ECE414/514
Electronics Packaging
Spring 2012 Lecture 8
Mechanical A
Basic Concepts / Vibration
See Dally et al: Chapters 11 & 13

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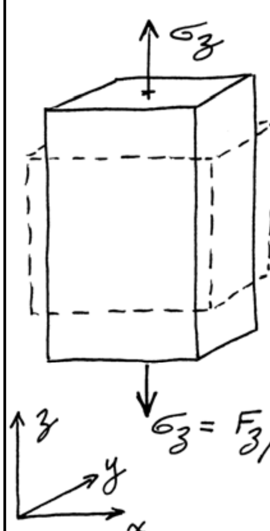
Basic Concepts



$\sigma_z = F_z/A_z$

Tensile stress σ & strain ϵ

$E = \sigma/\epsilon$

Young's Modulus (of elasticity) $= \dfrac{\text{stress}}{\text{strain}}$

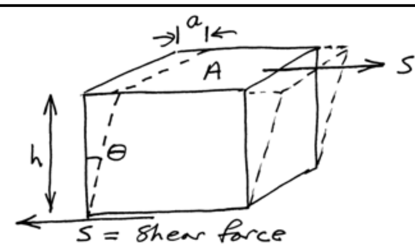
$\nu = -\dfrac{\epsilon_x}{\epsilon_z}$

(Poisson's Ratio)

Stress = F/A (force/unit area)
Strain = $\Delta\ell/\ell$ (extension/length)

a

Shear Stress



Shear stress $\tau = S/A$

Shear strain $\gamma = a/h = \tan\theta$

& for elastic shear $\tau = G\gamma$

elastic shear modulus $G = \tau/\gamma$

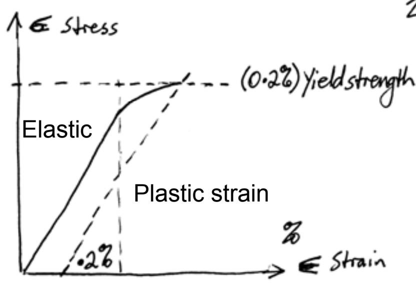
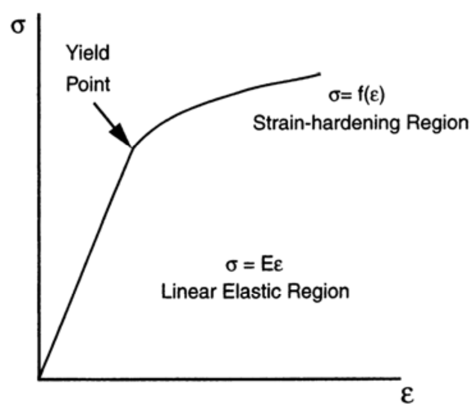


FIGURE 5.14 Stress–strain curve for an elastic-plastic material.



See also Dally Fig 11.3

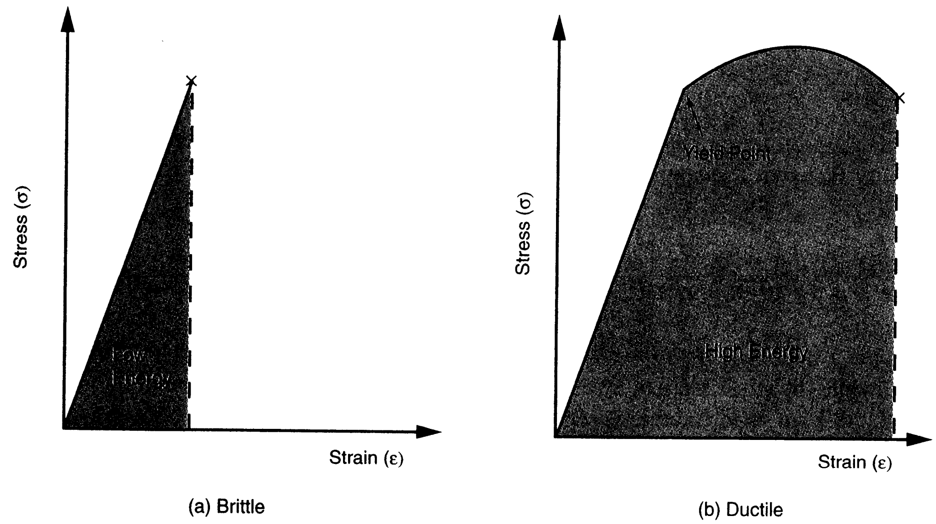


FIGURE 5.8 Stress–strain relation of (a) brittle and (b) ductile materials.

(a) Low energy (b) High energy

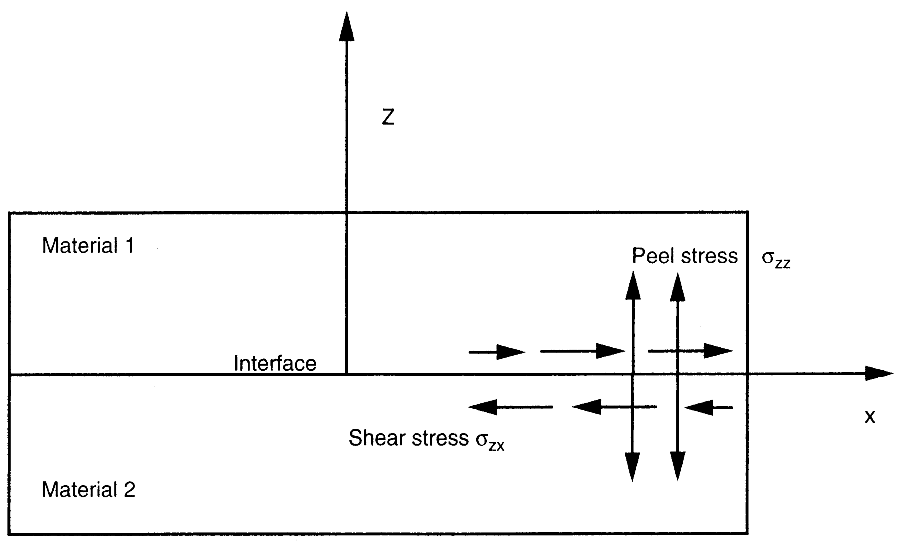


FIGURE 5.13 Schematic of interfacial stresses.

Basic Concepts

Stress : "Engineering" stress $= \sigma_{eng} = \frac{\text{Force}}{\text{c/s area}} = \frac{F}{A_0}$ ← initial

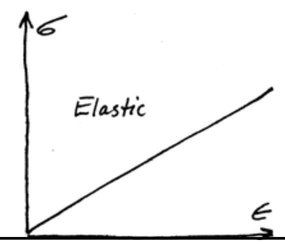
"True" stress $= \sigma_{true} = \frac{F}{A}$ ← instantaneous

Strain : "Engineering" strain $= \epsilon_{eng} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$ ← initial

"True" strain $\Rightarrow \Delta\epsilon_{true} = \frac{\Delta l}{l}$ ← instantaneous

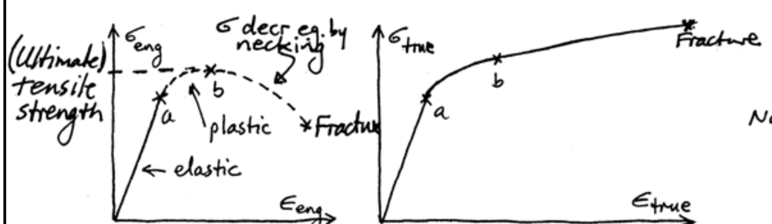
$$\therefore \epsilon_{true} = \int_{l_0}^{l_f} \frac{dl}{l} = \ln \frac{l_f}{l_0}$$

