ECE414/514
Electronics Packaging
Spring 2012 Lecture 4
Electrical B: Transmission lines
(Z_0 , velocity, lossless/lossy lines)

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Interconnect modeling

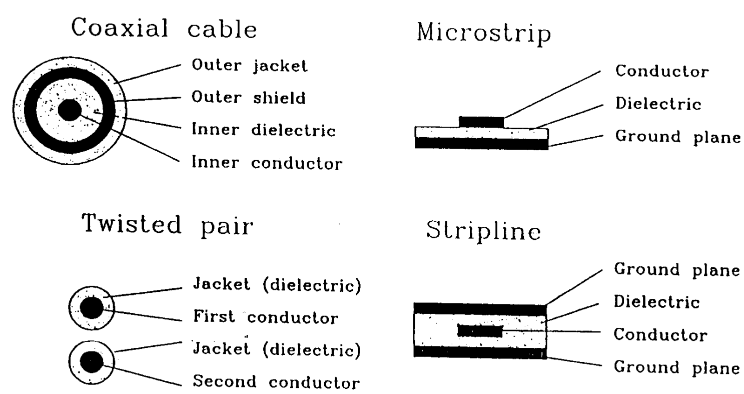


Figure 4.4 Cross sections of popular transmission line geometries.

4.2.1 Ideal Distortionless, Lossless Transmission Line

Transmission line effects

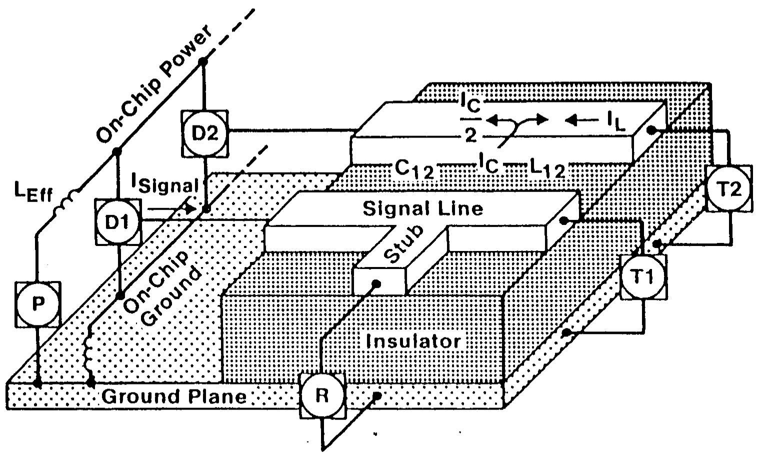
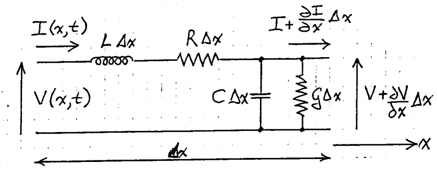


Figure 1-12. Causes of Reflected, Coupled, and Switching (ΔI) Noises.

Objectives

- Develop the concepts of:
  - Characteristic impedance Z_0
  - Pulse velocity and line delay
  - Lossless line
    - Distortionless transmission
    - Shape factor
  - Effects of loss
    - Distortion
  - PWB routing

1. Transmission line theory



$$V - \left(V + \frac{\partial V}{\partial x}\Delta x\right) = R\,\Delta x\, I + L\,\Delta x . \frac{\partial I}{\partial t}$$

ie. $$-\frac{\partial V}{\partial x} = R I + L \frac{\partial I}{\partial t}$$

Assume $$V(x,t) = V(x)\, e^{j\omega t}$$
$$I(x,t) = I(x)\, e^{j\omega t}$$

then $$-\frac{dV(x)}{dx} = (R + j\omega L) I(x)$$

And similarly
$$I - \left(I + \frac{\partial I}{\partial x}\Delta x\right) = G\,\Delta x\, V + C\,\Delta x \frac{\partial V}{\partial t}$$

gives $$-\frac{dI(x)}{dx} = (G + j\omega C) V(x)$$

These give
$$\frac{d^2V(x)}{dx^2} = (R+j\omega L)(G+j\omega C)V(x)$$
& $$\frac{d^2I(x)}{dx^2} = (G+j\omega C)(R+j\omega L)I(x)$$

