ECE414/514 Electronics Packaging

Spring 2012 Lecture 12 Thermal B Convection & Radiation

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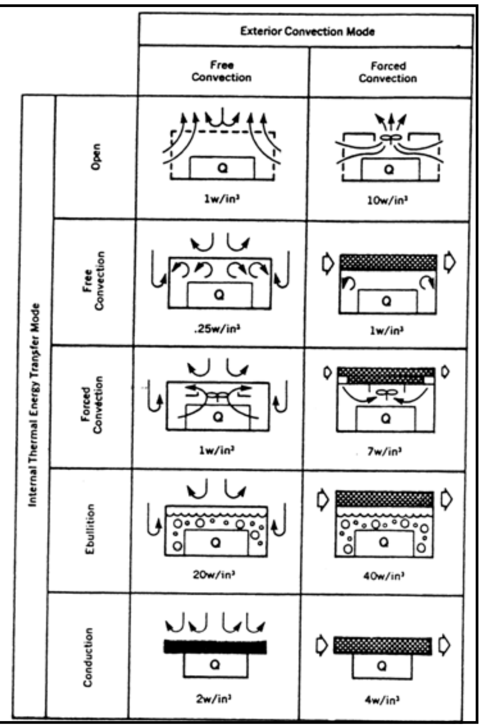
Natural and Forced Convection:

Heat transfer from a solid to a fluid via a boundary layer



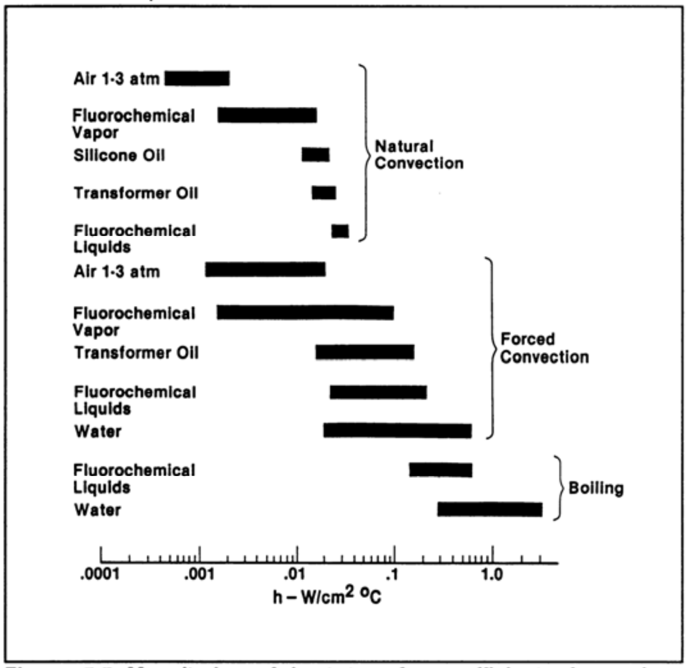


Figure 5.5 Magnitudes of heat transfer coefficients for various coolants and modes of convection.

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The boundary layer impedes thermal transfer

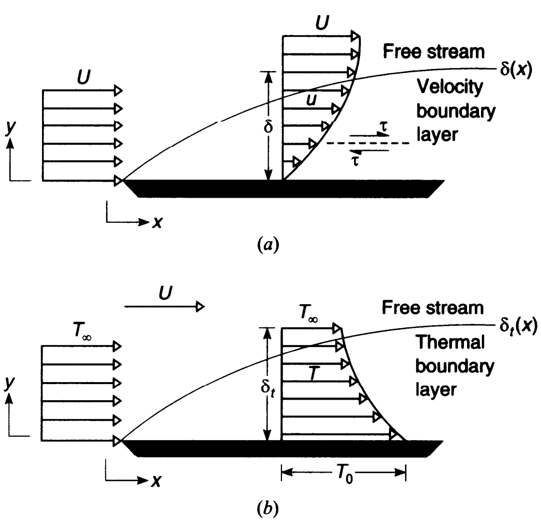


Figure 5.10 (*a*) Velocity and (*b*) thermal boundary layer development.

Laminar and turbulent flow

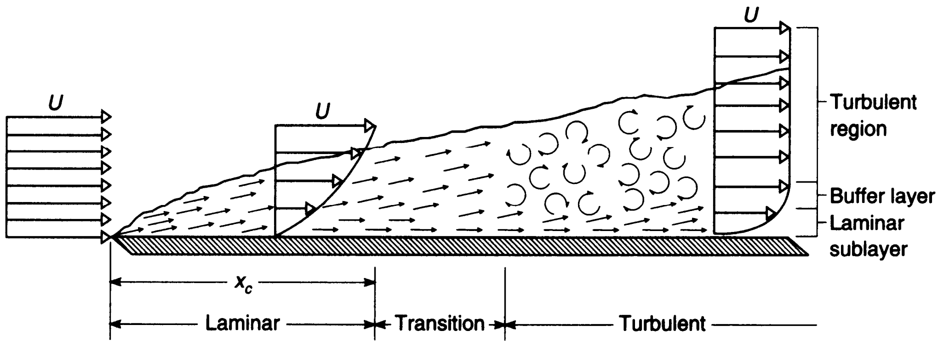
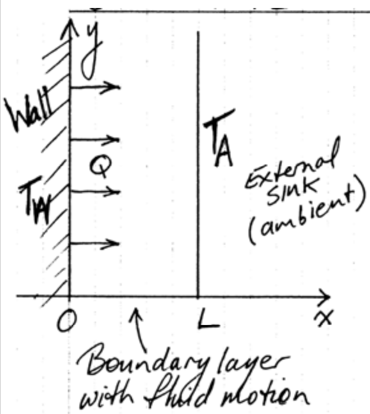


Figure 5.11 Laminar to turbulent boundary layer transition.

Characteristic Parameters



$Q_{CV} = h A (T_W - T_A)$

↑ convective heat transfer coefficient

$R_{conv} = 1/hA$

WITHOUT fluid motion in boundary layer:

Conduction heat transfer

$$Q_{CN} = \frac{kA}{L}(T_W - T_A) = \frac{T_W - T_A}{R_T}$$

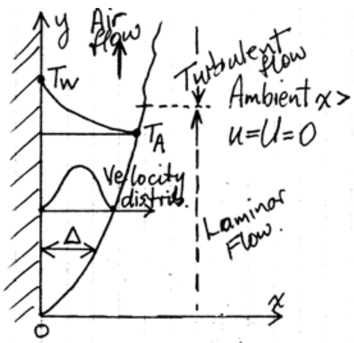
NUSSFELT NUMBER (Non-dimensional fluid heat transfer coefficient)

$$Nu = \frac{Q_{CV}}{Q_{CN}} = \frac{h}{k} L$$

For $Nu \sim 1$, negligible fluid motion contribution (low velocities)

$Nu \sim 100$, convection dominant (eg turbulent)

Characteristic parameters cont'd



Convection velocity $u_{av} = \left[gy \left(1 - \frac{\rho_w}{\rho_a}\right)\right]^{1/2}$

gravity accel

air densities

GRASHOF NUMBER $Gr_y = \frac{g\beta}{\nu^2}(T_W - T_A)y^3$

β coeff thermal expansion (vol)

ν kinematic viscosity

PRANDTL NUMBER $Pr = \frac{\mu c_p}{k} = \frac{\mu}{\rho \alpha} = \frac{\nu}{\alpha}$ ≈ 0.72 for air at normal temperatures

(non-dimensional fluid characteristic)

μ - dynamic viscosity

α - thermal diffusivity

RAYLEIGH NUMBER

$Ra = Gr . Pr$

(REYNOLDS NUMBER Re)

(non-dimensional fluid velocity)

$R_e = UL\rho/\mu$

$N_u = C R_e^m Pr^n$ (Nusselt number)

TABLE 6.3 Nomenclature.