Hooke's Law



$\sigma = E\epsilon$

↑ Young's modulus



Note: Yield strength < tensile strength

$$\epsilon_{eng} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1 \quad \& \quad \epsilon_{true} = \ln\left(\frac{l}{l_0}\right) = \ln(1 + \epsilon_{eng})$$

$$= \epsilon_{eng} - \frac{\epsilon_{eng}^2}{2} + \frac{\epsilon_{eng}^3}{3} - \cdots$$

$$\approx \epsilon_{eng} \text{ for small strain}$$

For constant volume $V = Al$, $\quad \epsilon_{true} = \ln\left(\frac{l_f}{l_0}\right) = \ln\left(\frac{A_0}{A}\right)$

$$\therefore \exp \epsilon_{true} = \frac{A_0}{A}$$

$$\& \ \sigma_{eng} = F/A_0 = (F/A)(A/A_0) = \sigma_{true} / \exp\epsilon_{true}$$

$$\sigma_{true} = \sigma_{eng} \cdot (\exp \epsilon_{true}) \approx \sigma_{eng}(1 + \epsilon_{true} + \epsilon_{true}^2/2 + \cdots) \gtrsim \sigma_{eng} \text{ small } \epsilon$$

Apply stress in y direction $\epsilon_{yy} = \frac{\sigma_{yy}}{E}$

Also transmitted to x, z

$$\epsilon_{xx} = \epsilon_{zz} = -\nu \frac{\sigma_{yy}}{E} = -\nu \epsilon_{yy}$$

$$\nu = \text{Poisson's ratio} = -\frac{\epsilon_{xx}}{\epsilon_{yy}}$$

For triaxial stress, $\epsilon_{xx} = \frac{\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})}{E}$ etc

and shear strains $\gamma_{xy} = \frac{\tau_{xy}}{G}$ ← shear stress, ← shear modulus etc

From general case, can also show $G = \frac{E}{2(1+\nu)}$ & $K = \frac{\sigma}{\epsilon_v} = \frac{2G(1+\nu)}{3(1-2\nu)}$

where $\sigma = \sigma_x = \sigma_y = \sigma_z$

& $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

Plane stress & plane strain

& $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

Plane stress & plane strain

For $\epsilon_x = \frac{\sigma_x - \nu(\sigma_y + \sigma_z)}{E}$

$\epsilon_y = \frac{\sigma_y - \nu(\sigma_z + \sigma_x)}{E}$

$\epsilon_z = \frac{\sigma_z - \nu(\sigma_x + \sigma_y)}{E}$

Plane stress: say $\sigma_z = 0$ (eg. thin plate)

$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$

$\epsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$

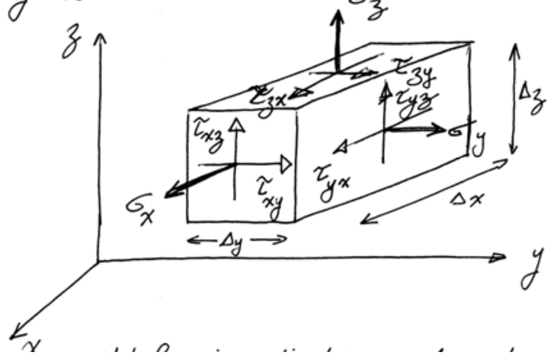
& $\epsilon_z = -\nu \frac{(\sigma_x + \sigma_y)}{E} \neq 0$

Plane strain: say $\epsilon_z = 0$ (& $\gamma_{zx} = \gamma_{zy} = 0$) (eg thick plate)

then $\sigma_z = \nu(\sigma_x + \sigma_y) \neq 0$ & $\epsilon_x = \frac{1+\nu}{E}\left[\sigma_x - \nu(\sigma_x + \sigma_y)\right]$

$\epsilon_y = \frac{1+\nu}{E}\left[\sigma_y - \nu(\sigma_y + \sigma_x)\right]$

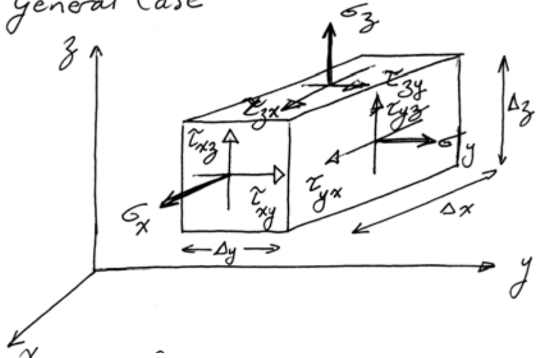
General Case



Net force in y direction on element

$$F_y = \left(\sigma_y + \frac{\partial \sigma_y}{\partial y}\Delta y\right)\Delta x \Delta z - \sigma_y \Delta x \Delta z$$
$$+ \left(\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z}\Delta z\right)\Delta x \Delta y - \tau_{zy}\Delta x \Delta y$$
$$+ \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\Delta x\right)\Delta y \Delta z - \tau_{xy}\Delta y \Delta z = 0$$

General Case



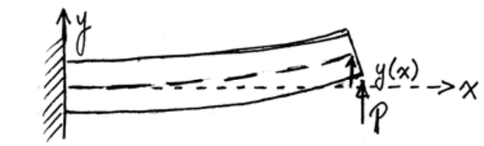
& net moment $M_y = 0$

$$= \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\Delta z\right)\Delta x \Delta y \frac{\Delta z}{2} - \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\Delta x\right)\Delta y \Delta z \frac{\Delta x}{2}$$
$$+ \tau_{zx}\Delta x \Delta y \frac{\Delta z}{2} - \tau_{xz}\Delta y \Delta z \frac{\Delta x}{2}$$

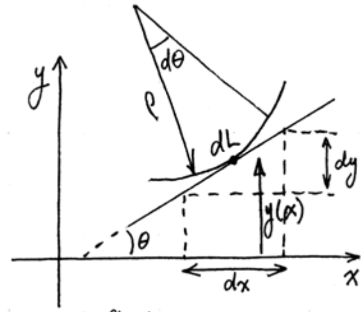
$$\div \Delta x \Delta y \Delta z \Rightarrow \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0$$ (from previous slide)

$$\Rightarrow \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{\Delta z}{2}\right) - \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\frac{\Delta x}{2}\right) \xrightarrow[\Delta x, \Delta z \to 0]{} \tau_{zx} = \tau_{xz}$$

BEAM DEFLECTION



Deflection $y(x)$ due to tip Load P



Deflection at x

Arc $dL = \rho\, d\theta$

$\frac{dy}{dx} = \tan\theta \approx \theta$ for small deflections

$dx/dL=\cos\theta$

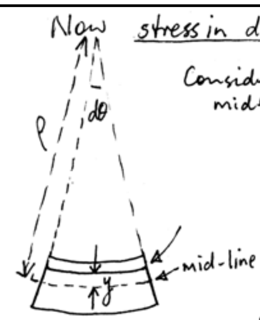
$$\frac{d^2y}{dx^2} = \sec^2\theta \cdot \frac{d\theta}{dx} \qquad \therefore\ d\theta = \frac{d^2y}{dx^2}\,\frac{1}{\sec^2\theta}\,dx = \frac{d^2y/dx^2}{1+\tan^2\theta}\,dx$$

$$= \frac{d^2y/dx^2}{1+(dy/dx)^2}\,dx = \frac{dL}{\rho}$$

$$\therefore \text{ Curvature } K = \frac{1}{\rho} = \frac{d\theta}{dL} = \frac{d^2y/dx^2}{1+(dy/dx)^2}\left(\frac{dx}{dL}\right) \approx \frac{d^2y}{dx^2}$$

for small deflections where θ small & $dL \sim dx$

Now stress in deflected beam:



Consider line y above midline.

