ECE414/514
Electronics Packaging
Spring 2012 Lecture 9
Mechanical B
Thermo-mechanical Stress; Underfill

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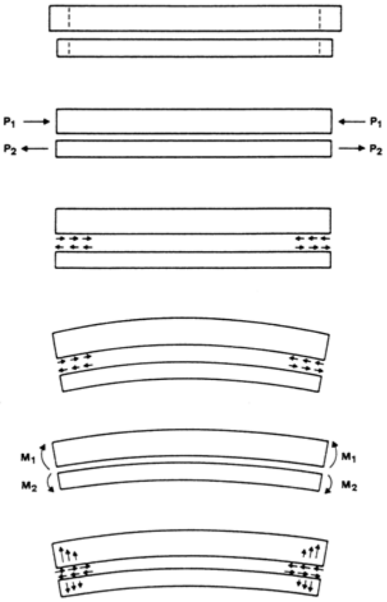


Figure 5-53. Bimetallic Beam Model. (a) Free expansion; (b) End forces to match; (c) Equivalent traction σ_{xy}; (d) Curling from traction; (e) End moments to match; (f) Interface stresses for compatibility.

Thermomechanical stresses

CTE mismatches $\Delta\alpha$

(also $T_{chip} > T_{board} \rightarrow \Delta T$)

A. Neglect Adhesive Layer
  1. Simple case — no bending
  — adhesive layer very thin, strong OR infinitely compliant (zero modulus)
  2. Stoney's thin film formula — 2 materials, so thin adhes.
  — Forces constant, no shear
  — Simple radius of curvature for benc
  3. Add bending moment in initial equations
  4. Timoshenko Bimetallic Strip — Bending
  — Curvature.

B. Include Adhesive Layer
  1. Basic Case — Stress varies with x, no bending
  2. Include bending

C. Peeling stress & Summary
  1. Peeling Stress
  2 Summary

D. More general case → solder joint attach, etc

E. Added comments and fracture applications

ELEMENTARY MODELS

$\epsilon = \frac{\Delta l}{l}$

Hooke's Law

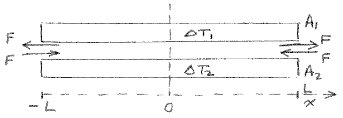
$E = \frac{\epsilon}{\sigma} = \frac{\Delta l / l}{F/A}$

Young's Modulus $= \frac{\text{stress}}{\text{strain}}$

BIMATERIAL STRAINS — CTE (or temp) mismatch.

A(1) 1D case – no bending

Case (i)
Rigid attachment



$$\epsilon_1 = \alpha_1 \Delta T_1 + \frac{F}{A_1 E_1}$$

$$\epsilon_2 = \alpha_2 \Delta T_2 - \frac{F}{A_2 E_2}$$

& for $\epsilon_1 = \epsilon_2$

$$F = \frac{\alpha_2 \Delta T_2 - \alpha_1 \Delta T_1}{\frac{1}{A_1 \epsilon_1} + \frac{1}{A_2 \epsilon_2}} > 0 \text{ if}$$

$\alpha_2 \Delta T_2 > \alpha_1 \Delta T_1$

Case (ii)
Flexible adhesive

For the plates to be separated/joined by a material (thickness h) of modulus $E << E_1, E_2$ eg. adhesive, solder. $F_1 \neq F_2$ but $F_1, F_2 \Rightarrow 0$

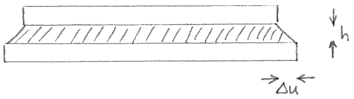
so $\epsilon_1 - \epsilon_2 = \alpha_1 \Delta T_1 - \alpha_2 \Delta T_2$

and the difference is taken up by shear stress in the separate layer, i.e. $\Delta u = (\epsilon_1 - \epsilon_2) L$ on each side of $x = 0$

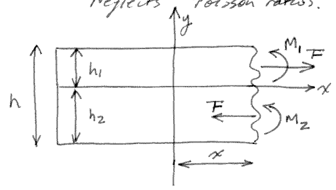
$= (\alpha_1 \Delta T_1 - \alpha_2 \Delta T_2) L$

and solder/adhesive shear strain is (for $\Delta T_1 = \Delta T_2$)

$$\gamma = \frac{\Delta u}{h} = \frac{L}{h} \Delta T (\alpha_1 - \alpha_2)$$



A② Stoney's formula (1909 for thin film on thin substrate)
Can apply for thin adhesive layer of high modulus
Neglects Poisson ratios.



Already set
$F_1 = F_2 = F$

Moments:

$$M_1 + M_2 - \frac{h_1 + h_2}{\cancel{2}} F = 0$$

where $M_i = E_i^o \frac{I_i}{\rho} = \frac{E_i^o h_i^3}{12\rho}$

from earlier beam theory.

$$\therefore \frac{E_1^o h_1^3}{12\rho} + \frac{E_2^o h_2^3}{12\rho} = \frac{h_1 + h_2}{2} F \quad \& \quad F = \frac{E_1^o h_1^3 + E_2^o h_2^3}{6\rho(h_1 + h_2)}$$

ρ radius of curvature

For thermal CTE mismatch $\Delta\alpha = \alpha_1 - \alpha_2$
and/or thermal mismatch $\Delta T = T_1 - T_2$

Strains must be equal (and opposite)

$$\therefore \epsilon_2 = \alpha_2 \Delta T + \frac{F}{E_2^o h_2} = \epsilon_1 = \alpha_1 \Delta T - \frac{F}{E_1^o h_1}$$

$$\& \quad F = \frac{\Delta\alpha \, \Delta T}{1/E_1^o h_1 + 1/E_2^o h_2}$$

Compare previous case, but now we have ρ in terms of F

$$\text{So } \frac{1}{\rho} = \frac{6(h_1 + h_2)}{E_1^o h_1^3 + E_2^o h_2^3} \cdot \frac{\Delta\alpha \Delta T}{1/E_1^o h_1 + 1/E_2^o h_2}$$

Note: For parabolic bending, $1/\rho \rightarrow \frac{d^2 \omega}{dx^2}$ where $\omega(x)$ is the "bow
Gives $\omega(x) = \omega_0 \left(1 - \left(\frac{x}{l}\right)^2\right)$ where max bow ω_0 at $x = 0$ & $l = 1/2$ length

Remember from last slide: $\epsilon_2 = \alpha_2 \Delta T + \frac{F}{E_2^{\circ} h_2} = \epsilon_1 = \alpha_1 \Delta T - \frac{F}{E_1^{\circ} h_1}$

A③ See Brown p241–242, except $M_i = \frac{D_i}{\rho} = \frac{E_i h_i^3}{12(1-\nu_i^2)\rho}$

and add also the bending moment, then

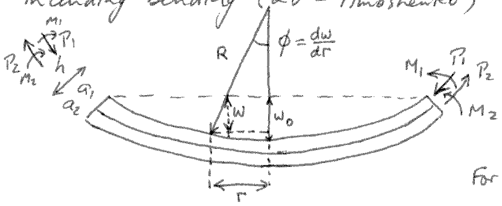
$$\epsilon_1 = \epsilon_2 \implies \frac{F}{E_1 h_1} + \alpha_1 \Delta T + \frac{(h_1/2)}{\rho} = -\frac{F}{E_2 h_2} + \alpha_2 \Delta T - \frac{(h_2/ \text{2)}}{\rho}$$

& the moment equation $M_R = M_1 + M_2 - \frac{h_1 + h_2}{2} F = 0$

Solve for F (substitute for ρ) i.e. $F = \frac{\Delta\alpha\, \Delta T}{\lambda_1 + \lambda_2 + \lambda_{12}} = \frac{\Delta\alpha\, \Delta T}{\lambda}$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_{12}$, $\lambda_i = \frac{1}{E_i t_i}$ $\lambda_{12} = \frac{(t_1 + t_2)^2}{4(D_1 + D_2)}$

A(4) Including bending (2D – Timoshenko)



1925 Bi-metallic strip, so still two materials, rigid the attachment.
Include Poisson ratio

Refer back to beams.