Symbol	Units	Description
A	m^2	Area
C_p	J/kgK	Specific heat
D	m	Diameter
g	m/s^2	Gravitational acceleration
h	W/m^2K	Heat transfer coefficient
H	m	Characteristic length (length or width)
k	W/mK	Thermal conductivity
L	m	Characteristic length (length or width or diameter)
m	kg	Mass
Nu		Nusselt number = (hL/k)
Pr		Prandtl number = $(\mu C_p/k)$
Q	W	Heat flow
R	°C/W or K/W	Thermal resistance
Ra		Rayleigh number
Re		Reynolds number $(\rho VL/\mu)$
t	s	Time
T	°C or K	Temperature
ΔT	°C or K	Temperature difference
V	m/s	Velocity
W	m	Characteristic length (length or width or diameter)
x	m	Length or distance
Greek symbols		
β	1/K	Volumetric expansion coefficient
δ	m	Thickness or gap width
ϵ		Emissivity
ρ	kg/m^3	Density
μ	$N\ s/m^2$	Dynamic viscosity
Subscripts		
amb		Ambient
c		Contact region
f		Fluid
o		Open space
r		Radiation
s		Surface
w		Wall

FREE CONVECTION VS FORCED CONVECTION
(vertical surface) (horizontal surface)
LAMINAR FLOW VS TURBULENT FLOW

1. Free Convection / Vertical Surface
(a) Laminar Flow

Experimental result: for laminar flow region $10^4 \le Gr_y \le 10^9$

$$Nu_y \approx \frac{0.75\, P_r^{1/2}\, Gr_y^{1/4}}{(2.435 + 4.884\, Pr^{1/2} + 4.953\, Pr)^{1/4}} \xrightarrow[\text{air } Pr \approx 0.72]{} 0.357\, Gr_y^{1/4}$$

$$\& \quad h_y = \frac{k}{y} Nu_y = 0.357 \frac{k}{y} Gr^{1/4}$$

$$= 0.357\, k \left[\frac{g\beta}{\nu^2}(T_W - T_A)\right]^{1/4} y^{-1/4}$$ =const.$y^{-1/4}$

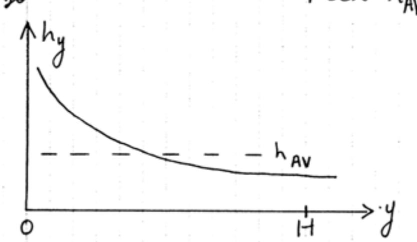
Decrease in h_y with y due to increasing boundary layer thickness

$$\text{Need } h_{AV} = \frac{1}{H}\int_0^H h_y\, dy$$

$$= \text{const } H^{-1} \int_0^H y^{-1/4}\, dy$$

$$= \frac{4}{3} \text{ const } H^{-1} \left[y^{3/4}\right]_0^H$$ =(4/3)const $H^{-1/4}$

$$= \frac{4}{3} \left[h_y\right]_{y=H}$$



1. Free Convection / Vertical Surface
(b) Turbulent Flow

Turbulent flow <u>begins</u> at $Gr \gtrsim 10^7$

& is <u>fully developed</u> at $Gr \gtrsim 10^9$ (where $Ra \sim 10^9$)

For $Gr > 10^9$, experimentally $Nu_y \approx 0.0221\, Gr^{2/5}$ ← for air $Pr \approx 0.72$

so $h_y = \frac{k}{y} Nu_y = \frac{\text{const}}{y}\, y^{6/5} \propto y^{1/5}$

gives $h_{AV} = \frac{1}{H}\int_0^H h_y\, dy = \frac{5}{6}\left[h_y\right]_{y=H}$

i.e. $\left.h_{AV}\right)_{turbulent} \sim 10\times \left.h_{AV}\right)_{laminar}$ for $Gr_H \sim 10^9$ for both.

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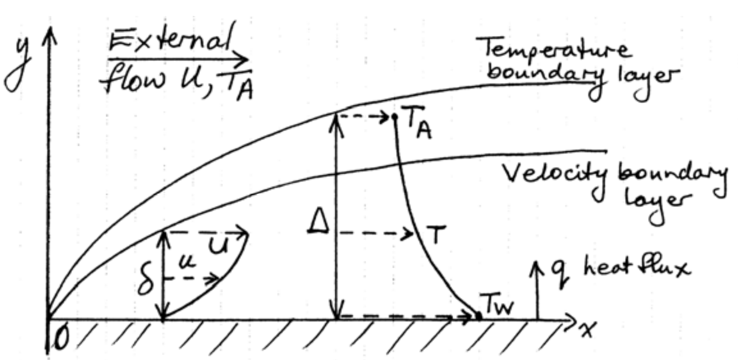
2. Forced Convection / Horizontal Surface
(a) Laminar Flow



$$\frac{u}{U} \approx 2\eta - 2\eta^3 + \eta^4 \quad \text{where } \eta(x,y) = \frac{y}{\delta(x)}$$

$$\& \quad \frac{T - T_W}{T_A - T_W} \approx 2\xi - 2\xi^3 + \xi^4 \quad \text{where } \xi(x,y) = \frac{y}{\Delta(x)}$$

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2(a) cont'd

Momentum integral equation:

Shear stress at plate $\tau_w = \frac{d}{dx}\int_0^{\infty} \rho u (U-u)\, dy = \mu \left[\frac{\partial u}{\partial y}\right]_{y=0}$

$= \mu U 2 \frac{1}{\delta} = \frac{2\mu U}{\delta}$

where $\delta = \frac{5.84\, x}{\sqrt{Re_x}}$ (theory) $\xrightarrow{expt} \frac{5x}{\sqrt{Re_x}}$ & $Re_x = \rho \frac{U x}{\mu} = \frac{U x}{\nu}$ Reynolds Number

$\therefore \quad \delta \propto x^{1/2}$

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2(a) cont'd

Energy integral equation:

$q_w/A_x = \frac{d}{dx}\int \rho c_p u (T-T_A)\, dy = -k\left[\frac{\partial T}{\partial y}\right]_{y=0}$