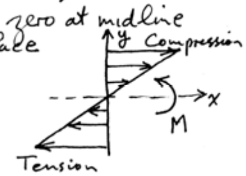
orig length is midline length

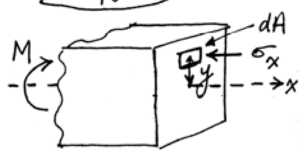
Strain along line $\epsilon_x = \frac{l_f - l_o}{l_o} = \frac{(\rho - y)d\theta - \rho\, d\theta}{\rho\, d\theta}$

$= -y/\rho$

& Stress along line $\sigma_x = E\left(-y/\rho\right)$

NB varies linearly from zero at midline to max at surface





$dM = -y(\sigma_x\, dA)$

& $M = -\int y\, \sigma_x\, dA$

$= \int_A -E\left(\frac{-y}{\rho}\right) y\, dA = \frac{E}{\rho}\int_A y^2 dA \triangleq \frac{EI}{\rho}$

M is torque

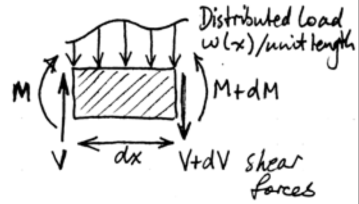
(I is area moment of inertia)

(EI is bending stiffness)

& hence $\sigma_x = -\left(\frac{E}{\rho}\right) y = -\frac{M}{I} y$

So now $K = \frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M}{EI}$ moment-curvature relationship

Now relate <u>shear force & deflection</u>



In equilibrium:

forces $\quad dV = -w\,dx$

anti-cw moments about RH edge

$$-M - V.dx + w\,dx.\frac{dx}{2} + (M+dM) = 0$$

$$\frac{dM}{dx} = V - \frac{w}{2}dx \xrightarrow[dx\to 0]{} V$$

So $\quad V = \frac{dM}{dx} = \frac{d}{dx}\left(EI\frac{d^2y}{dx^2}\right) = EI\frac{d^3y}{dx^3}$ $\quad$ shear force

$V(x)$

& $\quad w = -\frac{dV}{dx} = -EI\frac{d^4y}{dx^4}$ $\quad$ distributed load

$w(x)$

& $M(x)$

See Dally 11.2.2

(Dally 11.4)

Now consider <u>bending of plates</u>

For cylindrically bent plate, thin in z axis $\longrightarrow$ plane strain $\epsilon_z = 0$

assume $\sigma_y = 0$

$$\therefore \quad \epsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)$$
$$= \frac{1}{E}\left(\sigma_x - \nu\sigma_z\right)$$
$$= \frac{\sigma_x}{E}(1-\nu^2)$$

$$\left\{\begin{array}{l} \epsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right) \\ 0 = \frac{1}{E}\left(\sigma_z - \nu\sigma_x\right) \\ \therefore \sigma_z = \nu\sigma_x \end{array}\right.$$

& $\gamma_{xy} = \tau_{xy}/G$

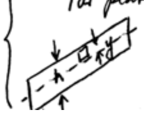
$\therefore$ In equations

$$M(x) = EI\frac{d^2y}{dx^2} \longrightarrow \left[Eh^3/12(1-\nu^2)\right]\frac{d^2y}{dx^2} = D\frac{d^2y}{dx^2}$$

$$w(x) = EI\frac{d^4y}{dx^4} \longrightarrow \quad " \quad \frac{d^4y}{dx^4} = D\frac{d^4y}{dx^4}$$

$$V(x) = EI\frac{d^3y}{dx^3} \longrightarrow \quad " \quad \frac{d^3y}{dx^3} = D\frac{d^3y}{dx^3}$$

$$\left\{ \text{For plate } I = \int_A y^2 dA = \left.\frac{y^3}{3}\right]_{-h/2}^{h/2} = h^3/12 \right\}$$



$D \triangleq$ flexural rigidity & $M = D/\rho$

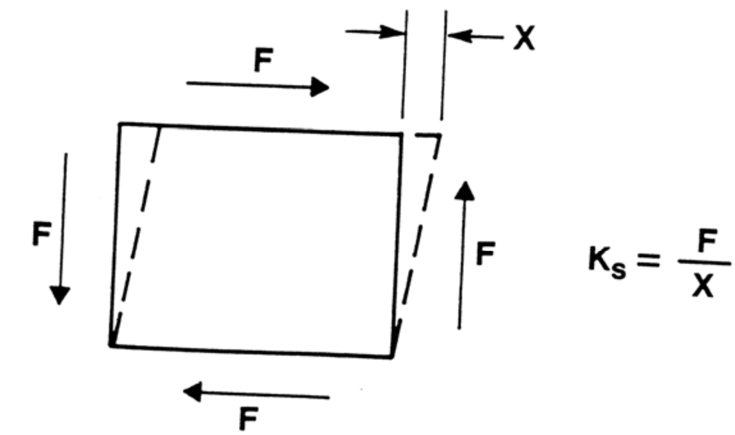


Figure 5-49. Terminal Stiffnesses. Definition of K_s.

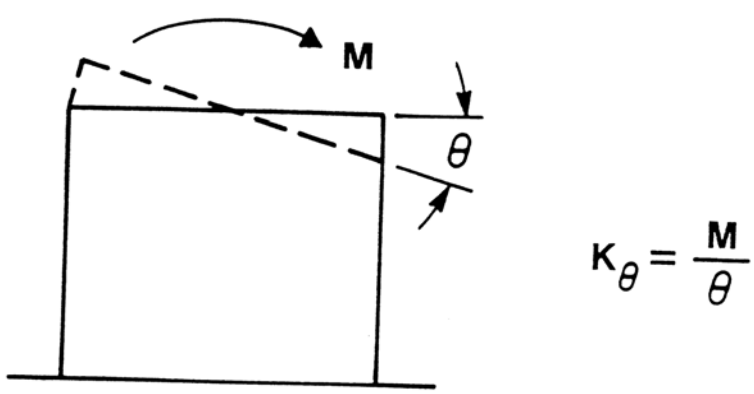


Figure 5-50. Terminal Stiffnesses. Definition of K_θ.

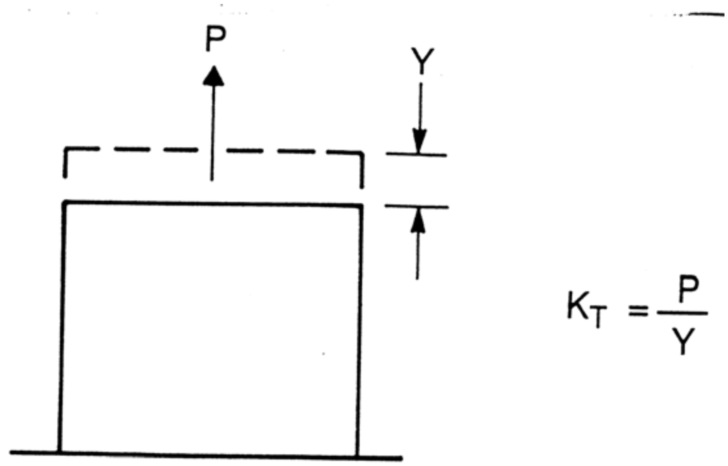


Figure 5-51. Terminal Stiffnesses. Definition of K_p.

