EE415/515 Fundamentals of Semiconductor Devices Fall 2012

Lecture 13: Bipolar Junction Transistor (Chapter 12.1-12.3)

NPN & PNP BJTs

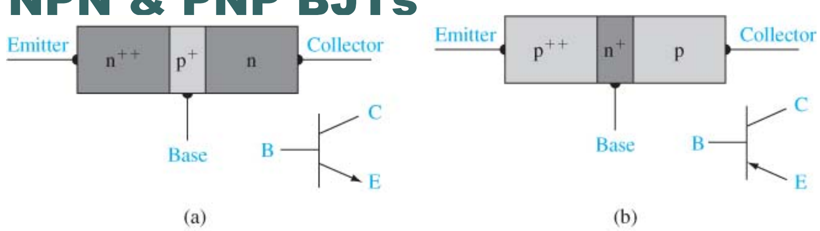


Figure 12.1 | Simplified block diagrams and circuit symbols of (a) npn and (b) pnp bipolar transistors.

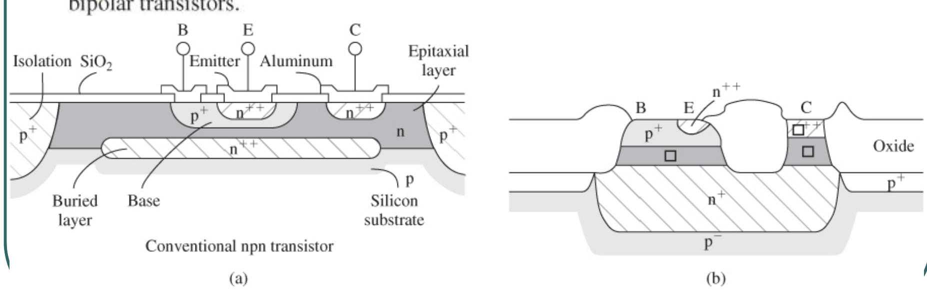


Figure 12.2 | Cross section of (a) a conventional integrated circuit npn bipolar transistor and (b) an oxide-isolated npn bipolar transistor.
(From Muller and Kamins [41].)

NPN BJT doping

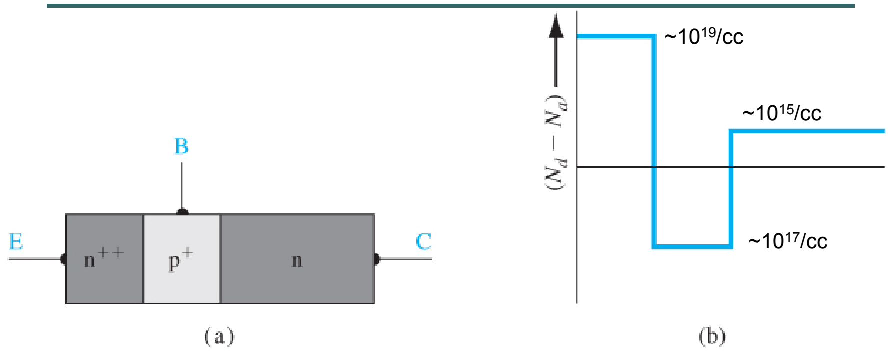


Figure 12.3 | Idealized doping profile of a uniformly doped npn bipolar transistor.

Forward-active NPN BJT operation

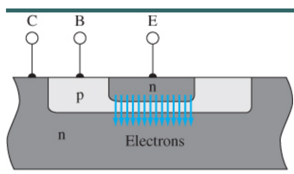


Figure 12.5 | Cross section of an npn bipolar transistor showing the injection and collection of electrons in the forward-active mode.

Electrons injected into B by fwd bias BE diode, diffuse across base, and swept into C by elec field in rev bias CB sp ch region. See back to fwd/rev bias diode for carrier concentrations at the space charge region edges.

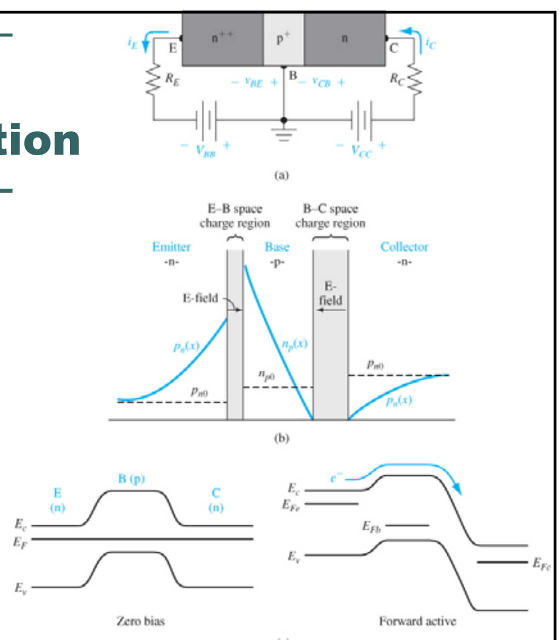


Figure 12.4 | (a) Biasing of an npn bipolar transistor in the forward-active mode, (b) minority carrier distribution in an npn bipolar transistor operating in the forward-active mode, and (c) energy-band diagram of the npn bipolar transistor under zero bias and under a forward-active mode bias.

First approximation: Linear $n_B(x)$, i.e. no recombination

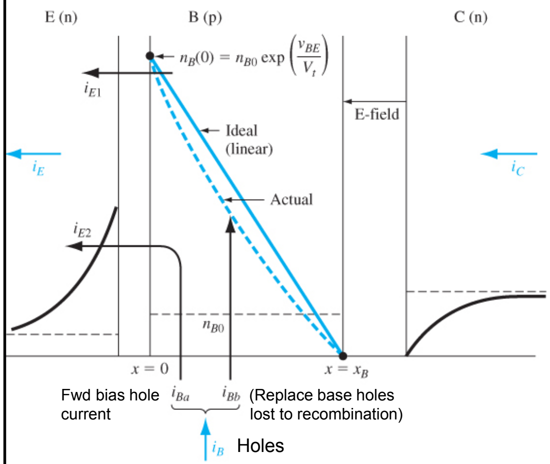


Figure 12.6 | Minority carrier distributions and basic currents in a forward-biased npn bipolar transistor.

