ECE414/514
Electronics Packaging
Spring 2012 Lecture 16

Reliability Theory

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Reliability

- *Reliability*: The reliability of a product is the probability that the product will perform satisfactorily for a given time at a desired confidence level under specified operating and environmental conditions.

Potential consequences of poor reliability

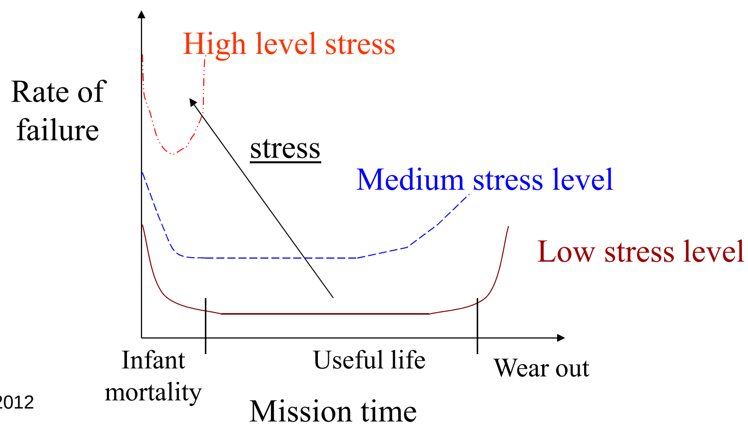
Customer	Supplier/Vendor
Loss of Product	Warranty claims
Loss of product capability	Production downtime
Production downtime	Test and repair cost
Spare parts and maintenance	Diminished confidence and image
Loss opportunities	Loss of future business

Classification of Failures

- *Failure* is defined as the loss of the ability of the product to perform a required operation in a specific environment.
- *Three types* according to when a failure occurred during a product's operating life:
  - Infant mortality
  - "Useful" life
  - Wearout failure

The Bathtub Curve

- If a plot of the rate failure of a product versus its operating life is constructed from data taken a large sample of identical products placed in operation at t=0



4. MIL-HDBK-217

Standards for microelectronics reliability evaluation under various device categories

Example:-

(M. Ohring)

188 The Mathematics of Failure and Reliability

MIL - HDBK-217F

6.4 TRANSISTORS, LOW FREQUENCY, Si FET

SPECIFICATION MIL-S-19500

DESCRIPTION N-Channel and P-Channel Si FET (Frequency ≤ 400 MHz)

$\lambda_p = \lambda_b \pi_T \pi_A \pi_Q \pi_E$ Failures/10^6 Hours

Base Failure Rate - λ_b

Transistor Type	λ_b
MOSFET	.012
JFET	.0045

Temperature Factor - π_T

T_J (°C)	π_T	T_J (°C)	π_T
25	1.0	105	3.9
30	1.1	110	4.2
35	1.2	115	4.5
40	1.4	120	4.8
45	1.5	125	5.1
50	1.6	130	5.4
55	1.8	135	5.7
60	2.0	140	6.0
65	2.1	145	6.4
70	2.3	150	6.7
75	2.5	155	7.1
80	2.7	160	7.5
85	3.0	165	7.9
90	3.2	170	8.3
95	3.4	175	8.7
100	3.7		

$$\pi_T = \exp\left(-1925\left(\frac{1}{T_J + 273} - \frac{1}{298}\right)\right)$$

T_J = Junction Temperature (°C)

Quality Factor - π_Q

Quality	π_Q
JANTXV	.70
JANTX	1.0
JAN	2.4
Lower	5.5
Plastic	8.0

Application Factor - π_A

Application (P_r, Rated Output Power)	π_A
Linear Amplification (P_r < 2W)	1.5
Small Signal Switching	.70
Power FETs (Non - linear, $P_r \geq$ 2W)	
$2 \leq P_r$ < 5W	2.0
$5 \leq P_r$ < 50W	4.0
$50 \leq P_r$ < 250W	8.0
$P_r \geq$ 250W	10

Environment Factor - π_E

Environment	π_E
G_B	1.0
G_F	6.0
G_M	9.0
N_S	9.0
N_U	19
A_{IC}	13
A_{IF}	29
A_{UC}	20
A_{UF}	43
A_{RW}	24
S_F	.50
M_F	14
M_L	32
C_L	320

6-8

Fig. 4-6 Failure rate for low-frequency field-effect transistors. Page 6-8 reproduced from MIL-HDBK-217F.