$\rightarrow q/A_w = 2k(T_w - T_A)/\Delta$

& $h_x = q_w/(T_w - T_A) = 2k/\Delta$

$\longrightarrow 0.298\, \rho^{1/2} U^{1/2} k / \mu^{1/2} x^{1/2}$

$\therefore \; h_{AV} = \frac{1}{L}\int_0^L h_x\, dx = const \frac{1}{L}\int_0^L x^{-1/2} dx = 2\, h_L$

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2(a) cont'd

Procedure to calculate heat transfer rate for U:

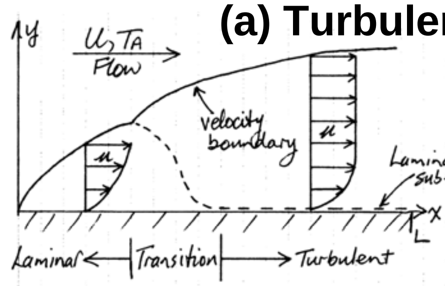
$$Re_L = UL/\nu$$

Substitute into $Nu_L = 0.298\sqrt{Re_L}$

Then $h_L = \frac{k}{L} Nu_L \longrightarrow h_{AV} = 2 h_L$

(Air parameters $\alpha, \nu, k, \rho, \mu$ etc vary with T; use $\frac{T_A + T_W}{2}$)

2. Forced Convection / Horizontal Surface
(a) Turbulent Flow



Turbulent for $x > x_c$

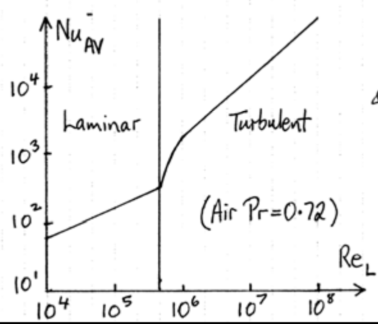
where $x_c = \left(\frac{5\nu}{U}\right) \times 10^5$

& $Re_{x_c} = \frac{U}{\nu} x_c = 5 \times 10^5$

For turbulent flow $Nu_x = 0.0296\, Re_x^{4/5} Pr^{1/3} \xrightarrow{Pr = 0.72} 0.0265\, Re_x^{4/5}$

& $Nu_{AV} = 0.0332\, Re_L^{4/5}$

$h_{AV} = \frac{k}{L} Nu_{AV}$ etc.



Turbulence increases heat flow!

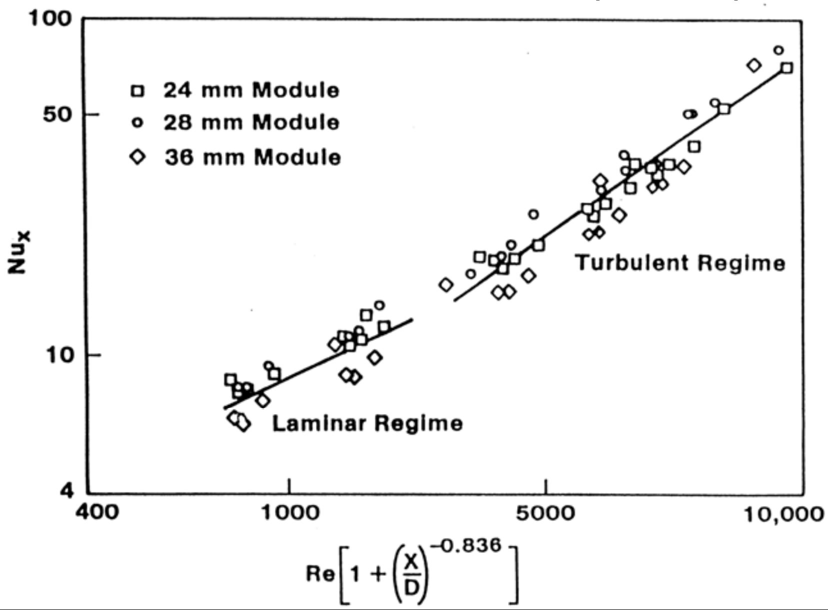
TABLE 6.7 Simplified equations for convective heat transfer coefficients for air at 50°C.

Mode of Convection	Correlation
Natural convection from an isothermal vertical surface	$h = 1.51 \left(\frac{\Delta T}{L}\right)^{1/4}$
Natural convection from a vertical isoflux surface	$h = 1.338 \, [q''H^4]^{1/5}$
Natural convection on an isothermal horizontal surface	$h = 1.381 \left(\frac{\Delta T}{L}\right)^{1/4}$
Forced convection on an isothermal flat plate	$h = 3.886 \left(\frac{V}{L}\right)^{0.5}$ for laminar flow $h = 0.099 \left(\frac{V^4}{L}\right)^{0.2}$ for turbulent flow
Forced convection on an isoflux flat plate	$h = 2.651 \left(\frac{V}{L}\right)^{0.5}$ for laminar flow $h = 0.103 \left(\frac{V^4}{L}\right)^{0.2}$ for turbulent flow
Laminar forced convection in a circular tube	$h = \frac{1}{D}\left(0.131 + \frac{1563\left(\frac{VD^2}{L}\right)}{1 + 32.50\left(\frac{VD^2}{L}\right)^{2/3}}\right)$
Turbulent forced convection in a circular tube	$h = 0.00071 \left(\frac{V^4}{D}\right)^{0.2}$ for heating $h = 0.000736 \left(\frac{V^4}{D}\right)^{0.2}$ for cooling

TABLE 6.6 Correlation for convective heat transfer coefficients [11,15].

