ is reverse - biased (saturation?) C - B current

For open base, B - E forward biased, current $I_{CEO}$

$$I_{CEO} = \alpha I_{CEO} + I_{CBO} \Rightarrow I_{CEO} = \frac{I_{CBO}}{1-\alpha} \approx \beta I_{CBO}$$

At C - B breakdown $I_{CBO} \rightarrow M I_{CBO}$ where $M = \frac{1}{1-\left(V_{CB}/BV_{CBO}\right)^n}$ where $n = 3 \rightarrow 6$

For open base at beakdown, $I_{CEO} = M(\alpha I_{CEO} + I_{CBO}) = \frac{M I_{CBO}}{1-\alpha M} \rightarrow \infty$ for $\alpha M = 1$

i.e. when $\frac{\alpha}{1-\left(BV_{CEO}/BV_{CBO}\right)^n} = 1$ (for $V_{CE} \approx V_{CB}$ at breakdown), i.e. $BV_{CEO} = BV_{CBO}\sqrt[n]{1-\alpha} \approx \frac{BV_{CBO}}{\sqrt[n]{\beta}}$

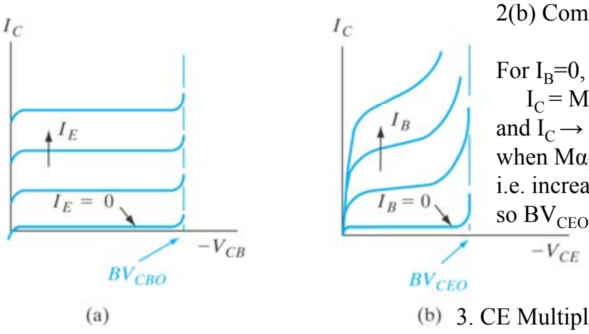
Three effects:

1. Punchthrough when $W_b=0$, i.e. $V_{BC}=(qN_dL_b/2\varepsilon)^2$, but usually CB avalanche first
2. Avalanche: $I_C = M(\alpha_N I_E + I_{CO})$ where $M = \left[1-\left(V_{BC}/BV_{CBO}\right)^n\right]^{-1}$
3. Current multiplication

2(a) Common-base (CB)

For $I_E=0$:
$I_C = MI_{CO}$
and $I_C \to \infty$
at $V_{BC}=BV_{CBO}$
when $M \to \infty$

(single rev bias diode)



2(b) Common-emitter (CE)

For $I_B=0$, $I_C=I_E$
$I_C = MI_{CO}/(1-M\alpha_N)$
and $I_C \to \infty$
when $M\alpha_N = 1$,
i.e. increase as $M \to 1$,
so $BV_{CEO} \ll BV_{CBO}$

3. CE Multiplication:
C-B EHP generation → hole to C, $e^-$ to B
$I_B$ incr → $I_E$ incr → $I_C$ incr

Figure 7.19

Avalanche breakdown in a transistor: (a) common-base configuration; (b) common-emitter configuration.

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**Ex 12.10 Metallurgical BW of a Si NPN BJT is $x_{B0}$=0.80μm, with $N_B$=5x10$^{16}$/cc & $N_C$=2x10$^{15}$/cc. Determine (a) punch-through voltage & (b) avalanche voltage.**

$$\text{(a)}\quad V_{pt} = \frac{ex_{B0}^2}{2\epsilon_s}\cdot\frac{N_B(N_C+N_B)}{N_C}$$

$$= \frac{(1.6\times10^{-19})(0.80\times10^{-4})^2}{2(11.7)(8.85\times10^{-14})} \times \frac{(5\times10^{16})(2\times10^{15}+5\times10^{16})}{(2\times10^{15})}$$

$$V_{pt} = 643\text{ V}$$

(b) From Figure 7.15, $BV \cong 180\text{ V}$

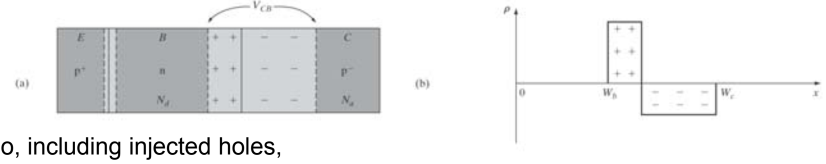
## Ex 12.11
**Uniformly doped Si BJT with $N_B=5\text{x}10^{16}$/cc & $N_C=2\text{x}10^{15}$/cc has β=125. Determine (a) $BV_{CBO}$ and (b) $BV_{CEO}$, for n=3.**

(a) From Figure 7.15, $BV_{CBO} \cong 125$ V

(b) $BV_{CEO} = \dfrac{BV_{CBO}}{\sqrt[n]{\beta}} = \dfrac{125}{\sqrt[3]{125}} = 25$ V

Kirk Effect

Normal C-B depletion region



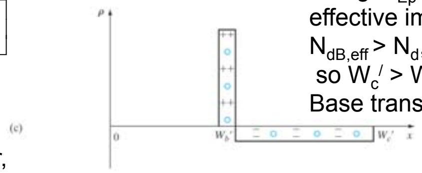
Also, including injected holes, write:

$$\frac{d\xi}{dx} = \frac{1}{\varepsilon}\left[q\left(N_d^+ - N_a^-\right) + \frac{I_C}{Av_d}\right]$$

($v_d$ drift velocity), and

$$V_{CB} = -\int_{W_b}^{W_c} \xi dx$$

$V_{CB}$ fixed, so as $I_C$ incr, base side depletion region → 0



At high $I_{Ep}$, injected holes change effective impurity densities:
$N_{dB,eff} > N_{d,0}$ & $N_{aC,eff} < N_{a,0}$
so $W_c^{/} > W_c$ & $W_b^{/} > W_b$
Base transit time $\tau_t$ incr, so β decr

Figure 7.23

Kirk effect: (a) cross section of p-n-p BJT; (b) space-charge distribution in the base-collector reverse-biased junction for very low currents; (c) space-charge distribution at the base-collector junction for higher current levels. We see that the injected mobile holes (shown in color) add to the space charge of the immobile donors on the base side of the depletion region, but subtract from the space charge of the immobile acceptors on the collector side. This leads to a widening of the neutral base width from $W_b$ to $W_b'$.

From *Solid State Electronic Devices,* Sixth Edition, by Ben G. Streetman and Sanjay Kumar Banerjee.