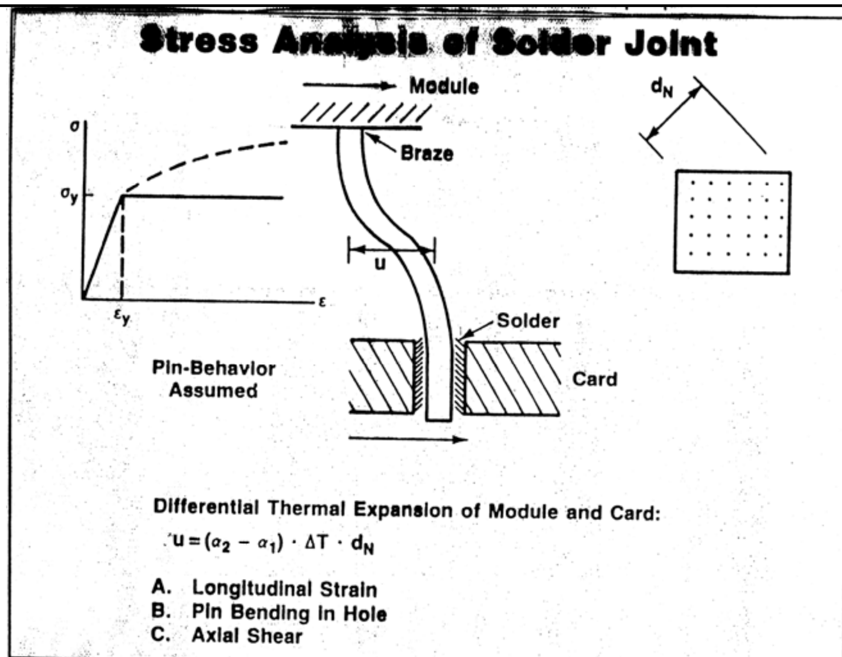
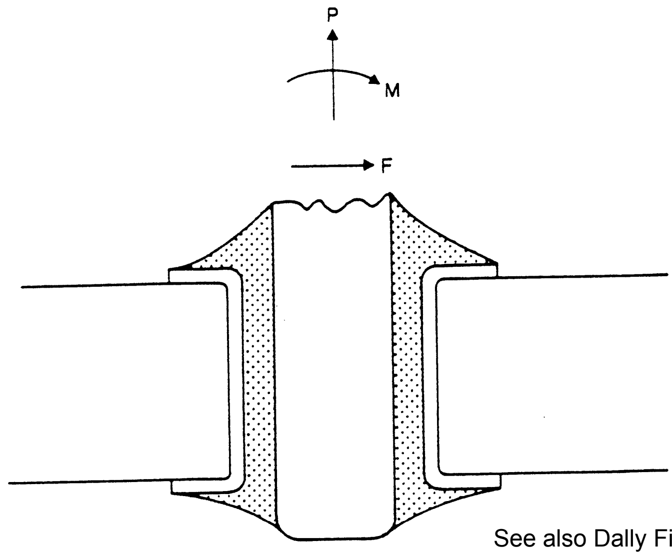


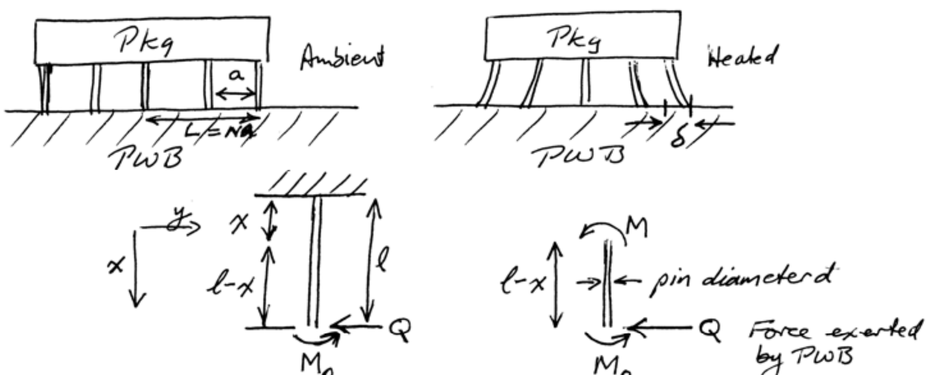
Figure 2.4 Schematic of pin grid array structure and thermal deformations.



See also Dally Fig 13.26(a)

Figure 5-52. Forces on PTH.

Stress in through-hole pins



Max stress in farthest pin from center

M_0 moment exerted by solder to stop end bending
(i.e. soldered at top of board; $M_0 = 0$ if soldered underneath)

$$\epsilon_x = \frac{F_x/A}{E} + \frac{M}{EI}y + \alpha\Delta T$$

$\rightarrow My/EI$ for pin $F_x = 0$ & $\alpha\Delta T$ does not contribute to stress

Hence find M

At x $\quad \Sigma M = M + M_0 - Q(\ell - x) = 0$

$\therefore \; M = Q(\ell - x) - M_0 \qquad Q, M_0$ unknown

But $\delta = (\alpha_p - \alpha_c)\,\Delta T\,(na)$

α_p ← PWB, α_c ← pkg

If displacement of beam at x is $y(x)$, then curvature $\quad 1/R = \dfrac{d^2y}{dx^2} \Big/ \left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2} \approx \dfrac{d^2y}{dx^2} = \dfrac{M}{EI}$

$$\therefore \; EI\,\frac{d^2y}{dx^2} = Q(\ell - x) - M_0$$

Integrate $\longrightarrow \; EI\,\dfrac{dy}{dx} = Q\left(\ell x - \dfrac{x^2}{2}\right) - M_0 x + C_1$

$$EI\,y = Q\left(\frac{\ell x^2}{2} - \frac{x^3}{6}\right) - M_0\frac{x^2}{2} + C_1 x + C_2$$

Bdy cond^{n}s : At $x = 0 \quad y = 0 \quad dy/dx = 0 \quad \therefore C_1 = C_2 = 0$

Also at $x = \ell \quad \dfrac{dy}{dx} = 0$ (for solder constraint, $M_0 \neq 0$)

$$\therefore \quad 0 = Q\,\frac{\ell^2}{2} - M_0\ell$$

i.e. $M_0 = Q\ell/2$

Also at $x = \ell \qquad y = \delta$

$$\therefore \; EI\delta = Q\ell^3/3 - (Q\ell/2)(\ell^2/2) = Q\ell^3/12$$

$$= EI\,(\alpha_p - \alpha_c)\,\Delta T\,(na)$$

$$\therefore \; Q = (12\,na)\,EI\,(\alpha_p - \alpha_c)\,\Delta T/\ell^3$$

Max value of $M = Q(\ell - x) - M_0 = M_{max}$ at $x = 0$

$$M_{max} = Q\ell - Q\ell/2 = 6na\,EI(\alpha_p - \alpha_c)\Delta T/\ell^2$$

Stress $\sigma = My/I$

Max stress $\frac{d\sigma}{dx} = \left(M\frac{dy}{dx} + y\frac{dM}{dx}\right)/I = 0$

when $x \gtrsim \ell/4$

Using $x = \ell/4$, $\sigma_{max} = \left(Q\frac{\ell}{4}\right)\left(\frac{Q}{EI}\frac{5\ell^3}{6.64}\right)/I$

$$= \frac{30}{64}\left[n a\,\Delta T(\alpha_p - \alpha_c)\right]^2 E\cancel{I}\ell^2$$

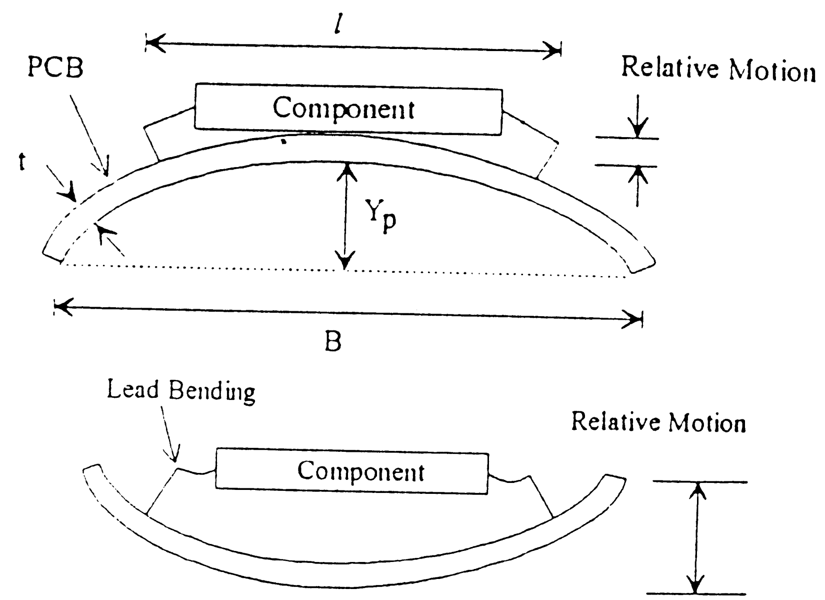


Figure 10.12 Component lead wire shape due to deflection of the PCB.