Mode of Convection	Correlation
Natural convection from an isothermal vertical surface	$h = C(\text{Ra})^n = C\left(\frac{k_f}{L}\right)\left(\frac{\rho_f^2 \beta g C_p \, \Delta T \, L^3}{\mu_f k_f}\right)^n$ $C = 0.59$, $n = 1/4$ for $1 < \text{Ra} < 10^9$ $C = 0.10$, $n = 1/3$ for $10^9 < \text{Ra} < 10^{14}$
Natural convection from a vertical isoflux surface	$h = 0.631\left(\frac{k_f}{L}\right)\left[\frac{C_p \rho^2 g \beta q'' H^4}{\mu k_f^2}\right]^{1/5}$
Natural convection on an isothermal horizontal surface	$h = C(\text{Ra})^n = C\left(\frac{k_f}{L}\right)\left(\frac{\rho_f^2 \beta g C_p \, \Delta T \, L^3}{\mu_f k_f}\right)^n$ $C = 0.54$, $n = 1/4$ for $10^4 \le \text{Ra}_L \le 10^7$ $C = 0.15$, $n = 1/3$ for $10^7 \le \text{Ra}_L \le 10^{11}$
Forced convection on an isothermal flat plate	$h = C(\text{Re})^n (\text{Pr})^{1/3} = C\left(\frac{k_f}{L}\right)\left(\frac{\rho_f V L}{\mu_f}\right)^n\left(\frac{\mu_f C_p}{k_f}\right)^{1/3}$ Laminar: $C = 0.664$, $n = 1/2$ Turbulent: $C = 0.0296$, $n = 4/5$
Forced convection on an isoflux flat plate	$h = C(\text{Re})^n (\text{Pr})^{1/3} = C\left(\frac{k_f}{L}\right)\left(\frac{\rho_f V L}{\mu_f}\right)^n\left(\frac{\mu_f C_p}{k_f}\right)^{1/3}$ Laminar: $C = 0.453$, $n = 1/2$ Turbulent: $C = 0.0308$, $n = 4/5$
Laminar forced convection in a circular tube	$h = \frac{k_f}{D}\left(3.66 + \frac{0.0668 \, (D/L) \, \text{Re}_D \, \text{Pr}}{1 + 0.04[(D/L) \, \text{Re}_D \, \text{Pr}]^{2/3}}\right)$ $\text{Re}_D = \left(\frac{\rho_f V D}{\mu_f}\right)$, $\text{Pr} = \left(\frac{\mu_f C_p}{k_f}\right)$ for $\text{Re}_D < 2000$
Turbulent forced convection in a circular tube	$\text{Nu} = 0.023 \, \text{Re}_D^{0.8} \, \text{Pr}^n \, (\text{Re}_D > 2000)$ $n = 0.4$ for heating $n = 0.3$ for cooling

External thermal resistance vs. flow in non-dimensional form (Tai & Lucas)



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Module array, with one odd-sized

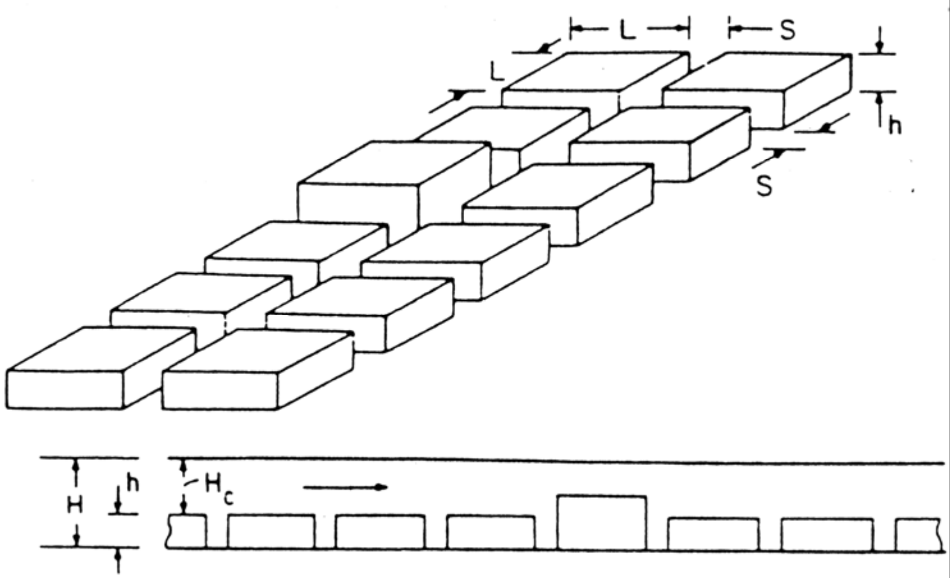
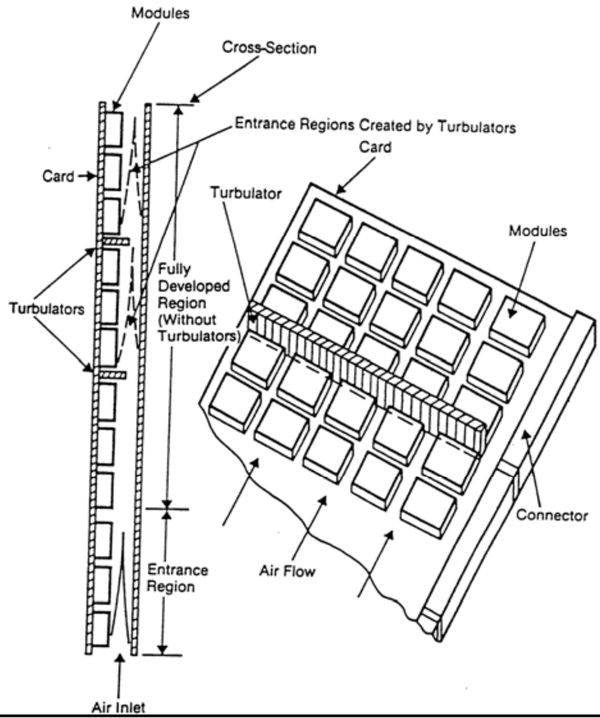


Figure 7.42 Array of modules with one odd-sized module



Turbulator

Note:
“Fully developed flow”
=
boundary layers meet and merge

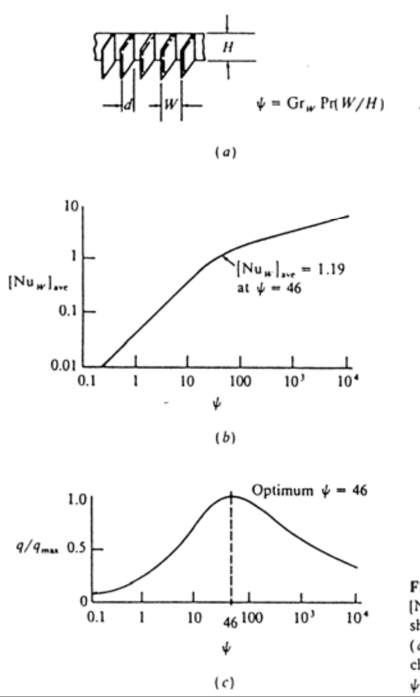


FIGURE 9.11
$[Nu_W]_{ave}$ and q/q_{max} as a function of ψ showing optimum spacing of the fins. (*a*) Geometry of a fin-type heat exchanger. (*b*) $[Nu_W]_{ave}$ as a function of ψ. (*c*) q/q_{max} as a function of ψ.