For parabolic displacement
$w(r) = w_0 - r^2/2R$

Assume same ΔT for both. $P_1 = P_2 = P$

$$M_1 + M_2 + P\frac{h}{2} = 0$$

Plate equation (radial plate moment) $M = -D\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)$ where plate rigidity $D = \frac{Eh^3}{12(1-\nu^2)}$

$$\therefore (D_1 + D_2)\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) = P\frac{h}{2}$$ if $\nu_1 \sim \nu_2 = \nu$

with $\frac{dw}{dr} = -\frac{r}{R}$, $\frac{d^2w}{dr^2} = -\frac{1}{R}$ so $(D_1 + D_2)\left(\frac{1}{R} + \frac{\nu}{\cancel{r}}\frac{\cancel{r}}{R}\right) = -P\frac{h}{2}$

i.e. $P\frac{h}{2} = (D_1 + D_2)(1 + \nu)\left(-\frac{1}{R}\right)$

$$\text{ie.} \quad P\frac{h}{2} = (D_1 + D_2)(1+\nu)\left(-\frac{1}{R}\right)$$

Strains due to temperature, axial forces, and bending

$$\epsilon_1 = \alpha_1 \Delta T - \frac{P_{\cancel{x}}}{a_1 D_1} + a_1/2R$$

$$\epsilon_2 = \alpha_2 \Delta T + \frac{P_{\cancel{x}}}{a_2 D_2} - a_2/2R$$

& $\epsilon_1 = \epsilon_2$, so

$$\frac{1}{R} = \frac{2}{a_1 + a_2}\left((\alpha_2 - \alpha_1)\Delta T + \left(\frac{1}{a_2 D_2} + \frac{1}{a_1 D_1}\right)P\right)$$

$$= \frac{2}{a_1+a_2}\left\{(\alpha_2 - \alpha_1)\Delta T + \left[\frac{1}{a_1 D_1} + \frac{1}{a_2 D_2}\right]\frac{2}{h}(D_1 + D_2)(1+\nu)\left(-\frac{1}{R}\right.\right.$$

$$= \frac{2}{a_1 + a_2} \quad \frac{(\alpha_2 - \alpha_1)\Delta T}{1 + \frac{2}{a_1+a_2}\left(\frac{1}{a_1 D_1} + \frac{1}{a_2 D_2}\right)\frac{2}{h}(D_1+D_2)(1+\nu)}$$

$$= \frac{(\alpha_2 - \alpha_1)\Delta T}{\frac{a_1+a_2}{2} + \frac{2}{h}(1+\nu)(D_1+D_2)\frac{(a_1 D_1 + a_2 D_2)}{a_1 a_2 D_1 D_2}} \qquad \text{etc.}\ P$$

OR for $\nu_1 \neq \nu_2$

$$P = \frac{2}{h}\left\{ D_1(1+\nu_1) + D_2(1+\nu_2)\right\}(-1/R)$$

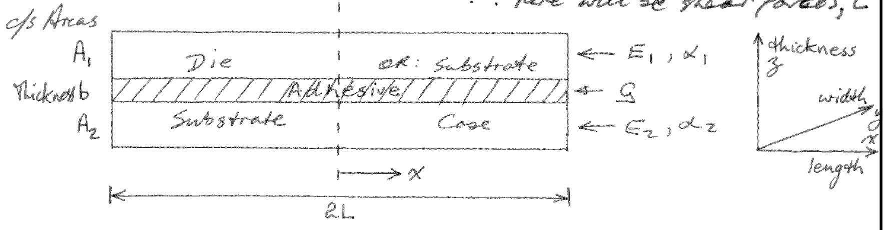
$$\& \quad \frac{1}{R} = \frac{\frac{2}{a_1+a_2}(\alpha_2 - \alpha_1)\Delta T}{1 + \frac{2}{a_1+a_2}\left(\frac{1}{a_1 D_1} + \frac{1}{a_2 D_2}\right)\frac{2}{h}\left(D_1(1+\nu_1) + D_2(1+\nu_2)\right)} \qquad \text{etc}$$

For intervening solder/adhesive layer
$D << D_1, D_2$
Thickness h ??

Need finite D or no curvature

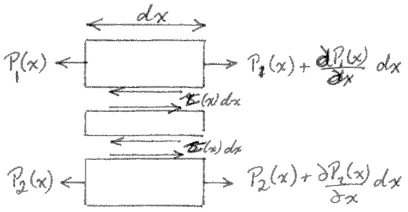
See Brown pp243-244 B① BASIC CASE

No bending, BUT Allow $P(x)$ to vary along the structure
∴ There will be shear forces, τ



P force/unit width

Stress assembly with temperature change ΔT



Assume:
- Adhesive elasticity << adherend materials
- No stress/strain variation in y direction (plane conditions), unit width
- Neglect bending.

Deformation in length of element dx $\quad d\delta_i = \frac{(1-\nu_i)P_i\,dx}{E_i A_i} + \alpha_i \Delta T\,dx \qquad i = 1,2$

For compliant adhesive, die/substrate stresses invariant in z
shear stress in adhesive

$$dP_1 - \tau\, dx = 0 \qquad dP_2 + \tau\, dx = 0$$

In adhesive $\underset{\text{shearstrain}}{d\gamma} = \frac{d\delta_1 - d\delta_2}{b} = \frac{d\tau}{G}$

$$\therefore \left[\frac{(1-\nu_1)P_1\, dx}{E_1 A_1} + \alpha_1 \Delta T\, dx\right] - \left[\frac{(1-\nu_2)P_2\, dx}{E_2 A_2} + \alpha_2 \Delta T\, dx\right] = b\frac{d\tau}{G}$$

ie. $\frac{d\tau}{dx} = \frac{G}{b}\left[\left(\frac{1-\nu_1}{E_1 A_1}\right)P_1 - \left(\frac{1-\nu_2}{E_2 A_2}\right)P_2 + (\alpha_1 - \alpha_2)\Delta T\right]$

$$\frac{d^2\tau}{dx^2} = \frac{G}{b}\left[\left(\frac{1-\nu_1}{E_1 A_1}\right)\frac{dP_1}{dx} - \left(\frac{1-\nu_2}{E_2 A_2}\right)\frac{dP_2}{dx}\right] = \frac{G}{b}\left[\frac{1-\nu_1}{E_1 A_1} + \frac{1-\nu_2}{E_2 A_2}\right]\tau$$

$$= A^2\tau \qquad \text{where } A^2 = \frac{G}{b}\left[\frac{1-\nu_1}{E_1 A_1} + \frac{1-\nu_2}{E_2 A_2}\right]$$

Assume general solution $\tau = c_1 \exp Ax + c_2 \exp -Ax$

with boundary conditions $\tau(0) = 0$
& $P_1(\pm L) = P_2(\pm L) = 0$

$$\therefore \quad c_1 + c_2 = 0 \qquad c_2 = -c_1$$

& $\frac{d\tau}{dx} = c_1 A \exp Ax + (-c_1)(-A)\exp -Ax$

$$= c_1 A (\exp Ax + \exp -Ax)$$

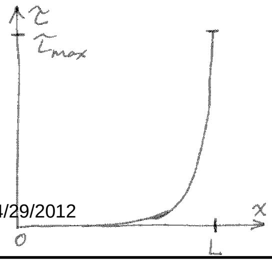
At $x = L$, $P_1 = P_2 = 0$ & $\frac{d\tau}{dx} = \frac{G}{b}(\alpha_1 - \alpha_2)\Delta T$

$$= c_1 A . 2\cosh AL$$

$$\therefore c_1 = \frac{G}{b}(\alpha_1 - \alpha_2)\Delta T / 2A \cosh AL$$

$$\therefore \tau = c_1 (\exp Ax - \exp -Ax)$$

$$= \frac{G}{b}\,\frac{(\alpha_1 - \alpha_2)\Delta T}{\cancel{2}A}.\frac{\cancel{2}\sinh(Ax)}{\cosh(AL)}$$