$$i_C = -eD_n A_{BE} \frac{dn_B(x)}{dx} \text{ in the base}$$

$$= -eD_n A_{BE} \left[\frac{n_B(0) - 0}{0 - x_B}\right]$$

$$= \frac{eD_n A_{BE}}{x_B} n_{B0} \exp\frac{ev_{BE}}{kT}$$

$$= I_S \exp\frac{ev_{BE}}{kT}$$

$$i_E = i_{E1} + i_{E2} = i_C + i_{E2}$$

$$= i_C + I_{S2} \exp\frac{ev_{BE}}{kT}$$

$$= eA_{BE}\left[\frac{D_n}{x_B} n_{B0} + \frac{D_p}{L_p} p_{E0}\right] \exp\frac{ev_{BE}}{kT}$$

$$= I_{SE} \exp\frac{ev_{BE}}{kT}$$

$$\alpha = \frac{i_C}{i_E} = \frac{1}{1 + i_{E2}/i_C} = \frac{1}{1 + D_p p_{E0} x_B / D_n n_{B0} L_p}$$

$$= \frac{1}{1 + \frac{D_p}{D_n} \frac{p_{B0}}{n_{E0}} \frac{x_B}{L_p}} = \frac{1}{1 + \frac{D_p}{D_n} \frac{N_{AB}}{N_{DE}} \frac{x_B}{L_p}}$$

$$\beta = \frac{i_C}{i_B} = \frac{i_C}{i_{Ba} + i_{Bb}} = \frac{i_C}{i_{E2} + 0} = \frac{D_n}{D_p} \frac{N_{DE}}{N_{AB}} \frac{L_p}{x_B}$$

Modes of Operation

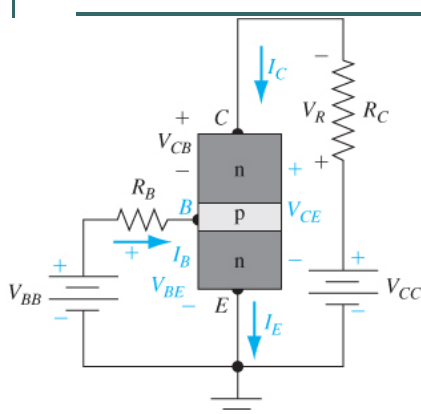


Figure 12.8 | An npn bipolar transistor in a common-emitter circuit configuration.

Load line $I_C=(V_{CC}-V_{CE})/R_C$

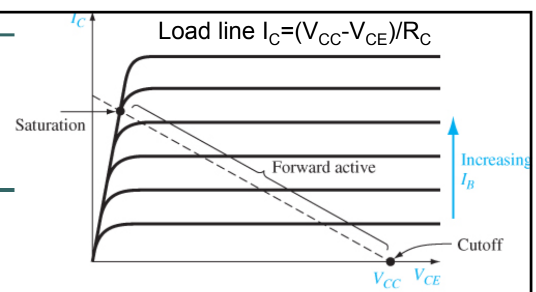


Figure 12.9 | Bipolar transistor common-emitter current–voltage characteristics with load line superimposed.

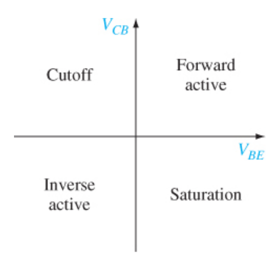


Figure 12.10 | Junction voltage conditions for the four operating modes of a bipolar transistor.

Forward active: Amplification

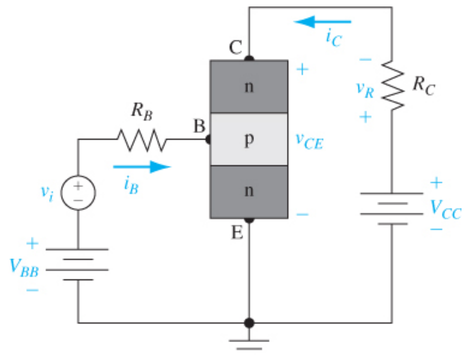


Figure 12.11 | Common-emitter npn bipolar circuit configuration with a time-varying signal voltage v_i included in the base–emitter loop.

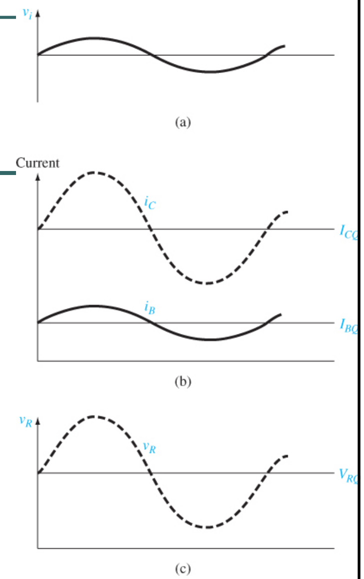


Figure 12.12 | Currents and voltages existing in the circuit shown in Figure 12.11. (a) Input sinusoidal signal voltage. (b) Sinusoidal base and collector currents superimposed on the quiescent dc values. (c) Sinusoidal voltage across the R_C resistor superimposed on the quiescent dc value.

Minority carrier distributions:

Consider each region in turn in forward-active mode

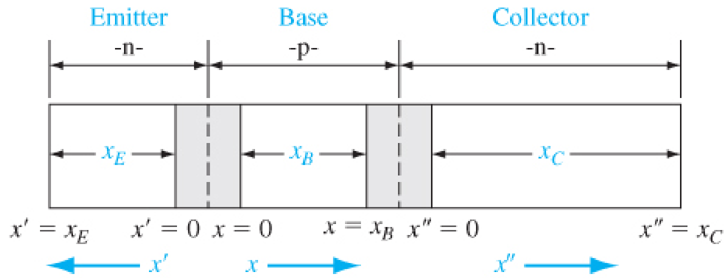
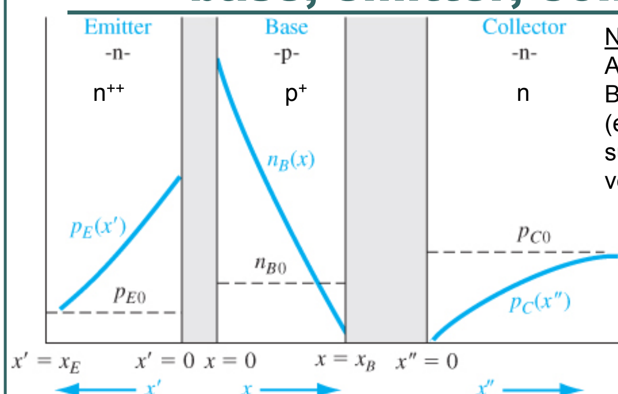


Figure 12.13 | Geometry of the npn bipolar transistor used to calculate the minority carrier distribution.

