

**ECE414/514**  
**Electronics Packaging**  
**Spring 2012 Lecture 3**  
**Electrical A:**  
**CMOS; R, L, & C;**  
**ground planes; delta-I noise**

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**Engineering**  
**Portland State University**

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## Lecture topics

1. Interconnect modeling
2. Resistance, inductance, & capacitance
  - R, L, & C
3. Skin effect
4. Ground planes
5. MOS devices and CMOS
6. Delta-I ( $\Delta I$ ) switching noise

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# Objectives

- Explain origins & effects of inductance
  - Review impedance concepts (inductance)
  - Introduce skin effect
  - Understand role of ground planes
- Relate CMOS requirements to package
  - Review MOS device physics
  - Understand CMOS current switching
  - Understand origins of delta-I noise

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## 1. Interconnect modeling

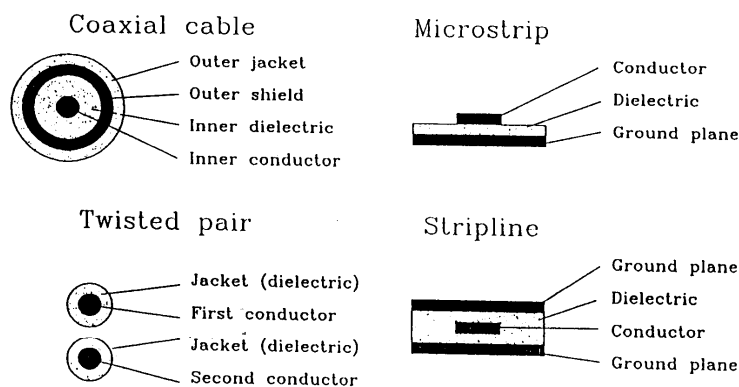


Figure 4.4 Cross sections of popular transmission line geometries.

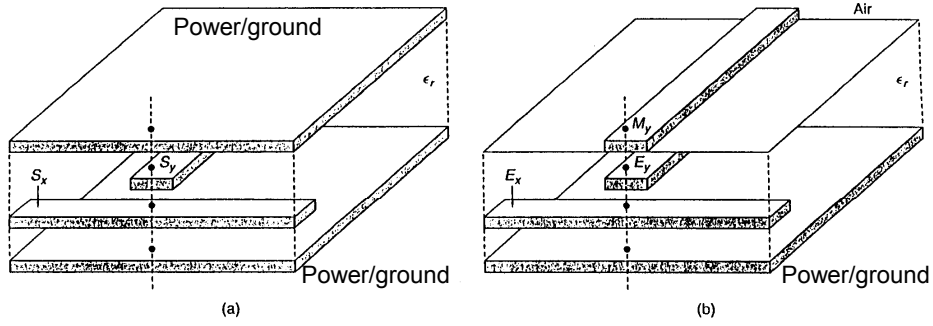
### 4.2.1 Ideal Distortionless, Lossless Transmission Line

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## Orthogonal line systems: stripline ↓ & microstrip ↓

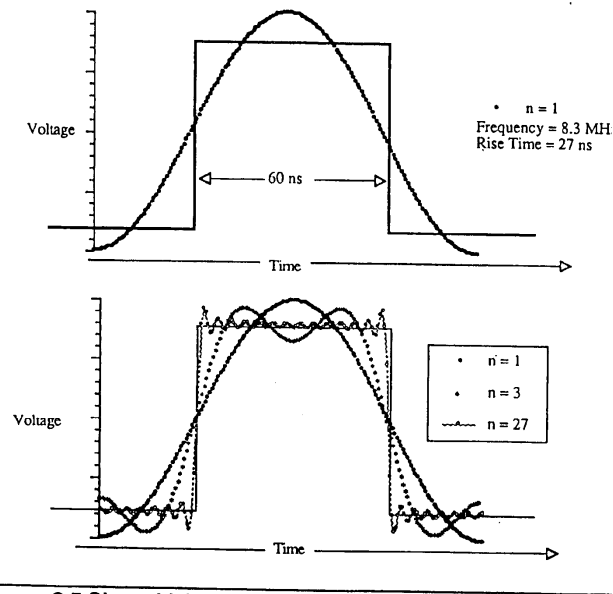


**Figure 4.18** Common strip types are (a) for internal layers: offset striplines  $S_x$  and  $S_y$ , and (b) for surface layers: microstrip  $M_y$  and embedded microstrips  $E_x$  and  $E_y$ . The same design may not use all of them. Metal edges are shaded.

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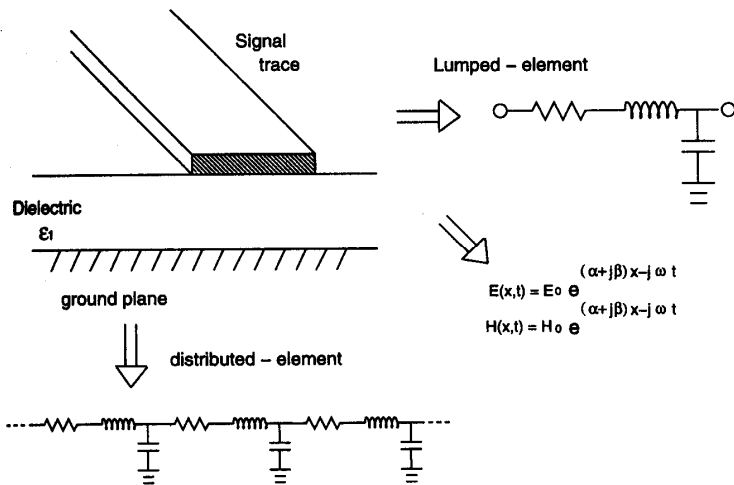
**Figure 8.5** Sinusoidal components of a square wave pulse.

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### Three modeling approaches:



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### (a) $\omega L \gg R$ , (b) $R \gg \omega L$ , (c) $R \approx 0$

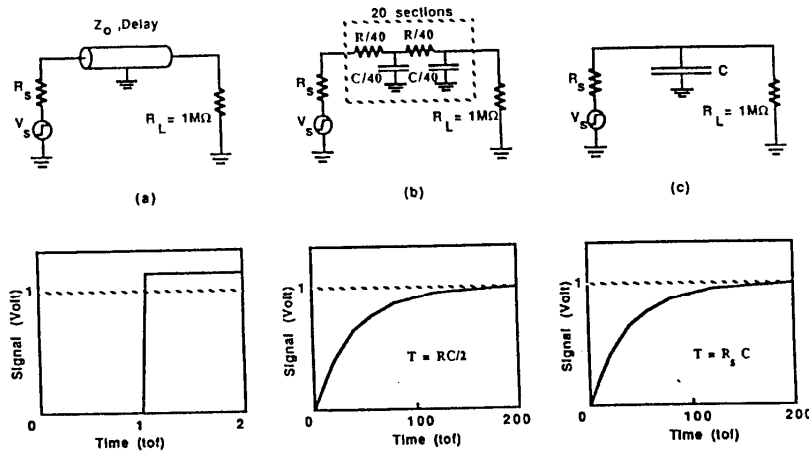


Figure 5-7 Simplified SPICE circuits & signal load responses for limiting propagation regimes. (a) Time-of-flight ( $R_s < Z_0$ ) (b) lossy-line diffusion (c)  $R_s$ -limited; time scales are in units of time-of-flights

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## Typical packaging line lengths & widths

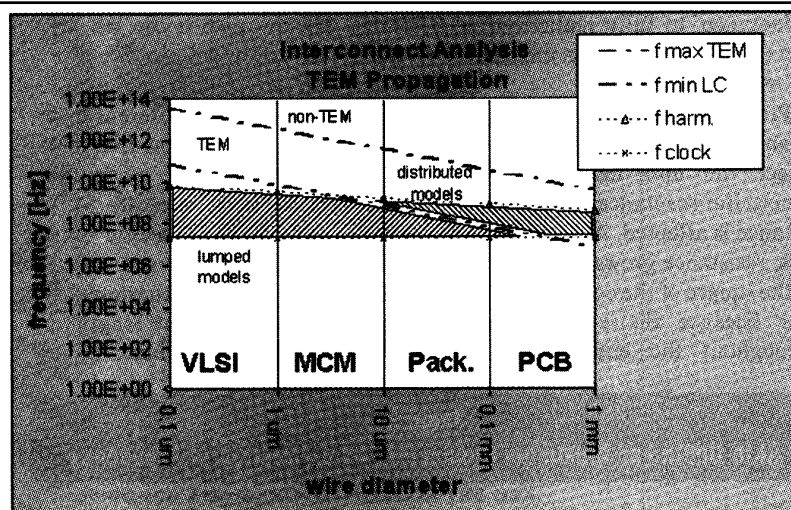
Table 2 Maximum Interconnect Length at Different Packaging Levels	
Component	Dimension
Die	2 cm
Chip carrier (single chip)	4 cm
Module (multichip)	10 cm
PCB	50 cm
Cables (LAN)	100 m

Widths  
 VLSI "wires" in Si:  
 0.1 to 1  $\mu\text{m}$   
 MCM trace widths:  
 1 to 10  $\mu\text{m}$   
 Packages:  
 10  $\mu\text{m}$  to 0.1 mm  
 PCBs:  
 0.1 to 1 mm

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**Figure 2**  
 Currently used clock rates ( $f_{\text{clock}}$ ) and their harmonics ( $f_{\text{harm}}$ ).