# Ebers-Moll equivalent circuit

$I_E + I_B + I_C = 0$ for current directions shown

$I_C = \alpha_F I_F - I_R$ in the forward-active mode

where $I_F = I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right]$, $I_{ES}$ is the BE reverse saturation current

and $I_R = I_{CS}\left[\exp\frac{eV_{BC}}{kT} - 1\right]$, $I_{CS}$ is the CB reverse saturation current

i.e. $I_C = \alpha_F I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right] - I_{CS}\left[\exp\frac{eV_{BC}}{kT} - 1\right]$

Similarly: $I_E = \alpha_R I_R - I_F$ in the reverse-active mode, and

$I_E = \alpha_R I_{CS}\left[\exp\frac{eV_{BC}}{kT} - 1\right] - I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right]$

Note: $\alpha_F I_{ES} = \alpha_R I_{CS}$ and $\alpha_F, \alpha_R, I_{CS}, I_{ES}$ not independent

Ebers-Moll valid for all four operating modes

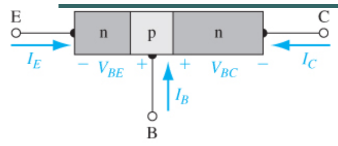


**Figure 12.36** | Current direction and voltage polarity definitions for the Ebers–Moll model.

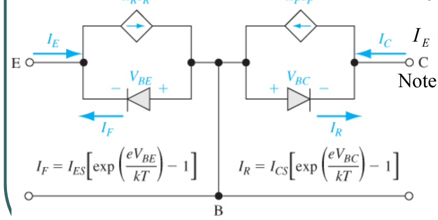


**Figure 12.37** | Basic Ebers–Moll equivalent circuit.

# Ebers-Moll: Saturation voltage

$V_{CE}(sat) = V_{BE} - V_{BC}$ where $V_{BE}$, $V_{BC}$ both > 0.

$$I_E = -(I_B + I_C) = \alpha_R I_{CS}\left[\exp\frac{eV_{BC}}{kT} - 1\right] - I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right]$$

i.e. $\left[\exp\frac{eV_{BC}}{kT} - 1\right] = \frac{1}{\alpha_R I_{CS}}\left(I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right] - (I_B + I_C)\right)$

Substitute into $I_C = \alpha_F I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right] - I_{CS}\left[\exp\frac{eV_{BC}}{kT} - 1\right]$

$$= \alpha_F I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right] - \frac{1}{\alpha_R}\left(I_{ES}\left[\exp\frac{eV_{BE}}{kT} - 1\right] - (I_B + I_C)\right)$$

gives $V_{BE} = \frac{kT}{e}\ln\left[\frac{I_C(1-\alpha_R) + I_B + I_{ES}(1-\alpha_F\alpha_R)}{I_{ES}(1-\alpha_F\alpha_R)}\right] \approx \frac{kT}{e}\ln\left[\frac{I_C(1-\alpha_R) + I_B}{I_{ES}(1-\alpha_F\alpha_R)}\right]$

## Ebers-Moll: Saturation voltage (cont'd)

$$\text{Similarly } \left[\exp\frac{eV_{BE}}{kT}-1\right]=\frac{1}{\alpha_F I_{ES}}\left(I_{CS}\left[\exp\frac{eV_{BC}}{kT}-1\right]+I_C\right)$$

$$\text{Substitute into } -(I_B+I_C)=\alpha_R I_{CS}\left[\exp\frac{eV_{BC}}{kT}-1\right]-I_{ES}\left[\exp\frac{eV_{BE}}{kT}-1\right]$$

$$=\alpha_R I_{CS}\left[\exp\frac{eV_{BC}}{kT}-1\right]-\frac{1}{\alpha_F}\left(I_{CS}\left[\exp\frac{eV_{BC}}{kT}-1\right]+I_C\right)$$

$$\text{gives } \mathrm{V_{BC}}=\frac{kT}{e}\ln\left[\frac{\alpha_F I_B-I_C(1-\alpha_F)+I_{CS}(1-\alpha_F\alpha_R)}{I_{CS}(1-\alpha_F\alpha_R)}\right]$$

$$\approx\frac{kT}{e}\ln\left[\frac{\alpha_F I_B-I_C(1-\alpha_F)}{I_{CS}(1-\alpha_F\alpha_R)}\right]$$

$$\text{so } \mathrm{V_{CE}}(sat)=V_{BE}-V_{BC}\approx\frac{kT}{e}\ln\left[\frac{I_C(1-\alpha_R)+I_B}{I_{ES}(1-\alpha_F\alpha_R)}\cdot\frac{I_{CS}(1-\alpha_F\alpha_R)}{\alpha_F I_B-I_C(1-\alpha_F)}\right]$$

$$=\frac{kT}{e}\ln\left[\frac{I_{CS}}{I_{ES}}\cdot\frac{I_B+(1-\alpha_R)I_C}{\alpha_F I_B-(1-\alpha_F)I_C}\right]$$

### Ex 12.12 Calculate $V_{CE}$(sat) for a BJT at 300K, with $\alpha_F$=0.992, $\alpha_R$=0.05, $I_C$=0.5mA, $I_B$=50µA.

We find

$$V_{CE}(sat)=V_t\ln\left[\frac{I_C(1-\alpha_R)+I_B}{\alpha_F I_B-(1-\alpha_F)I_C}\cdot\frac{\alpha_F}{\alpha_R}\right]$$

$$=(0.0259)\ln\left[\frac{(0.5)(1-0.05)+0.05}{(0.992)(0.05)-(1-0.992)(0.5)}\cdot\frac{0.992}{0.05}\right]$$

$$V_{CE}(sat)=0.141\,\mathrm{V}$$

# Gummel-Poon equivalent circuit

## (more physical - include non-uniform base doping)

In NPN base $J_n = e\mu_n n(x)E + eD_n \frac{dn(x)}{dx}$

$$\text{where E} = \frac{\text{kT}}{\text{e}} . \frac{1}{p(x)} . \frac{dp(x)}{dx}$$

$$\text{so } J_n = e\mu_n n(x)\frac{\text{kT}}{\text{e}} . \frac{1}{p(x)} . \frac{dp(x)}{dx} + eD_n \frac{dn(x)}{dx}$$

$$= \frac{eD_n}{p(x)}\left[ n(x)\frac{dp(x)}{dx} + p(x)\frac{dn(x)}{dx} \right] = \frac{eD_n}{p(x)} . \frac{d(pn)}{dx}$$

$$\frac{J_n}{eD_n}\int_0^{x_B} p(x)dx = \int_0^{x_B} d(pn) = p(x_B)n(x_B) - p(0)n(0)$$

For fwd bias BE, rev bias CB: $n(0) = n_{B0}\exp(eV_{BE}/kT),\ n(x_B) = 0,\ p = n_i^2/n_{B0}$

$$\text{so } J_n = \frac{-eD_n n_i^2 \exp(eV_{BE}/kT)}{\int_0^{x_B} p(x)dx} = \frac{-eD_n n_i^2 \exp(eV_{BE}/kT)}{Q_B}$$

where $\text{Q}_\text{B}$ = Base Gummel Number, total majority charge in base

$$\text{Similarly for the emitter, } J_p = \frac{-eD_p n_i^2 \exp(eV_{BE}/kT)}{\int_0^{x_E} n(x')dx'} = \frac{-eD_p n_i^2 \exp(eV_{BE}/kT))}{Q_E}$$

where $\text{Q}_\text{E}$ = Base Gummel Number, total majority charge in emitter

Note:
Gummel-Poon can include non-ideal effects like BW-modulation, high-level injection by including effects on charges (Gummel numbers)