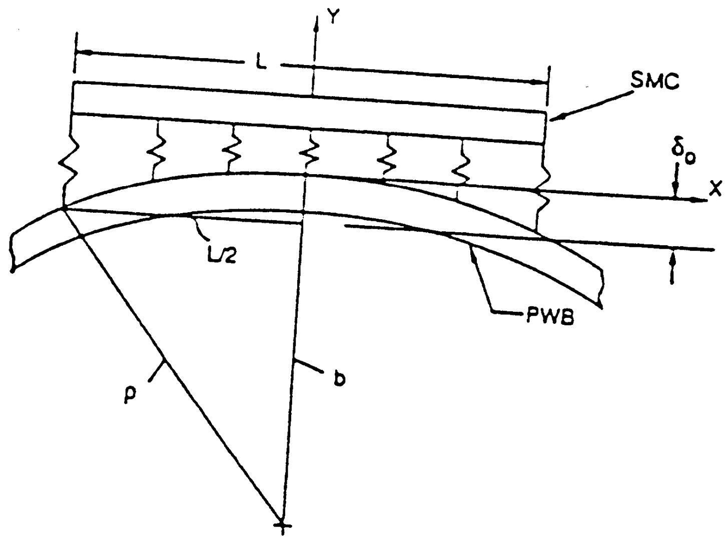


Figure 9.24 Component is modeled as a rigid body suspended above board with springs.

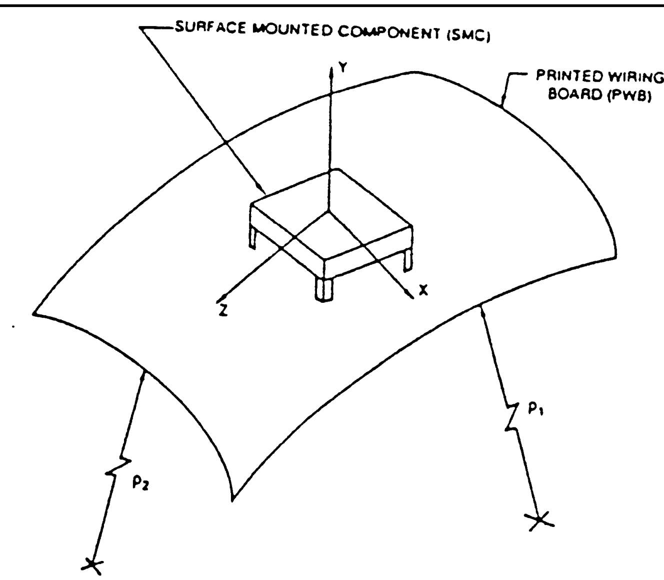


Figure 9.25 Stress in component leads and solder joints is a function of local radius of curvature.

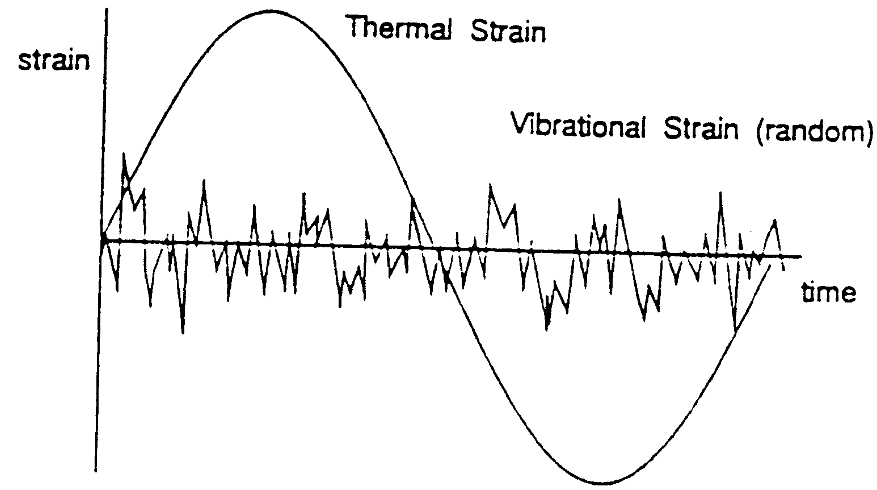
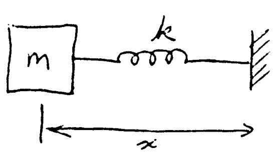


Figure 9.26 Schematic representation of vibrational and thermal strain.

Simple Vibrations Theory



$$m\ddot{x} = -kx \rightarrow m\ddot{x} + kx = 0$$

$$\text{SHM} \rightarrow x = A\sin\omega_n t + B\cos\omega_n t$$

$$\omega_n^2 = k/m$$

Initial conditions $t=0$ $\left\{ x = -X_m \quad \dot{x} = 0 \right. \Rightarrow x = -X_m \cos\omega_n t$

Add damping: (Figs 13.7, 13.9) $m\ddot{x} + \eta\dot{x} + kx = 0$

$$x = e^{\lambda t} \Rightarrow \lambda = \frac{-\eta}{2m} \pm \left[\left(\frac{\eta}{2m}\right)^2 - \left(\frac{k}{m}\right)\right]^{1/2}$$

Case 1. Critical damping $k/m = (\eta/2m)^2 \Rightarrow x = X_m(1 - e^{-\alpha t}(1+\alpha t))$

Case 2. Overdamped $(\eta/2m)^2 > k/m \Rightarrow x = X_m\left[1 - e^{-\alpha t}\left\{\frac{\alpha}{\beta}\sinh\beta t + \cosh\beta t\right\}\right]$

Case 3. Underdamped $(\eta/2m)^2 < k/m \Rightarrow x = X_m\left[1 - e^{-\alpha t}\left\{\frac{\alpha}{\beta}\sin\beta t + \cos\beta t\right\}\right]$

$\alpha = \eta/2m \qquad \beta^2 = (\eta/2m)^2 - (k/m)$

Concerned here with UNDERDAMPED system $\Rightarrow$ failures

Introduce critical damping $= \eta_c = (4mk)^{1/2} = 2m\omega_n$

& damping ratio $= d = \eta/\eta_c$

$\therefore$ Rewrite $x = X_m\left\{1 - e^{-\alpha t}\left[\frac{\alpha}{\beta}\sin\beta t + \cos\beta t\right]\right\}$

$= e^{-d\omega_n t}\left[A\sin\omega_n\sqrt{1-d^2}\,t + B\cos\omega_n\sqrt{1-d^2}\,t\right]$

$\underset{t=0}{\Rightarrow}$ $x = -X_m$, $\dot{x} = 0$ $\Rightarrow$ $A = 0$, $B = -X_m$ $\Rightarrow x = -\underbrace{X_m e^{-d\omega_n t}}_{\text{damping term}} \underbrace{\cos\omega_n\sqrt{1-d^2}\,t}_{\omega_{nd} = (1-d^2)^{1/2}\omega_n < \omega_n}$

Forced Vibrations

$$m\ddot{x} + \eta\dot{x} + kx = P\cos\omega t$$

Particular solution $x_p = A\cos\omega t + B\sin\omega t$ describes steady state motion due to external force