2nd order differential equation has solution:
$$V(x) = A\,e^{-\lambda x} + B\,e^{+\lambda x}$$
where $$\lambda = [(R+j\omega L)(G+j\omega C)]^{1/2}$$
ie. $$V(x,t) = V(x)\,e^{j\omega t} = \underbrace{A\,e^{j\omega t-\lambda x}}_{\text{Forward wave (damped }\lambda)} + \underbrace{B e^{j\omega t+\lambda x}}_{\text{Reverse wave (damped }\lambda)}$$

Similarly $$I(x) = -(R+j\omega L)^{-1}\frac{dV(x)}{dx}$$
$$= -\frac{[-A e^{-\lambda x} + B e^{+\lambda x}]\lambda}{(R+j\omega L)}$$
$$= \frac{A\,e^{-\lambda x} - B\,e^{+\lambda x}}{(R+j\omega L)^{1/2}}(G+j\omega C)^{1/2}$$
$$= \frac{A\,e^{-\lambda x} - B e^{+\lambda x}}{Z_0}$$

where the Characteristic Impedance is
$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Note: units Ω

Attenuation, dispersion, velocity :-
$$\lambda = [(R+j\omega L)(G+j\omega C)]^{1/2}$$
Write $\lambda = \alpha + j\beta$

Real part α is attenuation
Imag. part β is dispersion

$$\lambda^2 = \alpha^2 - \beta^2 + 2j\beta\alpha = (RG - \omega^2 LC) + j\omega(RC+GL)$$
$$\therefore\ \alpha^2 - \beta^2 = RG - \omega^2 LC$$
$$2\alpha\beta = \omega(RC+GL)$$

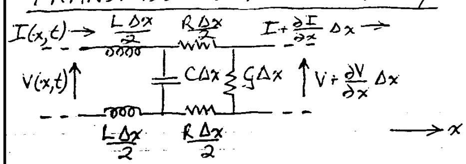
Substitute for $\beta = \frac{1}{2}\frac{\omega}{\alpha}(RC+GL)$
$$\alpha^2 - \frac{\omega^2}{4\alpha^2}(RC+GL)^2 = (RG-\omega^2 LC)$$
$$(\alpha^2)^2 - (\alpha^2)(RG-\omega^2LC) - \frac{\omega^2}{4}(RC+GL)^2 = 0$$
$$\alpha^2 = \frac{1}{2}\left\{(RG-\omega^2LC) \pm [(RG-\omega^2LC)^2 + \omega^2(RC+GL)^2]^{1/2}\right\}$$
& $$\beta^2 = \alpha^2 - (RG-\omega^2LC)$$
$$= \frac{1}{2}\left\{-(RG-\omega^2LC) \pm [(RG-\omega^2LC)^2+\omega^2(RC+GL)^2]^{1/2}\right\}$$

By comparison with standard wave equations:
$$V(x,t) = A e^{j\omega t - \lambda x} = A e^{j\omega t} e^{-(\alpha+j\beta)x}$$
$$= A e^{-\alpha x} e^{j\omega(t - \frac{\beta}{\omega}x)}$$

i.e. velocity $v = \omega/\beta$

2. Generalized theory summary

TRANSMISSION LINE THEORY



$$V(x,t) = \underbrace{A e^{j\omega t - \lambda x}}_{\text{Forward wave}\rightarrow} + \underbrace{B e^{j\omega t + \lambda x}}_{\leftarrow\text{Reverse wave}}$$

$$I(x,t) = \frac{A}{Z_o} e^{j\omega t - \lambda x} - \frac{B}{Z_o} e^{j\omega t + \lambda x}$$

where $Z_o = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$ (units Ω)

Characteristic Impedance

& $\lambda = \sqrt{(R + j\omega L)(G + j\omega C)}$

$= \underset{\text{Attenuation}}{\alpha} + j\underset{\text{Dispersion}}{\beta}$

Wave velocity $v = \omega/\beta$

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3. Lossless lines

In practice $G \ll \omega C$ in practical systems

i.e. set $G \approx 0$

$$Z_o \Rightarrow \sqrt{\frac{L}{C} + \left(\frac{R}{j\omega C}\right)}$$

$$\alpha^2 \Rightarrow \frac{1}{2}\omega^2 LC\left\{\left[1 + \left(\frac{R}{\omega L}\right)^2\right]^{1/2} - 1\right\}$$

$$\beta^2 \Rightarrow \frac{1}{2}\omega^2 LC\left\{\left[1 + \left(\frac{R}{\omega L}\right)^2\right]^{1/2} + 1\right\}$$

The usual approximation (not as good, but still normally valid) is that $R \ll \omega L$