# Second-order effects in Gummel-Poon

Early effect: Write $Q_B = \int_{0(V_{EB})}^{W_b(V_{CB})} \text{n}(\text{x}_\text{n})dx_n$ with bias dependent limits

High – level injection, when $\int_0^{W_b} \text{n}(\text{x}_\text{n})dx_n > \int_0^{W_b} \text{N}_\text{D}(\text{x}_\text{n})dx_n$ in the base:

$\text{I}_\text{C} \propto I_{Ep} \propto \exp(qV_{EB}/2kT)$ (for high - level injection in the base; see Neaman 8.2.2

and $\text{I}_\text{B} \propto I_{En} \propto \exp(qV_{EB}/kT)$ (since no high - level injection effects in $\text{p}^+$ emitter)

$$\text{so } \beta = \frac{\text{I}_\text{C}}{\text{I}_\text{B}} \propto \frac{\exp(qV_{EB}/2kT)}{\exp(qV_{EB}/kT)} \propto \exp-(qV_{EB}/2kT) \propto I_C^{-1}$$

B - E depletion region generation/recombination at low $\text{I}_\text{E}$:

$I_B \propto I_{En} \propto \exp(qV_{EB}/nkT)$ (see Neaman 8.2.1 non - ideality factor)

but $I_{Ep}$ large and unaffected, so $I_{Ep} \propto \exp(qV_{EB}/kT)$, and

$$\beta = \frac{\text{I}_\text{C}}{\text{I}_\text{B}} \propto \frac{\exp(qV_{EB}/kT)}{\exp(qV_{EB}/nkT)} \propto \exp-\left(1-\frac{1}{n}\right)(qV_{EB}/2kT) \propto I_C^{\left[1-\frac{1}{n}\right]}$$

# Hybrid-pi equivalent circuit

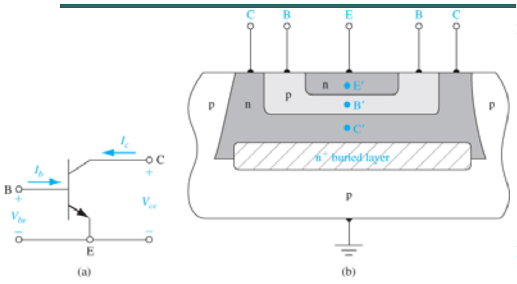


Figure 12.38 | (a) Common-emitter npn bipolar transistor with small-signal current and voltages. (b) Cross section of an npn bipolar transistor for the hybrid-pi model.

E$^/$, B$^/$, C$^/$ "ideal" internal regions

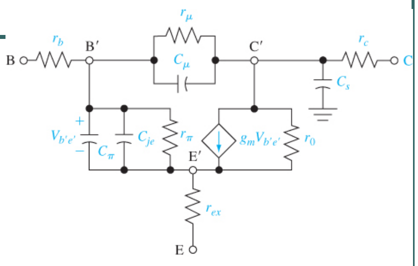


Figure 12.40 | Hybrid-pi equivalent circuit.

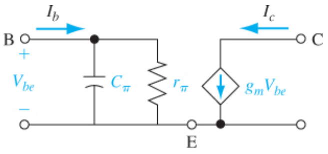
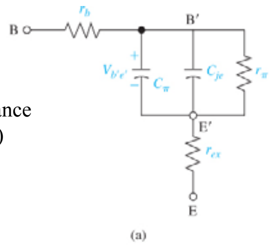


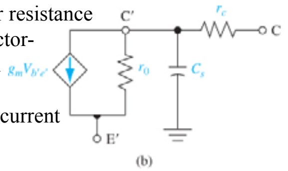
Figure 12.41 | Simplified hybrid-pi equivalent circuit.

# Hybrid-pi components (small signal)

$r_b$: series base resistance
$C_\pi$: fwd bias diffusion capacitance
$r_\pi$: small signal junction resistance
$C_{je}$: junction (space charge) capacitance
$r_{ex}$: series emitter resistance (~1-2Ω)



$r_c$: series collector resistance
$C_s$: rev bias collector-substrate junction
$r_O$: Early effect
$g_m V_{b/e/}$: collector current



$r_\mu$: rev bias small signal resistance (~MΩ, neglect)
$C_\mu$: rev bias CB junction capacitance

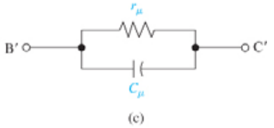


Figure 12.39 | Components of the hybrid-pi equivalent circuit between (a) the base and emitter, (b) the collector and emitter, and (c) the base and collector.

**Ex 12.13 Find the value of $C_\pi$ such that $|A_i|=h_{fe0}/2^{1/2}$ at 35MHz.**

$$|A_i| = \frac{h_{fe0}}{\sqrt{1+(2\pi f r_\pi C_\pi)^2}}$$

We have $f = \dfrac{1}{2\pi r_\pi C_\pi}$

Or $C_\pi = \dfrac{1}{2\pi r_\pi f} = \dfrac{1}{2\pi\left(2.6\times10^3\right)\left(35\times10^6\right)}$

$C_\pi = 1.75\times10^{-12}\,\mathrm{F} = 1.75\,\mathrm{pF}$

# Frequency limits

Need base transit time << signal period

E-C response time $\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$

$\tau_e$ = BE junction charging time $= r_e'(C_{je} + C_\pi)$, $r_e' \approx r_\pi = (kT/e)/I_e$

$\tau_d$ = collector depletion region transit time $= x_{dc}/v_s$ = CB space charge width/saturation velocity

$\tau_c$ = collector capacitance charging time $= r_c(C_\mu + C_s)$

$\tau_b$ = base transit time :

$$J_n = -en_B(x)v(x) = -en_B(x)\frac{dx}{dt} \text{ and } \tau_b = \int_0^{x_B} dt = \int_0^{x_B}\frac{dx}{v(x)} = \int_0^{x_B}\frac{-en_B(x)dx}{J_n}$$

$$\text{For } n_B(x) \approx n_{B0}\left[\exp\frac{eV_{BE}}{kT}\right]\left(1-\frac{x}{x_B}\right) \text{ and } J_n = eD_n\frac{dn_B(x)}{dx} = \frac{eD_n n_{B0}}{x_B}\left[\exp\frac{eV_{BE}}{kT}\right]$$

$$\tau_b = \int_0^{x_B}\frac{-en_B(x)dx}{J_n} = \frac{-en_{B0}\left[\exp\frac{eV_{BE}}{kT}\right]x_B}{eD_n n_{B0}\left[\exp\frac{eV_{BE}}{kT}\right]}\int_0^{x_B}\left(1-\frac{x}{x_B}\right) = \frac{x_B}{D_n}\left(x-\frac{1}{2}\frac{x^2}{x_B}\right)_0^{x_B} = \frac{x_B^2}{2D_n}$$

