ECE414/514 Electronics Packaging
Spring 2012 Lecture 10
Mechanical C
Viscoelasticity

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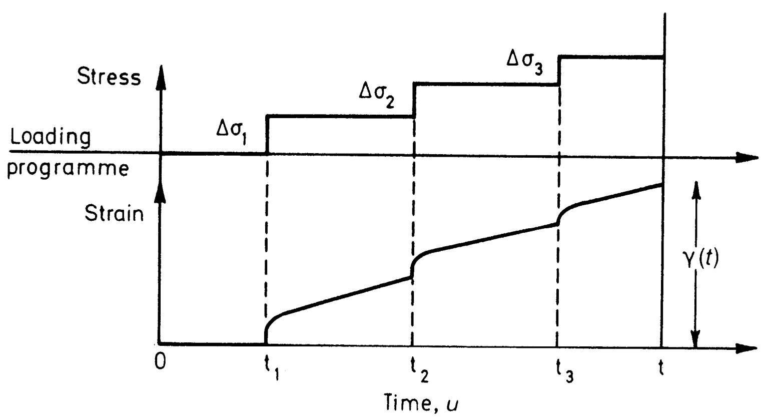
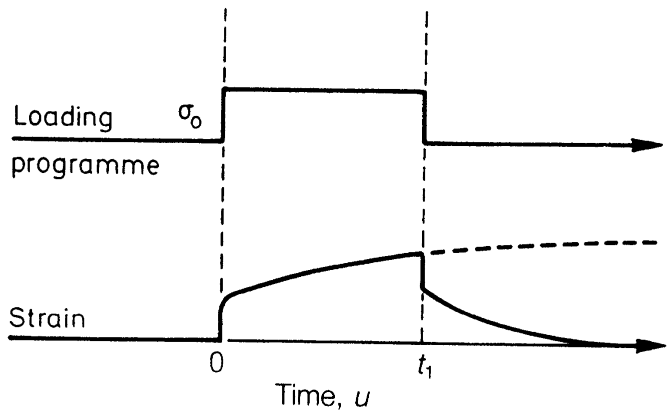
Creep



4.30 Three stress increments at a sequence of times (t_1, t_2, t_3) leading to a total strain $\gamma(t)$.

Polymers do not creep indefinitely and return fully to the initial conditions.



4.31 Response of a viscoelastic solid to loading at 0 and unloading at t_1.

Stress $= \sigma$ and Strain $= \gamma$

① Apply stresses σ_1 and σ_2

For any time t $\quad \frac{\gamma_1(t)}{\sigma_1} = \frac{\gamma_2(t)}{\sigma_2} = J(t)$

$J(t)$ is creep compliance $= \gamma(t)/\sigma$

② Apply strains γ_1 and γ_2

For any time t $\quad \frac{\sigma_1(t)}{\gamma_1} = \frac{\sigma_2(t)}{\gamma_2} = G(t)$

$G(t)$ is stress-relaxation modulus $= \sigma(t)/\gamma$

① → creep i.e. apply stress, $\gamma(t)$ increases

② → stress relaxation i.e. define a ~~stress~~ strain, $\sigma(t)$ decreases.

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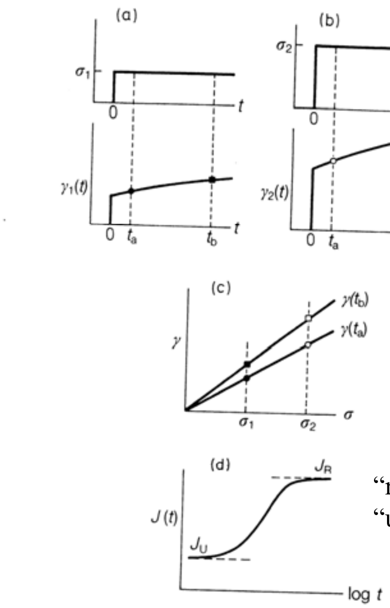
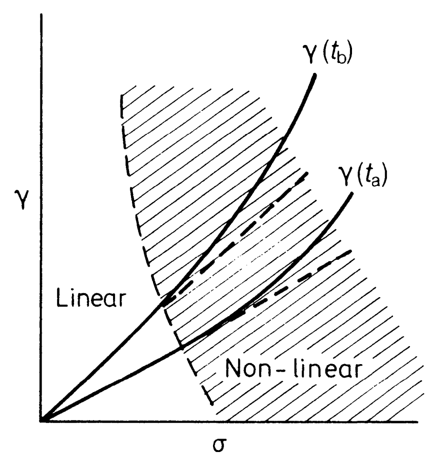