Table 12.1 | Notation used in the analysis of the bipolar transistor

Notation	Definition
For both the npn and pnp transistors	
N_E, N_B, N_C	Doping concentrations in the emitter, base, and collector
x_E, x_B, x_C	Widths of neutral emitter, base, and collector regions
D_E, D_B, D_C	*Minority carrier* diffusion coefficients in emitter, base, and collector regions
L_E, L_B, L_C	*Minority carrier* diffusion lengths in emitter, base, and collector regions
$\tau_{E0}, \tau_{B0}, \tau_{C0}$	*Minority carrier* lifetimes in emitter, base, and collector regions
For the npn	
p_{E0}, n_{B0}, p_{C0}	Thermal-equilibrium *minority carrier* hole, electron, and hole concentrations in the emitter, base, and collector
$p_E(x'), n_B(x), p_C(x'')$	Total *minority carrier* hole, electron, and hole concentrations in the emitter, base, and collector
$\delta p_E(x'), \delta n_B(x), \delta p_C(x'')$	Excess *minority carrier* hole, electron, and hole concentrations in the emitter, base, and collector
For the pnp	
n_{E0}, p_{B0}, n_{C0}	Thermal-equilibrium *minority carrier* electron, hole, and electron concentrations in the emitter, base, and collector
$n_E(x'), p_B(x), n_C(x'')$	Total *minority carrier* electron, hole, and electron concentrations in the emitter, base, and collector
$\delta n_E(x'), \delta p_B(x), \delta n_C(x'')$	Excess *minority carrier* electron, hole, and electron concentrations in the emitter, base, and collector

Consider forward-active minority carriers in each region: base, emitter, collector



Note:
Assume $x_C >> L_C = L_p$
But x_E finite and $p_E(x' = x_E) = p_{E0}$ (excess → 0 for infinite surface recombination velocity at contact.)

Figure 12.14 | Minority carrier distribution in an npn bipolar transistor operating in the forward-active mode.

Base region electrons (assume zero field)

$$\text{Ambipolar transport equation: } D_B \frac{\partial^2 \delta n_B(x)}{\partial x^2} = \frac{\delta n_B(x)}{\tau_{B0}} \quad \text{where } \delta n_B(x) = n_B(x) - n_{B0}$$

$$\text{General solution: } \delta n_B(x) = A \exp\frac{+x}{L_B} + B \exp\frac{-x}{L_B} \quad \text{where } L_B = \sqrt{D_B \tau_{B0}}$$

$$\text{Boundary conditions: } \delta n_B(0) = A + B = n_{B0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]$$

$$\delta n_B(x_B) = A \exp\frac{+x_B}{L_B} + B \exp\frac{-x_B}{L_B} = n_{B0}\left[\exp\frac{eV_{BC}}{kT} - 1\right] \approx 0 - n_{B0}$$

$$\text{which gives: } A = \frac{-n_{B0} - n_{B0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\exp\frac{-x_B}{L_B}}{2\sinh\frac{x_B}{L_B}}$$

$$\text{and } B = \frac{+n_{B0} + n_{B0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\exp\frac{+x_B}{L_B}}{2\sinh\frac{x_B}{L_B}}$$

$$\text{and hence } \delta n_B(x) = n_{B0}\frac{\left[\exp\frac{eV_{BE}}{kT} - 1\right]\sinh\frac{x_B - x}{L_B} - \sinh\frac{x}{L_B}}{\sinh\frac{x_B}{L_B}}$$

Linear approximation in base

Table 12.2 | Taylor expansions of hyperbolic functions

Function	Taylor expansion
$\sinh(x)$	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$
$\cosh(x)$	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$
$\tanh(x)$	$x - \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$

Typically $\sinh(x) \approx \tanh(x) \approx x$ & $\cosh(x) \approx 1$

$$\delta n_B(x) = n_{B0}\frac{\left[\exp\frac{eV_{BE}}{kT} - 1\right]\sinh\frac{x_B - x}{L_B} - \sinh\frac{x}{L_B}}{\sinh\frac{x_B}{L_B}}$$

$$\text{For } \sinh x \approx x \text{ for small } x, \text{ and } x_B << L_B$$

$$\delta n_B(x) \approx n_{B0}\frac{\left[\exp\frac{eV_{BE}}{kT} - 1\right]\frac{x_B - x}{L_B} - \frac{x}{L_B}}{\frac{x_B}{L_B}}$$

$$= \frac{n_{B0}}{x_B}\left(\left[\exp\frac{eV_{BE}}{kT} - 1\right](x_B - x) - x\right)$$

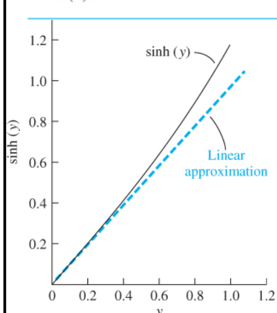


Figure 12.15 | Hyperbolic sine function and its linear approximation.

Emitter region holes

$$D_E \frac{\partial^2 \delta p_E(x^/)}{\partial x^{/2}} = \frac{\delta p_E(x^/)}{\tau_{E0}} \quad \text{where } \delta p_E(x^/) = p_E(x^/) - p_{E0}$$

$$\text{General solution: } \delta p_E(x^/) = C\exp\frac{+x^/}{L_E} + D\exp\frac{-x^/}{L_E} \quad \text{where } L_E = \sqrt{D_E \tau_{E0}}$$

$$\text{Boundary conditions: } \delta p_E(0) = C + D = p_{E0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]$$

$$\delta p_E(x_E) = C\exp\frac{+x_E}{L_E} + D\exp\frac{-x_E}{L_E} = 0 \quad \text{for infinite surface recombination velocity at contact}$$

$$\text{Solving for C \& D gives: } C = p_{E0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\frac{-\exp\frac{-x_E}{L_E}}{2\sinh\frac{x_E}{L_E}} \quad \text{and} \quad D = p_{E0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\frac{\exp\frac{x_E}{L_E}}{2\sinh\frac{x_E}{L_E}}$$

$$\text{and hence } \delta p_E(x^/) = p_{E0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\frac{\sinh\frac{x_E - x^/}{L_E}}{\sinh\frac{x_E}{L_E}}$$

$$\approx \frac{p_{E0}}{x_E}\left[\exp\frac{eV_{BE}}{kT} - 1\right](x_E - x^/) \quad \text{for } x_E << L_E$$