# BJT cut-off frequency

Alpha cutoff frequency $\alpha = \dfrac{\alpha_0}{1 + j\dfrac{f}{f_\alpha}}$ where $f_\alpha = \dfrac{1}{2\pi\tau_{ec}} = f_T$ and $\alpha = \dfrac{\alpha_0}{\sqrt{2}}$ $at\ f = f_T$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{\dfrac{\alpha_0}{1+j\dfrac{f}{f_T}}}{1 - \dfrac{\alpha_0}{1+j\dfrac{f}{f_T}}} = \frac{\alpha_0}{1-\alpha_0 + j\dfrac{f}{f_T}}$$

$$= \frac{\alpha_0}{1-\alpha_0} \cdot \frac{1}{1 + j\dfrac{f}{(1-\alpha_0)f_T}} \approx \frac{\beta_0}{1 + j\dfrac{f}{(f_T/\beta_0)}} = \frac{\beta_0}{1 + j\dfrac{f}{f_\beta}}$$

$f_\beta$ = Beta cutoff frequency

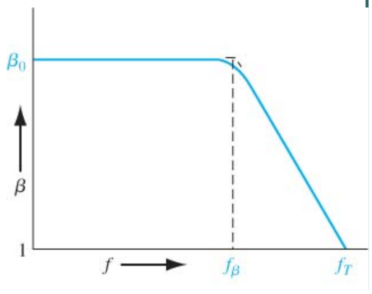


**Figure 12.42** | Bode plot of common-emitter current gain versus frequency.

## Ex 12.14 For BJT with $I_E$=50μA, $C_{je}$=0.40pF, $C_\mu$=0.05pF, $x_B$=0.5μm, $x_{dc}$=2.4μm, $D_n$=25cm²/s, $r_c$=20Ω, $C_s$=0.1pF, determine the E-to-C transit time, the cutoff frequency, and the beta cutoff frequency.

$$r_e' = \frac{V_t}{I_E} = \frac{0.0259}{50\times10^{-6}} = 518\,\Omega$$

$$\tau_e = r_e' \cdot C_{je} = (518)(0.40\times10^{-12})$$
$$= 2.072\times10^{-10}\,\text{s} = 207.2\,\text{ps}$$

$$\tau_b = \frac{x_B^2}{2D_n} = \frac{(0.5\times10^{-4})^2}{2(25)}$$
$$= 5\times10^{-11}\,\text{s} = 50\,\text{ps}$$

$$\tau_d = \frac{x_{dc}}{\upsilon_s} = \frac{2.4\times10^{-4}}{10^7}$$
$$= 2.4\times10^{-11}\,\text{s} = 24\,\text{ps}$$

$$\tau_c = r_c \cdot C_\mu = (20)(0.05\times10^{-12})$$
$$= 1\times10^{-12}\,\text{s} = 1\,\text{ps}$$

Now

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$
$$= 207.2 + 50 + 24 + 1 = 282.2\,\text{ps}$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi(282.2\times10^{-12})}$$
$$= 5.64\times10^8\,\text{Hz} = 564\,\text{MHz}$$

Also

$$f_\beta = \frac{f_T}{\beta} = \frac{564\times10^3}{100}$$
$$= 5.64\times10^6\,\text{Hz} = 5.64\,\text{MHz}$$

# Large-signal switching

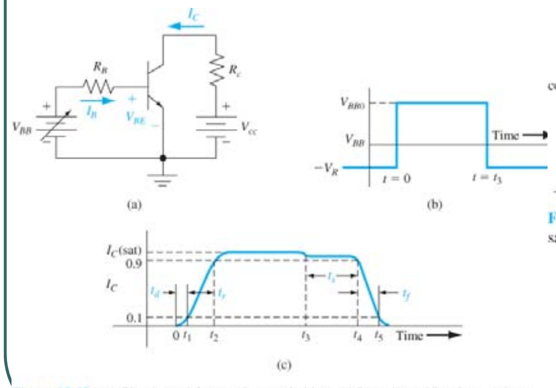


**Figure 12.43** | (a) Circuit used for transistor switching. (b) Input base drive for transistor switching. (c) Collector current versus time during transistor switching.

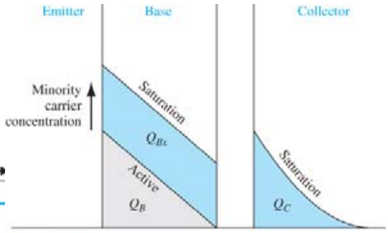


**Figure 12.44** | Charge storage in the base and collector at saturation and in the active mode.

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# Schottky-clamped BJT

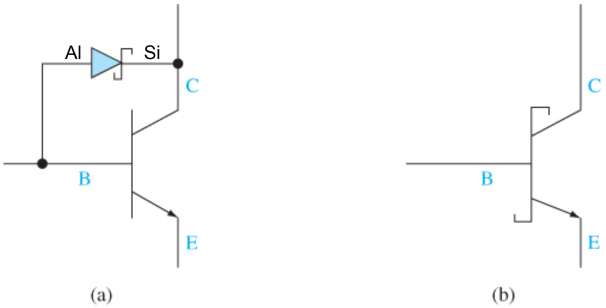
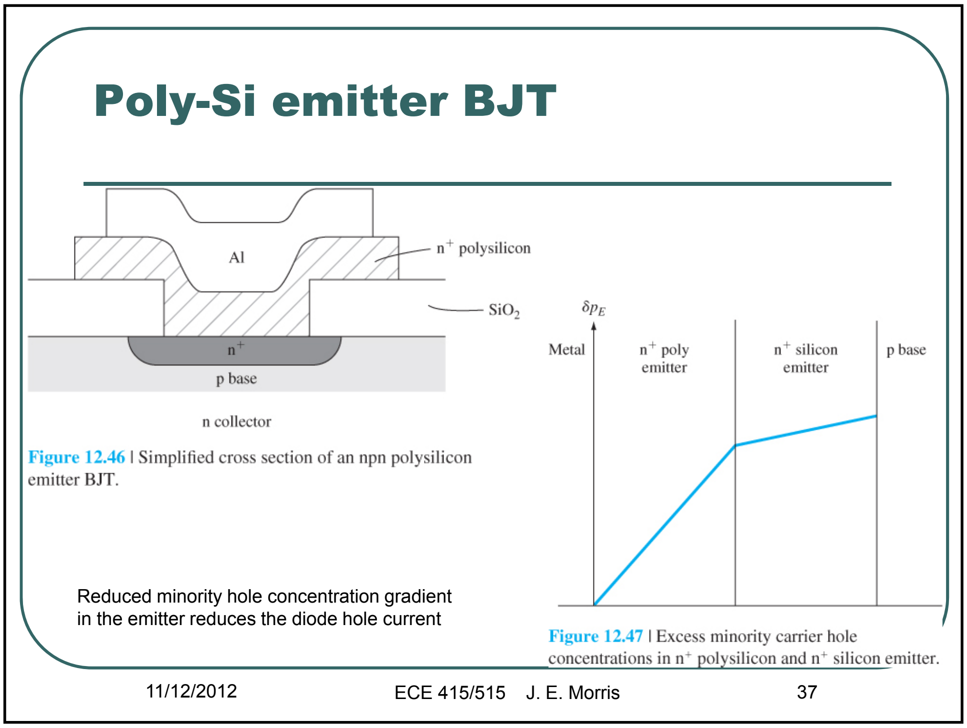