$Z_o \Rightarrow \sqrt{L/C}$ Note: "Real" but not dissipative

$\alpha \Rightarrow 0$ i.e. lossless

$\beta \Rightarrow [\omega^2 LC]^{1/2} = \omega\sqrt{LC}$

& velocity $v \Rightarrow \dfrac{\omega}{\omega\sqrt{LC}} = \dfrac{1}{\sqrt{LC}}$

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4. Z_0 calculations

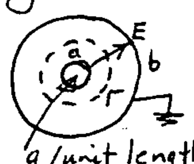
Need L, C

Also gives β, v

Easiest geometry is coaxial line
(radial symmetry)

(a) For dielectric: C/unit length

GAUSS'S LAW $\epsilon \underset{\sim}{E} \cdot \underset{\sim}{dS} = \sum q_i$



q/unit length
V_0

$$\epsilon E \; 2\pi r(\ell) = q(\ell)$$

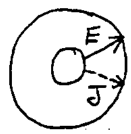
$$\therefore \; E = \frac{q}{2\pi\epsilon}\cdot\frac{1}{r}$$

$$\therefore \; V_0 = -\int_b^a E \cdot dr = \frac{q}{2\pi\epsilon}\ln\frac{b}{a}$$

Capacitance/unit length $C = q/V_0 = 2\pi\epsilon/\ln\left(\frac{b}{a}\right)$

(b) For leakage conductance: G/unit length

(b) FOR LEAKAGE CONDUCTANCE: G /unit length



Current density $\underset{\sim}{J} = \sigma \underset{\sim}{E}$

& using E, V_0 from above $E = \frac{1}{r}\frac{V_0}{\ln(b/a)}$

Conductance/unit length $G = I/V_0$

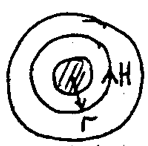
$$= \frac{J(r)\, 2\pi r \times \ell}{V_0} / \ell$$
$$= \frac{\sigma}{V_0}\, 2\pi r\, \frac{1}{r}\, \frac{V_0}{\ln(b/a)}$$
$$= 2\pi\sigma / \ln\left(\frac{b}{a}\right)$$

Note: for $G \ll \omega C$ need $\sigma \ll \omega\epsilon$

or $\omega \gg \sigma/\epsilon$

(c) For inductance: L/unit length

(c) FOR INDUCTANCE: L /unit length



$\underset{\sim}{B} = \mu \underset{\sim}{H} = \mu \frac{I}{2\pi r}$ $\qquad I = \oint \underset{\sim}{H} \cdot d\underset{\sim}{\ell}$

$\qquad \phi = \int \underset{\sim}{B} \cdot d\underset{\sim}{s}$

Magnetic flux $\phi = \int_a^b B(dr \times \ell) = \ell \frac{\mu}{2\pi} I \ln(b/a)$

$$L = \frac{\phi/\ell}{I} = \frac{\mu}{2\pi} \ln(b/a)$$

/unit length.

Coaxial line summary (lossless)

∴ Characteristic impedance

$$Z_0 = \sqrt{L/C} = \left\{ \frac{\mu}{2\pi} \ln(b/a) \left[2\pi\epsilon / \ln(b/a) \right]^{-1} \right\}^{1/2}$$

$$= \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\ln(b/a)}{2\pi} = \sqrt{\frac{\mu}{\epsilon}} \times \text{shape factor}$$

Velocity

$$v = (LC)^{-1/2} = \left(\frac{2\pi \ln(b/a)}{\mu \ln(b/a) 2\pi\epsilon} \right)^{1/2}$$

$$= 1/\sqrt{\mu\epsilon}$$

Note: independent of geometry

& for $\mu \approx \mu_0$, $\epsilon = \epsilon_r \epsilon_0$

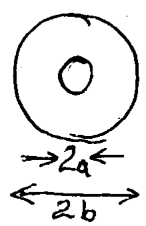
$$\Rightarrow \frac{(\mu_0 \epsilon_0)^{-1/2}}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$\text{Dispersion} = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon}$$

$$\Rightarrow \sqrt{\epsilon_r} \cdot \frac{\omega}{c} \quad \text{radians/meter}$$

$$Z_0 \Rightarrow 376.7 \left(\text{s.f.}/\sqrt{\epsilon_r} \right) \Omega$$

e.g. Coaxial line



$$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \text{ H/m}$$

$$C = 2\pi\epsilon / \ln(b/a) \text{ F/m}$$

$$\therefore Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{\ln(b/a)}{2\pi}$$