PRINTED CIRCUIT DESIGN / JULY 1995

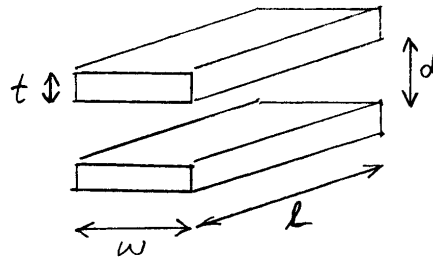
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## 2. R, G, $\ell$ , and C

- Conductor:
  - $R = \rho \ell / A_R = \rho \ell / wt = (\ell / w) (\rho / t)$ 
    - $R / \ell = \rho / wt$
- Dielectric:
  - $G = \sigma A_C / d = \sigma \ell w / d$ 
    - $G / \ell = \sigma w / d$
  - $C = \epsilon A_C / d = \epsilon \ell w / d$ 
    - $C / \ell = \epsilon w / d$



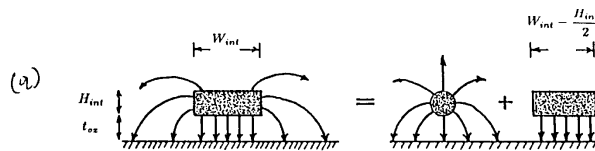
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## Capacitance/unit length

“Visual”  
approximations



**FIGURE 4.2** Modeling of the contribution of fringing fields to interconnection capacitance. Total wire capacitance can be thought of having two components: a parallel plate capacitance determined by the perpendicular field lines between the wire and the ground plane and a fringing field component, which can be approximated by the capacitance of a cylindrical wire with a diameter equal to interconnection thickness [4.3]. Reprinted by permission of Addison-Wesley Publishing Co.

reduced by  $H_{int}/2$  to account for some second-order effects [4.4]. This yields the interconnection capacitance per unit length  $C_{int}$  as

$$C_{int} = \epsilon_{ox} \left\{ \frac{W_{int}}{t_{ox}} - \frac{H_{int}}{2t_{ox}} + \frac{2\pi}{\ln \left[ 1 + \frac{2t_{ox}}{H_{int}} \left( 1 + \sqrt{1 + \frac{H_{int}}{t_{ox}}} \right) \right]} \right\}. \quad (4.5)$$

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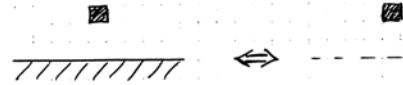
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## Other Capacitor Geometries

(a) "VISUAL" APPROXIMATIONS

(b) METHOD OF IMAGES



$$C_1 = 2C_2$$

$$\& \text{ since } Z_0 = \sqrt{\frac{L}{C}} \implies \frac{v^{-1}}{\sqrt{C}} \frac{1}{\sqrt{C}}$$

$$\& v^{-1} = \sqrt{L/C} \quad \therefore Z_0 = \frac{v^{-1}}{C}$$

$Z_0 C$  independent of geometry.

$$\text{so } 2Z_{01} \iff Z_{02}$$

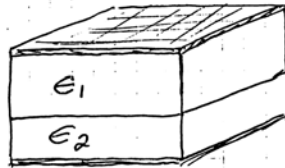
Can find one if know other.

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(c) MIXED DIELECTRICS



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

(d) Analytical field calculations  
(usually idealized structures)  
 $V = \sum v_i$  for charges  $q_i$

(e) Graphical approximations  
— curvilinear squares

(f) Experimental field plotting

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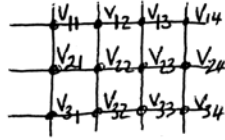
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(g) Numerical techniques

- mesh
- iteration

For 2D



$$V_{22} = \frac{1}{4} (V_{12} + V_{23} + V_{32} + V_{21})$$

(h) Method of Moments

Potential from known charge distribution simple by superposition  $V = \frac{1}{4\pi\epsilon} \sum \frac{Q_i}{R_i}$

Converse to find charge distribution to produce known potential.

## Method of Moments Example

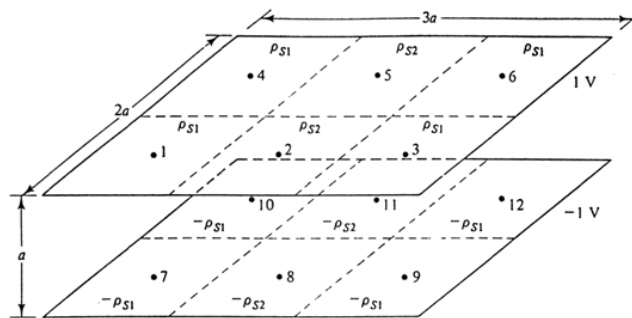


Figure 5.10. For finding the capacitance of a parallel-plate capacitor by the method of moments.

$$C = \epsilon(6a^2/a) = 6\epsilon a?$$



# Method of Moments Example

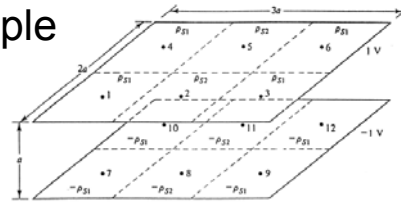


Figure 5.10. For finding the capacitance of a parallel-plate capacitor by the method of moments.

PARALLEL PLATE CAPACITOR

$C = \epsilon \frac{A}{d}$  neglects fringing fields field curvature  $\rightarrow$  non-uniform surface charge distribution  $\rho_s$

eg.  $3a \times 2a \times a$  capacitor. Charges  $\rho_{s1} a^2, \rho_{s2} a^2$  by symmetry. Assume charges at center of  $\square$ 's.

Potential at point 1 due to  $\rho_{s1}$  at point 4  $\rightarrow \frac{(\rho_{s1} a^2)}{4\pi\epsilon a}$   $\leftarrow$  distance

Potential at point 1 due to distributed charge  $\rho_{s1} a^2$  on square #1 =  $\frac{(\rho_{s1} a^2)}{\pi\epsilon a} \ln(1+\sqrt{2})$

# Method of Moments Example

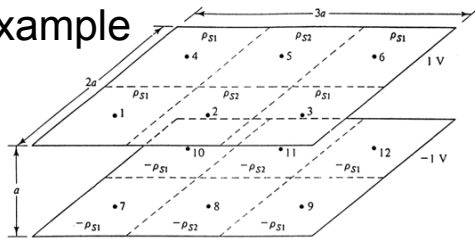


Figure 5.10. For finding the capacitance of a parallel-plate capacitor by the method of moments.

$\therefore$  For fixed plate potentials  $1v, -1v$

Square #1:  $V_1 = \frac{\rho_{s1} a}{\pi\epsilon} \ln(1+\sqrt{2}) + \frac{\rho_{s1} a}{4\pi\epsilon} \left( \frac{1}{2} + 1 + \frac{1}{\sqrt{5}} - 1 - \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}} - \frac{1}{16} \right) + \frac{\rho_{s2} a}{4\pi\epsilon} \left( \frac{1}{2} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right) = 1v$

Square #2:  $V_2 = \frac{\rho_{s2} a}{\pi\epsilon} \ln(1+\sqrt{2}) + \frac{\rho_{s1} a}{4\pi\epsilon} \left( 2 + \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{3}} \right) + \frac{\rho_{s2} a}{4\pi\epsilon} \left( 1 - 1 - \frac{1}{\sqrt{2}} \right) = 1v$

## Method of Moments Example

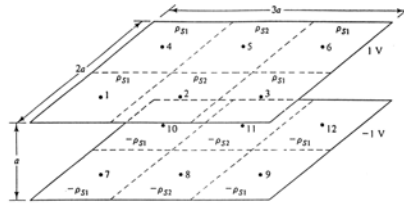


Figure 5.10. For finding the capacitance of a parallel-plate capacitor by the method of moments.