Substitute $\dot{x}_p = (-A\sin\omega_p t + B\cos\omega_p t)\omega_p$

$$\ddot{x}_p = (A\cos\omega_p t \mp B\sin\omega_p t)\omega_p^2$$

$$m\omega_p^2(A\cos\omega_p t - B\sin\omega_p t) + \eta\omega_p(B\cos\omega_p t - A\sin\omega_p t) + k(A\cos\omega_p t + B\sin\omega_p t) = P\cos\omega_p t.$$

$$\Rightarrow A = \frac{P[k - m\omega^2]}{(\eta\omega)^2 + (k - m\omega^2)^2} \qquad B = \frac{P\eta\omega}{(\eta\omega)^2 + (k - m\omega^2)^2}$$

$$\Rightarrow x = \frac{P\cos(\omega t - \phi)}{[(k - m\omega^2)^2 + (\eta\omega)^2]^{1/2}}, \quad \phi = \tan^{-1}\frac{\eta\omega}{k - m\omega^2}$$

$$= \frac{(P/k)\cos(\omega t - \phi)}{[(1 - r^2)^2 + (2dr)^2]^{1/2}}, \quad r = \omega/\omega_n$$

Consider static deflection for static force P, $x = P/k$

$\therefore$ Dynamic amplitude ratio $A = \dfrac{\text{Amplitude dynamic}}{\text{Amplitude static}} = [(1-r^2)^2 + (2dr)^2]^{-1/2}$

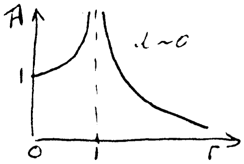
1. At natural resonant frequency ω (driving freq) $= \omega_n$ (natural reson) $r = 1$

$\therefore A = 1/2d$

ie. dynamic ratio at $\omega \sim \omega_n$ can be $>> 1$ if d small.

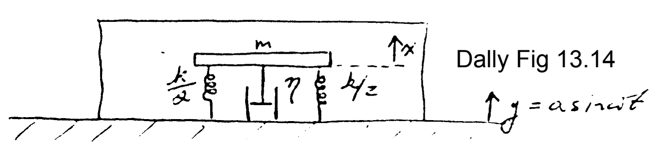
ie. dynamic amplitude $>>$ static amplitude

2. if damping $d \sim 0$, $A \sim 1/(1-r^2)$



Need to make sure $\omega_n > 2\omega$

or isolate



Dally Fig 13.14

ISOLATION: $m\ddot{x} + \eta(\dot{x}-\dot{y}) + k(x-y) = 0$

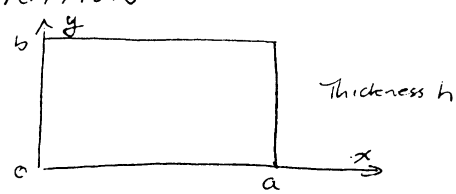
& for $y = y_0 e^{j\omega t}$, $x = x_\omega e^{j(\omega t - \phi)}$

$$\frac{x_\omega}{y_0} = \frac{(k+j\omega\eta)e^{j\phi}}{(k-m\omega^2)+j\omega\eta}$$

CIRCUIT BOARD VIBRATION

Vibrational modes.

All edges: clamped or unclamped.



$$\omega_n^2 = \left(\frac{\pi}{a}\right)^4 \frac{D}{\rho}\frac{C_1}{C_2}$$

ρ = mass/unit area

D = flexural rigidity $= \frac{Eh^3}{12(1-\nu^2)}$

e.g. clamp at $x = 0, a$ Deflection $W = W_o \cdot \sin(\pi\frac{x}{a}) \cdot \cos\omega t$

W_{max} (at $x = a/2$) $= W_{mn} \cos\omega_n t$ at $\omega = \omega_n$ for max

$\therefore$ Max out of plane deflection $\ddot{W}_{max} = -\omega_n^2 W_{mn} \cos\omega_n t$

VIBRATION STRESS

$W = W_0 \sin \pi \frac{x}{a} \rightarrow \max a\ x = a/2$

Moment/unit width $M = -D\frac{d^2W}{dx^2} = DW_0(\pi/a)^2 \sin(\pi x/a)$

& $M_{max} = DW_0(\pi/a)^2$

Bending moment → flexural stress $\sigma_b = 6KM/h^2$

K = stress concentration factor due to geometry, etc

Typ max $K \sim 3$ at plated holes.

For $K=3$ $\sigma_{max} = 18 D W_0 (\pi/ah)^2$

COPPER TRACE STRESS

x direction strain $\epsilon_x = \frac{\sigma_x - \nu\sigma_y}{E}$

$\rightarrow \frac{(1-\nu^2)\sigma_x}{E}$ for $\epsilon_y = 0$ so $\sigma_y = \nu\sigma_x$

Assume circuit board strain transmitted to Cu trace

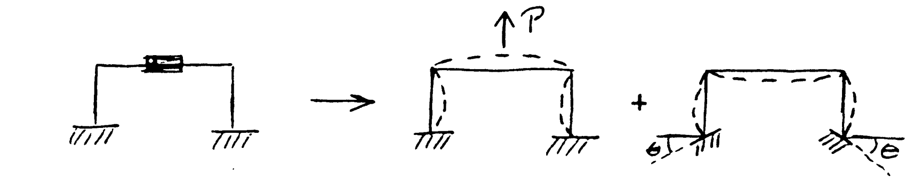
$\epsilon_{Cu} = \epsilon_{pwb}$

$\sigma_{Cu} = E_{Cu}\epsilon_{Cu} = \frac{E_{Cu}}{E_{PWB}}(1-\nu_{PWB}^2)\,\sigma_{xPWB}$

Dally Fig 13.23

LEAD FAILURES

Wire frame models



Fracture at solder joint
" " lead entry to package
" " bend.

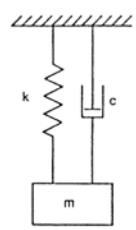


Figure 9.9 Spring-mass-damper single-degree-of-freedom system.

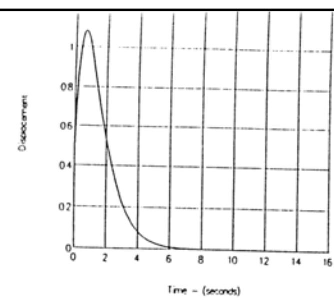


Figure 9.12 Free vibrational response for a critically damped system.

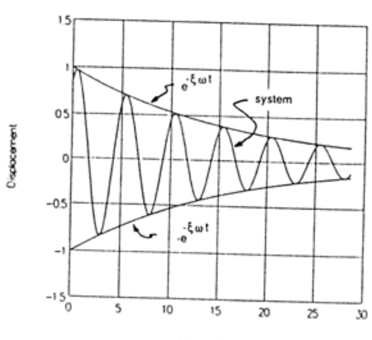


Figure 9.11 Free vibrational response for an underdamped system.

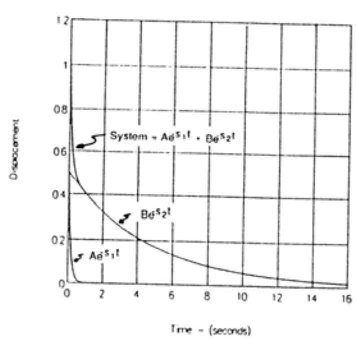


Figure 9.13 Free vibrational response for an overdamped system (two decaying exponentials).

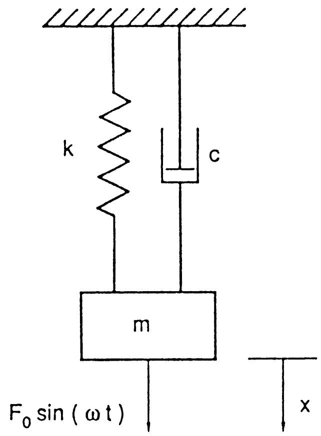


Figure 9.14 Simple single-degree-of-freedom system with forced excitation.