MIL - HDBK-217F

6.4 TRANSISTORS, LOW FREQUENCY, Si FET

SPECIFICATION
MIL-S-19500

DESCRIPTION
N-Channel and P-Channel Si FET (Frequency ≤ 400 MHz)

$\lambda_p = \lambda_b \pi_T \pi_A \pi_Q \pi_E$ Failures/10^6 Hours

Evaluate $\lambda_p = \lambda_b\, \pi_T\, \pi_A\, \pi_Q\, \pi_E$

Base Failure Rate - λ_b = 0.012

Transistor Type	λ_b
MOSFET →	.012
JFET	.0045

Temperature Factor - π_T

30°C →

$\pi_T = 1.1$

T_J (°C)	π_T	T_J (°C)	π_T
25	1.0	105	3.9
30	1.1	110	4.2
35	1.2	115	4.5
40	1.4	120	4.8
45	1.5	125	5.1
50	1.6	130	5.4
55	1.8	135	5.7
60	2.0	140	6.0
65	2.1	145	6.4
70	2.3	150	6.7
75	2.5	155	7.1
80	2.7	160	7.5
85	3.0	165	7.9
90	3.2	170	8.3
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100	3.7		

$$\pi_T = \exp\left(-1925\left(\frac{1}{T_J + 273} - \frac{1}{298}\right)\right)$$

T_J = Junction Temperature (°C)

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Quality Factor-π_Q = 8.0

Quality	π_Q
JANTXV	.70
JANTX	1.0
JAN	2.4
Lower	5.5
Plastic →	8.0

Application Factor - π_A = .70

Application (P_r, Rated Output Power)	π_A
Linear Amplification ($P_r < 2W$)	1.5
→ Smell Signal Switching	.70
Power FETs (Non - linear, $P_r \geq 2W$)	
$2 \leq P_r < 5W$	2.0
$5 \leq P_r < 50W$	4.0
$50 \leq P_r < 250W$	8.0
$P_r \geq 250W$	10

Environment Factor - π_E **.50**

Environment	π_E
G_B	1.0
G_F	6.0
G_M	9.0
N_S	9.0
N_U	19
A_{IC}	13
A_{IF}	29
A_{UC}	20
A_{UF}	43
A_{RW}	24
→ S_F	.50
M_F	14
M_L	32
C_L	320

Physics of Failure vs MIL-HDBK-217

Above: λ_p = .012 x 1.1 x 8.0 x .50 x .70

= 0.037 failures per 10^6 hours

i.e. 37 FITs (failures per 10^9 hours)

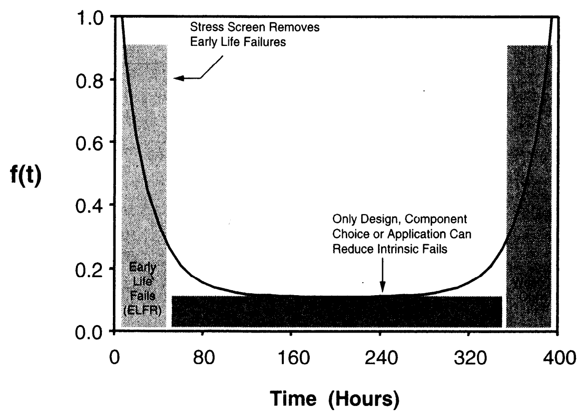
MIL-HDBK-217 is often orders of magnitude out in predictions

Physics of Failure:

Computer modeling based on experimental data

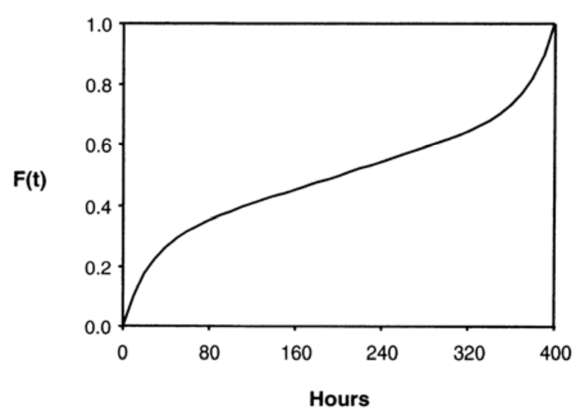
The Bathtub Curve

FIGURE 22.4 A typical bathtub curve representing a *failure density function, f(t).*



Cumulative Failure Function [Q(t) below]

FIGURE 22.3 A Typical cumulative failure function, *F(t).*



Reliability Theory

Sample size = n_o

Number failed at time t = $n_f(t)$

Number still operating satisfactorily = $n_s(t)$

$$n_f(t) + n_s(t) = n_o$$

- Reliability
- Probability device "ok"

$$R(t) = n_s(t)/n_o$$
$$= 1 - n_f(t)/n_o$$
$$= 1 - Q(t)$$

- “Unreliability” $Q(t)$

$$Q(t) = n_f(t)/n_o$$
$$= 1 - n_s(t)/n_o$$
$$= 1 - R(t)$$

Failure Probability Density Function

- Failure probability function $f(t)$

$$f(t) = (1/n_o)\frac{d}{dt}n_f(t) = \frac{dQ(t)}{dt} = -\frac{dR(t)}{dt}$$

$$\therefore Q(t) = \int_0^t f(\tau)\,d\tau = \frac{n_f(t)}{n_o}$$

Hazard Rate

- The Hazard Rate is defined as the number of failures per unit time per number of operational parts left.