$$\frac{\rho_{21}a}{\pi\epsilon_0} \ln(1 + \sqrt{2}) + \frac{\rho_{21}a^2}{4\pi\epsilon_0} \left( \frac{2}{a} + \frac{2}{\sqrt{2}a} - \frac{2}{\sqrt{2}a} - \frac{2}{\sqrt{3}a} \right) + \frac{\rho_{21}a^2}{4\pi\epsilon_0} \left( \frac{1}{a} - \frac{1}{a} - \frac{1}{\sqrt{2}a} \right) = 1 \quad (5.31b) \quad (\text{Square \#2 last slide})$$

or

$$2.9101\rho_{21} + 0.4226\rho_{22} = \frac{4\pi\epsilon_0}{a} \quad (5.32a)$$

$$0.8453\rho_{21} + 2.8184\rho_{22} = \frac{4\pi\epsilon_0}{a} \quad (5.32b)$$

Solve:  $\rho_{s1} = 3.8378\epsilon/a$      $\rho_{s2} = 3.3075\epsilon/a$

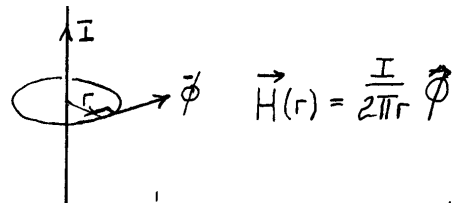
$\therefore$  Total  $Q = 21.9662\epsilon a$  and  $C = Q/V = Q/2v = 10.983\epsilon a$  (c.f.  $6\epsilon a$ )

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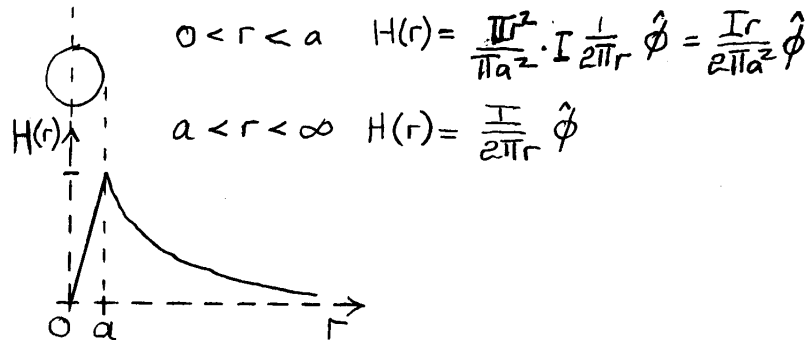
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## Inductance of a Straight Wire: Magnetic Field



$$\vec{H}(r) = \frac{I}{2\pi r} \hat{\phi}$$



$$0 < r < a \quad H(r) = \frac{\pi r^2}{\pi a^2} \cdot I \frac{1}{2\pi r} \hat{\phi} = \frac{I r}{2\pi a^2} \hat{\phi}$$

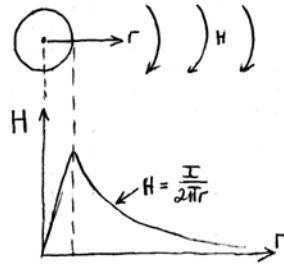
$$a < r < \infty \quad H(r) = \frac{I}{2\pi r} \hat{\phi}$$

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## Internal & External Inductances



Internal inductance  $L_i$       External inductance  $L_e \Rightarrow L_i$  usually  
 $l = L_i + L_e \approx L_e$

$g, c$  leakage conduct/capac  
 $\sigma, \epsilon, \mu$  for external medium

Also  $L_e g = \mu G$   
 $L_e c = \mu C$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{\text{enclosed}} \quad \xrightarrow{\text{Circular contour at } r} \quad H(r) \cdot 2\pi r = I$$

$$H(r) = \frac{I}{2\pi r}$$

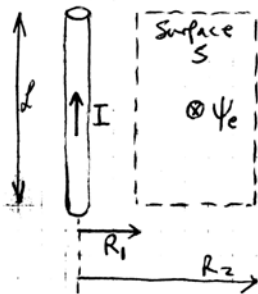
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## External Inductance: Basic formula

Need to find total external flux



$$\Psi_e = \int_S \mathbf{B}(r) \cdot d\mathbf{s}$$

$$= \int_{R_1}^{R_2} \mu_0 \frac{I}{2\pi r} dr$$

$$= \frac{\mu_0 I}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

Note: To find  $L_e = \frac{\Psi_e}{I} = \frac{\mu_0}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$

over all space ( $R_2 \rightarrow \infty$ )  
 $R_1 = r_w$

requires assumption of specific geometries

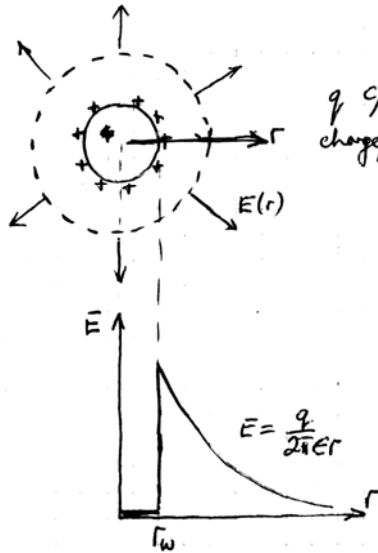
(Can't have isolated current, must be a return path)

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### Straight Wire: Radial Electric Field



$q$  c/m  
charge/unit length

$$\oint_S \vec{E} \cdot d\vec{s} = Q_{enc.}$$

$$\epsilon E (2\pi r \times L) = q \times L$$

$$\therefore E(r) = \frac{q}{2\pi\epsilon r}$$

$$\& V = -\int_C \vec{E} \cdot d\vec{l}$$

$$= -\int_{r_2}^{r_1} \frac{q}{2\pi\epsilon r} dr$$

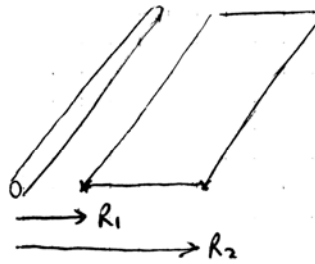
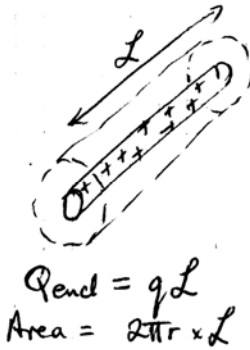
$$= \frac{q}{2\pi\epsilon} \ln\left(\frac{R_2}{R_1}\right)$$

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### $\Psi_e$ and $V$ : equate $(2\pi)^{-1}\ln(R_2/R_1)$



& note  $\frac{V \epsilon}{q} = \frac{\Psi_e}{\mu_0 I}$

$$L_e = \frac{\Psi_e}{I} = \mu_0 \epsilon \frac{V}{q} = \mu_0 \epsilon C^{-1}$$

↑ Capac/unit length

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## Internal Inductance

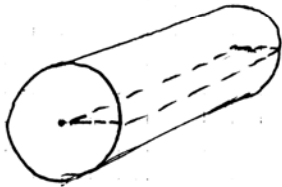
low freq - uniform current.

$$I_{\text{encl},i} = \pi r^2 \left( \frac{I}{\pi r_w^2} \right) = I \left( \frac{r}{r_w} \right)^2$$

$$\therefore H(r) \cdot 2\pi r = I \left( \frac{r}{r_w} \right)^2 \Rightarrow H(r) = \frac{I}{2\pi r_w^2} \cdot r$$

$$\& \Psi_i = \int_s \underline{B}(r) \cdot d\underline{s} \Rightarrow \Psi_i \cdot L = \mu_0 \int_0^{r_w} \frac{I}{2\pi r_w^2} r \cdot L dr$$

↑  
per unit length



$$\Psi_i = \frac{\mu_0 I}{4\pi}$$

$$L_i = \frac{\mu_0}{4\pi} \text{ H/m}$$

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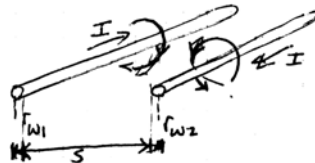
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## External Inductance: 2-wire line

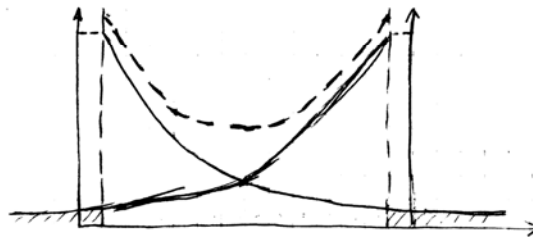
(a) 2-wire line

Total flux between wires



$$L_e = \frac{\Psi_e}{I} = \frac{\mu_0 I}{2\pi} \left[ \ln \left( \frac{s+r_{w1}}{r_{w1}} \right) + \ln \left( \frac{s+r_{w2}}{r_{w2}} \right) \right]$$

$$\Rightarrow s \gg r_{w1}, r_{w2} \Rightarrow \frac{\mu_0}{2\pi} \ln \frac{s^2}{r_w^2} = \frac{\mu_0}{\pi} \ln \frac{s}{r_w}$$