τ_{max} at $x = L$

$$\longrightarrow \frac{G}{bA}(\alpha_1 - \alpha_2)\Delta T . \tanh(AL)$$

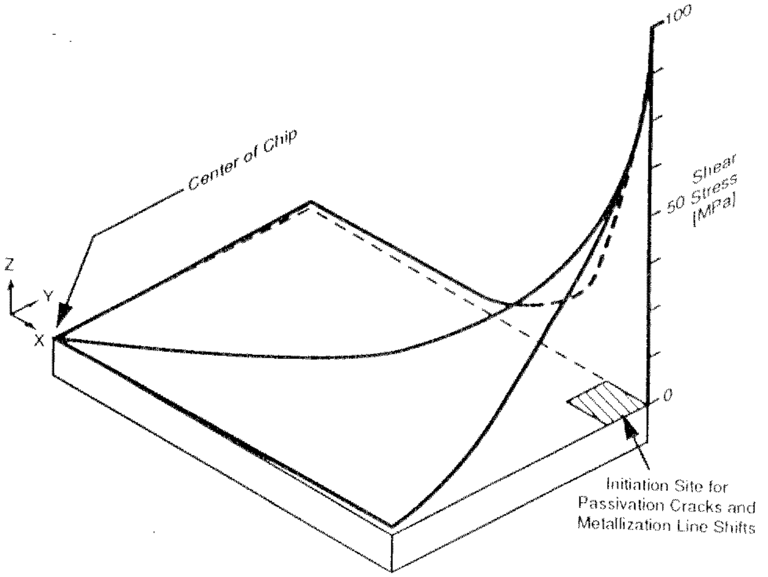


Figure 3.10 **Shear stress distribution across surface of die as calculated with FEA techniques.**

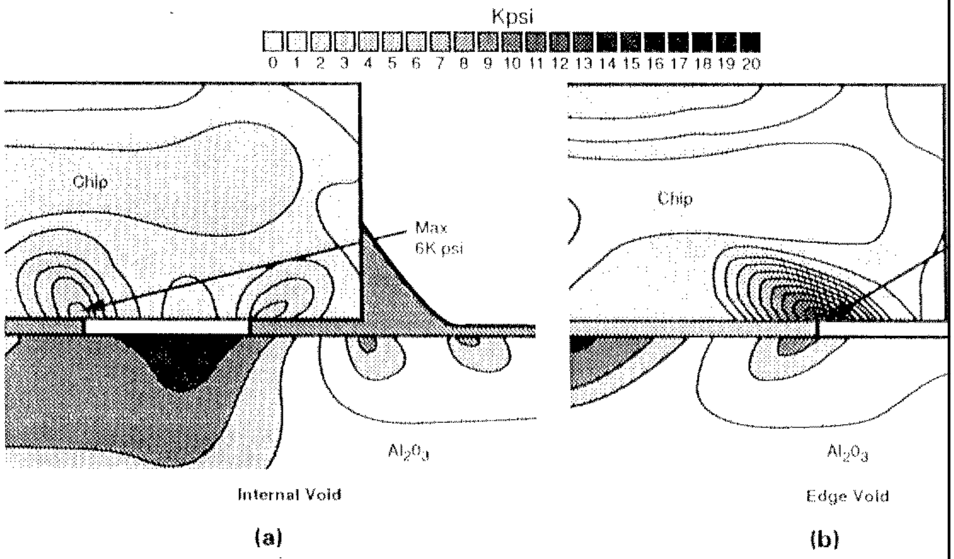
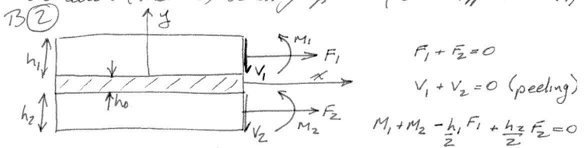


Figure 3.7 **Maximum principal (tensile) stresses calculated through FEA tec voids in eutectic attach (*a*) under die or (*b*) at die edge.**

Now allow (include) bending effects (Brown pp 245–247)

B ②



$F_1 + F_2 = 0$

$V_1 + V_2 = 0$ (peeling)

$M_1 + M_2 - \frac{h_1}{2} F_1 + \frac{h_2}{2} F_2 = 0$

with $M_i = \frac{E_i h_i^3}{12\rho} \Rightarrow \frac{1}{\rho} = \frac{h_1 F_1 - h_2 F_2}{2D}$

$D = D_1 + D_2 = \left[\frac{E_1 h_1^3}{12(1-\nu_1^2)} + \frac{E_2 h_2^3}{12(1-\nu_2^2)}\right]\rho^{-1}$

Equal strains (shear)

$$\frac{\partial u_1}{\partial x} = \alpha_1 \Delta T + \lambda_1 F_1 - h_1/2\rho - K_1 \frac{\partial \tau}{\partial x}$$

$$\frac{\partial u_2}{\partial x} = \underbrace{\alpha_2 \Delta T + \lambda_2 F_2}_{\text{As before}} + \underset{\substack{\uparrow \\ \text{Bending} \\ \text{strain}}}{h_2/2\rho} + \underset{\substack{\uparrow \\ \text{Correction} \\ \text{for non-uniform} \\ F \text{ across c/s}}}{K_2 \frac{\partial \tau}{\partial x}}$$

where $K_i = \frac{2(1+\nu_i)h_i}{3E_i}$ interfacial compliance $\rightarrow \left(\begin{matrix}\text{bond layer} \\ \text{stiffness}\end{matrix}\right)^{-1}$

$$u_1 - u_2 = \tau h_0 / G$$

$\partial u_1/\partial x = \partial u_2/\partial x$

Solve: $(\alpha_1 - \alpha_2)\Delta T + \lambda_1 F_1 - \lambda_2 F_2 + \frac{h_1}{2\rho} + \frac{h_2}{2\rho} - (K_1 + K_2 + K)\frac{\partial \tau}{\partial x} = 0$

i.e. $(K_1 + K_2 + K)\frac{\partial^2 \tau}{\partial x^2} - \left[\lambda_1 + \lambda_2 + \frac{(h_1 + h_2)^2}{4D}\right]\tau = 0$

$$\Rightarrow \frac{\partial^2 \tau}{\partial x^2} - k^2 \tau = 0$$

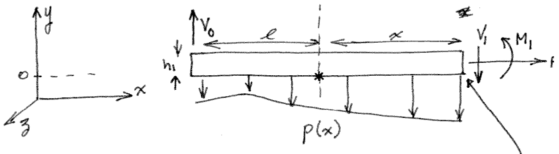
as before, but with $k^2 = \frac{\lambda_1 + \lambda_2 + (h_1 + h^2)^2/4D}{K_1 + K_2 + K}$

instead of $k^2 = \frac{1}{K}(\lambda + \lambda_2)$

C ①

PEELING STRESS

Consider section of chip (or substrate)



Taking moments about z axis at x, y = 0

$$M_1 - V_0(x+\ell) - F_1 \frac{a_1}{2} + \int_{-\ell}^{x}\int_{-\ell}^{\xi} P(\xi)\,d\xi = 0$$

$$M_1 - F_1 \frac{a_1}{2} \rightsquigarrow \frac{D_1 a_2 - D_2 a_1}{2(D_1 + D_2)} F$$

$$\& \frac{d}{dx}\left\{V_0(x+\ell) - \int_{-\ell}^{x}\int_{-\ell}^{\xi} P(\xi)\,d\xi\right\} \longrightarrow V_0 - \int_{-\ell}^{x} P(\xi)\,d\xi$$

$$= \frac{D_1 a_2 - D_2 a_1}{2(D_1 + D_2)} \frac{\partial F}{\partial x}$$

$$\& \frac{d}{dx} \rightsquigarrow P(x) = \frac{D_1 a_2 - D_2 a_1}{2(D_1 + D_2)} \frac{\partial^2 F}{\partial x} \text{ or } \frac{D_1 a_2 - D_2 a_1}{2(D_1 + D_2)} \cdot \frac{\partial \tau}{\partial x}$$

eg. For shear stress $\tau = \frac{(\alpha_1 - \alpha_2)\Delta T}{K k \cosh k\ell} \sinh kx$

$$P(x) = \frac{(\alpha_1 - \alpha_2)\Delta T}{K \cancel{k} \cosh k\ell} \cosh kx$$