$$\lambda(t) = \frac{1}{n_s(t)}\frac{dn_f(t)}{dt} = \frac{1}{R(t)}\frac{dQ(t)}{dt} = \frac{-1}{R(t)}\frac{d[R(t)]}{dt} = \frac{f(t)}{R(t)}$$

Cumulative Hazard Rate: $H(t) = \int_0^t \lambda(\tau)d\tau = -\ln R(t)$

Note difference between Failure Probability Density Function *f(t)* and Hazard Rate *λ(t)*

$$f(t) = (1/n_o)\frac{d}{dt}n_f(t) \qquad \lambda(t) = \frac{1}{n_s(t)}\frac{dn_f(t)}{dt}$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$

Example: Exponential Distribution

- Most commonly used with wide application in electronic systems. It is a single parameter distribution.

$$f(t) = \lambda_o \exp^{(-\lambda_o t)} . u(t)$$

$$\longrightarrow \; 0 \quad for \;\; t = 0$$

where $u(t)$ is the Heavyside step function and

λ_o is called the chance hazard rate.

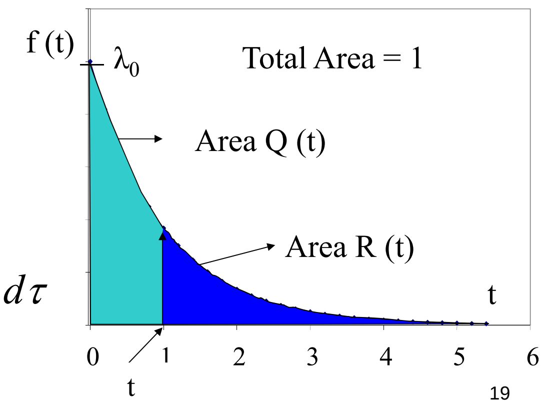
Exponential Distribution

$$Q(t) = \frac{n_f(t)}{n_o} = \int_0^t f(\tau)\,d\tau$$

$$\int_0^\infty f(\tau)\,d\tau = 1$$

and

$$R(t) = \frac{n_s(t)}{n_0} = \int_t^\infty f(\tau)\,d\tau$$



Exponential Distribution

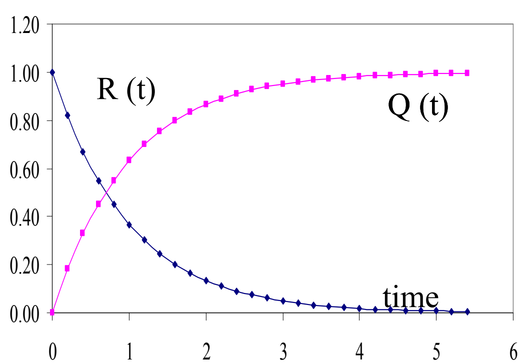
$$R(t) = \int_t^\infty \lambda_o \exp(-\lambda_o \tau).d\tau = -\exp(-\lambda_0 \tau)\Big]_t^\infty$$

$$= \exp-(\lambda_0 t)$$

Check: $R(0) = 1$

$$Q(0) = 0$$

$$Q(t) = 1 - R(t)$$



Hazard Rate ("force of mortality")

General results $\lambda(t) = \frac{1}{n_s(t)} \frac{dn_f(t)}{dt} \xrightarrow{(\div n_0)}$

$$\Rightarrow \frac{1}{R(t)} \frac{dQ(t)}{dt} = -\frac{dR(t)}{dt} \Big/ R(t)$$

for exponential $\Rightarrow -\frac{\left(-\lambda_0 \exp^{-\lambda_0 t}\right)}{\exp^{-\lambda_0 t}} = \lambda_0$

The failure frequency at time t is constant

Conditional Reliability

- Probability of operating for time t, after (non repairable) system has operated for time T

For exponential distribution:

$$R(t,T) = \frac{R(t+T)}{R(T)} = \frac{\exp -\lambda(t+T)}{\exp(-\lambda T)} = \exp(-\lambda t)$$

Mean Time to Failure MTTF

$$MTTF = \int_0^{\infty} t.f(t)dt$$

$$\Rightarrow \int_0^{\infty} R(t)dt \qquad \text{(General Result)}$$

$$= \frac{1}{\lambda_0} \exp\left(\frac{-\lambda_0}{t}\right)\Big]_0^{\infty} = \frac{1}{\lambda_0}$$ For exponential distribution

i.e. interpret Reliability as $R(t) = \exp\left(\frac{-t}{MTTF}\right)$

Weibull Distribution

$f_w(t)$ is Weibull probability distribution function

$$f_w(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \exp-\left(\frac{t-\gamma}{\eta}\right)^{\beta} \longrightarrow$$ "3-parameter distribution"

$\gamma = 0 \longrightarrow$ "2-parameter Weibull"