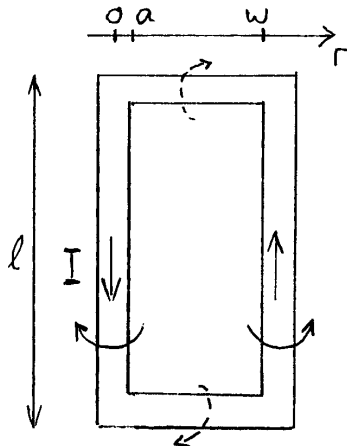


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Compare:-



$$L = \frac{\mu}{I} \int \vec{H} \cdot d\vec{s}$$

$$\approx \frac{2\mu l}{I} \left[ \int_0^a \frac{I r}{2\pi a^2} dr + \int_a^w \frac{I}{2\pi r} dr \right]$$

$$L/l = 2 \left[ \frac{\mu}{4\pi} \left( 1 + 2 \ln \frac{w}{a} \right) \right]$$

Hence "partial" inductance

$$L = \frac{\mu_0}{4\pi} + \frac{\mu_0}{2\pi} \ln \frac{w}{a}$$

$$= L_i + L_e \approx L_e$$

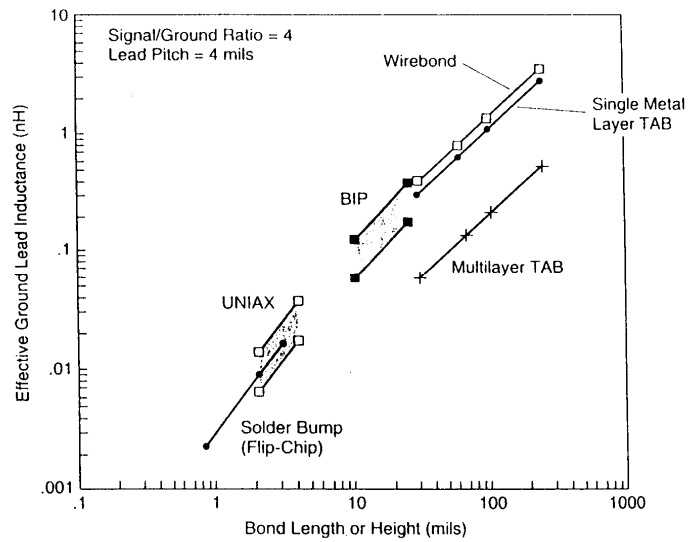
$$L = \frac{\Phi}{I} = \frac{1}{I} \int_S \vec{B} \cdot d\vec{s}$$

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## Lead inductances



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### 3. Skin Effect

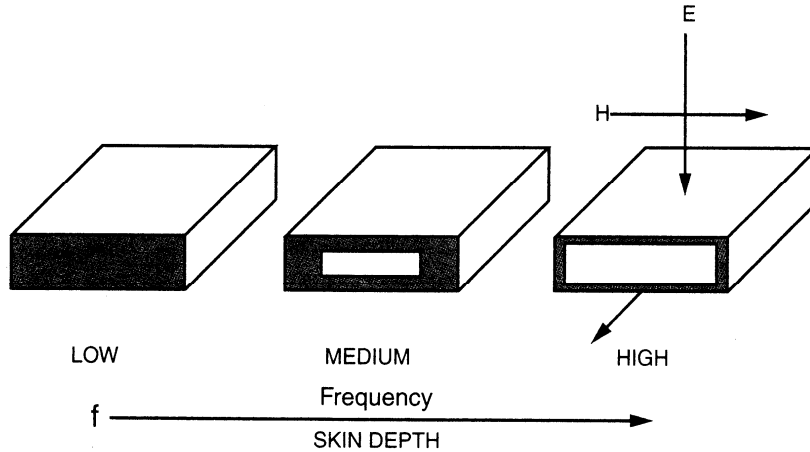


FIGURE 13.19 Effect of frequency on conductor cross section.

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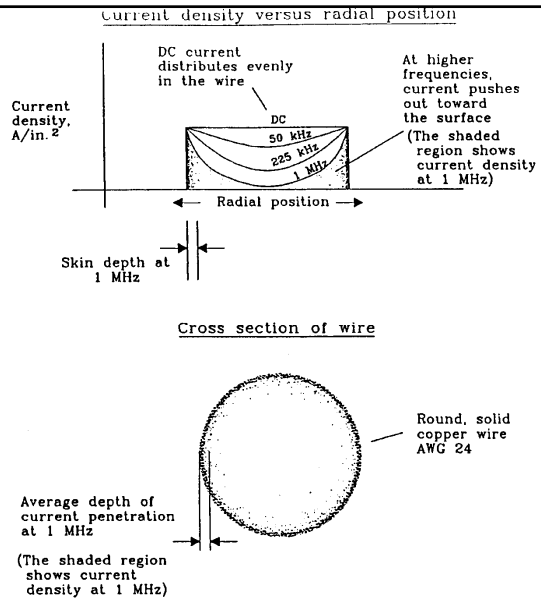


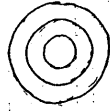
Figure 4.12 Distribution of current in a round wire.

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### 8. SKIN EFFECT



Consider wire to consist of concentric cylinders

Inductance of outer rings < Inductance of center rings

At high frequency, current path seeks lower inductance → outer "skin"

$$\text{SKIN DEPTH } \delta = \left( \frac{2\rho}{\omega\mu} \right)^{1/2}$$

As current area decreases, resistance increases

$$(R = \rho/A)$$

$$R \propto 1/\delta \propto \text{freq}^{1/2}$$

$$\text{and } L_{\text{SKIN}} \propto \text{freq}^{-1/2}$$

$$\text{Skin Reactance } \omega L_{\text{SKIN}} \propto \text{freq}^{1/2}$$

$$(L_{\text{internal}} \ll L_{\text{external}})$$

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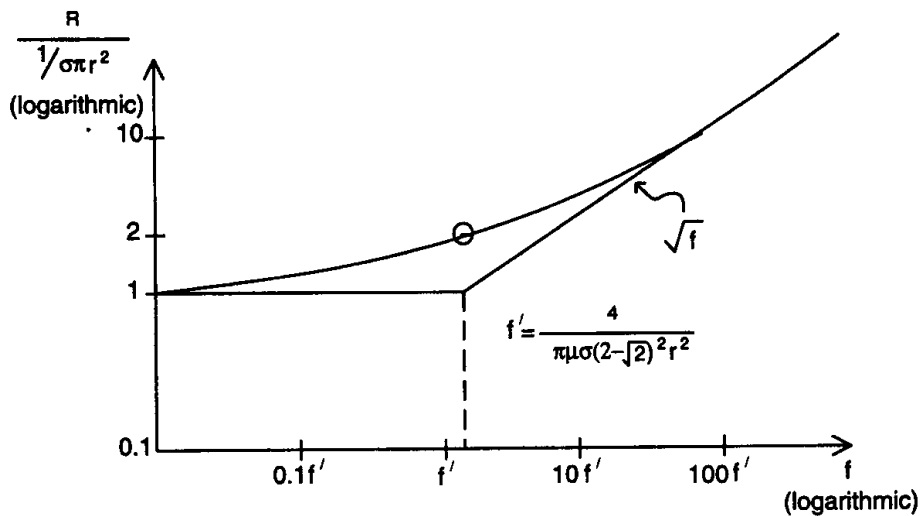


Figure 2.25  $R(f)$  for a wire of circular cross section, radius  $r$ .

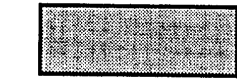
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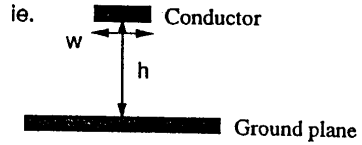
8. Skin Effect.



Current Distribution in the conductor at low frequencies.



Current distribution in the conductor at high frequencies (skin effect.) when  $w/h \rightarrow 0$

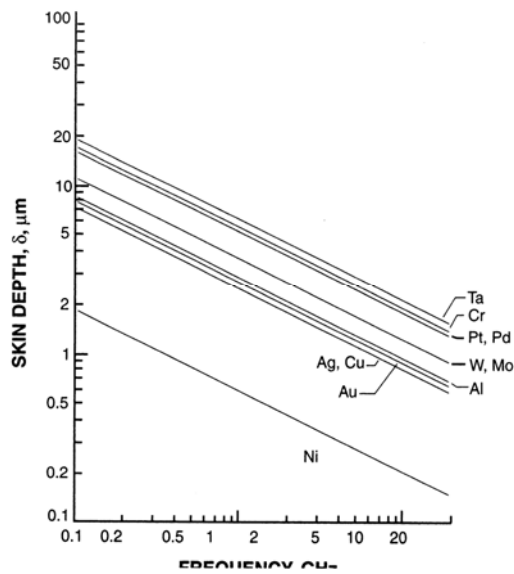


Current distribution in the conductor at high frequencies (skin effect.) when  $w/h \rightarrow \text{infinity}$



4/8 Figure 7.5 The skin effect.