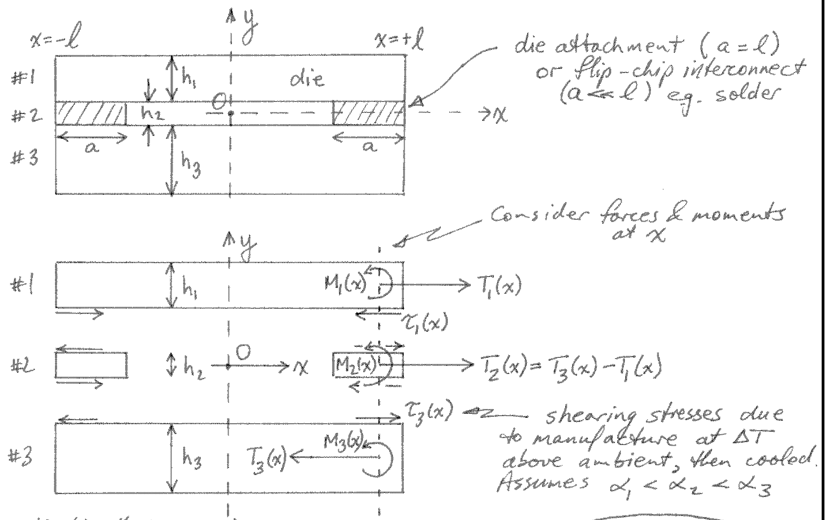
All solutions are of the general form:

~~Max~~ Peeling stress $\propto \Delta\alpha\, \Delta T \frac{\cosh kx}{\cosh kl} \longrightarrow$ Max at $x = l$

Shear stress $\propto \Delta\alpha\, \Delta T$ ~~tanh~~ $\frac{\sinh kx}{\cosh kl}$

$\xrightarrow[\text{max at } x = l]{} \propto$ ~~Δ~~ $\alpha \Delta T \tanh kl$

D Refs: E. Suhir Proc ISHM Sympos 1986 p 383
Proc ECTC 1987 p 508
Brown pp 248–250

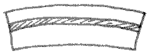


die attachment $(a = l)$ or flip-chip interconnect $(a \ll l)$ eg. solder

Consider forces & moments at x

shearing stresses due to manufacture at ΔT above ambient, then cooled. Assumes $\alpha_1 < \alpha_2 < \alpha_3$

(z-direction thickness = 1)



[E1] Linear displacements equal, if assembly remains attached.

$$u_1^-(x) = u_2^+(x) \qquad u_2^-(x) = u_3^+(x)$$

lower side of #1 ↑ ↖ upper side of #3.

where

$$u_1^-(x) = \alpha_1 \Delta T . x - \frac{1-\nu_1^2}{E_1 h_1}\int_0^x T_1(\xi)\,d\xi + \frac{h_1}{3G_1}\tau_1(x) + \frac{h_1}{2}\int_0^x \frac{d\xi}{\rho(\xi)}$$

$$u_2^+(x) = \alpha_2 \Delta T . x + \frac{1-\nu_2^2}{E_2 h_2}\int_0^x [T_1(\xi)-T_3(\xi)]\,d\xi - \frac{h_2}{3G_2}\tau_1(x) - \frac{h_2}{2}\int_0^x \frac{d\xi}{\rho(\xi)}$$

$$u_2^-(x) = \alpha_2 \Delta T . x + \frac{1-\nu_2^2}{E_2 h_2}\int_0^x [T_1(\xi)-T_3(\xi)]\,d\xi + \frac{h_2}{3G_2}\tau_3(x) + \frac{h_2}{2}\int_0^x \frac{d\xi}{\rho(\xi)}$$

$$u_3^+(x) = \alpha_3 \Delta T . x + \frac{1-\nu_3^2}{E_3 h_3}\int_0^x T_3(\xi)\,d\xi - \frac{h_3}{3G_3}\tau_3(x) - \frac{h_3}{2}\int_0^x \frac{d\xi}{\rho(\xi)}$$

thermal expansion ↗ Assumes T constant through thickness Shear stress bending

[E2] Substitute to get:

$$(\alpha_2-\alpha_1)\Delta T . x = \frac{1}{3}\left(\frac{h_1}{G_1}+\frac{h_2}{G_2}\right)\tau_1(x) + \frac{h_1+h_2}{2}\int_0^x \frac{d\xi}{\rho(\xi)} - \left(\frac{1-\nu_1}{E_1 h_1}+\frac{1-\nu_2}{E_2 h_2}\right)\int_0^x T_1(\xi)\,d\xi + \frac{1-\nu_2^2}{E_2 h_2}\int_0^x T_3(\xi)\,d\xi$$

$$(\alpha_3-\alpha_2)\Delta T . x = \frac{1}{3}\left(\frac{h_2}{G_2}+\frac{h_3}{G_3}\right)\tau_3(x) + \frac{h_2+h_3}{2}\int_0^x \frac{d\xi}{\rho(\xi)} - \left(\frac{1-\nu_3^2}{E_3 h_3}+\frac{1-\nu_2^2}{E_2 h_2}\right)\int_0^x T_3(\xi)\,d\xi + \frac{1-\nu_2^2}{E_2 h_2}\int_0^x T_1(\xi)\,d\xi$$

[E3] Taking moments about point $(x, 0)$

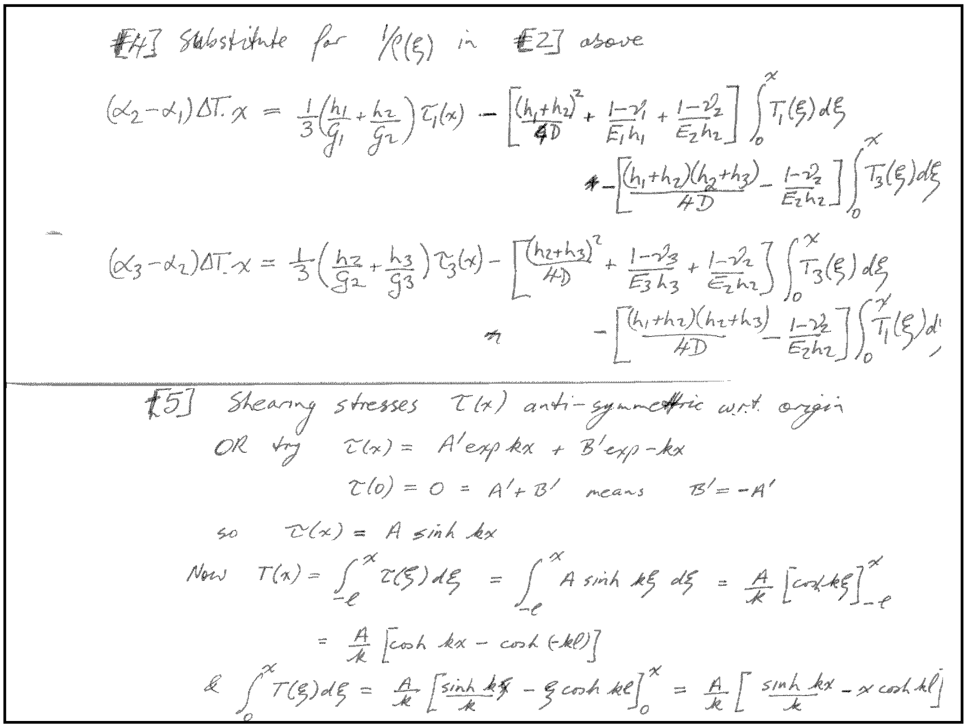
$$M_1(x) + M_2(x) + M_3(x) = \frac{h_1+h_2}{2}T_1(x) + \frac{h_2+h_3}{2}T_3(x)$$