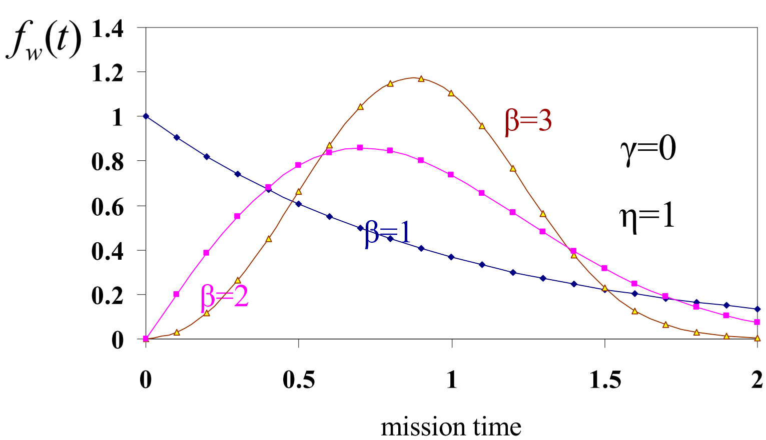
$\beta = 1, \gamma = 0,\ \eta = \lambda_o^{-1} \longrightarrow$ exponential

3 - parameter distribution $R(t) = \exp^{-\frac{(t-\gamma)^{\beta}}{\eta}}$

Weibull Distribution

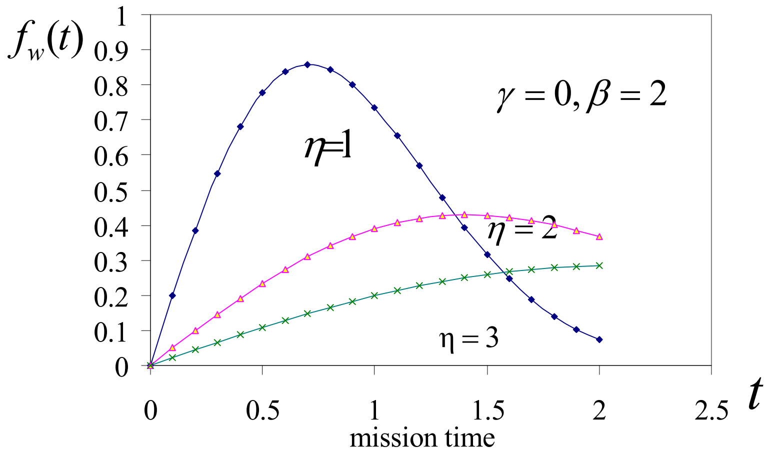
- $\beta > 0$ is "shape factor"
  - $0 < \beta < 1$ early failure
  - $\beta = 1$ constant rate
  - $\beta > 1$ wear out
- $\eta > 0$ is "scale factor"
- γ is "location parameter"
  - $\gamma < 0$ Pre-existing failure (storage, transport)
  - $\gamma = 0$ failure begins t = 0
  - $\gamma > 0$ failure free period t = 0

Weibull Distribution



Effect of shape parameter

Weibull Distribution



Effect of scale parameter

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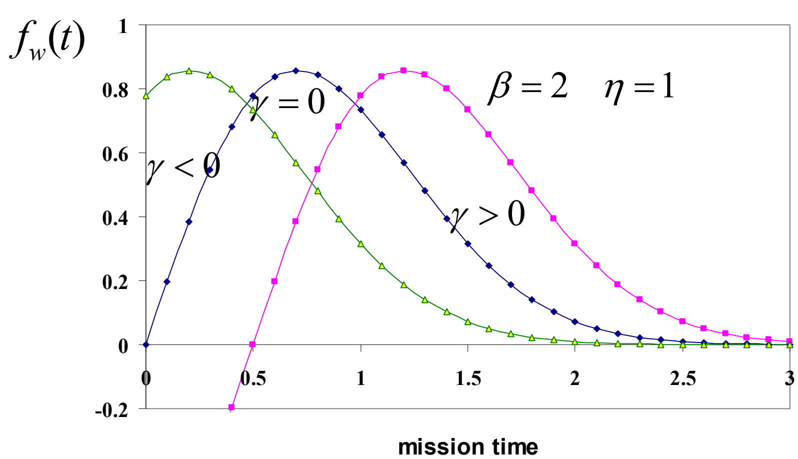
Weibull Distribution