### Skin Depth vs Frequency

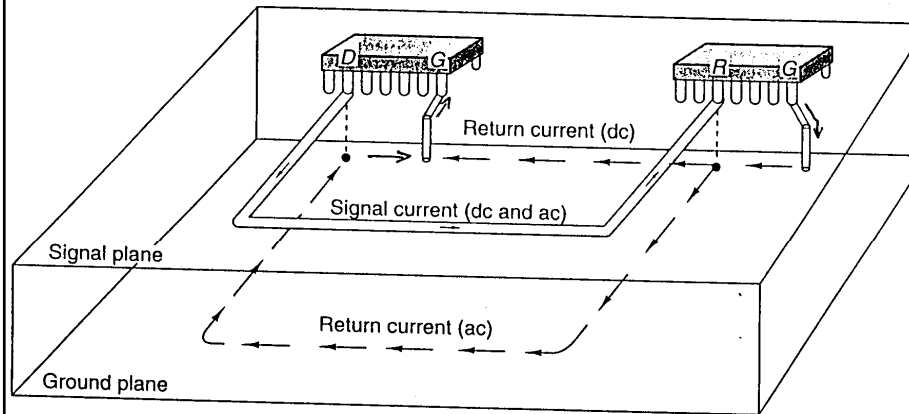


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# 4. Ground planes



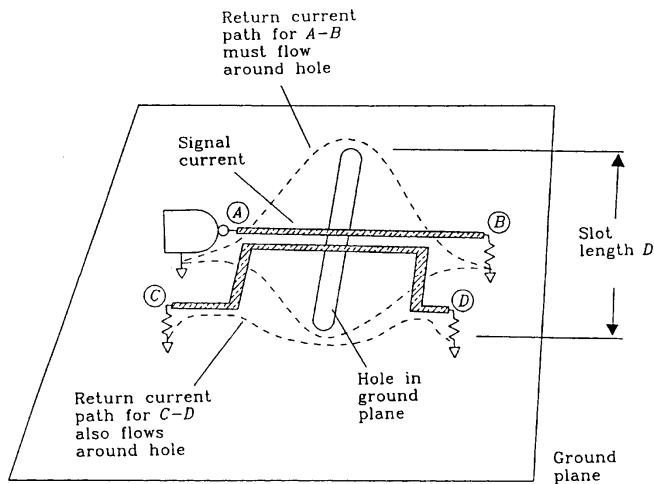
**Figure 8.16** Paths for dc and ac ground returns on a multilayer board.

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# Loss of coupled return → inductance



**Figure 5.8** Crosstalk in a slotted ground plane.

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## 2. DISCRETE IMPEDANCES

If a signal wire is separated from its return-current path (ground plane), it (& the return path) is modeled as a discrete component  $R+j\omega L$  rather than by a transmission line.  
(eg. common in high current systems)

[Note skin effect:

$$\text{Skin depth } \delta = \left(\frac{\rho}{\pi \mu f}\right)^{1/2} \text{ m}$$

& current distribution  $I(x) = I(0) \exp(-x/\delta)$

1. Round wire: diameter  $D$ , length  $l$

(a) Straight

(i) Low frequency  $f \ll 16\sigma/\pi \mu D^2$ , i.e.  $S \gg D/4$

$$R = \rho l / \pi (D/2)^2$$

$$L \approx \frac{\mu_0 l}{2\pi} \left[ \ln\left(\frac{4l}{D}\right) + \left(\frac{\mu_r}{4}\right) - 1 + 0.389\left(\frac{D}{l}\right) \right] \text{ H}$$

(ii) High frequency

$$R \approx \rho l / (\pi D \delta) \quad \text{Skin effect}$$

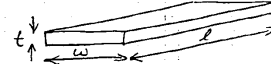
$$L \approx \frac{\mu_0 l}{2\pi} \left[ \ln\left(\frac{4l}{D}\right) + \left(\frac{\mu_r}{4}\right) - 1 + 0.5\left(\frac{D}{2l}\right) \right] \text{ H}$$

(b) Circular wire, circumference  $l$

$$\text{low freq } L \approx \frac{\mu_0 l}{2\pi} \left[ \ln\left(\frac{2}{\pi} \frac{4l}{D}\right) + \left(\frac{\mu_r}{4}\right) - 2 \right]$$

2. PCB land

$t \ll w$



(i) Low frequency  $f \ll 4\rho(w+t)^2/\pi \mu w^2 t^2$   
i.e.  $S \gg \frac{wt}{2(w+t)}$

$$R = \rho l / wt$$

"Skin area"  $\frac{2(w+t)\delta}{2} > \frac{wt}{2}$   
c/s area

$$L \approx \frac{\mu_0 l}{2\pi} \left[ \ln\left(\frac{2l}{w+t}\right) + 0.5 + 0.2235\left(\frac{w+t}{l}\right) \right]$$

(ii) High frequency

$$R = \rho l / 2(w+t)\delta \quad \text{Skin effect}$$

$$L \approx \frac{\mu_0 l}{2\pi} \left[ \ln(X) - 1 + \left(\frac{X}{l}\right) \right] \quad X = (wt) \left[ \frac{0.2729}{-0.0618} \frac{w+t}{wt} \right]$$

3. Ground plane: infinite ground plane, thickness  $t$

$$R = \frac{\rho}{S} \left[ \frac{1 - \exp(-t/\delta)}{1 - 2 \exp(-t/\delta) \cos(t/\delta) + \exp(-2t/\delta)} \right] \Omega$$

$$L = \frac{\rho}{2\pi f S} \left[ \frac{1 - \exp(-t/\delta) [\cos(t/\delta) - \sin(t/\delta)]}{1 - 2 \exp(-t/\delta) \cos(t/\delta) + \exp(-2t/\delta)} \right] \text{ H}$$

(i) low freq  $f < \rho/\pi \mu t^2$ , i.e.  $S \gg t$   $R \approx \rho/S$   $L \approx \rho/2\pi f t$

(ii) High freq,  $R \approx \rho/\delta$ ,  $L \approx \rho/2\pi f \delta$  (Skin effect)

Notes:  $\frac{\mu_0}{2\pi} = 2 \text{ nH/cm} = 5.08 \text{ nH/in}$

Self-inductance  $L_s = L_{si} + L_{se}$

internal  $\rightarrow 0$  as  $f \rightarrow \infty$  external

$$L_{si} = \frac{\mu_0}{2\pi} \cdot \frac{l}{4}$$

$L_{se}$  = Mutual inductance at  $s=r$  from center i.e. at surface

for round wire radius  $r =$  length  $l$

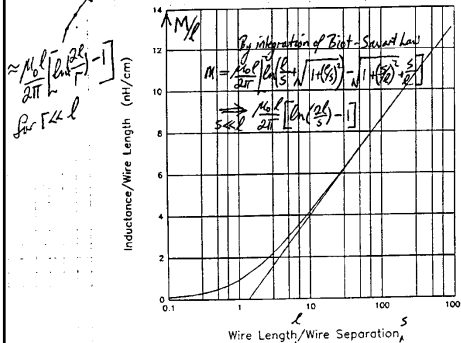


Figure AS.1 Mutual inductance  $M$  per unit wire length. The lower line is the long-wire approximation. See text to adapt for total self-inductance per unit wire length.

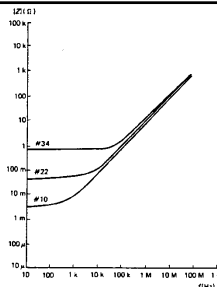


Figure D-2 Impedance of 1-in-long round copper wires

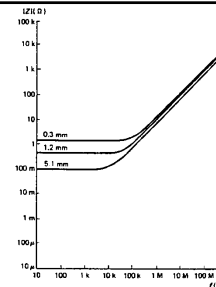


Figure D-4 Impedance of 1-in-long 35-um-thick PCB lands

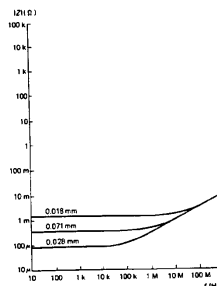


Figure D-6 Impedance of copper groundplanes

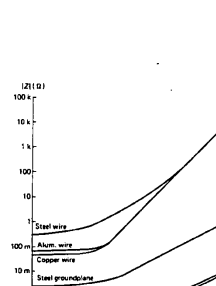
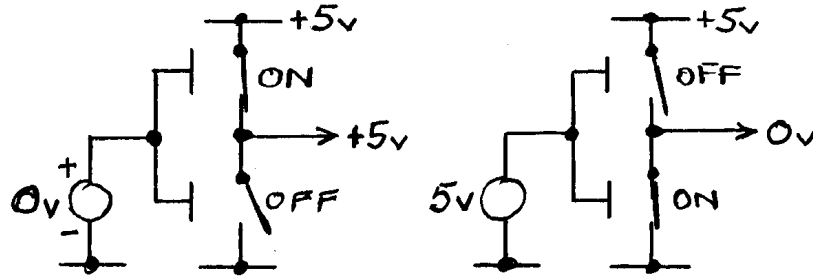