**Figure 12.45** | (a) The Schottky-clamped transistor. (b) Circuit symbol of the Schottky-clamped transistor.

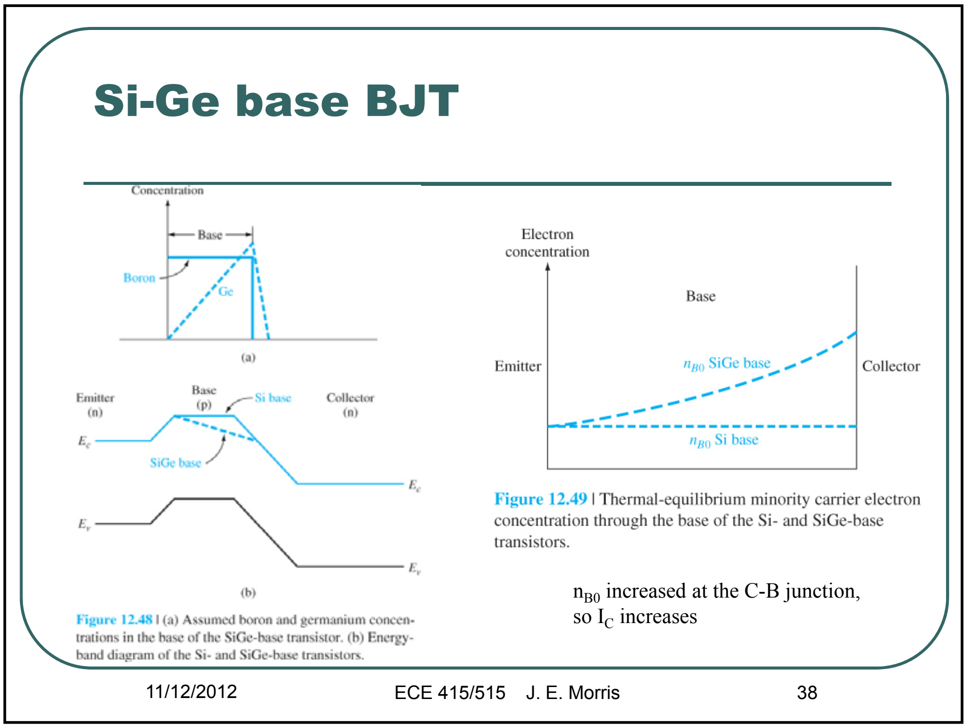


# Poly-Si emitter BJT

Figure 12.46 | Simplified cross section of an npn polysilicon emitter BJT.

Reduced minority hole concentration gradient in the emitter reduces the diode hole current

Figure 12.47 | Excess minority carrier hole concentrations in n$^+$ polysilicon and n$^+$ silicon emitter.



# Si-Ge base BJT

Figure 12.48 | (a) Assumed boron and germanium concentrations in the base of the SiGe-base transistor. (b) Energy-band diagram of the Si- and SiGe-base transistors.

Figure 12.49 | Thermal-equilibrium minority carrier electron concentration through the base of the Si- and SiGe-base transistors.

$n_{B0}$ increased at the C-B junction, so $I_C$ increases

# Heterojunction BJT

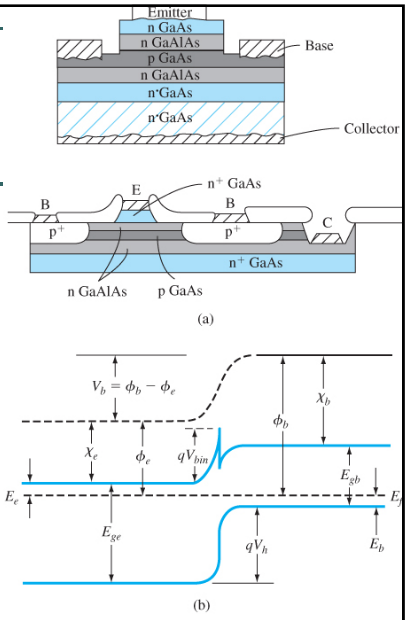


**Figure 12.50** | (a) Cross section of AlGaAs/GaAs hetero-junction bipolar transistor showing a discrete and integrated structure. (b) Energy-band diagram of the n-AlGaAs emitte and p-GaAs base junction.
*(From Tiwari et al. [20].)*

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## Heterojunction BJT

e.g. NPN wide band-gap emitter:
$qV_p$ (holes) >> $qV_n$ ($e^-$s)
Increases $\gamma$ as hole current $I_{Ep}$ suppressed

$$\frac{I_n}{I_p} \propto \frac{N_d^E}{N_a^B} \exp\frac{\Delta E_g}{kT}$$

where $\Delta E_g$ is band gap difference

so exponential effect

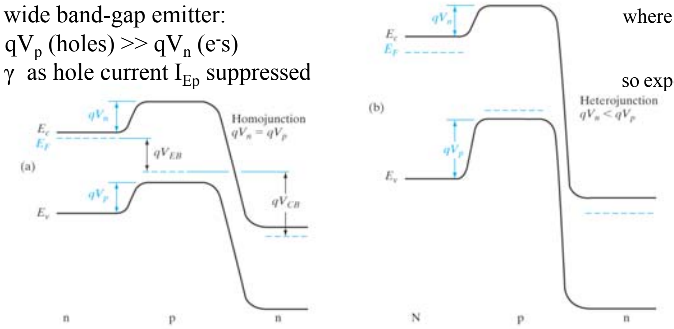

Figure 7.26

Contrast of carrier injection at the emitter of (a) a homojunction BJT and (b) a heterojunction bipolar transistor (HBT). In the forward-biased homojunction emitter, the electron barrier $qV_n$ and the hole barrier $qV_p$ are the same. In the HBT with a wide band gap emitter, the electron barrier is smaller than the hole barrier, resulting in the preferential injection of electrons across the emitter junction.