Effect of location parameter

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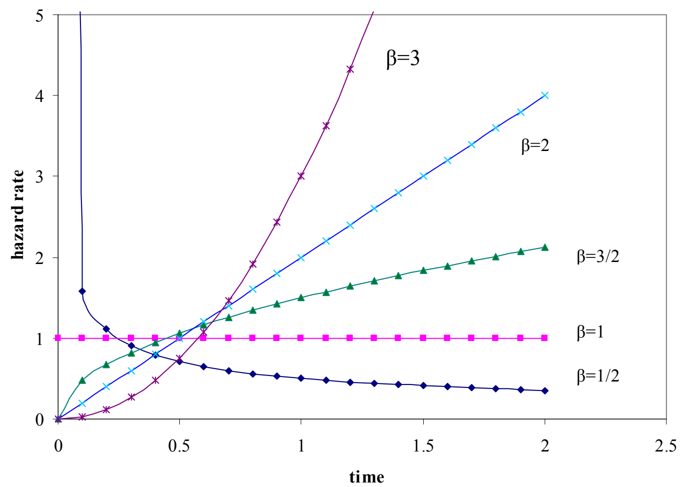
Weibull Distribution



Effects of shape parameter β

Series Reliability

- Failure of a system if any component fails. It is a function of the weakest link in the system.

$$R_{ss} = \prod_{i=1}^{n} R_i = R_1.R_2.R_3\ldots R_n,$$

$$\lambda_{ss} = \sum_{1}^{n} \lambda_i$$

$$MTTF_{ss} = \frac{1}{\lambda_{ss}}$$

Effect of Complexity on System Reliability

No. of Components in series	System reliability for individual component reliability of			
	99.999%	99.99%	99.90%	99.00%
10	99.99%	99.90%	99.004%	90.44%
100	99.90%	99.01%	90.48%	36.60%
250	99.50%	99531%	77.87%	8.1%
500	99.50%	99.12%	60.64%	0.66%
1000	99.01%	99.48%	36.77%	0.004%

Parallel Reliability

- If there are n redundant components

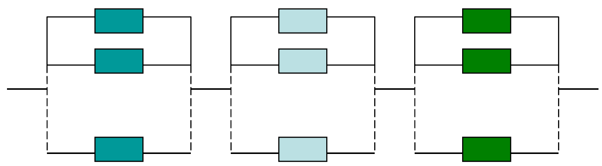
$$R_{ps} = 1 - Q_{ps} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

note: for n=2

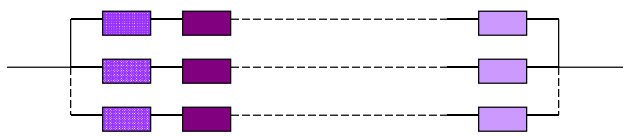
$$\lambda_{ps} = \frac{\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} - (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2) t}}{e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2) t}}$$

Different Systems.

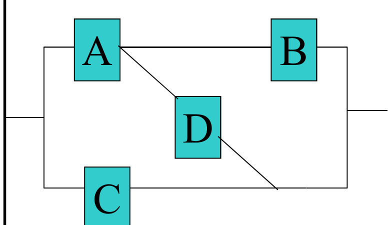
Series-Parallel



Parallel-Series



Complex System

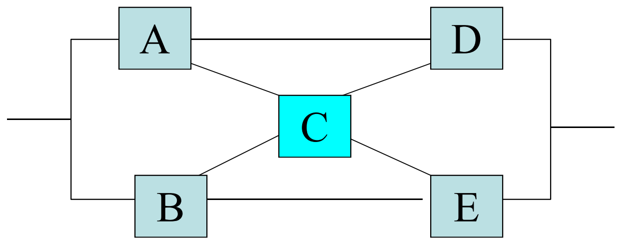


$$R = R_C + R_A R_B - R_A R_B R_C$$
$$+ R_A R_D - R_A R_D (R_B R_C)$$
$$- R_A R_D R_C (1 - R_B)$$
$$- R_A R_B R_D (1 - R_C)$$

Operating states	Non operating states
$C \, . \, ABD$	$\overline{C} \, . \, \overline{A} \, . \, BD$
$C \, . \, AB \overline{D}$	$\overline{C} \, . \, \overline{A} \, . \, \overline{B} D$
$C \, . \, A \overline{B} D$	$\overline{C} \, . \, \overline{A} \, . \, B \overline{D}$
$C \, . \, A \overline{B} \overline{D}$	$\overline{C} \, . \, \overline{A} \, . \, \overline{B} \overline{D}$
$C \, . \, A \overline{B} D$	$\overline{C} \, . \, A \, . \, \overline{B} \overline{D}$
$C \, . \, \overline{A} BD$	
$C \, . \, \overline{A} B \overline{D}$	
$C \, . \, \overline{A} \overline{B} D$	
$C \, . \, \overline{A} \overline{B} \overline{D}$	
$\overline{C} \, . \, ABD$	
$\overline{C} \, . \, A \overline{B} D$	
$\overline{C} \, . \, AB \overline{D}$	

$$R = R_A R_B R_C R_D + R_A R_B R_C (1 - R_D) + \ldots$$

Complex System



Key component ⟶ C and using decomposition rules of probability gives

R=R (C). R(AD or BE or AE or BD) + R (AD or BE) [1-R(C)] etc

Accelerated Testing

- Failure Mechanism (diffusion, corrosion, etc)

$$r = r_0 \exp^{-E_0/kT}$$

So, accelerated at higher temperature

time to failure α 1/r

So, times to failure t_1 at T_1 , t_2 at T_2

$$\frac{t_1}{t_2} = \exp\frac{E_0}{k}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