Optimum PWB spacing

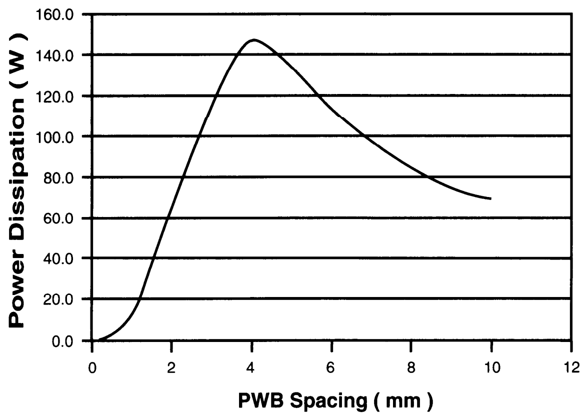
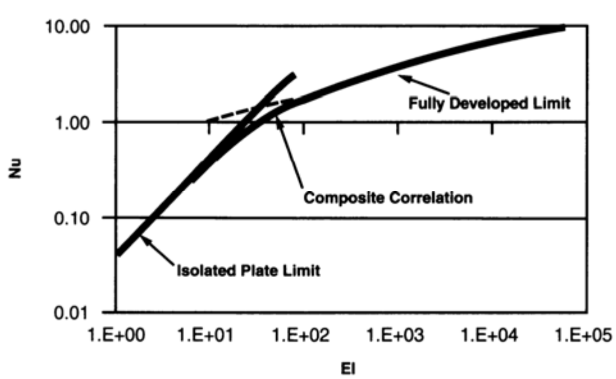


FIGURE 6.18 Variation of total power dissipation from a PWB array with PWB spacing.

Nussfelt Number and PWB Spacing

FIGURE 6.17 Composite correlation for parallel isothermal plates.



Relative heat transfer downstream from a finned heatsink in row 6

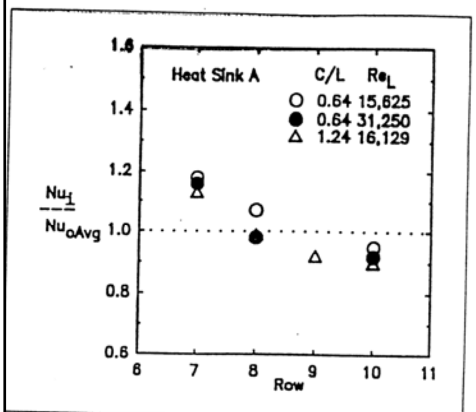


Figure 6.13 Relative heat transfer downstream of a finned heat sink in row 6; turbulent channel flow. Reproduced from [8]

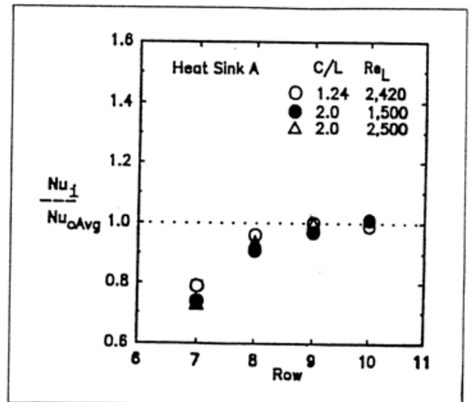


Figure 6.12 Relative heat transfer downstream of a finned heat sink in row 6: laminar channel flow. Reproduced from [8]

Turbulent **Laminar**

Heat Pipe

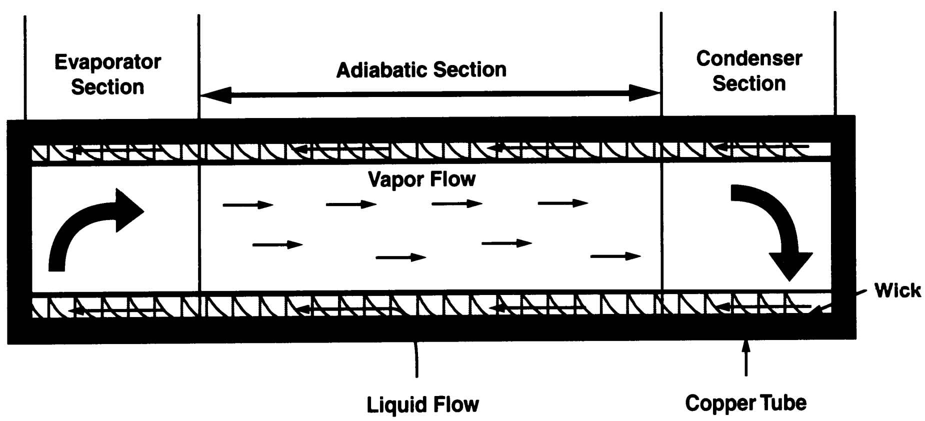


FIGURE 6.23 Longitudinal cross section of a heat pipe.

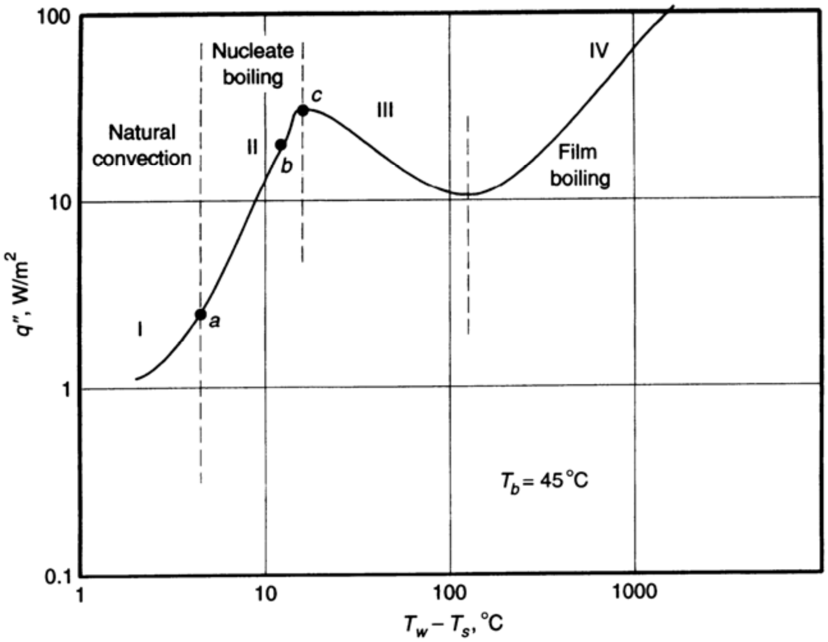
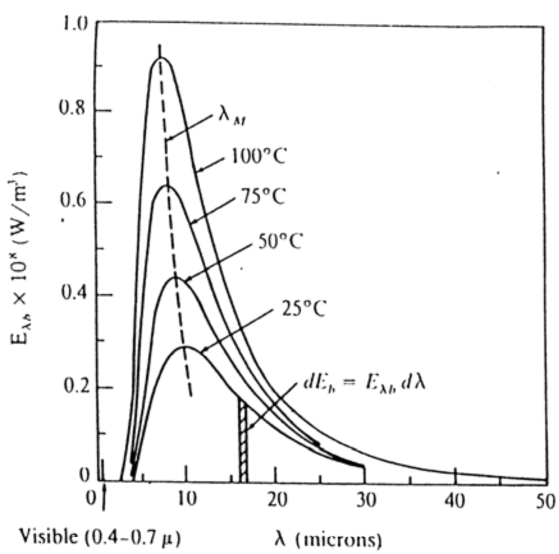


Figure 5.13 Boiling curve for refrigerant 113 at atmospheric pressure.