Figure D-9 Effect of materials on impedance

# 5. MOS devices and CMOS



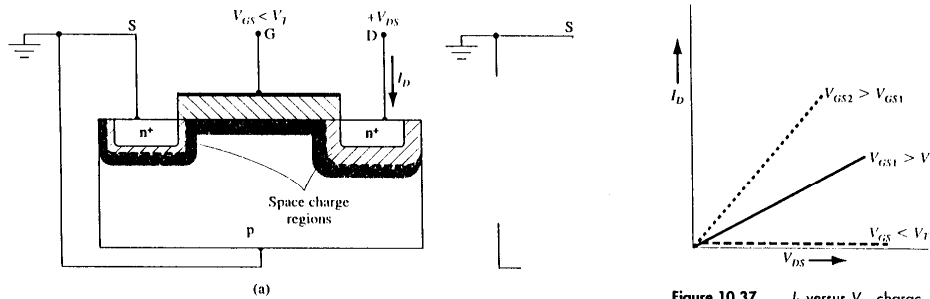
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## N-channel enhancement MOSFET:

$$V_{DS} \approx 0$$



**Figure 10.36** The n-channel enhancement mode MOSFET (a) with an applied  $v_g$  and (b) with an applied gate voltage  $V_{GS} > V_T$

**Figure 10.37**  $I_D$  versus  $V_{DS}$  characteristics for small values of  $V_{DS}$  at three  $V_{GS}$  voltages

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# MOSFET operation

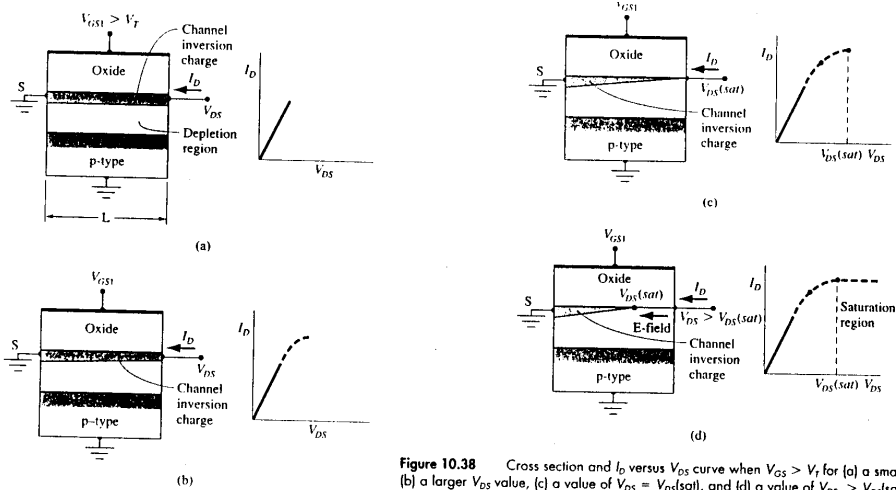


Figure 10.38 Cross section and  $I_D$  versus  $V_{DS}$  curve when  $V_{GS} > V_T$  for (a) a small  $V_{DS}$  value, (b) a larger  $V_{DS}$  value, (c) a value of  $V_{DS} = V_{DS(sat)}$ , and (d) a value of  $V_{DS} > V_{DS(sat)}$

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# Enhancement MOSFET characteristics

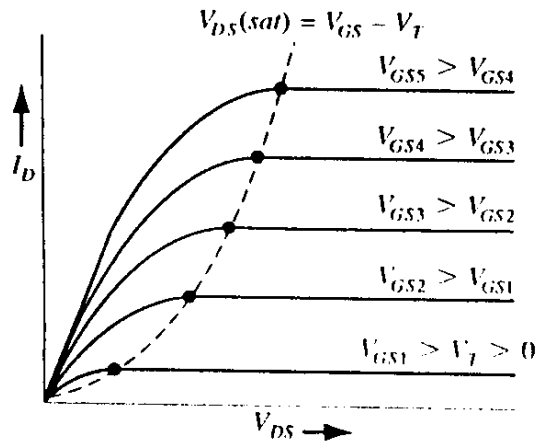


Figure 10.39 Family of  $I_D$  versus  $V_{DS}$  curves for an n-channel enhancement mode MOSFET

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# Derivation of MOSFET characteristic equations

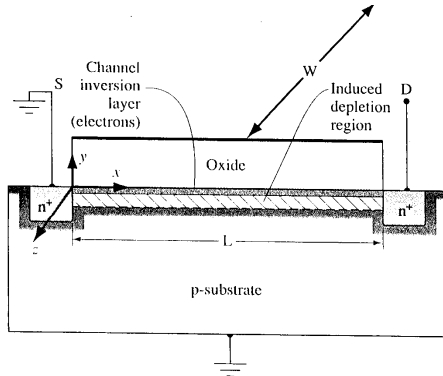


Figure 10.42 Geometry of a MOSFET for  $I_D$  versus  $V_{DS}$  derivation

$$I(x) = \int_y^y I(x) dy dz$$

$$= W \cdot \mu \int_y \sigma(x) \frac{dV(x)}{dx} dy$$

$$I(x) dx = \mu W q(x) dV(x)$$

$$\text{and } q(x) = \frac{C_{ox}}{e} [V_g - V(x)]$$

$$= C_{ox} [(V_g - V_T) - V(x)]$$

$$I_{DS} \int_0^L dx = \mu W C_{ox} \int_0^{V_{DS}} [(V_g - V_T) - V(x)] dV_x$$

$$I_{DS} \cdot L = \mu W C_{ox} [(V_g - V_T)V_{DS} - V_{DS}^2/2]$$

$$I_{DS} = \frac{\mu}{2} \frac{W}{L} C_{ox} [2(V_g - V_T) - V_{DS}] V_{DS}$$

Now set  $(\mu W/2L)C_{ox} = k \rightarrow$

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \mu \frac{W}{L} C_{ox} [(V_g - V_T) - V_{DS}]$$

$$\rightarrow 0 \text{ when } V_{DS} = V_g - V_T$$

$$\text{Substitute: } I_{DS} ]_{SAT} = \frac{\mu}{2} \frac{W}{L} C_{ox} (V_g - V_T)^2$$

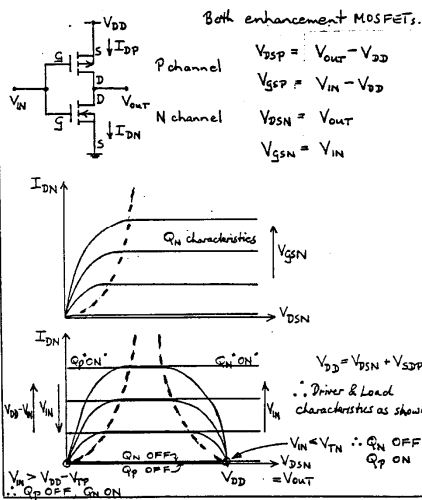
$$= \frac{\mu}{2} \frac{W}{L} C_{ox} V_{DS}^2$$

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# CMOS dynamic characteristics (part 1)

## CMOS INVERTER



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## CMOS INVERTER (cont)

As  $V_{in}$  increases from 0 volts :-

(i) For  $V_{in} < V_{TN}$ ,  $Q_N \text{ OFF} \therefore I = 0$   
 $\therefore V_{GSP} = V_{in} - V_{DD} = -V_{DD}$ ,  $Q_P$  not saturated.  
 $V_{out} = V_{DD}$  - see diagram.

(ii) For  $V_{in} > V_{TN}$ ,  $Q_N$  begins to turn ON  
 (for  $Q_N$  saturated,  $Q_P$  not saturated - see diagram  
 for  $V_{in} < V_{DD} - V_{TP}$ )  
 $\therefore I_{DN} = I_{DP}$   
 $k_N (V_{GSN} - V_{TN})^2 = k_P [2(V_{GSP} - V_{TP})V_{DSP} - V_{DSP}^2]$   
 $\Rightarrow k_P [2(V_{GSP} + V_{TP})V_{DSP} - V_{DSP}^2]$   
 $k_N (V_{in} - V_{TN})^2 = k_P [2(V_{DD} - V_{in} + V_{TP})(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2]$   
 $\therefore V_{out} = V_{DD} + (V_{DD} - V_{in} + V_{TP}) - \sqrt{[(V_{DD} - V_{in} + V_{TP})^2 - \frac{k_N}{k_P} (V_{in} - V_{TN})^2]}$

NB.  $V_{TN} > 0$   
 $V_{TP} < 0$  } for enhancement

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# CMOS dynamic characteristics (part 2)

## CMOS INVERTER (cont)

(iii) Middle region,  $Q_N$  &  $Q_P$  both saturated.