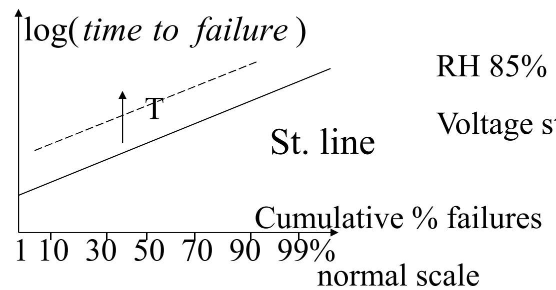
Acceleration factor

$t_1 / t_2 > 1$ *for* $T_1 < T_2$

Accelerated Testing

$$time\ to\ failure = r_0^{-1} \exp^{E_o / kT}$$

$$\log(time\ to\ failure) = -\log r_0 + \frac{E_0}{k}\frac{1}{T}$$



RH 85% & 85^{0}C

Voltage stress

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Physical Basis of Distribution

- *Exponential Distribution:* $\lambda(t) = const.$(bottom of the standard bath tub curve)

Example: Many different failure modes mask variations with time,

Apparent λ_0 constant.

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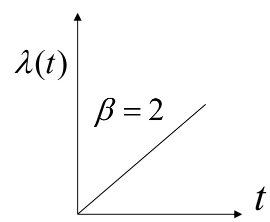
Physical Basis of Distribution

- *Weibull Distribution:*

  example: Rayleigh distribution $\beta = 2$ and

  $\lambda(t)$ is directly proportional to t

  ⟶ Time accumulated damage.

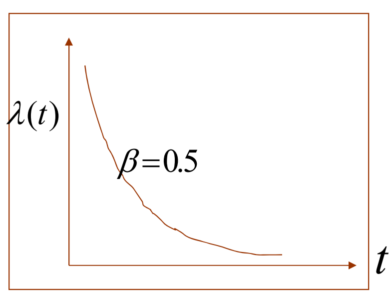


Physical Basis of Distribution

- *Weibull Distribution:* $\beta = 0.5$

  many competing defects sites, only one causes the failure (e.g. capacitor with weak dielectric points)

Damage susceptible population removed.



Physical Basis of Distribution

- *Lognormal Distribution:*

Example: Filament burnout i.e. single weak point.

Resistivity at site increases with time i.e $\rho_0 < \rho_1 < \rho_2 < .....\rho_n$ failure as time ⟶

Assume the increase

$\rho_{i+1} \quad \alpha \quad \rho_i$

$$\therefore \rho_{i+1} = \rho_i (1+\delta)$$

$$\therefore \rho_n = \rho_0 \prod_{i=0}^{n} (1+\delta_i)$$

where δ_i is an independent distributed random variable.

$$\ln \rho_n = \ln \rho_0 + \sum_{i=0}^{n} \ln(1+\delta_i)$$

Therefore, $\ln \rho_n$ is normally distributed and ρ_n (end of life resistivity) is log-normally distributed.

and for $\delta_i << 1$

$$\approx \ln \rho_0 + \sum_{i=0}^{n} \delta_i$$

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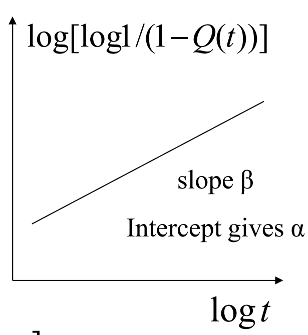
Plotting :Weibull Distribution