Black-body Radiation



(*b*) Low temperatures

FIGURE 9.1
Distribution of emissive power from a black body as a function of wave length λ of the radiation source.

Radiation Heat Transfer

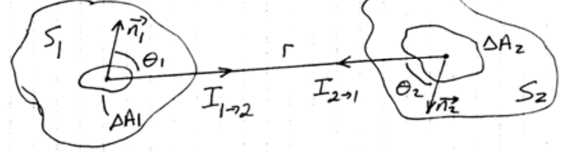
Integrate Planck's Law $\int_0^\infty E_\lambda^b \, d\lambda \rightarrow$ Stefan-Boltzmann Law

Black body $E^b = \sigma T^4$ $\quad \sigma = 5.670 \times 10^{-8} W/m^2K^4$ Stefan-Boltzmann constant

T absolute temperature.

Radiation heat transfer coefficient $h_r = \frac{q^b}{A} / \Delta T = \frac{E^b}{\Delta T}$

In practice, energy radiated to another body & absorbed from other bodies.



$\Delta q_{1\to2} = E_1^b (\Delta A_1 \cos\theta_1)(\cos\theta_2 \Delta A_2)/\pi r^2$

$\Delta q_{2\to1} = E_2^b (\Delta A_2 \cos\theta_2)(\Delta A_1 \cos\theta_1)/\pi r^2$

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$\therefore$ Net transfer $\Delta q = \Delta q_{1\to2} - \Delta q_{2\to1}$

$= (E_1^b - E_2^b)\, \Delta A_1 \Delta A_2 \cos\theta_1 \cos\theta_2 / \pi r^2$

$\& \iint_{S_1 S_2} \rightarrow \quad q^b = (E_1^b - E_2^b) A_1 F_{1\text{-}2} \leftarrow$ shape factor where $A_2 F_{2\text{-}1} = A_1 F_{1\text{-}2}$

$= \sigma (T_1^4 - T_2^4) A_1 F_{1\text{-}2}$

Note: This result for perfect "black" surfaces. Modify for practical surface emissivity ϵ

For any surface & incident radiation: $\alpha + \rho + \tau = 1$ (α absorbed, ρ reflected, τ transmitted)

$\tau = 0$ for opaque $\quad \alpha = \epsilon \quad \& \therefore \rho = 1 - \epsilon$

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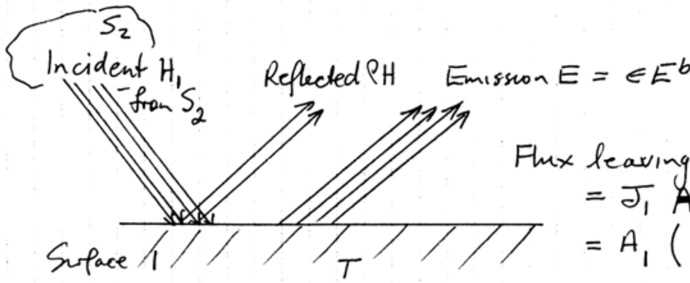


Flux leaving surface

$= J_1 A_1$

$= A_1 \left(\epsilon_1 E_1^b + \rho_1 H_1 \right)$

$H_1 = F_{2\text{-}1} A_2 J_2 \qquad \therefore J_1 A_1 = \epsilon_1 E_1^b A_1 + \rho_1 A_1 A_2 F_{2\text{-}1} J_2$

$J_1 = \epsilon_1 E_1^b + \rho_1 F_{2\text{-}1} J_2 A_2$

so $F_{2\text{-}1} J_2 A_2 = (J_1 - \epsilon_1 E_1^b)/\rho_1 = F_{1\text{-}2} A_1 J_2$

& net heat flux from surface 1 $(S_1) = q_1 = (J_1 - H_1) A_1$

& net heat flux from surface 1 $(S_1) = q_1 = (J_1 - H_1) A_1$

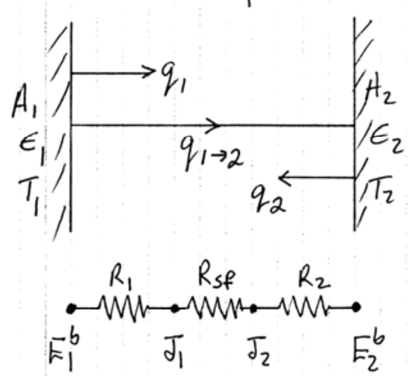
$= J_1 A_1 - A_1 A_2 F_{2\text{-}1} J_2$

$= J_1 A_1 - A_1 A_1 F_{1\text{-}2} J_2$

$= J_1 A_1 - A_1 (J_1 - \epsilon_1 E_1^b)/\rho_1$

$= \frac{\epsilon_1 A_1 E_1^b}{1-\epsilon_1} + J_1 A_1 \left(1 - \frac{1}{1-\epsilon_1}\right)$

$q_1 = A_1 \left(\frac{\epsilon_1}{1-\epsilon_1}\right)\left(E_1^b - J_1\right)$



← Use in electrical analog

$R_1 = \frac{1-\epsilon_1}{A_1 \epsilon_1} \qquad R_2 = \frac{1-\epsilon_2}{A_2 \epsilon_2} \qquad R_{sf} = \frac{1}{A_1 F_{1\text{-}2}} = \frac{1}{A_2 F_{2\text{-}1}}$

$$R_1 = \frac{1-\epsilon_1}{A_1\epsilon_1} \qquad R_2 = \frac{1-\epsilon_2}{A_2\epsilon_2} \qquad R_{sf} = \frac{1}{A_1 F_{1-2}} = \frac{1}{A_2 F_{2-1}}$$

$$q_1 = \frac{J_1 - J_2}{R_{sf}}$$

$$\& \; q_{1\to 2} = (E_1^b - E_2^b)/(R_1 + R_{sf} + R_2) = h_r A_1 (T_1 - T_2)$$

$$\uparrow \; \sigma(T_1^4 - T_2^4)$$

$$\therefore h_r = \frac{\sigma(T_1^4 - T_2^4)}{A_1(T_1 - T_2)\Sigma R} = \frac{\sigma}{A_1 \Sigma R}(T_1^3 + T_1^2 T_2 + T_1 T_2^2 + T_2^3)$$

$$R_{rad} = 1/h_{rad}A$$

Examples

1. A 6mm x 6mm chip (k_{chip}=150W/m.K) is 4mm thick and placed horizontally on a small 25mm x 25mm board that is 5mm thick with thermal conductivity of 1W/m.K. There is a contact conductance of 10^4W/m^2.K between the chip and the board. The top of the chip is cooled by air at 20°C with h=500W/m^2.K. The bottom is also cooled by air at 20°C, but the heat transfer coefficient is only 200W/m^2.K. If the chip operates at a heat flux of 20W/cm^2, what will be the temperature of the chip circuits if (a) the circuits are on the top, or (b) on the bottom?
2. Two 10cm x 40cm boards are mounted 2.5cm apart with the 40cm edges vertical. Natural convection causes air at 20°C to provide cooling by flowing vertically through the passage formed by the boards. The chips on the boards produce a heat flux of 0.5W/cm^2 from each board. (a) What is the surface temperature of the chips at the top of the boards? (b) What is the temperature if a fan is added that creates an upward velocity of 1m/s?