$$= \sqrt{\frac{\mu}{\epsilon}} \times \text{Shape factor}$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0}} \times \frac{\text{Shape factor}}{\sqrt{\epsilon_r}}$$

$$= 376.7\,\Omega \left(\text{S.F.}/\epsilon_r^{1/2} \right)$$

$$v = 1/\sqrt{\mu\epsilon}$$

$$\approx \frac{(\mu_0 \epsilon_0)^{-1/2}}{\sqrt{\epsilon_r}}$$

$$= \frac{c}{\sqrt{\epsilon_r}}$$

TABLE 5.2 CONDUCTANCE, CAPACITANCE, AND INDUCTANCE PER UNIT LENGTH FOR SOME STRUCTURES CONSISTING OF INFINITELY LONG CONDUCTORS HAVING THE CROSS SECTIONS SHOWN IN FIG. 5.12

Description	Capacitance per unit length, $\mathscr{C}$	Conductance per unit length, $\mathscr{G}$	Inductance per unit length, $\mathscr{L}$
Parallel-plane conductors, Fig. 5.12(a)	$\varepsilon \frac{w}{d}$	$\sigma \frac{w}{d}$	$\mu \frac{d}{w}$
Coaxial cylindrical conductors, Fig. 5.12(b)	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$
Parallel cylindrical wires, Fig. 5.12(c)	$\frac{\pi\varepsilon}{\cosh^{-1}(d/a)}$	$\frac{\pi\sigma}{\cosh^{-1}(d/a)}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{a}$
Eccentric inner conductor, Fig. 5.12(d)	$\frac{2\pi\varepsilon}{\cosh^{-1}\left(\frac{a^2+b^2-d^2}{2ab}\right)}$	$\frac{2\pi\sigma}{\cosh^{-1}\left(\frac{a^2+b^2-d^2}{2ab}\right)}$	$\frac{\mu}{2\pi}\cosh^{-1}\frac{a^2+b^2-d^2}{2ab}$
Shielded parallel cylindrical wires, Fig. 5.12(e)	$\frac{\pi\varepsilon}{\ln\frac{d(b^2-d^2/4)}{a(b^2+d^2/4)}}$	$\frac{\pi\sigma}{\ln\frac{d(b^2-d^2/4)}{a(b^2+d^2/4)}}$	$\frac{\mu}{\pi}\ln\frac{d(b^2-d^2/4)}{a(b^2+d^2/4)}$

5. Dielectric effects

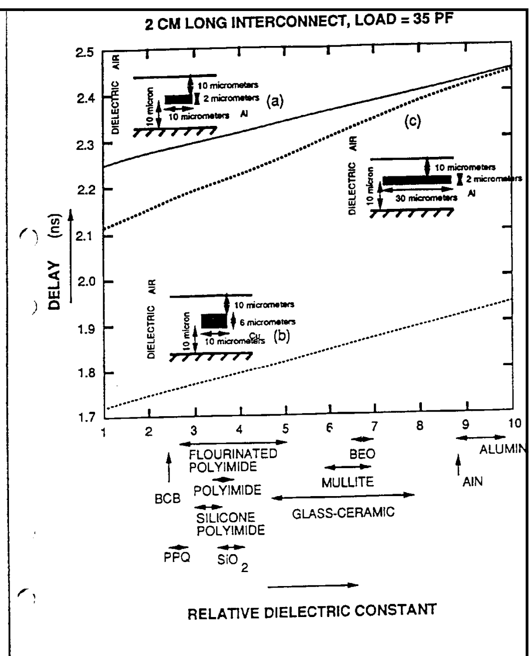
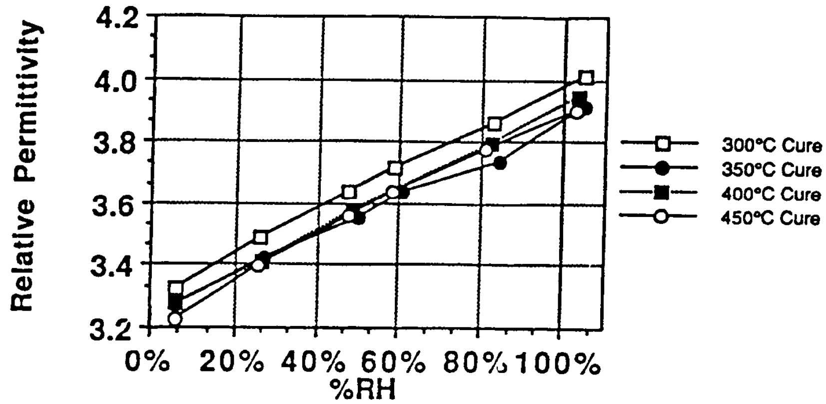