& with $M_i(x) = -D_i/\rho(x)$ where $D_i = \frac{E_i h_i^3}{12(1-\nu_i^2)}$

$$\frac{-D_1}{\rho(x)} + \frac{-D_2}{\rho(x)} + \frac{-D_3}{\rho(x)} = \frac{h_1+h_2}{2}T_1(x) + \frac{h_2+h_3}{2}T_3(x)$$

ie. $\frac{1}{\rho(x)} = -T_1(x)\left(\frac{h_1+h_2}{2D}\right) - T_3(x)\left(\frac{h_2+h_3}{2D}\right)$ where $D = D_1 + D_2 + D_3$

ie. $D = \frac{E_1 h_1^3}{12(1-\nu_1^2)} + \frac{E_2 h_2^3}{12(1-\nu_2^2)} + \frac{E_3 h_3^3}{12(1-\nu_3^2)}$ is total assembly flexural rigidity



[4] Substitute for $1/\rho(\xi)$ in [2] above

$$(\alpha_2-\alpha_1)\Delta T.x = \frac{1}{3}\Big(\frac{h_1}{G_1}+\frac{h_2}{G_2}\Big)\tau_1(x) - \Big[\frac{(h_1+h_2)^2}{4D}+\frac{1-\nu_1}{E_1h_1}+\frac{1-\nu_2}{E_2h_2}\Big]\int_0^x T_1(\xi)\,d\xi$$

$$\cancel{+}-\Big[\frac{(h_1+h_2)(h_2+h_3)}{4D}-\frac{1-\nu_2}{E_2h_2}\Big]\int_0^x T_3(\xi)\,d\xi$$

$$(\alpha_3-\alpha_2)\Delta T.x = \frac{1}{3}\Big(\frac{h_2}{G_2}+\frac{h_3}{G_3}\Big)\tau_3(x) - \Big[\frac{(h_2+h_3)^2}{4D}+\frac{1-\nu_3}{E_3h_3}+\frac{1-\nu_2}{E_2h_2}\Big]\int_0^x T_3(\xi)\,d\xi$$

$$-\Big[\frac{(h_1+h_2)(h_2+h_3)}{4D}-\frac{1-\nu_2}{E_2h_2}\Big]\int_0^x T_1(\xi)\,d$$

[5] Shearing stresses $\tau(x)$ anti-symmetric w.r.t. origin

OR try $\tau(x) = A'\exp kx + B'\exp -kx$

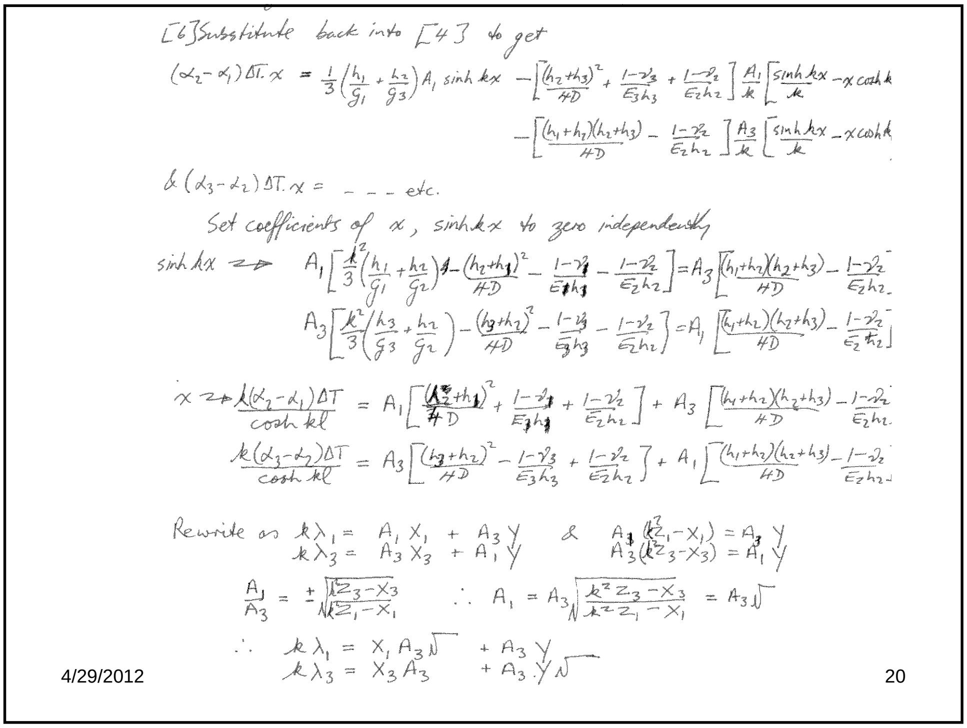
$\tau(0) = 0 = A' + B'$ means $B' = -A'$

so $\tau(x) = A\sinh kx$

Now $T(x) = \int_{-\ell}^{x}\tau(\xi)\,d\xi = \int_{-\ell}^{x} A\sinh k\xi\,d\xi = \frac{A}{k}\big[\cosh k\xi\big]_{-\ell}^{x}$

$= \frac{A}{k}\big[\cosh kx - \cosh(-k\ell)\big]$

& $\int_0^x T(\xi)\,d\xi = \frac{A}{k}\Big[\frac{\sinh k\xi}{k} - \xi\cosh k\ell\Big]_0^x = \frac{A}{k}\Big[\frac{\sinh kx}{k} - x\cosh k\ell\Big]$



[6] Substitute back into [4] to get

$$(\alpha_2-\alpha_1)\Delta T.x = \frac{1}{3}\Big(\frac{h_1}{G_1}+\frac{h_2}{G_3}\Big)A_1\sinh kx - \Big[\frac{(h_2+h_3)^2}{4D}+\frac{1-\nu_3}{E_3h_3}+\frac{1-\nu_2}{E_2h_2}\Big]\frac{A_1}{k}\Big[\frac{\sinh kx}{k} - x\cosh k$$

$$-\Big[\frac{(h_1+h_2)(h_2+h_3)}{4D} - \frac{1-\nu_2}{E_2h_2}\Big]\frac{A_3}{k}\Big[\frac{\sinh kx}{k} - x\cosh k$$

& $(\alpha_3-\alpha_2)\Delta T.x = - - -$ etc.

Set coefficients of x, $\sinh kx$ to zero independently

$$\sinh kx \Rightarrow A_1\Big[\frac{k^2}{3}\Big(\frac{h_1}{G_1}+\frac{h_2}{G_2}\Big) - \frac{(h_2+h_1)^2}{4D} - \frac{1-\nu_1}{E_1h_1} - \frac{1-\nu_2}{E_2h_2}\Big] = A_3\Big[\frac{(h_1+h_2)(h_2+h_3)}{4D} - \frac{1-\nu_2}{E_2h_2}$$

$$A_3\Big[\frac{k^2}{3}\Big(\frac{h_3}{G_3}+\frac{h_2}{G_2}\Big) - \frac{(h_3+h_2)^2}{4D} - \frac{1-\nu_3}{E_3h_3} - \frac{1-\nu_2}{E_2h_2}\Big] = A_1\Big[\frac{(h_1+h_2)(h_2+h_3)}{4D} - \frac{1-\nu_2}{E_2h_2}\Big]$$

$$x \Rightarrow \frac{k(\alpha_2-\alpha_1)\Delta T}{\cosh k\ell} = A_1\Big[\frac{(h_2+h_1)^2}{4D} + \frac{1-\nu_1}{E_1h_1} + \frac{1-\nu_2}{E_2h_2}\Big] + A_3\Big[\frac{(h_1+h_2)(h_2+h_3)}{4D} - \frac{1-\nu_2}{E_2h_2}$$

$$\frac{k(\alpha_3-\alpha_2)\Delta T}{\cosh k\ell} = A_3\Big[\frac{(h_3+h_2)^2}{4D} - \frac{1-\nu_3}{E_3h_3} + \frac{1-\nu_2}{E_2h_2}\Big] + A_1\Big[\frac{(h_1+h_2)(h_2+h_3)}{4D} - \frac{1-\nu_2}{E_2h_2}$$