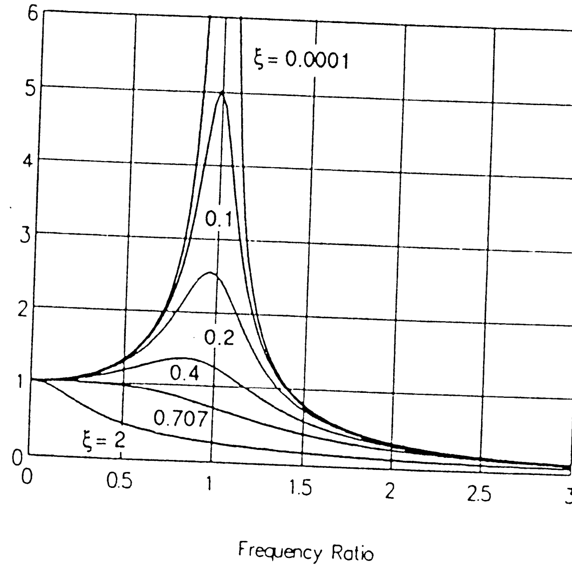


Figure 9.15 Dynamic magnification factor as a function of frequency ratio and damping factor.

Design for Vibration and Shock

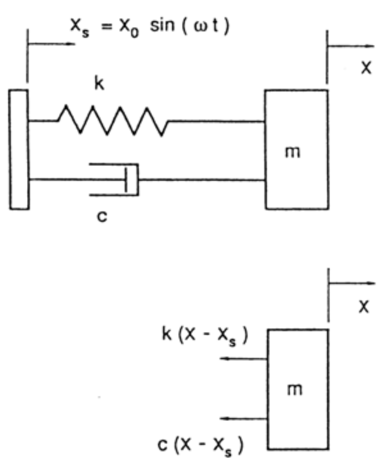


Figure 9.17 Simple system in which support has some motion.

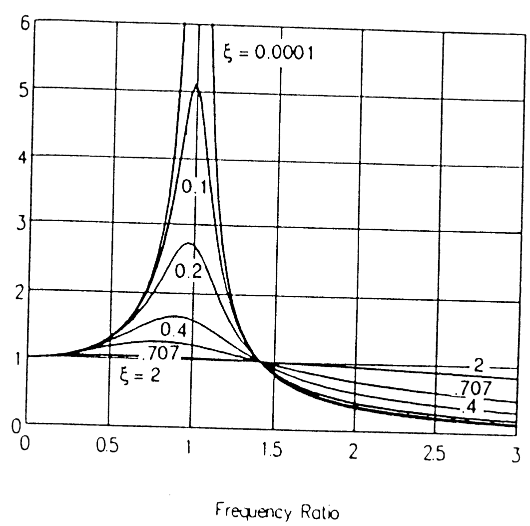


Figure 9.18 Transmissibility as a function of frequency ratio and damping factor.

Dally Table 13.3

Parameters c_1 and c_2 for rectangular plates with different boundary conditions used to compute the natural frequency $\omega_n^2 = (\pi^4 D c_1)/(a^4 \rho c_2)$

Boundary conditions	Deflection function or mode shape	c_2	c_1
1.	$\left(\cos\frac{2\pi x}{a} - 1\right)\left(\cos\frac{2\pi y}{b} - 1\right)$	2.25	$12 + 8\left(\frac{a}{b}\right)^2 + 12\left(\frac{a}{b}\right)^4$
2.	$\left(\cos\frac{3\pi x}{2a} - \cos\frac{\pi x}{2a}\right)\left(\cos\frac{2\pi y}{b} - 1\right)$	1.50	$3.85 + 5\left(\frac{a}{b}\right)^2 + 8\left(\frac{a}{b}\right)^4$
3.	$\left(1 - \cos\frac{\pi x}{2a}\right)\left(\cos\frac{2\pi y}{b} - 1\right)$	0.340	$0.0468 + 0.340\left(\frac{a}{b}\right)^2 + 1.814\left(\frac{a}{b}\right)^4$
4.	$\left(\cos\frac{2\pi x}{a} - 1\right)\sin\frac{\pi y}{b}$	0.75	$4 + 2\left(\frac{a}{b}\right)^2 + 0.75\left(\frac{a}{b}\right)^4$
5.	$\left(\cos\frac{2\pi x}{a} - 1\right)\frac{y}{b}$	0.50	$2.67 + 0.304\left(\frac{a}{b}\right)^2$
6.	$\cos\frac{2\pi x}{a} - 1$	1.50	8
7.	$\left(\cos\frac{3\pi x}{2a} - \cos\frac{\pi x}{2a}\right)\left(\cos\frac{3\pi y}{2b} - \cos\frac{\pi y}{2b}\right)$	1.00	$2.56 + 3.12\left(\frac{a}{b}\right)^2 + 2.56\left(\frac{a}{b}\right)^4$
8.	$\left(\cos\frac{3\pi x}{2a} - \cos\frac{\pi x}{2a}\right)\left(1 - \cos\frac{\pi y}{2b}\right)$	0.227	$0.581 + 0.213\left(\frac{a}{b}\right)^2 + 0.031\left(\frac{a}{b}\right)^4$
9.	$\left(1 - \cos\frac{\pi x}{2a}\right)\left(1 - \cos\frac{\pi y}{2b}\right)$	0.0514	$0.0071 + 0.024\left(\frac{a}{b}\right)^2 + 0.0071\left(\frac{a}{b}\right)^4$
10.	$\left(\cos\frac{3\pi x}{2a} - \cos\frac{\pi x}{2a}\right)\sin\frac{\pi y}{b}$	0.50	$1.28 + 1.25\left(\frac{a}{b}\right)^2 + 0.50\left(\frac{a}{b}\right)^4$
11.	$\left(\cos\frac{3\pi x}{2a} - \cos\frac{\pi x}{2a}\right)\frac{y}{b}$	0.333	$0.853 + 0.190\left(\frac{a}{b}\right)^2$
12.	$\cos\frac{3\pi x}{2a} - \cos\frac{\pi x}{2a}$	1.00	2.56
13.	$\left(1 - \cos\frac{\pi x}{2a}\right)\frac{\pi^2}{b^2}\sin\frac{\pi y}{b}$	0.1134	$0.0156 + 0.0852\left(\frac{a}{b}\right)^2 + 0.1134\left(\frac{a}{b}\right)^4$
14.	$\left(1 - \cos\frac{\pi x}{2a}\right)\frac{y}{b}$	0.0756	$0.0104 + 0.0190\left(\frac{a}{b}\right)^2$
15.	$1 - \cos\frac{\pi x}{2a}$	0.2268	0.0313
16.	$\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$	0.25	$0.25 + 0.50\left(\frac{a}{b}\right)^2 + 0.25\left(\frac{a}{b}\right)^4$
17.	$\left(\sin\frac{\pi x}{a}\right)\frac{y}{b}$	0.1667	$0.1667 + 0.0760\left(\frac{a}{b}\right)^2$
18.	$\sin\frac{\pi x}{a}$	0.50	0.50