Figure 7.8 Materials and cross section effects on the delay of a l loaded MCM interconnect. (Source : [9])

Humidity affects performance



Maximum velocity in air

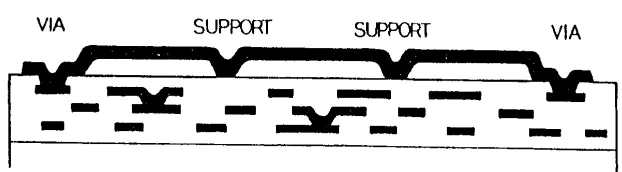


FIGURE 8.32 Air bridges for high-speed interconnections. Periodic supports enhance the strength of the interconnections.

For $Z_0 = [376.7/\sqrt{\epsilon_r'}] \times s.f.\ \Omega$

6. Z_0

(1)

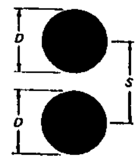


Figure E-2 Cross section of two-wire transmission line

The shape factor of the transmission line, *sf*, depends only on the shapes, sizes, and positions of the conductors. Two-wire transmission line (Figure E-2) has shape factor

$$sf = \frac{1}{\pi} \ln \left[\frac{S}{D} + \left(\frac{S^2}{D^2} - 1 \right)^{1/2} \right]$$

when both wires have diameter D. When the wires have diameters $D1$ and $D2$, the shape factor becomes

$$sf = \frac{1}{2\pi} \ln [x + (x^2 - 1)^{1/2}]$$

where

$$x = \frac{4S^2 - D1^2 - D2^2}{2D1D2}$$

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(2) Twisting a two-wire transmission line to form a twisted-pair increases the average relative permittivity $\epsilon_{r'}$. For hard insulation with relative permittivity ϵ_r,

$$\epsilon_{r'} \approx 1 + (0.25 + 0.0004\Theta^2)(\epsilon_r - 1),$$

and for soft insulation (Teflon, polyvinyl chloride)

$$\epsilon_{r'} \approx 1 + (0.25 + 0.001\Theta^2)(\epsilon_r - 1),$$
$$\Theta = \arctan(\pi S n) \text{ degrees},$$

and n is the number of twists per meter.

(3) A wire near a ground plane (Figure E-3) has shape factor

$$sf = \frac{1}{2\pi} \ln \left[\frac{2S}{D} + \left(\frac{4S^2}{D^2} + 1 \right)^{1/2} \right].$$

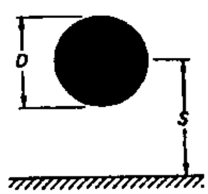


Figure E-3 Cross section of wire near a groundplane

(4) A laminar bus (Figure E-4) has shape factor

$$sf \approx \frac{1}{\pi} \ln \left(\frac{4S}{W} + \frac{W}{2S} \right) \quad \text{for } W \leq \frac{S}{2},\ T << W,$$

$$sf \approx \frac{2}{\frac{2W}{S} + 2.42 - \frac{0.22S}{W} + \left(1 - \frac{S}{2W}\right)^6} \quad \text{for } W > \frac{S}{2},\ T << W.$$

If an insulator separates the conductors (Figure E-1(c)),

$$\epsilon_{r'} \approx \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\left(4 + 20\frac{S}{W}\right)^{1/2}}.$$

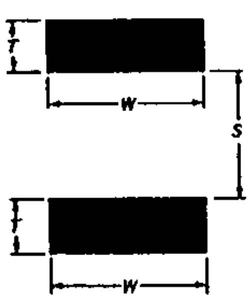


Figure E-4 Cross section of a laminar bus

Fig E5
Cross-section of
a microstrip line

(5) Microstrip (Figure E-5) has shape factor

$$sf \approx \frac{1}{2\pi} \ln\left(\frac{8S}{W} + \frac{W}{4S}\right) \qquad \text{for } W \leq S,\ T << W,$$

$$sf \approx \frac{1}{\dfrac{W}{S} + 2.42 - \dfrac{0.44S}{W} + \left(1 - \dfrac{S}{W}\right)^6} \qquad \text{for } W > S,\ T <<$$

If an insulator separates the conductors (Figure E-1(c)),

$$\epsilon_{r'} \approx \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\left(4 + 40\dfrac{S}{W}\right)^{1/2}}.$$