Rewrite as $k\lambda_1 = A_1X_1 + A_3Y$ & $A_1(k^2Z_1 - X_1) = A_3Y$

$k\lambda_3 = A_3X_3 + A_1Y$ $A_3(k^2Z_3 - X_3) = A_1Y$

$\frac{A_1}{A_3} = \pm\sqrt{\frac{k^2Z_3 - X_3}{k^2Z_1 - X_1}}$ $\therefore A_1 = A_3\sqrt{\frac{k^2Z_3 - X_3}{k^2Z_1 - X_1}} = A_3\sqrt{\ }$

$\therefore k\lambda_1 = X_1A_3\sqrt{\ } + A_3Y$

$k\lambda_3 = X_3A_3 + A_3Y\sqrt{\ }$

$$\frac{\lambda_1}{\lambda_3} = \frac{X_1\sqrt{\ } + Y}{X_3 + Y\sqrt{\ }} \quad . \text{ if } k \neq 0$$

$$\lambda_1 X_3 + \lambda_1 Y\sqrt{\ } = \lambda_3 X_1\sqrt{\ } + \lambda_3 Y$$

$$\therefore \sqrt{\ } = \frac{Y\lambda_3 - \lambda_1 X_3}{Y\lambda_1 - \lambda_3 X_1} = \sqrt{\frac{k^2 Z_3 - X_3}{k^2 Z_1 - X_1}}$$

$$\therefore \frac{k^2 Z_3 - X_3}{k^2 Z_1 - X_1} = \frac{(Y\lambda_3 - \lambda_1 X_3)^2}{(Y\lambda_1 - \lambda_3 X_1)^2}$$

$$k^2\left[Z_3(Y\lambda_1 - \lambda_3 X_1)^2 - Z_1(Y\lambda_3 - \lambda_1 X_3)^2\right]$$

$$= X_3(Y\lambda_1 - \lambda_3 X_1)^2 - X_1(Y\lambda_3 - \lambda_1 X_3)^2$$

$$\therefore k^2 = \frac{X_3(Y\lambda_1 - \lambda_3 X_1)^2 - X_1(Y\lambda_3 - \lambda_1 X_3)^2}{Z_3(Y\lambda_1 - \lambda_3 X_1)^2 - Z_1(Y\lambda_3 - \lambda_1 X_3)^2}$$

$$A_3 = k\lambda_1 / (X_1\sqrt{\ } + Y) = \frac{k\lambda_1}{X_1\sqrt{\frac{k^2 Z_3 - X_3}{k^2 Z_1 - X_1}} + Y}$$

$$A_1 = A_3\sqrt{\ } = \frac{k\lambda_1}{X_1 + Y/\sqrt{\ }} = \frac{k\lambda_1}{X_1 + Y\sqrt{\frac{k^2 Z_1 - X_1}{k^2 Z_3 - X_3}}}$$

etc.

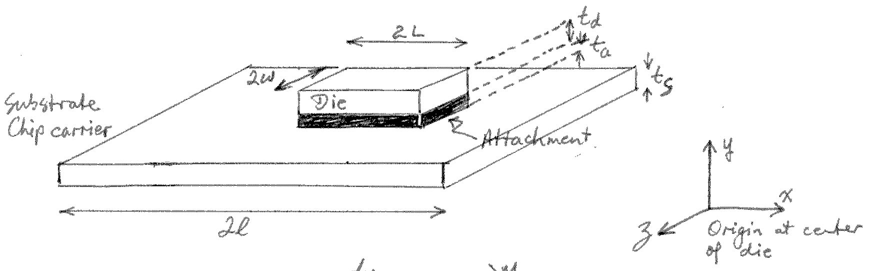
These are the general solutions.

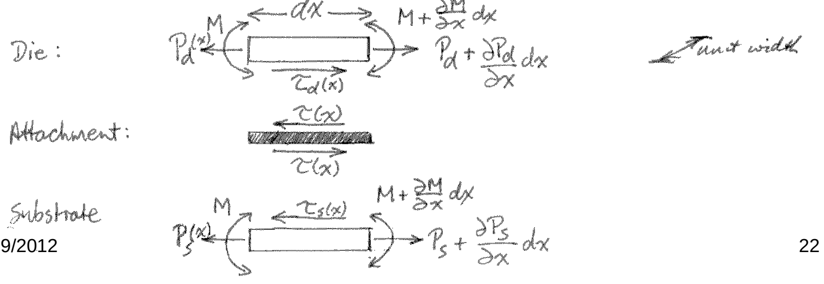
Generally simplified for $t_2 \to 0$ or $E_2 \to 0$ (thin adhesive)

or $E_2 \to \infty$ (rigid attach) etc.

THERMOMECHANICAL/ATTACHMENT STRESSES

The "Suhir" model (Christou)





Assumptions: (1) Attachment shear-stress varies with y, but equal & opp at interface

(2) Die & substrate stresses linearly distributed along y.

(3) No interface slippage

Also: Peeling stresses do not affect other stress determinations
There is an attachment moment (neglected)
All layers have same radius of curvature
Influence of z axis constraints on x axis negligible.

$$P_d(x) = \int_{-L}^{x} \tau_d(\xi)\, d\xi \qquad P_s(x) = \int_{-L}^{x} \tau_s(\xi)\, d\xi$$

G_s, G_d, G_a shear moduli of elasticity of substrate, die, attachment
E_s, E_d, E_a tensile " " " " " " "
ν_s, ν_d, ν_a Poisson's ratio of " " "
α_s, α_d, α_a coefficients of thermal expansion " " " (CTE)

Thermomechanical Cycling

Brown Equation 4.2 (Section 4.4.2)

Stress Intensity Factor $K_S = \sigma_{cyc} \sqrt{\pi L}\; Y$

where σ_{cyc} due to thermal cycling

$$= (\alpha_{substr} - \alpha_{ele})\Delta T \times \left[1 - \frac{1}{\cosh A_c l_{att, D, ele}}\right]$$

l ← half diagonal length of element attached to substrate.

$$\times \left[\frac{\dfrac{1}{t_{ele}} + \dfrac{3(t_{ele} + t_{substr}) E_{att} t_{ele}}{12\,(1-\nu_{att}^{2})\,D}}{\dfrac{1-\nu_{att}}{E_{att} t_{ele}} + \dfrac{1-\nu_{subs}}{E_{substr} t_{substr}} + \dfrac{(t_{ele}+t_{subst})^2}{4D}}\right]$$

$$A_c = \frac{\left[\dfrac{1-\nu_{substr}}{E_{substr} t_{substr}} + \dfrac{1-\nu_{die}}{E_{die} t_{die}} + \dfrac{t_{die} + t_{subst}}{4D}\right]^{1/2}}{\dfrac{t_{die}}{3G_{die}} + \dfrac{2\,t_{att}}{3G_{att}} + \dfrac{t_{subs}}{3G_{subs}}}$$