Examples (cont'd)

3. A 10mm x 10mm silicon chip is cooled with water circulated through 50 parallel etched microchannels of width W=50μm and height H=200 μm on the back of the chip. The heat flux produced by the chip maintains the surface of the microchannels at 350K. All heat generated by the circuits is transferred to the water. What are the water exit temperature and heat flux if water enters the channels at 300K at a rate of 10^{-4}kg/s?

Thermophysical Properties for Heat Transfer Calculations

Thermal Conductivities of Select Materials

Material	k(W/m°C)
Aluminum	
Pure	126
2024 T4	121
6061 T6	156
7075 T6	121
Beryllium copper	82.7
Brass 70 Cu-30 Zn	100
Copper	
Pure	381
Drawn wire	287
Gold	296
Iron wrought	58.8
Kovar	15.6
Lead	32.7
Magnesium	157
Silicon	153
Steel 1020	55.4
Tin	62.3
Titanium	15.6
Zinc	102
Alumina	
95% pure	29.4
90%	12.1
Beryllia	
99.5%	242
95%	156
Glass	
Soft	0.98
Pyrex	1.26
Mica	0.59
Epoxy	
Unfilled	0.21
Filled	2.16
Fiberglass	0.26
Mylar	0.19
Nylon	0.24
Phenolic paper	0.28
Plexiglass	0.19
Polyvinyl chloride	0.16
Rubber	
Butyl	0.26
Silicone	0.19
Silicone grease	0.21
Teflon	0.19
Air	0.026
Water	0.658

Source: James W. Dalley, *Packaging of Electronic Systems*, McGraw-Hill, 1990

Water at Saturation Pressure

Temperature T K	°C	Density ρ (kg/m^3)	Coefficient of Thermal Expansion $\beta \times 10^4$ (1/K)	Specific Heat c_p (J/kg·K)	Thermal Conductivity k (W/m·K)	Thermal Diffusivity $\alpha \times 10^6$ (m^2/s)	Absolute Viscosity $\mu \times 10^6$ (N·s/m^2)	Kinematic Viscosity $\nu \times 10^6$ (m^2/s)	Prandtl Number Pr	$\frac{g\beta}{\nu^2} \times 10^{-9}$ (1/K·m^3)
273	0	999.3	−0.7	4226	0.558	0.131	1794	1.789	13.7	
293	20	998.2	2.1	4182	0.597	0.143	993	1.006	7.0	2.035
313	40	992.2	3.9	4175	0.633	0.151	658	0.658	4.3	8.833
333	60	983.2	5.3	4181	0.658	0.159	472	0.478	3.00	22.75
353	80	971.8	6.3	4194	0.673	0.165	352	0.364	2.25	46.68
373	100	958.4	7.5	4211	0.682	0.169	278	0.294	1.75	85.09
473	200	862.8	13.5	4501	0.665	0.170	139	0.160	0.95	517.2
573	300	712.5	29.5	5694	0.564	0.132	92.3	0.128	0.98	1766.

Source: K. Raznjevic, *Handbook of Thermodynamic Tables and Charts*, McGraw-Hill Book Company, New York, 1976.

Dry Air at Atmospheric Pressure

Temperature T K	°C	Density ρ (kg/m^3)	Coefficient of Thermal Expansion $\beta \times 10^3$ (1/K)	Specific Heat c_p (J/kg·K)	Thermal Conductivity k (W/m·K)	Thermal Diffusivity $\alpha \times 10^6$ (m^2/s)	Absolute Viscosity $\mu \times 10^6$ (N·s/m^2)	Kinematic Viscosity $\nu \times 10^6$ (m^2/s)	Prandtl Number Pr	$\frac{g\beta}{\nu^2} \times 10^{-8}$ (1/K·m^3)
273	0	1.252	3.66	1011	0.0237	19.2	17.456	13.9	0.71	1.85
293	20	1.164	3.41	1012	0.0251	22.0	18.240	15.7	0.71	1.36
313	40	1.092	3.19	1014	0.0265	24.8	19.123	17.6	0.71	1.01
333	60	1.025	3.00	1017	0.0279	27.6	19.907	19.4	0.71	0.782
353	80	0.968	2.83	1019	0.0293	30.6	20.790	21.5	0.71	0.600
373	100	0.916	2.68	1022	0.0307	33.6	21.673	23.6	0.71	0.472
473	200	0.723	2.11	1035	0.0370	49.7	25.693	35.5	0.71	0.164
573	300	0.596	1.75	1047	0.0429	68.9	39.322	49.2	0.71	0.0709
673	400	0.508	1.49	1059	0.0485	89.4	32.754	64.6	0.72	0.0350
773	500	0.442	1.29	1076	0.0540	113.2	35.794	81.0	0.72	0.0193
1273	1000	0.268	0.79	1139	0.0762	240	48.445	181	0.74	0.00236

Source: K. Raznjevic, *Handbook of Thermodynamic Tables and Charts*, McGraw-Hill Book Company, New York, 1976.

TABLE 9.2
Properties of dry air at atmospheric pressure

T °C	T °F	ρ (g / in.^{3})	c_p (J / g °C)	μ (10^{-4}g / in. s)	ν (in.^{2} / s)	k (10^{-4} W / m °C)	Pr	β (10^{-3} / °C)	$g\rho^2/\mu^2$ (10^6 / in.^{3})	$g\beta\rho^2/\mu^2$ (10^3 / in.^{3} °C)
− 18	0	0.0227	1.000	4.195	0.0187	5.803	0.73	3.916	1.12	4.38
0	32	0.0212	1.003	4.403	0.0209	6.168	0.72	3.661	0.899	3.29
19	50	0.0204	1.004	4.519	0.0220	6.374	0.72	3.528	0.812	2.90
38	100	0.0186	1.004	4.856	0.0259	6.945	0.72	3.216	0.569	1.83
66	150	0.0171	1.008	5.150	0.0301	7.473	0.72	2.952	0.446	1.36
93	200	0.0158	1.008	5.442	0.0344	8.000	0.72	2.729	0.324	0.885
121	250	0.0147	1.012	5.780	0.0392	8.440	0.71	2.538	0.260	0.674
149	300	0.0137	1.012	6.085	0.0441	8.968	0.71	2.370	0.195	0.462
177	350	0.0129	1.016	6.305	0.0491	9.495	0.70	2.214	0.161	0.366
204	400	0.0121	1.024	6.614	0.0544	9.979	0.69	2.094	0.128	0.269