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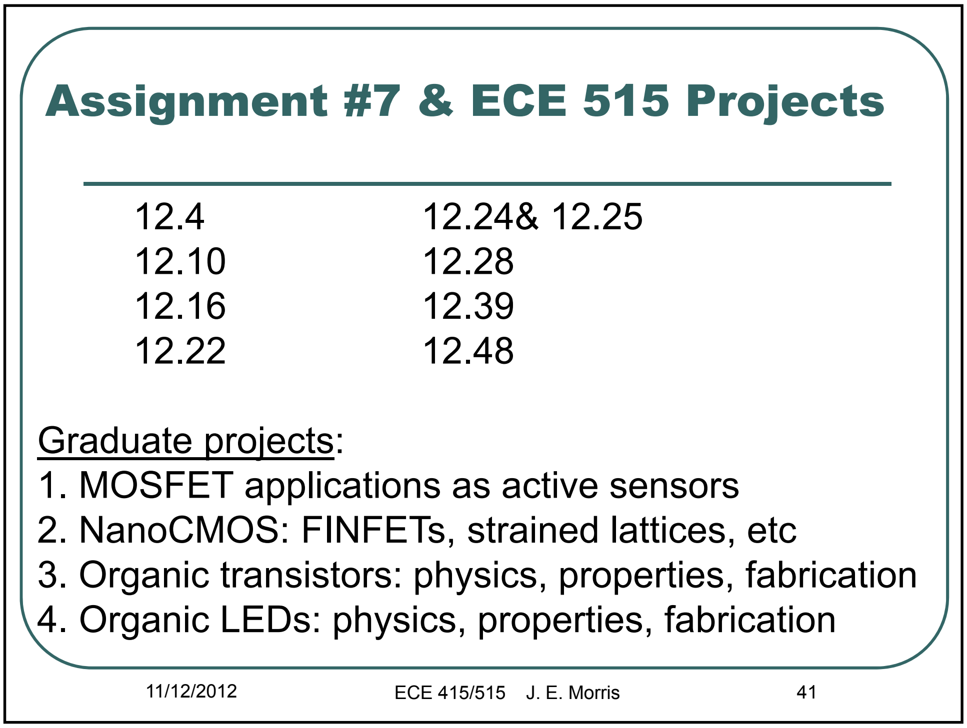
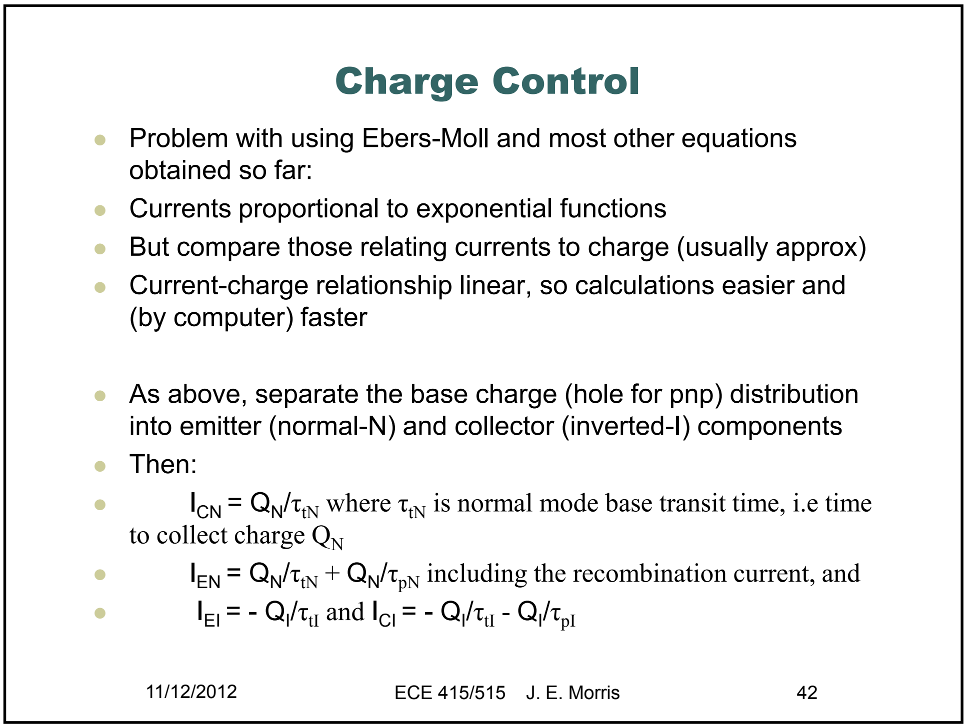


## Assignment #7 & ECE 515 Projects

12.4
12.10
12.16
12.22
12.24& 12.25
12.28
12.39
12.48

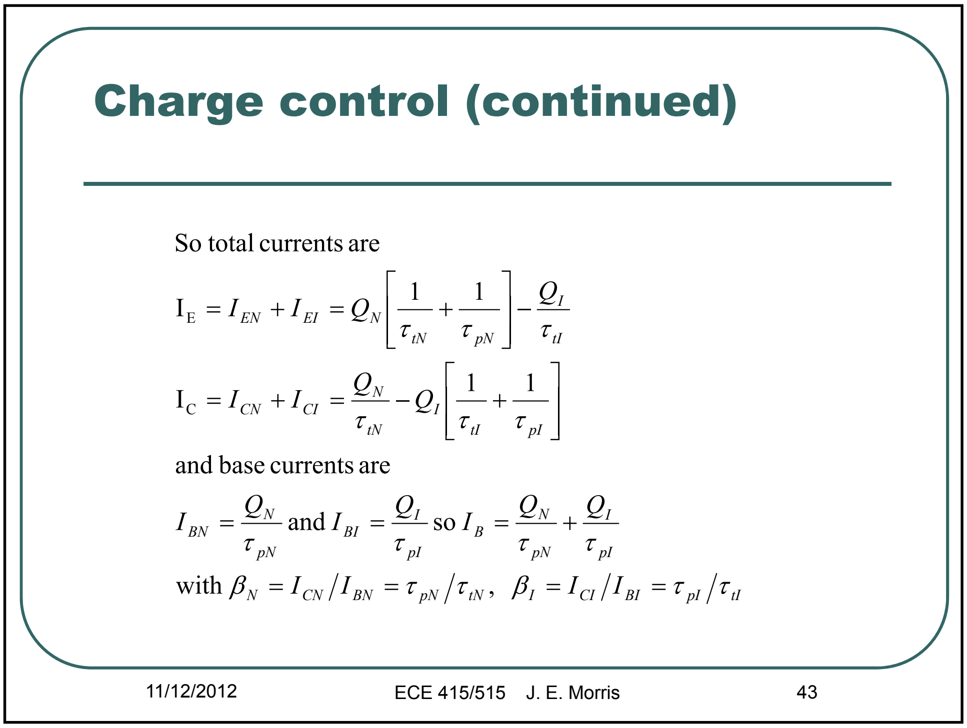
Graduate projects:
1. MOSFET applications as active sensors
2. NanoCMOS: FINFETs, strained lattices, etc
3. Organic transistors: physics, properties, fabrication
4. Organic LEDs: physics, properties, fabrication