$Q_P$  saturates when  $V_{S_{SP}} = V_{S_{DP}} - V_{TP}$

ie. when  $V_{DD} - V_{OH} = V_{DD} - V_{out} - V_{TP}$   
 $V_{IN} = V_{out} + V_{TP}$

$Q_N$  goes out of saturation when  $V_{S_{SN}} = V_{S_{SN}} + V_{TN}$

ie. when  $V_{IN} = V_{out} + V_{TN}$

In region (iii) with both transistors saturated  $I_{DP} = I_{DN}$

$$k_p (V_{S_{SP}} + V_{TP})^2 = k_n (V_{S_{SN}} - V_{TN})^2$$

$$k_p (V_{DD} - V_{in} + V_{TP})^2 = k_n (V_{in} - V_{TN})^2$$

$$V_{in} = \frac{V_{DD} + V_{TP} + (\frac{k_n}{k_p})^{1/2} V_{TN}}{1 + (\frac{k_n}{k_p})^{1/2}}$$

NB. only  $V_{in}$  defined,  $V_{out}$  changes abruptly.

NB. Need this transition at  $V_{in} = V_{DD}/2$  for symmetry ie. need:

$k_n = k_p$  ← Since  $\mu_n \neq \mu_p$ , need different geometries (P channel area > N channel area)  
 &  $V_{TP} = -V_{TN}$

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## CMOS INVERTER (cont)

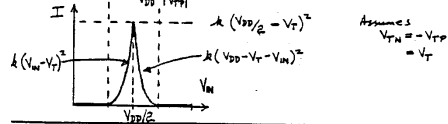
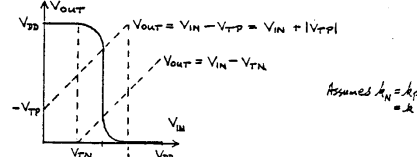
(iv)  $Q_N$  goes out of saturation  
 $Q_P$  still saturated

Complement of region (iii). Following same sequence:

$$V_{out} = (V_{in} - V_{TN}) - \left[ (V_{in} - V_{TN})^2 - \left( \frac{k_p}{k_n} \right) (V_{DD} - V_{in} + V_{TP})^2 \right]^{1/2}$$

(v) When  $V_{in} > V_{DD} - V_{TP}$ ,  $Q_P$  turned OFF

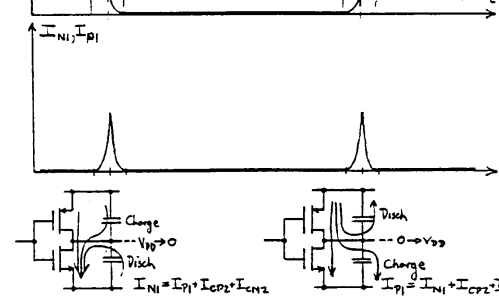
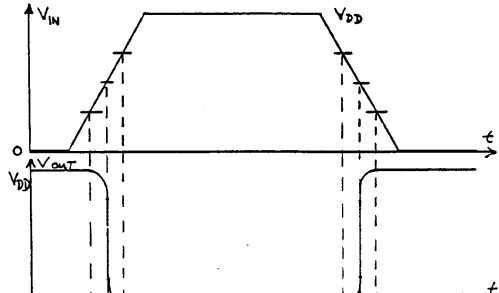
$\therefore V_{out} = 0$



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## Switching current (1)

### CMOS TRANSIENTS



$$I_{N1} = I_{P1} + I_{CP2} + I_{CN2}$$

$$I_{P1} = I_{N1} + I_{CP2} + I_{CN2}$$

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## Switching current (2)

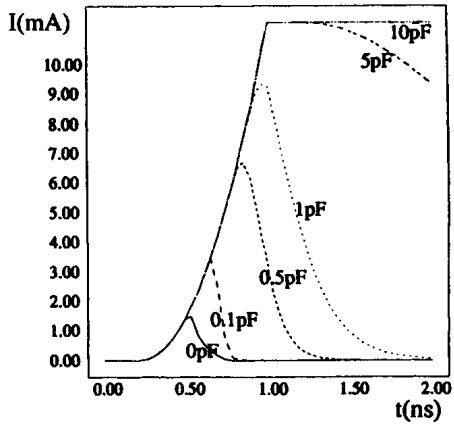
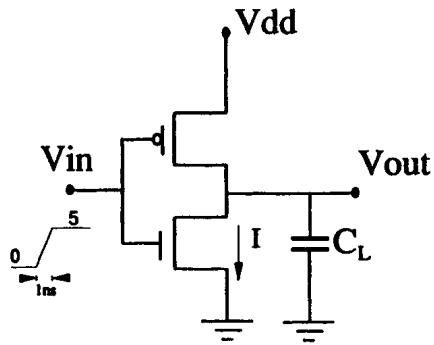


Figure 2.75 N-channel MOSFET current for various sizes of  $C_L$ .

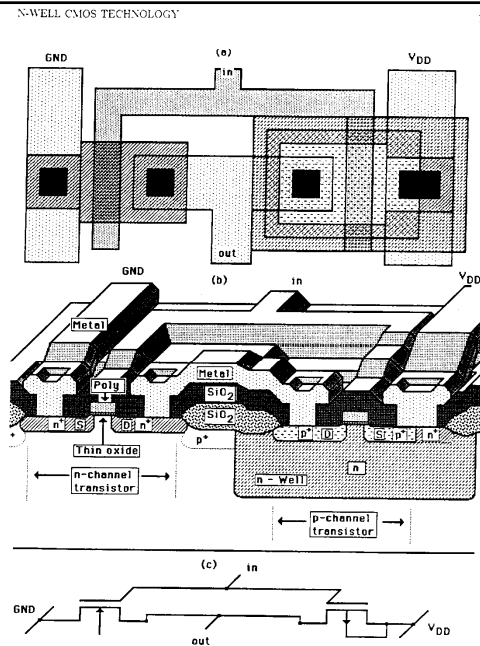
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CMOS  
inverter:

VLSI layout  
& structure



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Figure 5-12: CMOS inverter. Composite layout (a), cross-section (b), and electrical diagram (c).



### MOS SCALING

$[2-15\mu]$  NMOS ( $5\mu$ )  $\rightarrow$  HMOS ( $3\mu$ )  
 $[2-15\mu]$  CMOS ( $5\mu$ )  $\rightarrow$  HCMOS ( $3\mu$ ) [ $2-6v$ ]  
 1K RAM  $\xrightarrow{\alpha=8}$  64K RAM

(i) Say all dimensions  $L, W, t \rightarrow L/\alpha, W/\alpha, t/\alpha$  where  $\alpha > 1$ .

(ii) all voltages  $V \rightarrow V/\alpha$

(iii)  $I_D = \frac{\epsilon_0 \epsilon_o \mu_n}{t_{ox}} \frac{W}{L} [(V_g - V_T)V_D - V_D^2/2]$   
 $\propto V^2/t_{ox}$   
 ie. all currents  $I \rightarrow I/\alpha$

(iv) For all linear dimensions in proportion, depletion widths  $X_D \rightarrow X_D/\alpha$   
 Now  $X_D \approx (2\epsilon_{s1}\epsilon_o V/eN_A)^{1/2} \propto (V/N_A)^{1/2}$   
 ie.  $N_A \rightarrow \alpha N_A$   
 for  $X_D \propto (V/\alpha \cdot \alpha N_A)^{1/2} \rightarrow X_D/\alpha$

### MOS SCALING (cont) $\alpha > 1$

Dimension	$L$	$1/\alpha$
	$W$	$1/\alpha$
	$t_{ox}$	$1/\alpha$
Doping concentration	$N_A, N_D$	$\alpha$
Voltage	$V$	$1/\alpha$
Current	$I$	$1/\alpha$
Packing density (gates/area)		$\alpha^2$
Capacitance ( $C = \epsilon LW/t_{ox}$ )		$1/\alpha^2$
Power/gate ( $V I$ )		$1/\alpha^2$
Power/area ( $(1/\alpha^2)/(1/\alpha^2)$ )		1
Gate delay ( $T = C V/I$ )		$1/\alpha$
Power x delay product		$1/\alpha^3$
$R_l$ Interconnection line resistance ( $CL/A$ )		$\alpha$
Relative local interconnect response ( $RC/\tau$ )		$\alpha^2$
$R_g$ Global interconnection line resistance ( $\rho/A$ )		$\alpha^3$
Relative global interconnect response ( $RC/\tau$ )		$\alpha^3$
Relative line voltage drop ( $IR/V$ ) local		$\alpha^2$
" " " " ( $IR/V$ ) global		$\alpha^2$
Interconnection line current density ( $I/Wt$ )		$\alpha^2$
Contact resistance $R_c$		$\alpha^2$
Contact voltage drop ( $IR_c$ )		$\alpha$

Constant voltage  $\rightarrow I \propto V^2/t \rightarrow I/\alpha$   
 Power/unit area  $\rightarrow P \propto \alpha^3$

Short-channel effects  
 3-D device models.