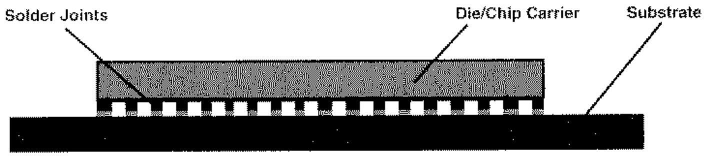
$$D = \frac{E_{die}\, t_{die}^3}{12(1-\nu_{die}^2)} + \frac{E_{att}\, t_{att}^3}{12(1-\nu_{att}^2)} + \frac{E_{subs}\, t_{subs}^3}{12\,(1-\nu_{subs}^2)}$$

$$Y = F_{cor} = \frac{2.85\left(0.953 - 2.369[a/t_{subs}] + 2.74 \tan[a/t_{subs}]\right)}{3 + (a/c)^2}$$

where α = TCE
ΔT = temperature variation
t = thickness
ν = Poisson's ratio
E = Young's modulus
G = shear modulus
a = crack depth
$c = L$ = crack half length

of: die / element
attachment material
substrate

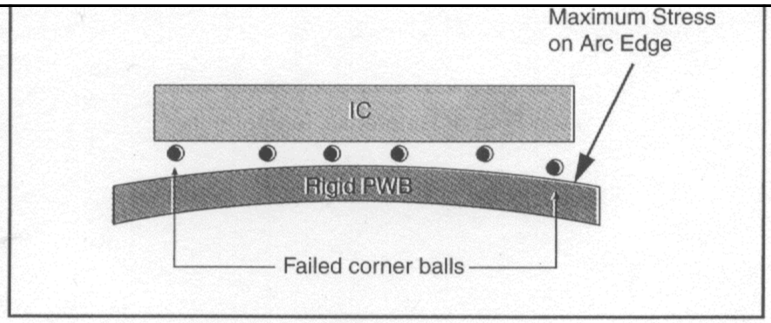
See also pages:
117/119/120
239/249
255/277/386



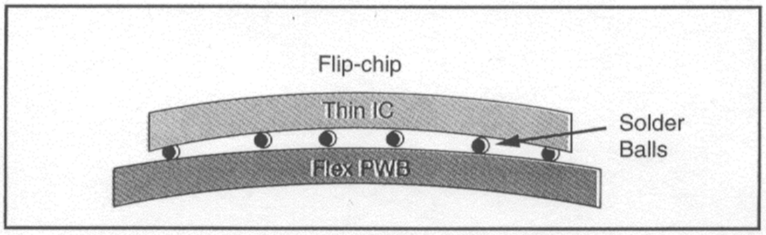
(a) Stress-Free or Reference Temperature (T_0)

(b) Deformation upon Heating (T_{max})

(c) Deformation upon Cooling (T_{min})



a) Rigid



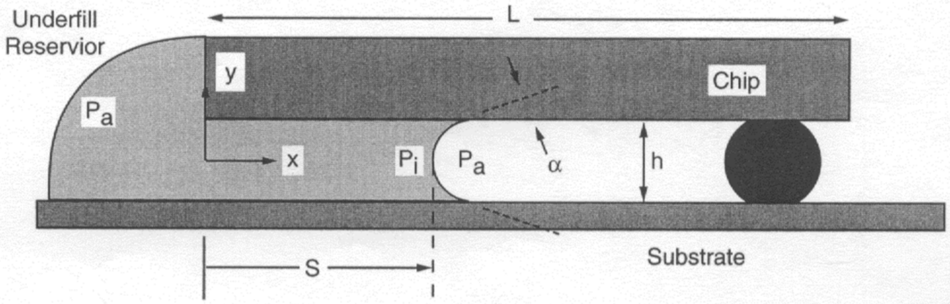
b) Thin or Compliant

Underfill

Thermomechanical stresses

Strengthen flip-chip adhesion by underfill

1. Capillary flow
2. Injection flow
3. "No flow" underfill



IGURE 9.19 Capillary flow underfill processing configuration and analysis schematic

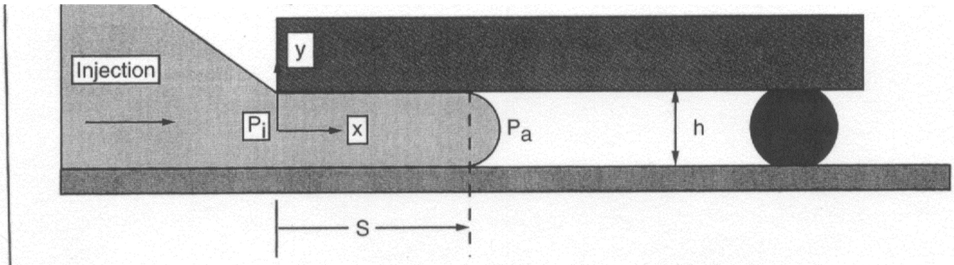
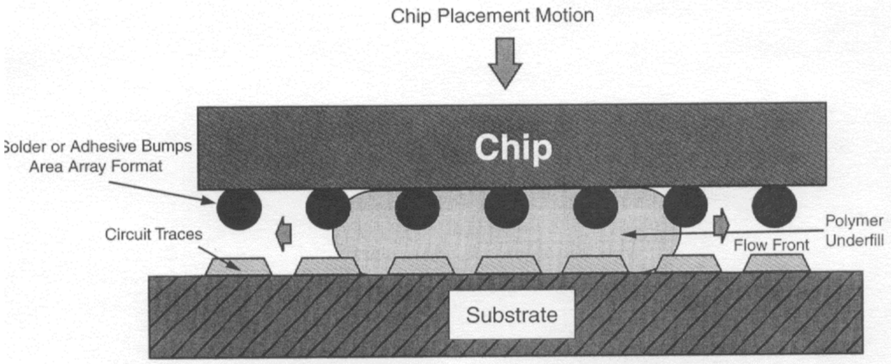


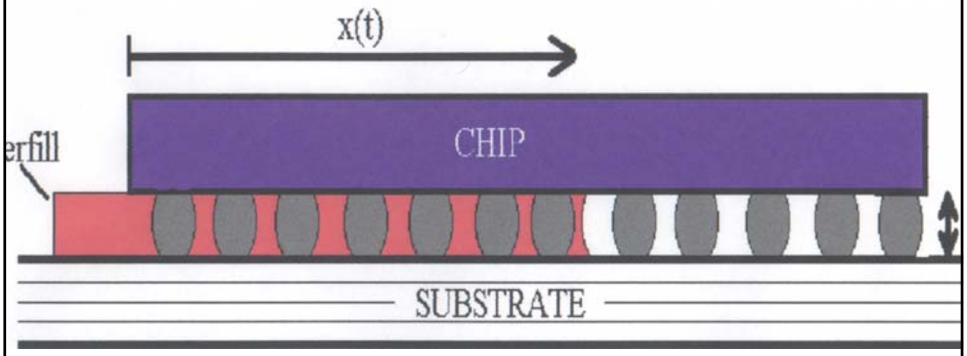
FIGURE 9.20 Schematic of injection flow underfill processing.

'UNDAMENTALS OF MICROSYSTEMS PACKAGING

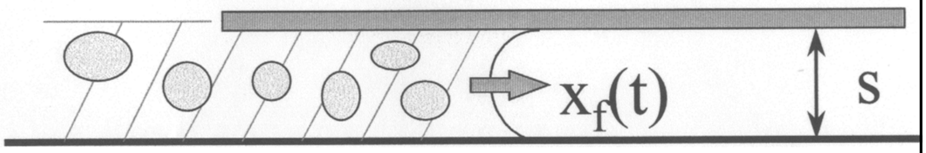


'IGURE 9.21 Compression flow underfill processing.