Collector region holes

$$D_C \frac{\partial^2 \delta p_C(x^{//})}{\partial x^{//2}} = \frac{\delta p_C(x^{//})}{\tau_{C0}} \quad \text{where } \delta p_C(x^{//}) = p_C(x^{//}) - p_{C0}$$

$$\text{General solution: } \delta p_C(x^{//}) = G\exp\frac{+x^{//}}{L_C} + H\exp\frac{-x^{//}}{L_C} \quad \text{where } L_C = \sqrt{D_C \tau_{C0}}$$

$$\text{Boundary conditions: } G = 0 \text{ for long collector region and finite } \delta p_C(\infty)$$

$$\delta p_C(0) = H = p_{C0}\exp\frac{eV_{BC}}{kT} - p_{C0} \approx 0 - p_{C0} = p_{C0} \quad \text{for } V_{BC} << 0$$

$$\text{and hence } \delta p_C(x^{//}) = p_{C0}\exp\frac{-x^{//}}{L_C} \quad \text{(compare reverse-biased diode)}$$

Other operation modes

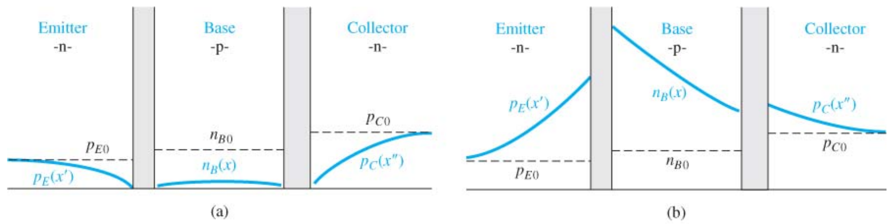


Figure 12.16 | Minority carrier distribution in an npn bipolar transistor operating in (a) cutoff and (b) saturation.

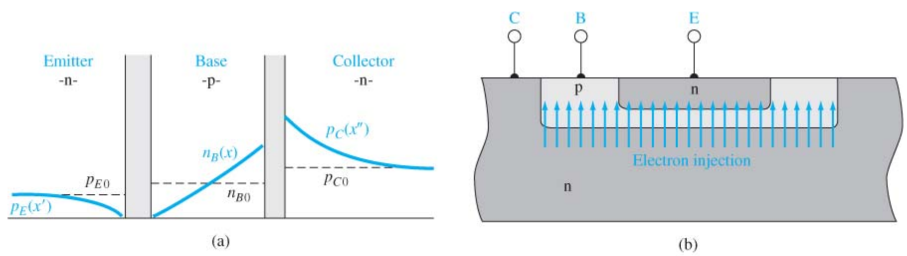


Figure 12.17 | (a) Minority carrier distribution in an npn bipolar transistor operating in the inverse-active mode. (b) Cross section of an npn bipolar transistor showing the injection and collection of electrons in the inverse-active mode.

Forward-active current gain I_C/I_E

Current components: electron and hole fluxes → J^-_{nE} & J^-_{nC} for BJT action
$n^{++}p^{+}n$ → $J^-_{nE} >> J^+_{pE}$; space charge recombination & generation currents J_R & J_G
J^+_{pC0} ideal B-C reverse saturation current; J^+_{RB} base recombination current

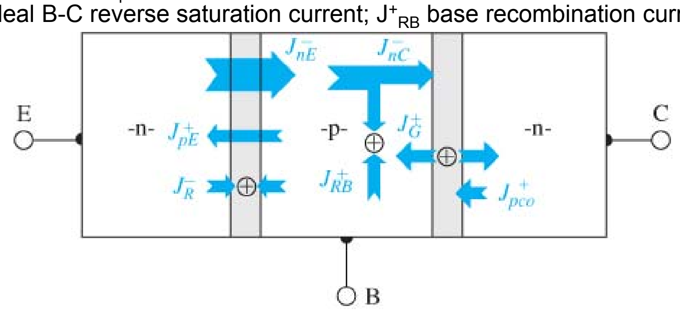
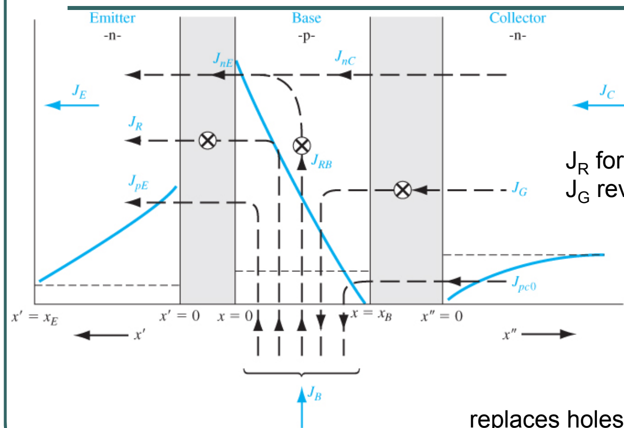


Figure 12.18 | Particle current density or flux components in an npn bipolar transistor operating in the forward-active mode.

Evaluation of current components



J_R forward bias BE recombination
J_G reverse bias BC generation

J_{pE} at $x'=0$
J_{pC0} at $x''=0$

J_{nE} at $x=0$
J_{nC} at $x=x_B$
$J_{RB}=J_{nE}-J_{nC}$
replaces holes lost by base recombination

Figure 12.19 | Current density components in an npn bipolar transistor operating in the forward-active mode.