$$Q(t) = 1 - \exp^{-(t/\alpha)^{\beta}}$$

$$\ln\{1 - Q(t)\} = -(t/\alpha)^{\beta}$$

$$\ln\{-\ln[1 - Q(t)] = \beta \ln t - \beta \ln \alpha$$



or use Cumulative Hazard rate

$$H(t) = \int_0^t \lambda(t)dt = -\ln[R(t)] = -\ln[1 - Q(t)]$$

$\therefore$ plot $\log H(t)$ vs $\log t$

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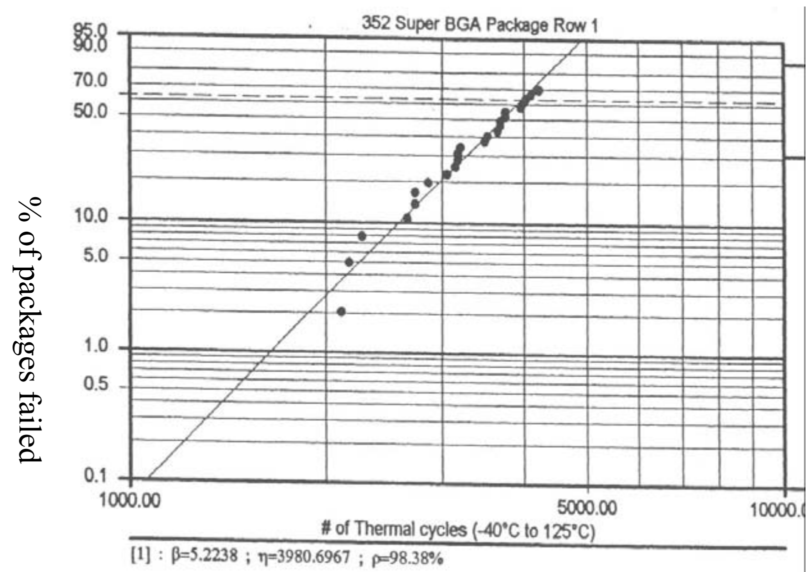
Weibull Plot Of The Accelerated Temperature Cycling



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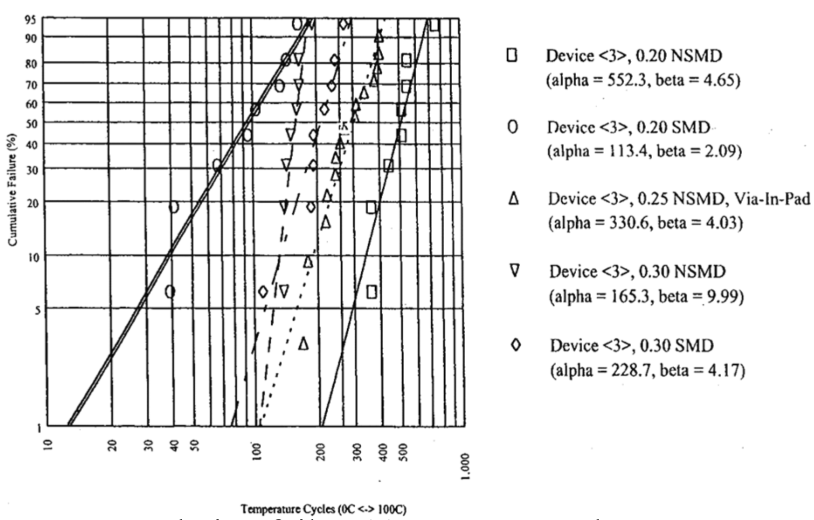
Weibull Plot



cumulative failure% vs temp. cycles

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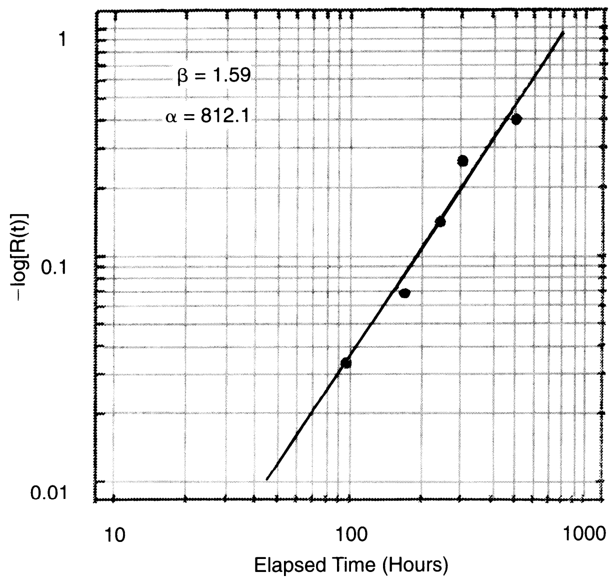
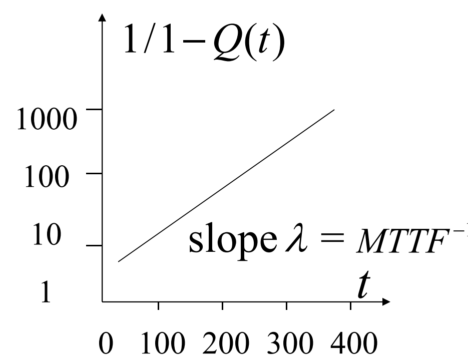


Fig. 4-11 Weibull plot of bonding-pad corrosion failures. Increasing failure rate is indicated by the value of $\beta = 1.59$. From K. D. Hong, S. S. Lee, J. Y. Kim, S. Daniel, and C. K. Yoon, *33rd Annual Proceedings of the IEEE Reliability Physics Symposium*, p. 85 (1995).

Plotting: Exponential Distribution

$$Q(t) = 1 - \exp^{-\lambda t}$$

$$\therefore H(t) = \ln \frac{1}{1 - Q(t)} = \lambda t$$



Example 4-3 A collection of 60 laser diodes were tested, and 7 failures occurred after 1000 hours. The lifetimes obtained are tabulated below.