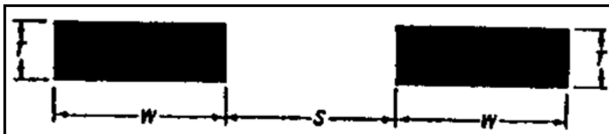


Figure E-6 Cross section of coplanar lines

(6) Coplanar lines (Figure E-6) have shape factor

$$sf \approx \frac{1}{\pi} \ln\left[2\frac{\left(\dfrac{2W}{S} + 1\right)^{1/2} + 1}{\left(\dfrac{2W}{S} + 1\right)^{1/2} - 1}\right] \qquad \text{for } W \leq 2.414S,\ T <<$$

$$sf \approx \frac{\pi}{4 \ln\left[2\left(\dfrac{2W}{S} + 1\right)^{1/2}\right]} \qquad \text{for } W > 2.414S,\ T << W.$$

If the lines are supported by an insulator on one side,

$$1 < \epsilon_{r'} < \frac{\epsilon_r + 1}{2}.$$

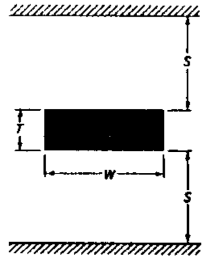


Figure E-7 Cross section of triplate stripline

(7) Triplate stripline (Figure E-7) has shape factor

$$sf \approx \frac{1}{2\pi} \ln \left[2 \frac{\exp\left(\frac{\pi W}{4S}\right) + 1}{\exp\left(\frac{\pi W}{4S}\right) - 1} \right] \quad \text{for } W \leq 1.117S,\ T << S,$$

$$sf \approx \frac{1}{1.765 + \frac{2W}{S}} \quad \text{for } W > 1.117S,\ T << S.$$

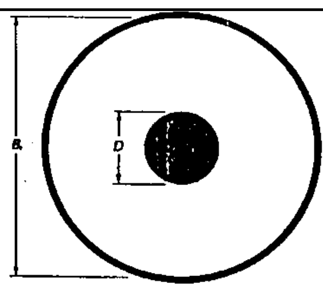


Figure E-8 Cross section of coaxial cable

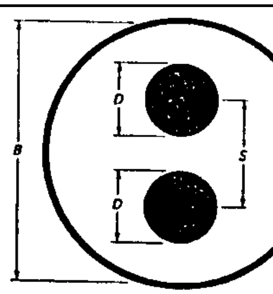


Figure E-9 Cross section of shielded twisted-pair

(8) Coaxial cable (Figure E-8) has shape factor

$$sf = \frac{1}{2\pi} \ln \left(\frac{B}{D}\right).$$

(9) Shielded twisted-pair (Figure E-9) has shape factor

$$sf = \frac{1}{\pi} \ln \left(\frac{2S}{D} \frac{B^2 - S^2}{B^2 + S^2}\right).$$

7. Lossy lines

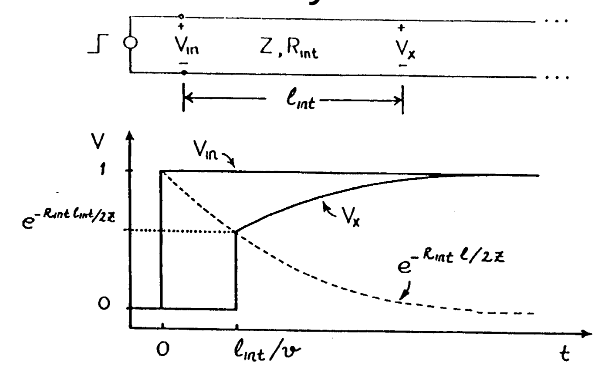
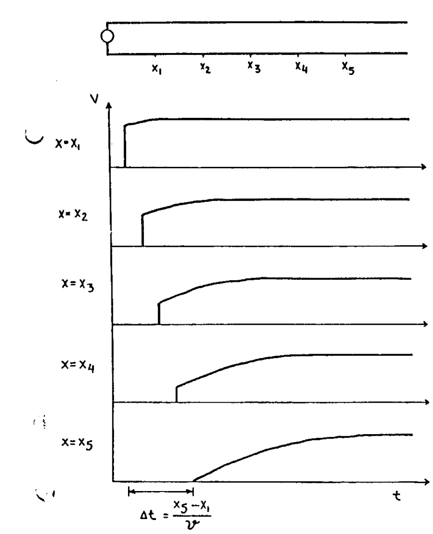