Current gain components

$$\alpha_0 = \frac{I_C}{I_E} \Rightarrow \frac{J_C}{J_E} \text{ if B - E and C - B junctions same area}$$

$$= \frac{J_{nC} + J_G + J_{pC0}}{J_{nE} + J_R + J_{pE}}$$

Small signal (ac) current gain $\alpha = \dfrac{J_{nC}}{J_{nE} + J_R + J_{pE}}$ since J_G & J_{pC0} independent of i_E

$$= \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right)\left(\frac{J_{nC}}{J_{nE}}\right)\left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}\right) = \gamma \alpha_T \delta$$

where $\gamma = \dfrac{J_{nE}}{J_{nE} + J_{pE}}$ = emitter injection efficiency $\rightarrow 1$ if $J_{nE} >> J_{pE}$

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \text{base transport factor } \rightarrow 1 \text{ if } J_{RB} \rightarrow 0$$

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} = \text{recombination factor } \rightarrow 1 \text{ if } J_R \rightarrow 0$$

Emitter injection efficiency γ

$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}} = \frac{1}{1 + \frac{J_{pE}}{J_{nE}}} = \text{emitter injection efficiency} \rightarrow 1 \text{ if } J_{nE} >> J_{pE}$$

$$J_{pE} = -eD_E \frac{d[\delta p_E(x')]}{dx'} \text{ at } x' = 0 \quad \text{where } \delta p_E(x') = p_{E0}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\frac{\sinh\frac{x_E - x'}{L_E}}{\sinh\frac{x_E}{L_E}}$$

$$= \frac{-eD_E p_{E0}}{\sinh\frac{x_E}{L_E}}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\left(\frac{-1}{L_E}\right)\cosh\frac{x_E}{L_E} = \frac{eD_E p_{E0}}{L_E}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\frac{1}{\tanh(x_E/L_E)}$$

$$J_{nE} = -eD_B \frac{d[\delta n_B(x)]}{dx} \text{ at } x = 0 \quad \text{where } \delta n_B(x) = n_{B0}\frac{\left[\exp\frac{eV_{BE}}{kT} - 1\right]\sinh\frac{x_B - x}{L_B} - \sinh\frac{x}{L_B}}{\sinh\frac{x_B}{L_B}}$$

$$= \frac{-eD_B n_{B0}}{\sinh\frac{x_B}{L_B}}\left(\left[\exp\frac{eV_{BE}}{kT} - 1\right]\left(\frac{-1}{L_B}\right)\cosh\frac{x_B}{L_B} - \left(\frac{1}{L_B}\right)\right) = \frac{eD_B n_{B0}}{L_B}\left(\left[\frac{\exp\frac{eV_{BE}}{kT} - 1}{\tanh\frac{x_B}{L_B}}\right] + \frac{1}{\sinh\frac{x_B}{L_B}}\right)$$

Emitter injection efficiency γ (cont'd)

$$J_{pE} = \frac{eD_E p_{E0}}{L_E}\left[\exp\frac{eV_{BE}}{kT} - 1\right]\frac{1}{\tanh(x_E/L_E)} \quad \& \quad J_{nE} = \frac{eD_B n_{B0}}{L_B}\left(\left[\frac{\exp\frac{eV_{BE}}{kT} - 1}{\tanh\frac{x_B}{L_B}}\right] + \frac{1}{\sinh\frac{x_B}{L_B}}\right)$$

For $V_{BE} >> kT/e$, $\exp\frac{eV_{BE}}{kT} >> 1$, and for $x_B << L_B$, $\tanh\frac{x_B}{L_B} \approx \sinh\frac{x_B}{L_B}$

$$J_{pE} \approx \frac{eD_E p_{E0}}{L_E}\left[\exp\frac{eV_{BE}}{kT}\right]\frac{1}{\tanh(x_E/L_E)} \quad \text{and} \quad J_{nE} \approx \frac{eD_B n_{B0}}{L_B}\left[\exp\frac{eV_{BE}}{kT}\right]\frac{1}{\tanh(x_B/L_B)}$$

$$\gamma \approx \frac{1}{1 + \frac{J_{pE}}{J_{nE}}} = \frac{1}{1 + \frac{eD_E p_{E0}}{L_E}\frac{L_B}{eD_B n_{B0}}\frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}} \Rightarrow \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{x_B}{x_E}} \quad \text{for } x_B, x_E << L_B, L_E$$

Base transport factor α_T

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \text{base transport factor} \rightarrow 1 \text{ if } J_{RB} \rightarrow 0$$

$$J_{nC} = -eD_B \frac{d[\delta n_B(x)]}{dx} \text{ at } x = x_B \quad \text{where} \quad \delta n_B(x) = n_{B0} \frac{\left[\exp\frac{eV_{BE}}{kT} - 1\right]\sinh\frac{x_B - x}{L_B} - \sinh\frac{x}{L_B}}{\sinh(x_B/L_B)}$$

$$= \frac{-eD_B n_{B0}}{\sinh(x_B/L_B)}\left(\left[\exp\frac{eV_{BE}}{kT} - 1\right]\left(\frac{-1}{L_B}\right) - \left(\frac{1}{L_B}\right)\cosh\frac{x_B}{L_B}\right) = \frac{-eD_B n_{B0}}{L_B}\left(\frac{\left[\exp\frac{eV_{BE}}{kT} - 1\right]}{\sinh(x_B/L_B)} + \frac{1}{\tanh(x_B/L_B)}\right)$$

$$J_{nE} = \frac{eD_B n_{B0}}{L_B}\left(\left[\frac{\exp\frac{eV_{BE}}{kT} - 1}{\tanh\frac{x_B}{L_B}}\right] + \frac{1}{\sinh\frac{x_B}{L_B}}\right) \text{ from above, so } \alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{\left[\exp\frac{eV_{BE}}{kT} - 1\right] + \cosh\frac{x_B}{L_B}}{\left[\exp\frac{eV_{BE}}{kT} - 1\right]\cosh\frac{x_B}{L_B} + 1}$$

For $V_{BE} >> kT/e$, $\exp\frac{eV_{BE}}{kT} >> 1$, and for $x_B << L_B$, $\cosh\frac{x_B}{L_B} \approx 1$

$$\alpha_T \approx \frac{\exp\frac{eV_{BE}}{kT} + \cosh\frac{x_B}{L_B}}{\exp\frac{eV_{BE}}{kT}\cosh\frac{x_B}{L_B} + 1} \approx \frac{\exp\frac{eV_{BE}}{kT}}{\exp\frac{eV_{BE}}{kT}\cosh\frac{x_B}{L_B}} \approx \frac{1}{\cosh\frac{x_B}{L_B}} \approx \frac{1}{1 + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2} \approx 1 - \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2 \rightarrow 1 \text{ if } x_B << L_B$$