TABLE 9.1
Emissivity ε of different materials $T = 100°C$

Material	Condition	
Alleghany metal	No. 4 polish	.13
Alleghany alloy No. 66	Polished	.11
Aluminum	Commercial sheet	.09
Aluminum	Polished	.095
Aluminum	Rough polish	.18
Brass	Polished	.059
Carbon	Rough plate	.77
Carbon, graphitized	Rough plate	.76
Chromium	Polished	.075
Copper	Polished	.052
Copper–nickel	Polished	.059
Iron	Dark gray surface	.31
Iron	Roughly polished	.27
Lampblack	Rough deposit	.84
Molybdenum	Polished	.071
Nickel	Polished	.072
Nickel–silver	Polished	.135
Paint, white	Clean	.79
Paint, cream	Clean	.77
Paint, black	Clean	.84
Paint, bronze	Clean	.51
Silver	Polished	.052
Stainless steel	Polished	.074
Steel	Polished	.066
Tin	Polished	.069
Tin	Commercial coat	.084
Tungsten	Polished coat	.066
Zinc	Commercial coat	.21
Fuzed quartz	1.96 mm thick	.775
Covex D (glass)	3.40 mm thick	.83
Nonex (glass)	1.57 mm thick	.835
Aluminum paint		.29

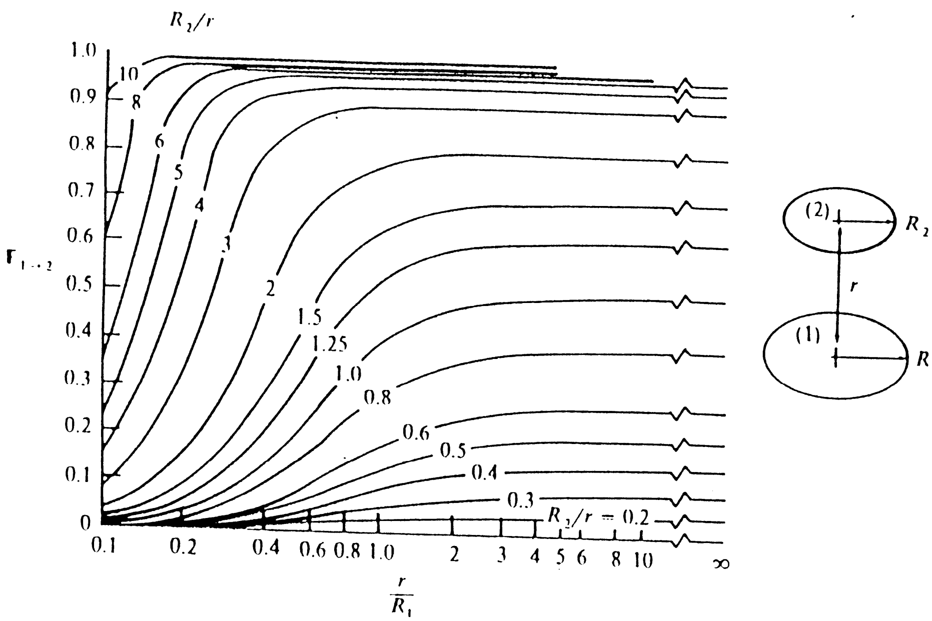


FIGURE 9.3
Shape factor for parallel coaxial disks [1]

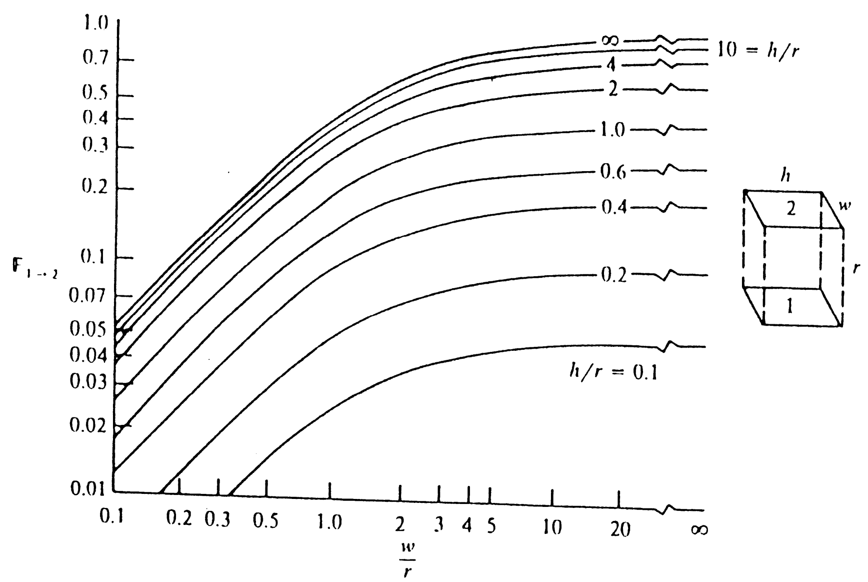


FIGURE 9.4
Shape factor for equal size opposing rectangles [1].

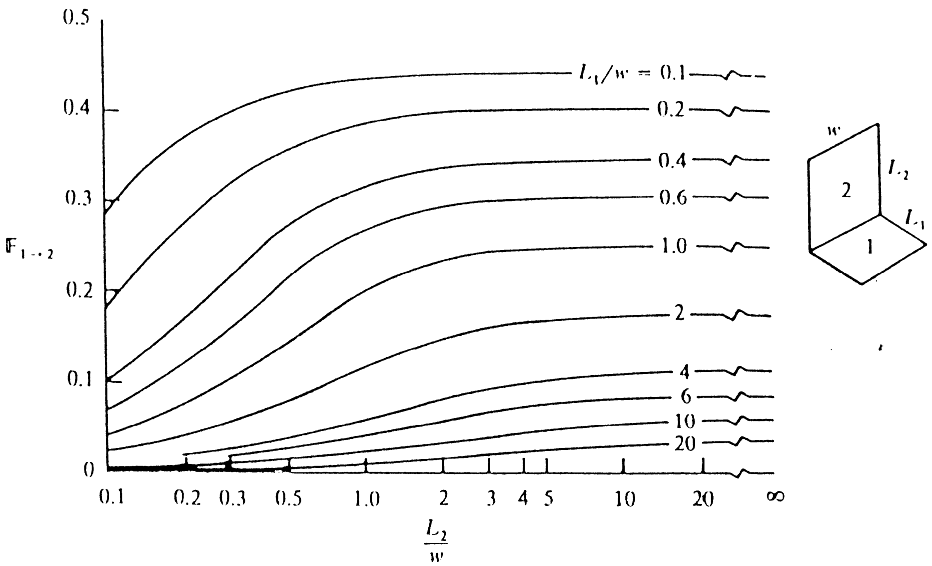


FIGURE 9.5
Shape factor for perpendicular rectangles [1].

Assignment #6

Dally et al:

9.7	9.16	9.17	9.25
10.1	10.16	10.17	10.37