FIGURE 6.10 Waveforms in a lossy transmission line

The response of this line to a unit step input at a distance x from the beginning of the line is [6.1]:

$$V(x,t) = \left\{ e^{-\mathcal{R}x/2Z_0} + \frac{\mathcal{R}x}{2Z_0}\int_{t=x\sqrt{\mathcal{LC}}}^{t} \left[\frac{e^{-\mathcal{R}t/2\mathcal{L}}}{\sqrt{t^2 - x\sqrt{\mathcal{LC}}}} \frac{I_1 \mathcal{R}}{2\mathcal{L}} \sqrt{t^2 - x\sqrt{\mathcal{LC}}} \right] dt \right\}$$

$$\times u\left(t - x\sqrt{\mathcal{LC}}\right).$$



URE 6.11 Waveform propagating along a lossy transmission line. The ıt to the transmission line is a unit voltage step, and the response is captured uccessive points along the line. The size of the step is attenuated as the eform travels down the line, and at a point far enough from the source, the onse is like that of a distributed RC line.

9. PWB Routing

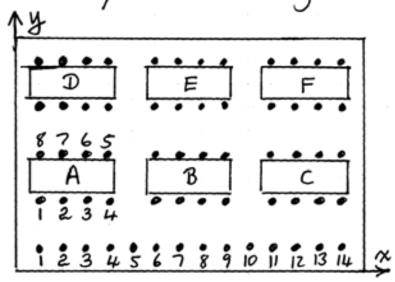
COMPONENT PLACEMENT (ref: Dally)

A. WIRABILITY i.e. minimization of wiring length

Example

6 8-pin packages

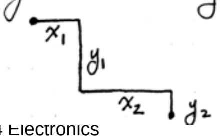
Board edge connections



Number of possible placements of 8 packages $= \lfloor 8 = 40{,}320$

Note: "Manhattan" wiring length $s = x + y$

or $\sum_i x_i + \sum_j y_j$



Step 1 WIRE LIST

Device	Pin	Device	Pin	Device	Pin	Device	Pin
U	1	V	5	X	1	W	6
	2	V	6		2	W	7
	3	Y	5		3	Z	6
	4	EC	1		4	EC	2
	5	Y	6		5	Z	7
	6	W	3		6	Z	1
	7	W	5		7	Z	2
	8	EC	13		8	EC	14
V	1	EC	3	Y	1	EC	8
	2	EC	4		2	EC	9
	3	EC	5		3	EC	10
	4	EC	1		4	EC	2
	5	U	1		5	U	3
	6	U	2		6	U	5
	7	W	1		7	W	2
	8	EC	13		8	EC	14
W	1	V	7	Z	1	X	6
	2	Y	7		2	X	7
	3	U	6		3	Z	5
	4	EC	1		4	EC	2
	5	U	7		5	Z	3
	6	X	1		6	X	3
	7	X	2		7	X	5
	8	EC	13		8	EC	14

Number of EC connections (excl. Power/Ground EC-1, 2, 13, 14)

Device:	U	V	W	X	Y	Z
Number:	0	3	0	0	3	0

Step 1
Wiring algorithm based on "wire list" of electrical interconnections

Notes: (a) U/V/W-4 to EC-1 & -8 to EC-13 } Power & Ground
X/Y/Z-4 to EC-2 & -8 to EC-14

(b) Each connection to device pins appears twice (EC once)

(c) Z pins 3 & 5

(d) These connections set by electronic design

Simplifications here:
(a) EC connections assumed fixed (I/O to board)
(b) Only one connection per device pin
(c) EC connections EC-3,4,5 EC-8,9,10 EC-1,2,13,14
EC-6,7,11,12 N.C.
(d) Ignores possible interchangeable circuits in devices

Devices V & Y have most EC connections ∴ place near EC
V connects to EC-3,4,5 → position A
Y connects to EC-8,9,10 → position B

Step 2 Choose device for position C – adjacent EC & Y
From table → place device U in position C.