Recombination factor δ

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} = \text{recombination factor} \rightarrow 1 \text{ if } J_R \rightarrow 0$$

$$\approx \frac{J_{nE}}{J_{nE} + J_R} \quad \text{for } J_{nE} >> J_{pE}$$

$$J_R = \frac{ex_{BE}n_i}{2\tau_0}\exp\frac{eV_{BE}}{2kT} = J_{r0}\exp\frac{eV_{BE}}{2kT} \quad \text{forward bias B-E recombination current}$$

$$\text{and from above } J_{nE} = \frac{eD_B n_{B0}}{L_B}\left(\left[\exp\frac{eV_{BE}}{kT} - 1\right]\cosh\frac{x_B}{L_B} + 1\right)\frac{1}{\sinh(x_B/L_B)}$$

$$\approx \frac{eD_B n_{B0}}{L_B\tanh(x_B/L_B)}\exp\frac{eV_{BE}}{kT} = J_{s0}\exp\frac{eV_{BE}}{kT}$$

$$\text{so } \delta = \frac{J_{s0}\exp\frac{eV_{BE}}{kT}}{J_{s0}\exp\frac{eV_{BE}}{kT} + J_{r0}\exp\frac{eV_{BE}}{2kT}} = \frac{1}{1 + \frac{J_{r0}}{J_{s0}}\exp\frac{-eV_{BE}}{2kT}}$$

$\rightarrow 1$ as V_{BE} increases to >> kT/e

Recombination factor: additional surface recombination

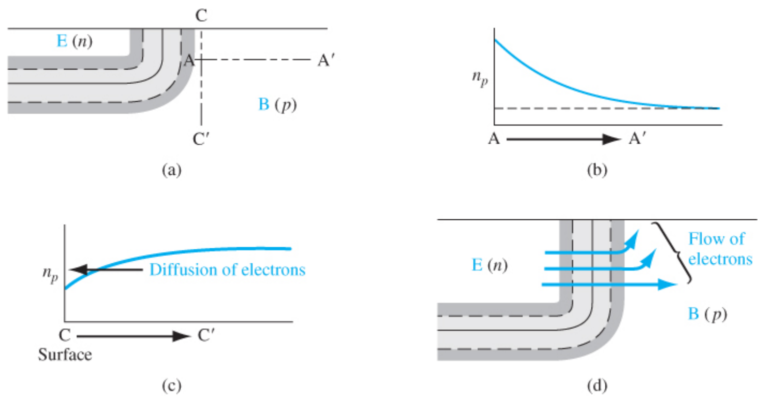


Figure 12.20 | The surface at the E–B junction showing the diffusion of carriers toward the surface.

Current gain α=γα$_T$δ, β=α/(1-α)

Common base current gain $\alpha = \dfrac{J_{nC}}{J_{nE}+J_R+J_{pE}} = \left(\dfrac{J_{nE}}{J_{nE}+J_{pE}}\right)\left(\dfrac{J_{nC}}{J_{nE}}\right)\left(\dfrac{J_{nE}+J_{pE}}{J_{nE}+J_R+J_{pE}}\right) = \gamma\alpha_T\delta$

$$= \frac{1}{1+\dfrac{D_E}{D_B}\dfrac{N_B}{N_E}\dfrac{x_B}{x_E}} \cdot \frac{1}{1+\dfrac{1}{2}\left(\dfrac{x_B}{L_B}\right)^2} \cdot \frac{1}{1+\dfrac{J_{r0}}{J_{s0}}\exp\dfrac{-eV_{BE}}{2kT}}$$

$$\approx \frac{1}{1+\dfrac{D_E}{D_B}\dfrac{N_B}{N_E}\dfrac{x_B}{x_E}+\dfrac{1}{2}\left(\dfrac{x_B}{L_B}\right)^2+\dfrac{J_{r0}}{J_{s0}}\exp\dfrac{-eV_{BE}}{2kT}} = \frac{\beta}{1+\beta} = \frac{1}{1+\dfrac{1}{\beta}}$$

Common emitter current gain $\beta = \dfrac{\alpha}{1-\alpha}$

$$= \frac{1}{\dfrac{D_E}{D_B}\dfrac{N_B}{N_E}\dfrac{x_B}{x_E}+\dfrac{1}{2}\left(\dfrac{x_B}{L_B}\right)^2+\dfrac{J_{r0}}{J_{s0}}\exp\dfrac{-eV_{BE}}{2kT}}$$

Ex 12.1 Calculate the emitter injection efficiency for BJT parameters N_B=5x10^{15}/cc, N_E=10^{18}/cc, D_E=10cm^2/s, D_B=20cm^2/s, x_B=0.80μm, x_E=0.60μm.

$$\gamma = \frac{1}{1+\left(\frac{N_B}{N_E}\right)\left(\frac{D_E}{D_B}\right)\left(\frac{x_B}{x_E}\right)}$$
$$= \frac{1}{1+\left(\frac{5\times10^{15}}{10^{18}}\right)\left(\frac{10}{20}\right)\left(\frac{0.80}{0.60}\right)}$$
$$= 0.9967$$

Ex 12.2 Calculate the BJT base transport factor for x_B=1.2μm and L_B=10.0μm.

$$\alpha_T = \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)} = \frac{1}{\cosh\left(\frac{1.2}{10.0}\right)}$$
$$= 0.9928$$

Ex 12.3 Calculate the BJT recombination factor for N_B=5x10^{15}/cc, D_B=20cm^2/s, L_B=10µm, x_B=0.80µm, x_{BE}=0.10µm, τ_0=10^{-7}s, V_{BE}=0.65V.

$$\delta = \frac{1}{1+\frac{J_{r0}}{J_{s0}}\cdot\exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1+\left(\frac{1.2\times10^{-7}}{1.804\times10^{-9}}\right)\cdot\exp\left(\frac{-0.65}{2(0.0259)}\right)}$$

$$= 0.99976$$

Ex 12.4 Determine base doping concentration for NPN BJT emitter injection efficiency γ=0.9950, for D_E=D_B, L_E=L_B, x_E=x_B, N_E=6x10^{18}/cc.

$$\gamma = \frac{1}{1+\left(\frac{N_B}{N_E}\right)\left(\frac{D_E}{D_B}\right)\left(\frac{x_B}{x_E}\right)}$$

$$0.9950 = \frac{1}{1+\left(\frac{N_B}{6\times10^{18}}\right)(1.0)(1.0)}$$

$$\Rightarrow N_B = 3.02\times10^{16}\,\mathrm{cm}^{-3}$$

Ex 12.5 Determine the minimum BJT base width for a base transport factor of α_T=0.9980, for D_B=10cm²/s and τ_{B0}=10⁻⁷s.

$$\alpha_T = \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)}$$

From Example 12.5, $L_B = 10\ \mu\text{m}$.