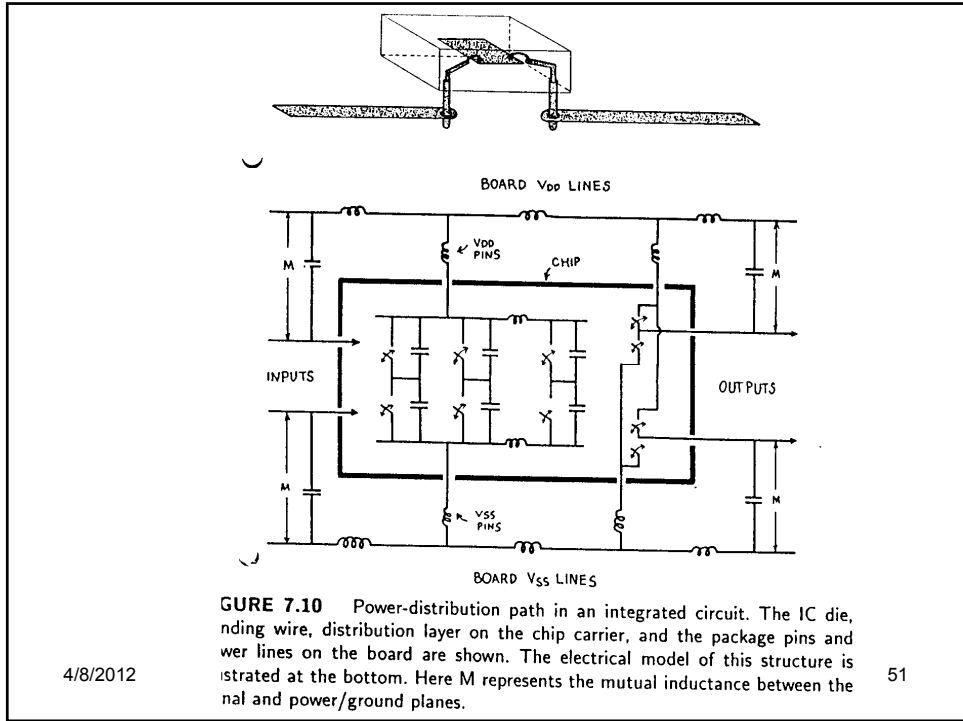
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## 6. Delta-I ( $\Delta I$ ) switching noise

### $\Delta V = L di/dt$

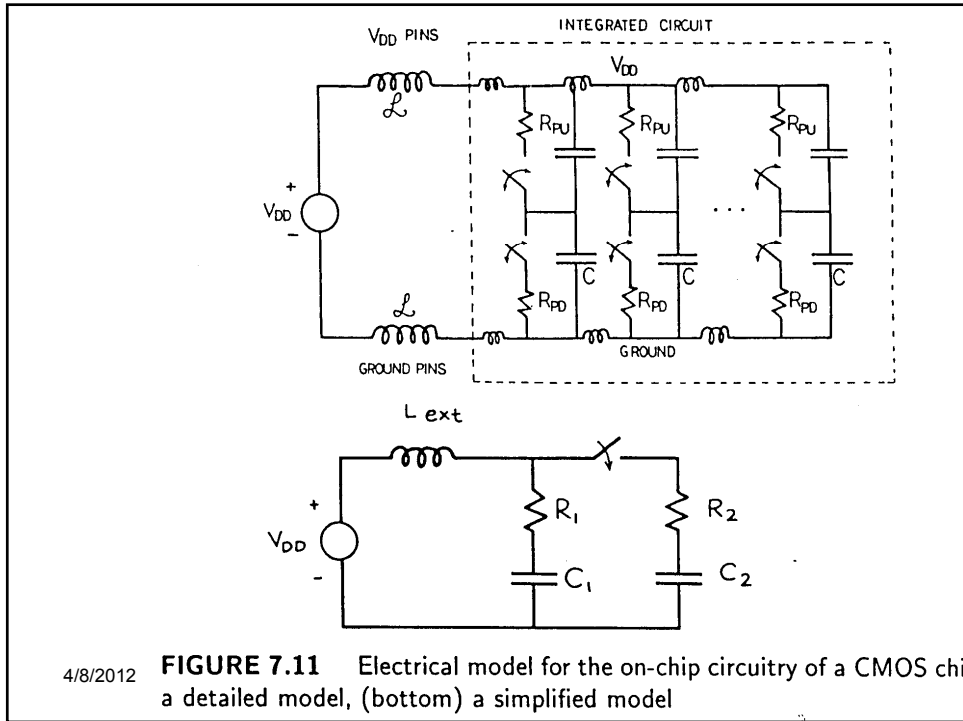
**Figure 2.83** Driver circuit for the calculation of the mutual effect between  $L_{vss}$  and  $L_{vdd}$  on  $V_n$ .

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## Example to show typical values

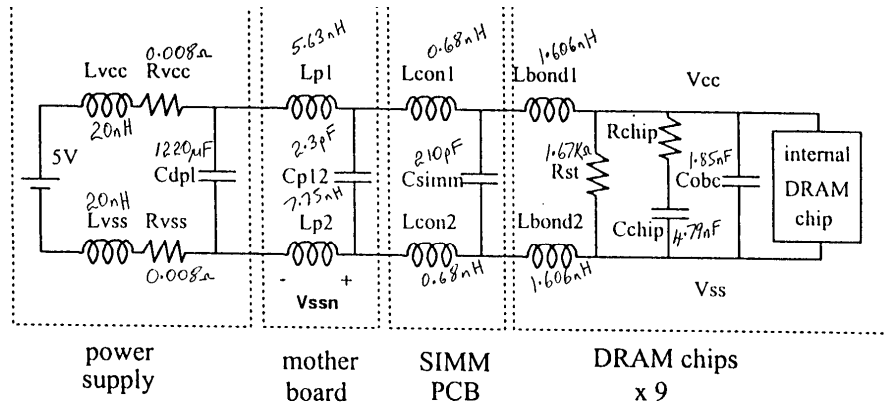


Fig. 15. Simplified equivalent circuit of the test setup shown in Fig. 14.

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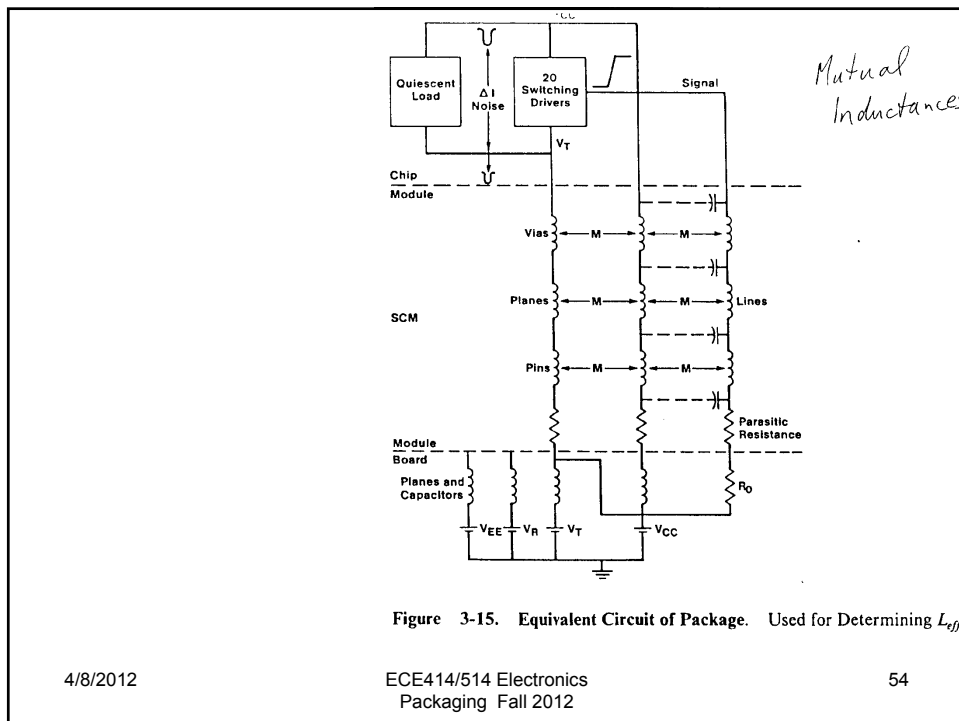


Figure 3-15. Equivalent Circuit of Package. Used for Determining  $L_{eff}$ .

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