Step 3 Distribute W, X, Z between D, E, F → (See connectivity
(1) X & Z adjacent (2) W between U & X) matrix

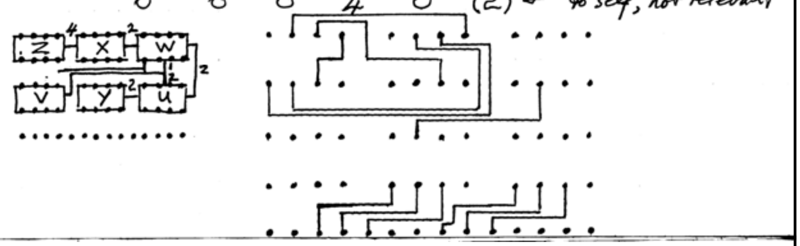
Step 2 Remaining Devices U, W, X, Z

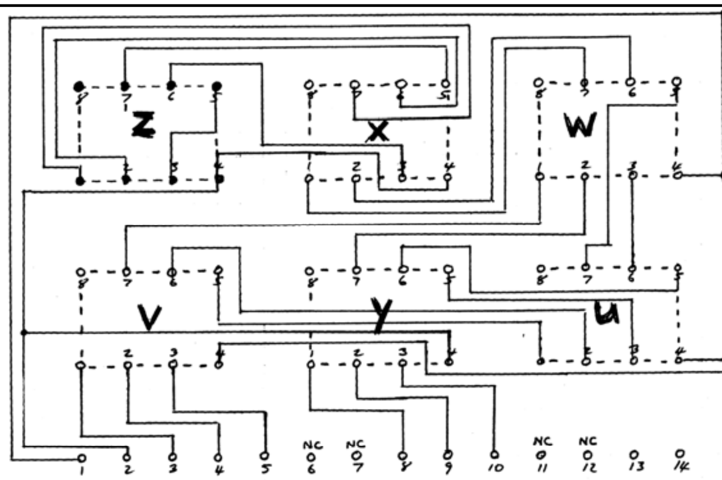
Device:	U	W	X	Z
Connections to EC (excl. Pwr/Gd)	0	0	0	0
& to Device Y (Position B)	2	1	0	0

Step 3 Connectivity matrix: Devices W, X, Z

Devices → ↓	U	V	W	X	Y	Z
W	2	1	–	2	1	0
X	0	0	2	–	0	4
Z	0	0	0	4	0	(2) ← to self, not relevant

Final result





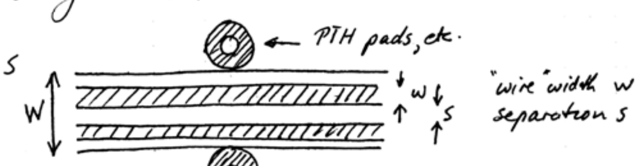
U, V, W - 8 to EC-13
X, Y, Z - 8 to EC-14

Reassign pins 4 between EC-1,2 ?
& pins 8 between EC-13,14

ROUTING

(1) Meet wire list requirements
(2) Topology to minimize crossovers
(3) Avoid long parallel runs → crosstalk
(4) Automatic algorithm + manual

WIRING CHANNELS

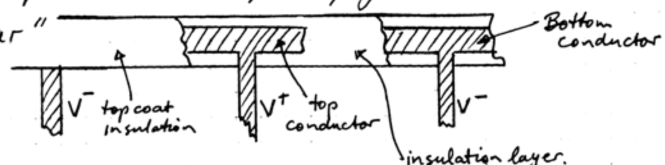


For i conductors: $W = s + i(s+w)$

POWER DISTRIBUTION

(1) Power & ground traces — as for signal lines (independent)
Lower power, low frequency systems.

(2) "Bus bar"



Transmission line structure for low inductance
low ΔI noise

(3) Multilayer ground-plane / power plane (cf single ground plane)
Stripline / microstrip structures for signal lines
→ "controlled" impedance

AUTOMATIC ROUTING

(1) Wire list
(2) Assign traces to specific layers ?
(3) Assign connection sequence
(a) shorter first ?
(b) critical paths first ?
etc.

(4) Lay out traces .

Assignment #2

1. An interconnect of width 10μm and length 100μm is above a ground plane, separated by 3μm thick SiO_2 ($\varepsilon_r = 3.9$.) Calculate the interconnect capacitance (a) ideally, and (b) including fringing effects.
2. A digital signal of risetime 2ns crosses a 1cm long Al interconnect embedded in polyimide ($\varepsilon_r = 3.9$). Should this interconnect be treated as a transmission line? Justify your answer.
3. The skin depths for Ag, Au, and Al are 0.64μm, 0.79μm, and 0.81μm respectively at 10GHz. Which conductor material would you choose for a 10GHz current signal application? Justify your answer.
4. An MCM consists of 10 drivers switching simultaneously. The characteristic impedance of the transmission line is 50Ω with a 50Ω matched load. The power supply is 5V with an effective L = 0.2nH, while the effective ground-plane L = 1nH. Find the magnitude of the delta-I noise in the power and ground planes.