Then

$$0.9980 = \frac{1}{\cosh\left(\frac{x_B}{10}\right)}$$

$$x_B = (10)\cosh^{-1}\left(\frac{1}{0.9980}\right) \quad \Rightarrow x_B = 0.633\ \mu\text{m}$$

Ex 12.6 Determine V_{BE} for δ=0.9950 if J_{r0}=10⁻⁸A/cm² and J_{s0}=10⁻¹¹A/cm².

$$\delta = \frac{1}{1+\frac{J_{r0}}{J_{s0}}\cdot\exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$0.9950 = \frac{1}{1+\left(\frac{10^{-8}}{10^{-11}}\right)\cdot\exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$\exp\left(\frac{-V_{BE}}{2V_t}\right) = \frac{0.005025}{\left(\frac{10^{-8}}{10^{-11}}\right)} = 5.025\times10^{-6}$$

or

$$V_{BE} = 2V_t\ln\left(\frac{1}{5.025\times10^{-6}}\right) = 2(0.0259)(12.201)$$

$$V_{BE} = 0.6320\ \text{V}$$

Ex 12.7 Assume $\gamma=\alpha_T=0.9980$, $J_{r0}=5\text{x}10^{-9}\text{A/cm}^2$ & $J_{r0}=5\text{x}10^{-9}\text{A/cm}^2$. Find CE current gain β for (a) $V_{BE}=0.550\text{V}$ and (b) $V_{BE}=0.650\text{V}$.

(a) $$\delta=\frac{1}{1+\dfrac{J_{r0}}{J_{s0}}\cdot\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}=\frac{1}{1+\left(\dfrac{5\times10^{-9}}{2\times10^{-11}}\right)\cdot\exp\left(\dfrac{-0.55}{2(0.0259)}\right)}=0.99392$$

$$\alpha=\gamma\alpha_T\delta=(0.9980)(0.9980)(0.99392)=0.98995$$

Then $$\beta=\frac{\alpha}{1-\alpha}=\frac{0.98995}{1-0.98995}=98.5$$

(b) $$\delta=\frac{1}{1+\left(\dfrac{5\times10^{-9}}{2\times10^{-11}}\right)\cdot\exp\left(\dfrac{-0.65}{2(0.0259)}\right)}=0.99911$$

$$\alpha=\gamma\alpha_T\delta=(0.9980)(0.9980)(0.99911)=0.99512$$

$$\beta=\frac{\alpha}{1-\alpha}=\frac{0.99512}{1-0.99512}=204$$

Alternative analysis: Carrier distribution in base

- Consider In the N base, minority carrier (hole) concentrations
  - At edge of EB depletion region: $\Delta p_E = p_n\left(e^{qV_{EB}/kT}-1\right)\approx p_n e^{qV_{EB}/kT}$
  - At edge of CB depletion region: $\Delta p_C = p_n\left(e^{qV_{CB}/kT}-1\right)\approx -p_n$
- Solution of diffusion equation : $d^2\delta p(x_n)/dx_n^2=\delta p(x_n)/L_p^2$

  is $\delta p(x_n)=C_1e^{x_n/L_p}+C_2e^{-x_n/L_p}$
- Boundary conditions:

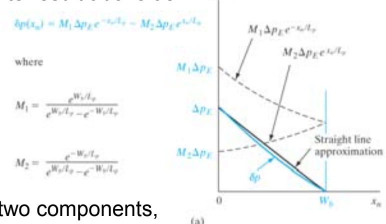
$$\delta p(x_n=0)=C_1+C_2=\Delta p_E$$

$$\delta p(x_n=W_b)=C_1e^{W_b/L_p}+C_2e^{-W_b/L_p}=\Delta p_C$$

$$\text{give } C_1=\frac{\Delta p_C-\Delta p_E e^{-W_b/L_p}}{e^{W_b/L_p}-e^{-W_b/L_p}} \text{ and } C_2=\frac{\Delta p_E e^{W_b/L_p}-\Delta p_C}{e^{W_b/L_p}-e^{-W_b/L_p}}$$

$$\text{i.e. } \delta p(x_n) = \Delta p_E \frac{e^{W_b/L_p} e^{-x_n/L_p} - e^{-W_b/L_p} e^{x_n/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}} \text{ if } \Delta p_C \approx p_n \approx 0$$

Re-write result above as:



See two components, net δp, and linear approximation

Minority carrier distributions

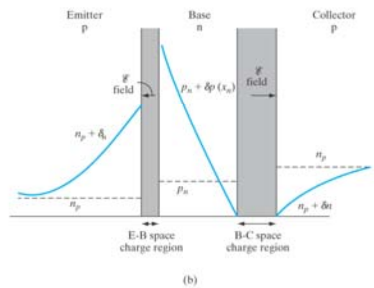


Figure 7.7

(a) Sketch of the terms in Eq.(7–14), illustrating the linearity of the hole distribution in the base region. In this example, $W_b/L_p = 1/2$. (b) Electron distributions in the emitter and collector.

Analysis: Terminal currents

- From diode theory:

$$I_{Ep} = I_p(x_n = 0) = -qAD_p \, d\delta p(x_n)/dx_n$$

$$= -qAD_p \frac{d}{dx_n}(C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}) = qA\frac{D_p}{L_p}(C_2 - C_1)$$

$$= qA\frac{D_p}{L_p}\left(\frac{\Delta p_E (e^{W_b/L_p} + e^{-w_b/L_p}) - 2\Delta p_C}{(e^{W_b/L_p} - e^{-w_b/L_p})}\right) = qA\frac{D_p}{L_p}\left(\Delta p_E ctnh\frac{W_b}{L_p} - \Delta p_C \operatorname{csc} h\frac{W_b}{L_p}\right)$$

$$\text{and } I_C = I_p(x_n = W_b) = -qAD_p \, d\delta p(x_n)/dx_n$$

$$= -qAD_p \frac{d}{dx_n}(C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}) = qA\frac{D_p}{L_p}(C_2 e^{-W_b/L_p} - C_1 e^{w_b/L_p})$$

$$= qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csc} h\frac{W_b}{L_p} - \Delta p_C ctnh\frac{W_b}{L_p}\right)$$

$$\text{so } I_B = I_E - I_C = qA\frac{D_p}{L_p}\left((\Delta p_E + \Delta p_C)(ctnh\frac{W_b}{L_p} - \operatorname{csc} h\frac{W_b}{L_p})\right)$$

$$= qA\frac{D_p}{L_p}\left((\Delta p_E + \Delta p_C)\tanh\frac{W_b}{2L_p}\right)$$