## Charge Control

- Problem with using Ebers-Moll and most other equations obtained so far:
- Currents proportional to exponential functions
- But compare those relating currents to charge (usually approx)
- Current-charge relationship linear, so calculations easier and (by computer) faster

- As above, separate the base charge (hole for pnp) distribution into emitter (normal-N) and collector (inverted-I) components
- Then:
- $I_{CN} = Q_N/\tau_{tN}$ where $\tau_{tN}$ is normal mode base transit time, i.e time to collect charge $Q_N$
- $I_{EN} = Q_N/\tau_{tN} + Q_N/\tau_{pN}$ including the recombination current, and
- $I_{EI} = - Q_I/\tau_{tI}$ and $I_{CI} = - Q_I/\tau_{tI} - Q_I/\tau_{pI}$



## Charge control (continued)

So total currents are

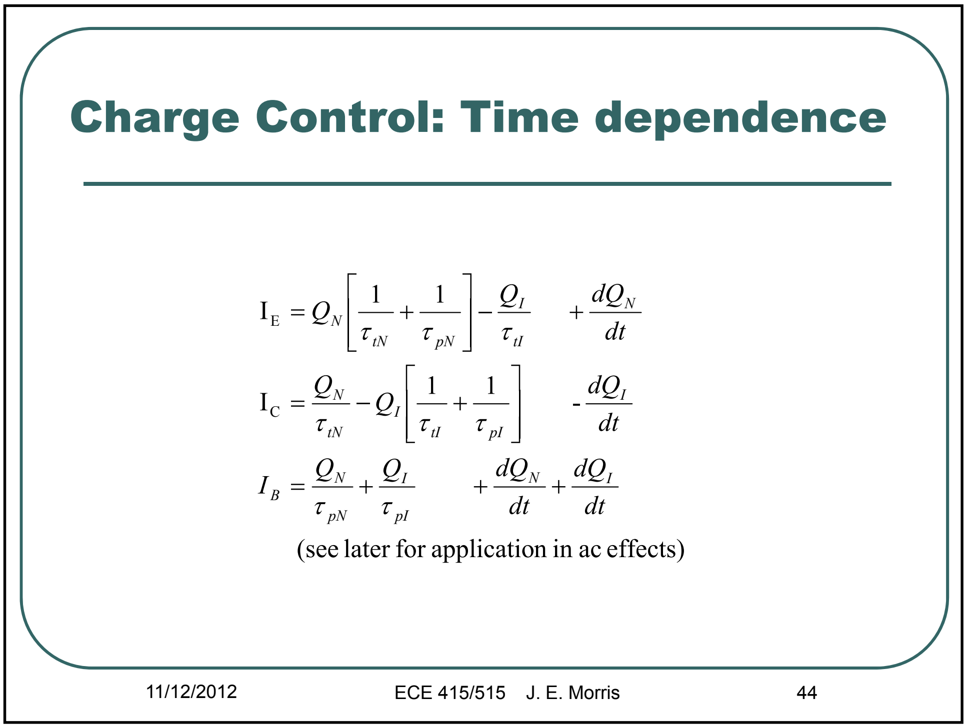
$$\mathrm{I_E} = I_{EN} + I_{EI} = Q_N\left[\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}\right] - \frac{Q_I}{\tau_{tI}}$$

$$\mathrm{I_C} = I_{CN} + I_{CI} = \frac{Q_N}{\tau_{tN}} - Q_I\left[\frac{1}{\tau_{tI}} + \frac{1}{\tau_{pI}}\right]$$

and base currents are

$I_{BN} = \frac{Q_N}{\tau_{pN}}$ and $I_{BI} = \frac{Q_I}{\tau_{pI}}$ so $I_B = \frac{Q_N}{\tau_{pN}} + \frac{Q_I}{\tau_{pI}}$

with $\beta_N = I_{CN}/I_{BN} = \tau_{pN}/\tau_{tN}$, $\beta_I = I_{CI}/I_{BI} = \tau_{pI}/\tau_{tI}$



## Charge Control: Time dependence

$$\mathrm{I_E} = Q_N\left[\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}\right] - \frac{Q_I}{\tau_{tI}} \qquad + \frac{dQ_N}{dt}$$

$$\mathrm{I_C} = \frac{Q_N}{\tau_{tN}} - Q_I\left[\frac{1}{\tau_{tI}} + \frac{1}{\tau_{pI}}\right] \qquad - \frac{dQ_I}{dt}$$

$$I_B = \frac{Q_N}{\tau_{pN}} + \frac{Q_I}{\tau_{pI}} \qquad + \frac{dQ_N}{dt} + \frac{dQ_I}{dt}$$

(see later for application in ac effects)

# Charge Control: Ebers-Moll equivalence

$$\text{Writing } Q_N = q_N \Delta p_E / p_n \;\&\; Q_I = q_I \Delta p_C / p_n \,;\left[\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}\right] = \frac{I_{ES}}{q_N} \;\&\; \left[\frac{1}{\tau_{tI}} + \frac{1}{\tau_{pI}}\right] = \frac{I_{CS}}{q_I}$$

$$\text{so } \mathrm{I_E} = Q_N\left[\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}\right] - \frac{Q_I}{\tau_{tI}} = \frac{q_N \Delta p_E}{p_n}\frac{I_{ES}}{q_N} - \frac{1}{\tau_{tI}}\frac{q_I \Delta p_C}{p_n}$$

$$= I_{ES}(e^{qV_{EB}/kT} - 1) - \alpha_I I_{CS}(e^{qV_{EC}/kT} - 1)$$

$$\mathrm{I_C} = \frac{Q_N}{\tau_{tN}} - Q_I\left[\frac{1}{\tau_{tI}} + \frac{1}{\tau_{pI}}\right] = \frac{1}{\tau_{tN}}\frac{q_N \Delta p_E}{p_n} - \frac{q_I \Delta p_C}{p_n}\frac{I_{CS}}{q_I}$$

$$= \alpha_N I_{ES}(e^{qV_{EB}/kT} - 1) - I_{CS}(e^{qV_{EB}/kT} - 1)$$

$$I_B = \frac{Q_N}{\tau_{pN}} + \frac{Q_I}{\tau_{pI}} = \frac{I_{ES}}{\tau_{tN}}(e^{qV_{EB}/kT} - 1)\left[\frac{1}{\tau_{tN}} + \frac{1}{\tau_{pN}}\right]^{-1} + \frac{I_{CS}}{\tau_{tI}}(e^{qV_{EC}/kT} - 1)\left[\frac{1}{\tau_{tI}} + \frac{1}{\tau_{pI}}\right]^{-1}$$