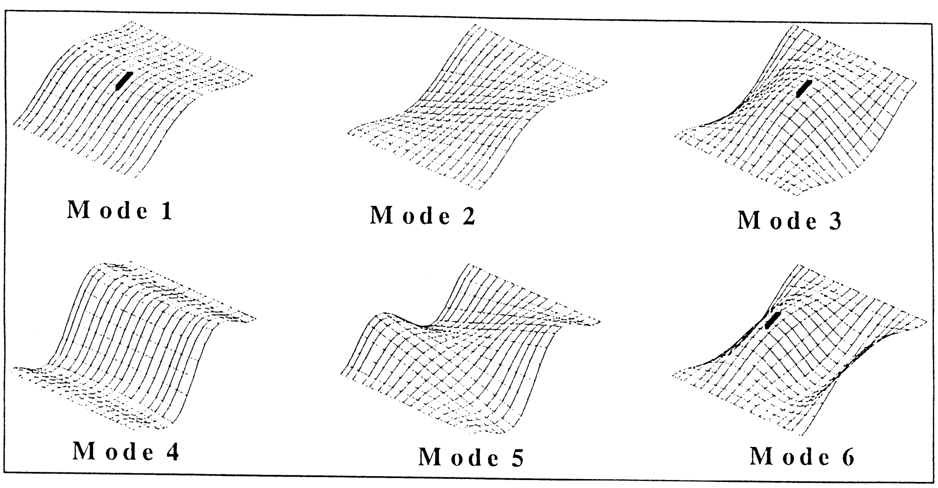


Figure 5 Modal Analysis

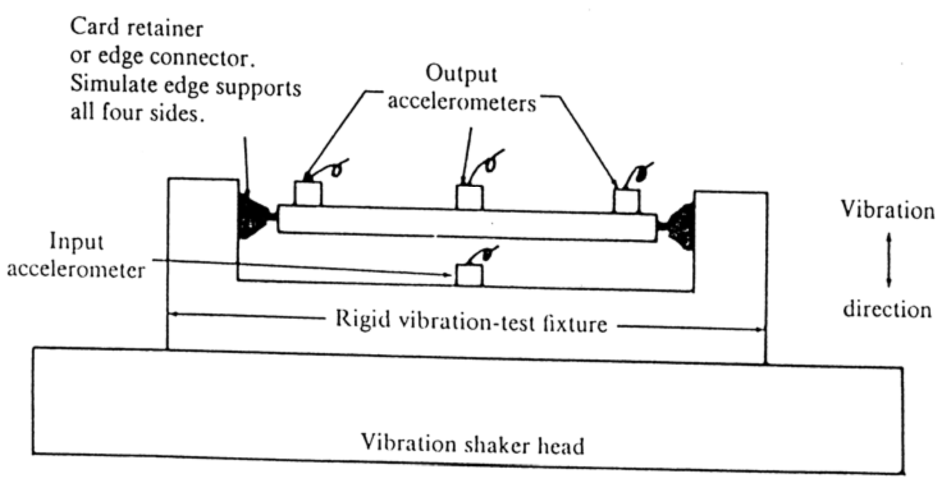
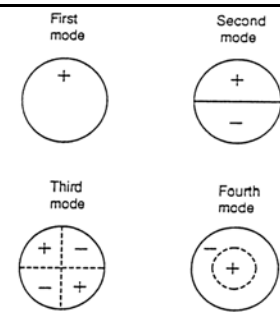
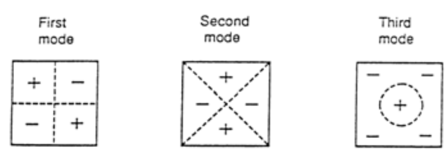


FIGURE 10.21
Vibration test arrangement for printed circuit boards.

See also Dally Fig 13.17



First four harmonic modes for a circular membrane



First three harmonic modes of a square plate

Figure 9.8 First four harmonic modes of a circular plate and a square plate.

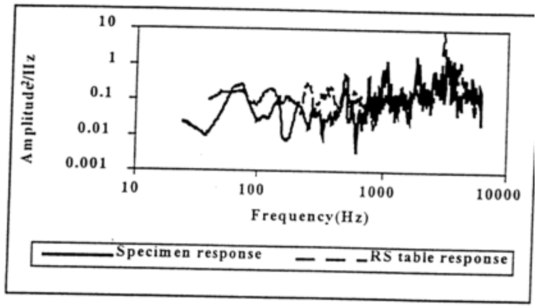
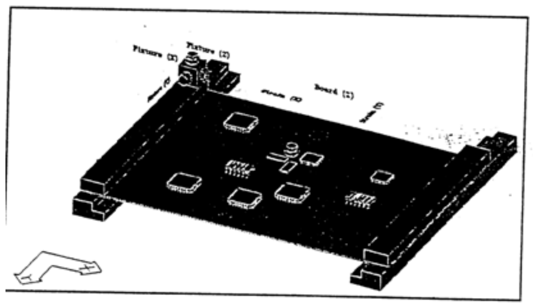
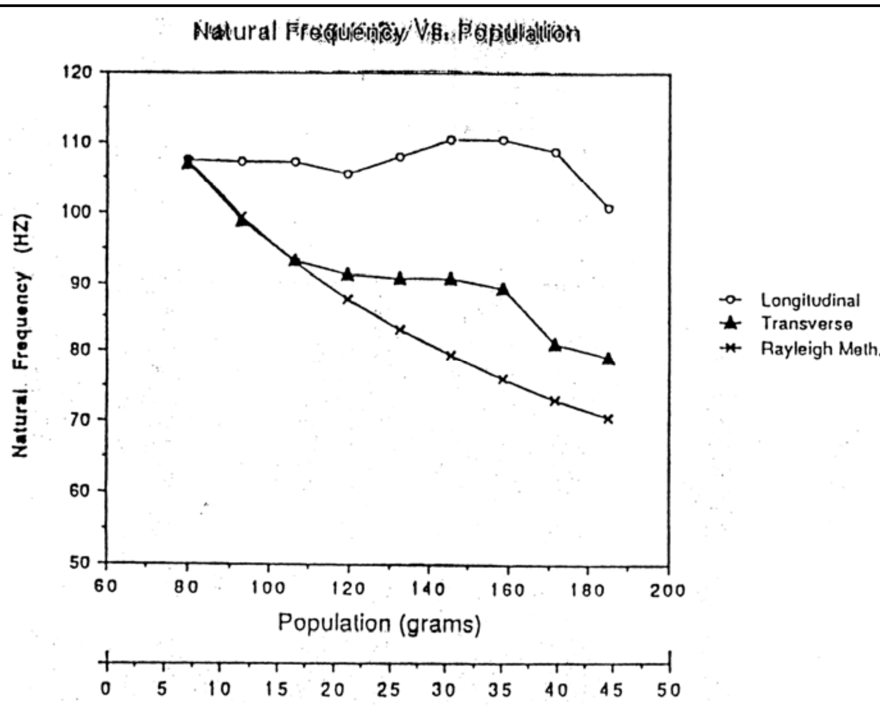


Figure 3. Typical Specimen and RS table Response Spectrum



11/9/05 47

Figure 4. Schematic of Vibration Instrumentation and Mounting Fixtures



Assignment #4

A. (1) Calculate the crosstalk signals in a quiet transmission line adjacent to an identical active line with a 1volt ramp step input of 1ps risetime on a 10cm active transmission line with inductance 15nH/m and capacitance 10pF/m, with both terminated in Z_0 at both source and load. Assume mutual inductance and capacitance 10nH/m and 5pF/m. (2) Use any version of SPICE (or equivalent) to simulate this system, and comment on your results.

B. Dally et al, Problems:
11.16, 11.19, 11.20, 11.21, 11.24
13.24, 13.26, 13.36

Grad Projects

- Sigrity (or other) EM modeling (demonstrate applications)
- Through silicon vias (Cu TSVs)
- Embedded components (R, L, & C)
- Compare reliability of no-Pb solder vs Sn-Pb
- Tin whiskers (cause & cure, reliability issues)
- Inkjet printed conductors (e.g. for flex electronics)
- Surface activated bonding (SAB)
- Life cycle modeling for the environment
- Drop test relationship to vibration testing
- Optoelectronic packaging
- MEMS packaging
- Biomedical electronics packaging
- High power LED packaging