Direct-Chip Attachment (DCA)



Underfill Flow



CONSTRUCT THE WASHBURN MODEL

Let the front speed equal the mean speed

$$u_m = \frac{d}{dt}x_f = \frac{s^2}{12\mu}\frac{\Delta p}{x_f(t)}$$

Model the interface pressure jump under capillary action as

$$\Delta p = \frac{2\cos(\theta)}{s}\sigma$$

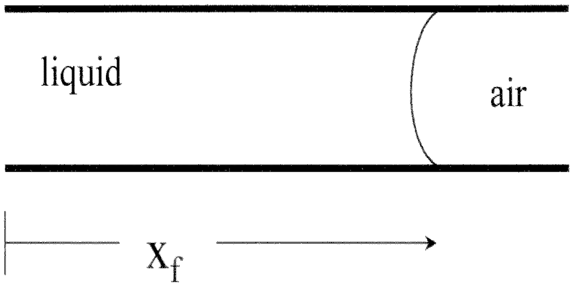
Combining

$$x_f \frac{dx_f}{dt} = \frac{\sigma\cos(\theta_w)}{6\mu}s$$

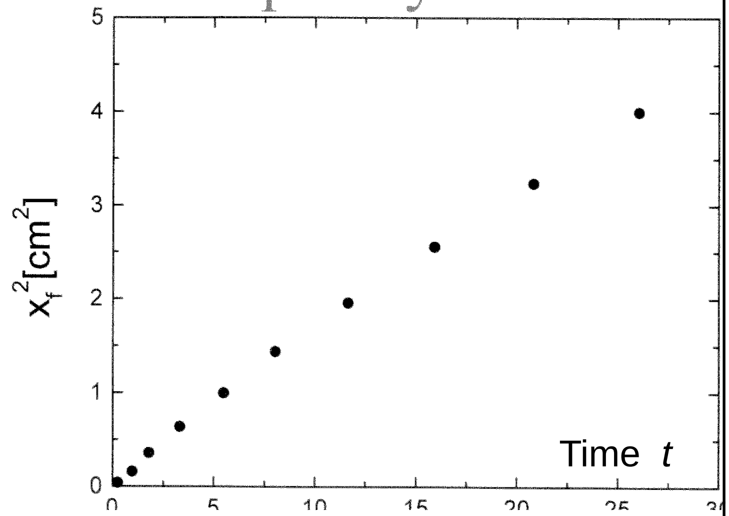
and $x_f(t=0) = 0$

FLOW BETWEEN PARALLEL PLATES

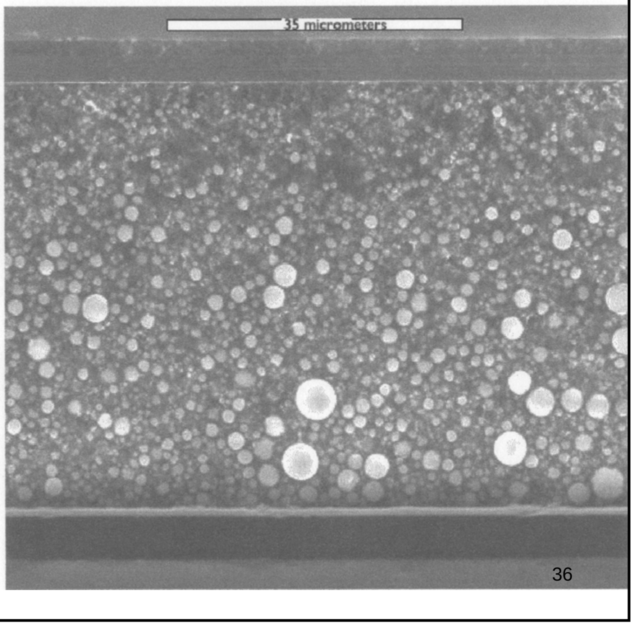
- Washburn (1921) model $x^2 = \sigma.\cos\theta_w s.t/3\mu$

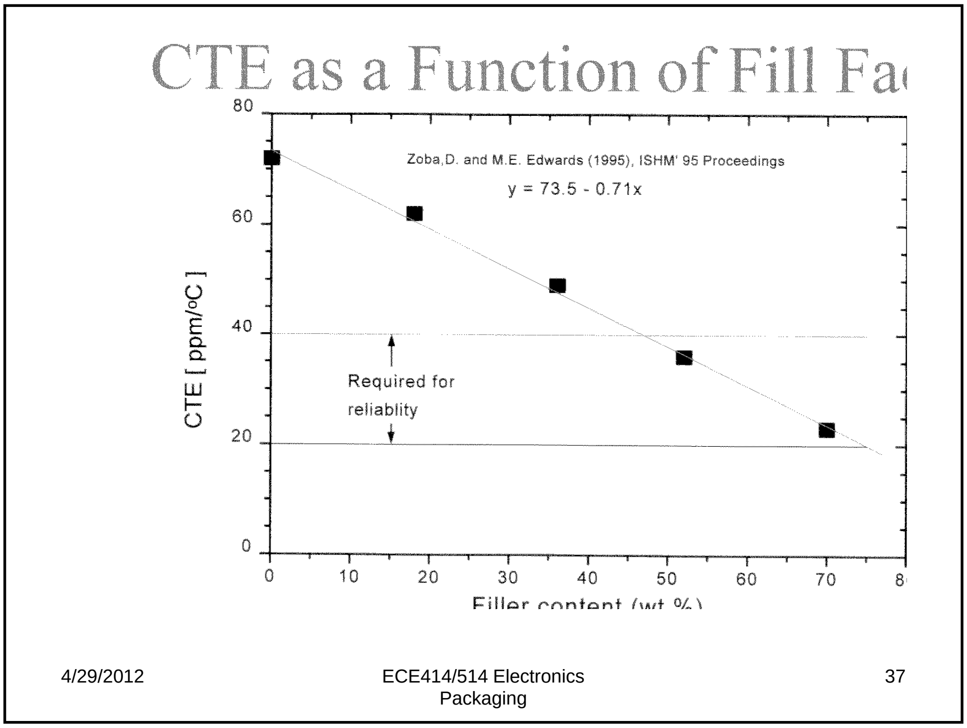


Capillary Flow



Electron Microscope of Underfill A





CTE as a Function of Fill Fa

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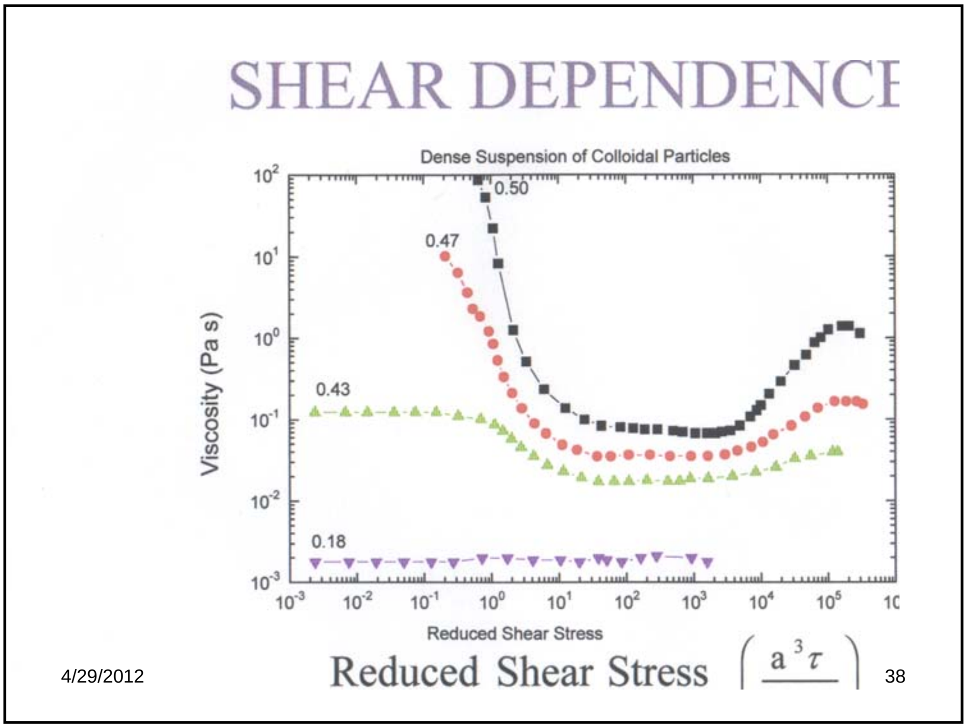

SHEAR DEPENDENCE

Reduced Shear Stress $\left(\frac{a^3\tau}{}\right.$

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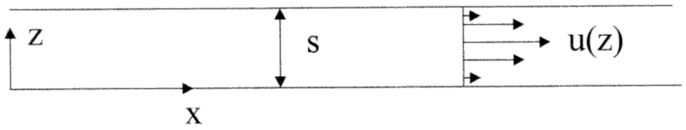
Pressure-driven channel flow

Laminar flow of a Newtonian fluid



$$\frac{d}{dz}\left(\mu\frac{du}{dz}\right)=\frac{dp}{dx}$$

u(z=0) = 0

u(z=s) = 0