Rank Number (i)	Lifetime (h)	$F(t_i) = (i - 0.3)/(60 + 0.4)$
1	181	0.012 (1.2%)
2	299	0.028 (2.8%)
3	389	0.045 (4.5%)
4	430	0.061 (6.1%)
5	535	0.078 (7.8%)
6	610	0.094 (9.4%)
7	805	0.111 (11.1%)

1. Plot the results in both Weibull and lognormal fashion.
2. What is the mean time to failure for each?
3. For the lognormal plot, what is the value of σ?
4. For the Weibull plot, what is α?

Answer 1. First, $F(t_i)$ is calculated for $n = 60$ and $i = 1, 2, 3, \ldots, 7$ and entered in the table. The F values are plotted versus log time on Weibull and lognormal paper as shown in Figs. 4-13a and b.
2. At a value of $F = 0.5$, MTTF = 2170 h (Weibull) and 3600 h (lognormal).
3. It was shown above that $\sigma = \ln [t_{0.5}/t_{0.159}]$. Substituting, $\sigma = \ln [3600/980] = 1.30$.
4. From Eq. 4-16 we note that when $t = \alpha$, $\ln [1 - F(t)] = -1$. Thus, $1 - F(t) = 0.368$, and $F(t) = 0.632$. At this value, which is noted on the ordinate axis, $\alpha = t = 2990$ h.

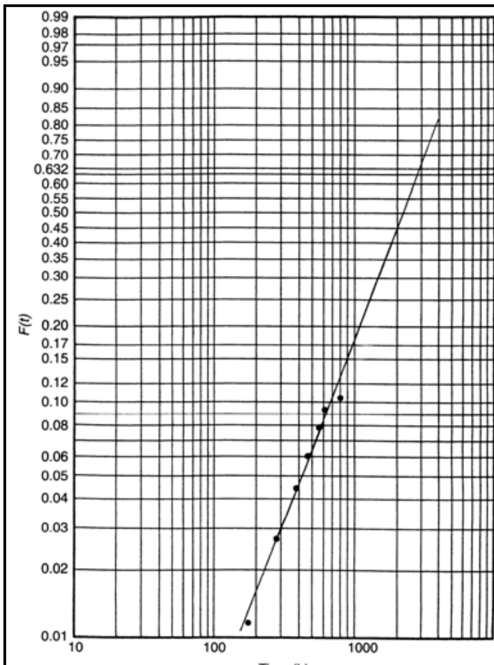


Fig. 4-13 Laser diode failure times from Example 4-3 (a) plotted on Weibull probability paper.

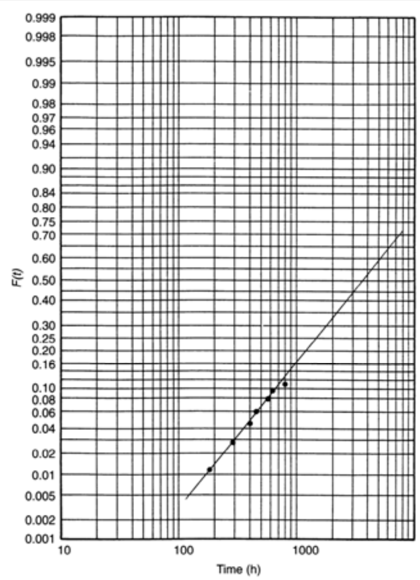


Fig. 4-13 (b) plotted on Lognormal probability paper.

"sport," or freak, population that exhibits untimely failures at short times. Each component behavior in this plot is linear, and a superposition of the two weighted according to the individual relative proportions yields the observed bimodal failure behavior. While the sport population had a median time to failure of ~1.7 h, the value for the main distribution was ~180 h.

	Weibull α>0 β>1t>0	Exponential β=1	Rayleigh β=2
probability density function $f(t)$	$\beta \frac{t^{\beta-1}}{\alpha^{\beta}} \exp-(\frac{t}{\alpha})^{\beta}$	$\lambda \exp-\lambda t$ $= \frac{1}{\alpha} \exp-(\frac{t}{\alpha})$	$kt \exp-\frac{kt^2}{2}$ $= 2\frac{t}{\alpha^2} \exp(-\frac{t^2}{\alpha^2})$
survivor function $R(t) = \int_t^{\infty} f(t)dt$	$\exp-\left(\frac{t}{\alpha}\right)^{\beta}$		$\exp - \frac{kt^2}{2}$ $= \exp - (\frac{t}{\alpha})^2$
cumulative failures $Q(t) = 1 - R(t)$	$1 - \exp-\left(\frac{t}{\alpha}\right)^{\beta}$		
hazard rate $\lambda(t) = f(t)/R(t)$	$\beta \frac{t^{\beta-1}}{\alpha^{\beta}}$	$\frac{1}{\alpha}(= \lambda)$	$\frac{2t}{\alpha^2}(= kt)$

Assignment #8

- Dally:
  - 11.32/33 14.14/15
  - 11.35 14.16
  - 11.36 14.32
  - 14.3/4/7 14.34