$$= \alpha_N I_{ES}(e^{qV_{EB}/kT} - 1) + \alpha_I I_{CS}(e^{qV_{EC}/kT} - 1)$$

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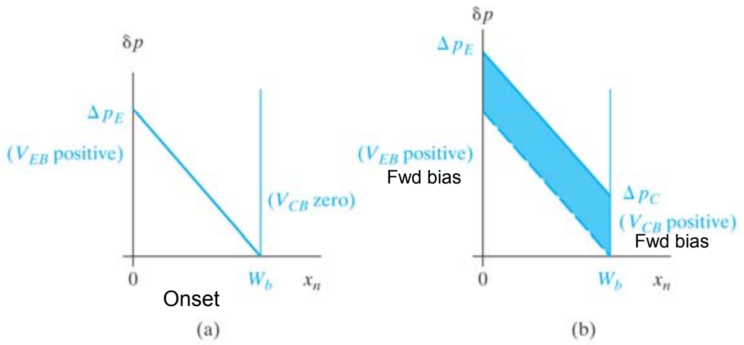
<u>Saturation</u>: BE forward biased,
CB forward biased (0 bias at onset)



Oversaturation: time to remove excess charge during switching

Figure 7.14

Excess hole distribution in the base of a saturated transistor: (a) the beginning of saturation; (b) oversaturation.

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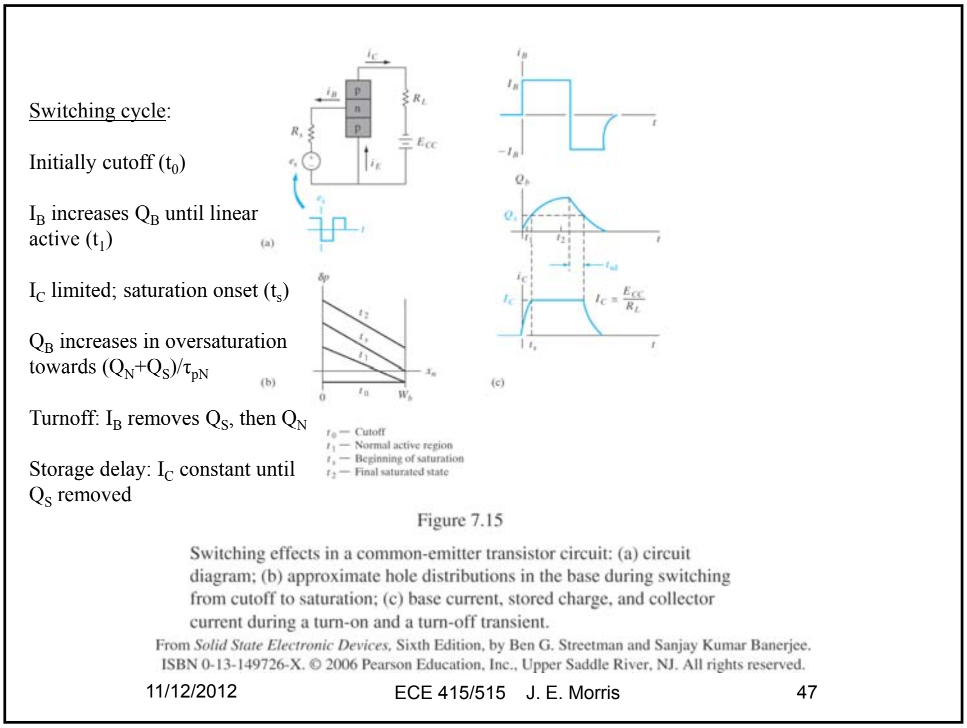

<u>Switching cycle</u>:

Initially cutoff ($t_0$)

$I_B$ increases $Q_B$ until linear active ($t_1$)

$I_C$ limited; saturation onset ($t_s$)

$Q_B$ increases in oversaturation towards $(Q_N+Q_S)/\tau_{pN}$

Turnoff: $I_B$ removes $Q_S$, then $Q_N$

Storage delay: $I_C$ constant until $Q_S$ removed

Figure 7.15

Switching effects in a common-emitter transistor circuit: (a) circuit diagram; (b) approximate hole distributions in the base during switching from cutoff to saturation; (c) base current, stored charge, and collector current during a turn-on and a turn-off transient.

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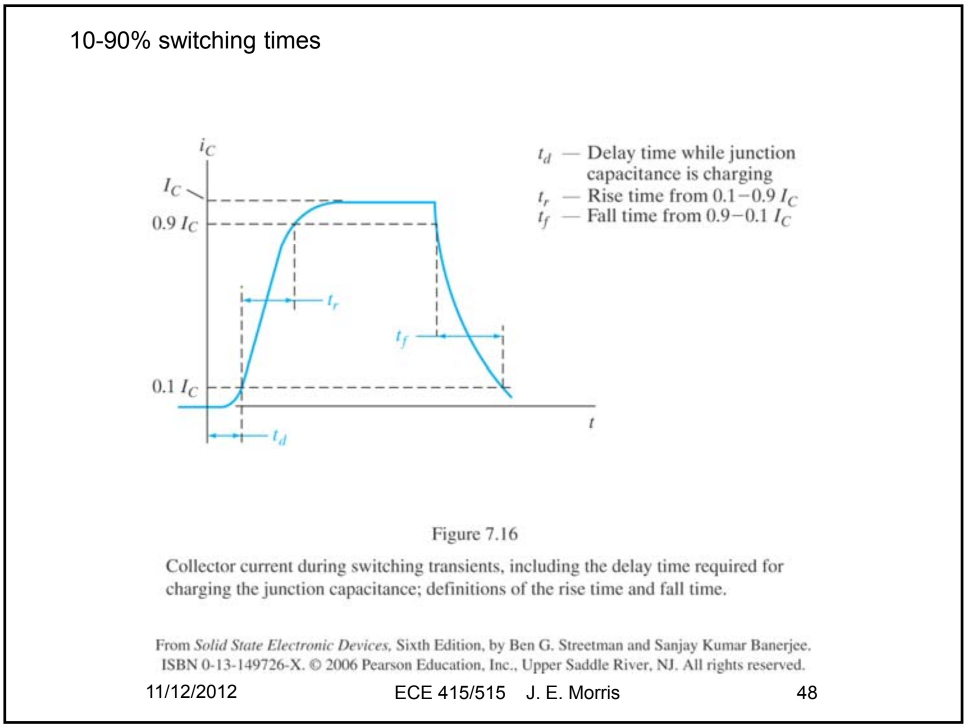

10-90% switching times

$t_d$ — Delay time while junction capacitance is charging
$t_r$ — Rise time from $0.1-0.9\ I_C$
$t_f$ — Fall time from $0.9-0.1\ I_C$

Figure 7.16

Collector current during switching transients, including the delay time required for charging the junction capacitance; definitions of the rise time and fall time.

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