<u>Streetman Example 7.2</u>

Find currents I_E, I_C, I_B for V_{CB}=0 (as shown), so

$$\Delta p_C = p_n\left(e^{qV_{CB}/kT} - 1\right) = 0$$

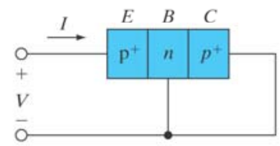


Figure 7.7.1 EX7-2

$$I_E = qA\frac{D_p}{L_p}\left(\Delta p_E ctnh\frac{W_b}{L_p} - \Delta p_C \operatorname{csc}h\frac{W_b}{L_p}\right) = qA\frac{D_p}{L_p}\left(\Delta p_E ctnh\frac{W_b}{L_p}\right)$$

$$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csc}h\frac{W_b}{L_p} - \Delta p_C ctnh\frac{W_b}{L_p}\right) = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csc}h\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left((\Delta p_E + \Delta p_C)\tanh\frac{W_b}{2L_p}\right) = qA\frac{D_p}{L_p}\left(\Delta p_E \tanh\frac{W_b}{2L_p}\right)$$

Refer back to slide 5 and Streetman Fig 7.7(a) (slide 33) for the linear approximation of the base hole concentration ($\Delta p_C = - p_n \approx 0$) so I_B, I_C, I_E as in Streetman Ex 7.2 (last slide)

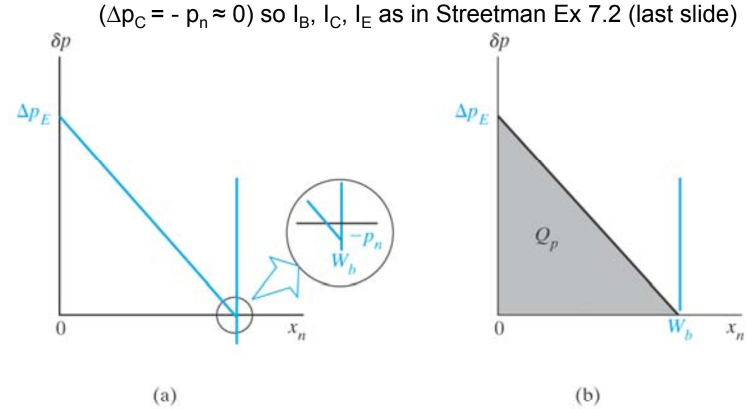


Figure 7.8

Approximate excess hole distributions in the base: (a) forward-biased emitter, reverse-biased collector; (b) triangular distribution for $V_{CB} = 0$ or for negligible *pn*.

Approximate base current

$$I_E = qA\frac{D_p}{L_p}\left(\Delta p_E ctnh\frac{W_b}{L_p}\right) \approx qA\frac{D_p}{L_p}\Delta p_E(\frac{L_p}{W_b}+\frac{W_b}{3L_p}) \approx qA\frac{D_p}{W_b}\Delta p_E$$

$$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \csc h\frac{W_b}{L_p}\right) \approx qA\frac{D_p}{L_p}\Delta p_E(\frac{L_p}{W_b}-\frac{W_b}{6L_p}) \approx qA\frac{D_p}{W_b}\Delta p_E$$

which matches $I_E = I_C$ in the limit for $W_b << L_p$

$$I_B = qA\frac{D_p}{L_p}\left(\Delta p_E \tanh\frac{W_b}{2L_p}\right) \approx qA\frac{D_p}{L_p}\Delta p_E\frac{W_b}{2L_p} = qA\frac{D_pW_b}{2L_p^2}\Delta p_E = \frac{qAW_b\Delta p_E}{2\tau_p}$$

$$\text{or } I_B = I_E - I_C = qA\frac{D_p}{L_p}\Delta p_E\left[(\frac{L_p}{W_b}+\frac{W_b}{3L_p})-(\frac{L_p}{W_b}-\frac{W_b}{6L_p})\right] = \frac{qAD_pW_b\Delta p_E}{2L_p^2} = \frac{qAW_b\Delta p_E}{2\tau_p}$$

$$\text{Also } Q_p = \frac{1}{2}qAW_b\Delta p_E \text{ so } I_B = \frac{Q_p}{\tau_p} = \frac{qAW_b\Delta p_E}{2\tau_p}$$

Current ratios

$$\text{Base Transport Factor } B = \frac{I_C}{I_{Ep}} = \csc h\frac{W_b}{L_p}\Big/ ctnh\frac{W_b}{L_p} = \sec h\frac{W_b}{L_p} \approx 1-\left(\frac{W_b}{L_p}\right)^2$$

$$\text{Emitter efficiency } \gamma = \frac{I_{Ep}}{I_E} = \frac{I_{Ep}}{I_{En}+I_{Ep}} = \left[1+\frac{I_{En}}{I_{Ep}}\right]^{-1}$$

$I_{En} \approx \frac{qAD_n^p}{L_n^p}n_p e^{qV_{EB}/kT}$ for $qV_{EB} >> kT$, and L_n^p is electron diffusion length in P - type emitter

$$\text{so } I_E = I_{En} + I_{Ep} = qA\left[n_p\frac{D_n^p}{L_n^p} + p_n\frac{D_p^n}{L_p^n}ctnh\frac{W_b}{L_p^n}\right]$$

$$\gamma = \left[1+\frac{I_{En}}{I_{Ep}}\right]^{-1} = \left[1+n_p\frac{D_n^p}{L_n^p}\Big/ p_n\frac{D_p^n}{L_p^n}ctnh\frac{W_b}{L_p^n}\right]^{-1} = \left[1+\frac{n_p}{p_n}\frac{D_n^p}{D_p^n}\frac{L_p^n}{L_n^p}\tanh\frac{W_b}{L_p^n}\right]^{-1}$$

$$= \left[1+\frac{n_n}{p_p}\frac{\mu_n^p}{\mu_p^n}\frac{L_p^n}{L_n^p}\tanh\frac{W_b}{L_p^n}\right]^{-1} \approx \left[1+\frac{n_n}{p_p}\frac{\mu_n^p}{\mu_p^n}\frac{W_b}{L_n^p}\right]^{-1}$$

since $\frac{n_p}{p_n} = \frac{n_i^2/p_p}{n_i^2/n_n} = \frac{n_n}{p_p}$, $D \propto \mu$, and $\tanh\frac{W_b}{L_p^n} \approx \frac{W_b}{L_p^n}$ for $W_b << L_p^n$