$J(t)=\gamma/\sigma$

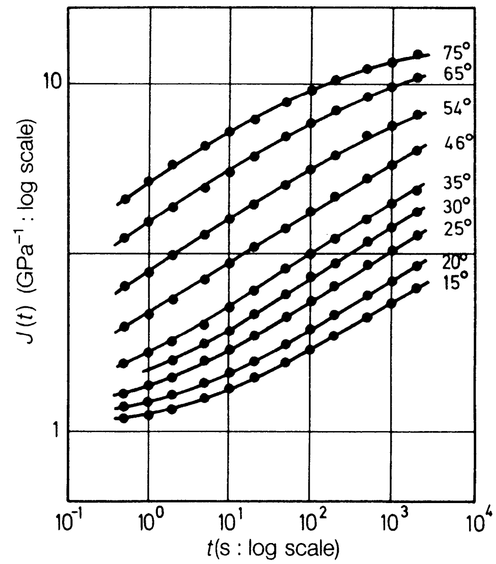
"relaxed" when $t\longrightarrow\infty$

"unrelaxed" $\longrightarrow$elastic response

4.1 Linear viscoelastic creep: (a) constant stress σ_1 applied at $t=0$ leads to time-dependent strain $\gamma_1(t)$; (b) a higher stress σ_2 applied at $t=0$ leads to time dependent strain $\gamma_2(t)$; (c) from (a) and (b) the strains at t_a, $\gamma(t_a)$, and at time t_b, $\gamma(t_b)$, are linear in the stress; (d) the observed dependence of $J(t)$ (eqn 4.3) on log t through one complete relaxation. J_U and J_R are the unrelaxed and relaxed compliances, respectively



4.2 Isochronals taken at t_a (see Figure 4.1) after the initiation of the creep experiment, $\gamma(t_a)$, and at t_b, $\gamma(t_b)$. The diagram illustrates the transition from linear to non-linear viscoelastic behaviour. Note that this is not the γ–σ plot that would be obtained in a conventional stress–strain test; the data are taken from creep experiments at different stresses.



4.4 Shear compliance $J(t)$ of linear polyethylene at different temperatures in the region of the α-relaxation. Measurements are taken at times between 0.8 and 2000 s. The data are plotted against log t, as a plot against t will not reveal the significant differences in the shape of the curves at different temperatures (after McCrum and Morris).

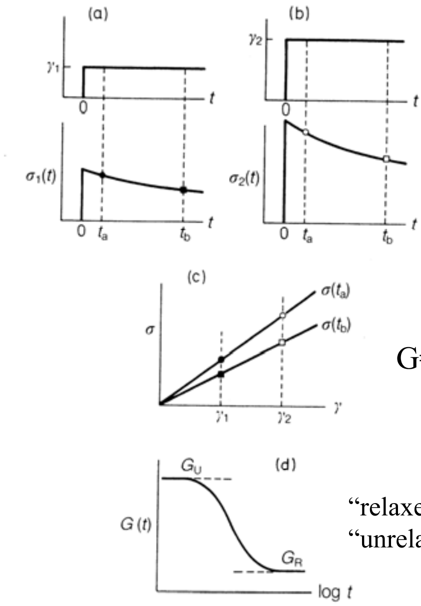
VISCOELASTICITY

CREEP → Molecular rearrangement

Creates "back stress" which leads to full recovery.

If applied stress σ_A for long time, back stress σ_B increases to σ_A as $t \to \infty$, at which time $\sigma_A - \sigma_B \to 0$ and no further extension.

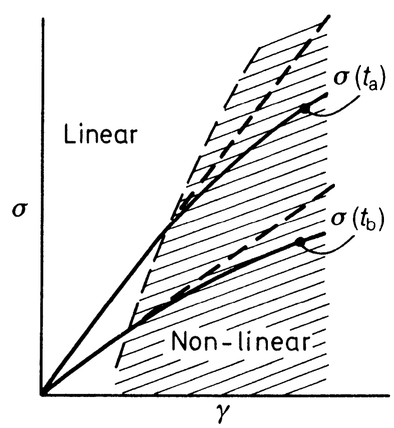
When σ_A removed $(=0)$ σ_B restores original state.



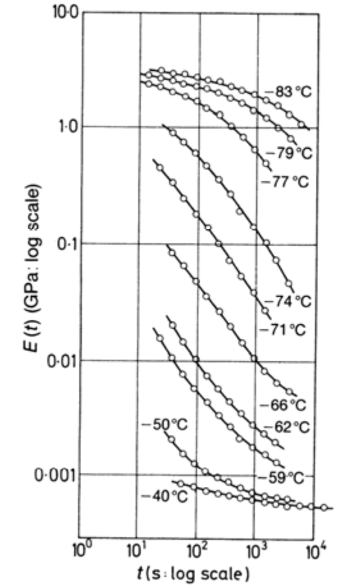
G=σ/γ

"relaxed" when t⟶∞
"unrelaxed" ⟶elastic response

4.5 Linear viscoelastic stress relaxation: (a) constant strain γ_1 applied at $t=0$ leads to a time-dependent stress $\sigma_1(t)$; (b) a higher strain γ_2 applied at $t=0$ leads to time-dependent stress $\sigma_2(t)$; (c) from (a) and (b) the stresses at time t_a, $\sigma(t_a)$, and at time t_b, $\sigma(t_b)$, are linear in the strain; (d) the observed dependence of $G(t)$ on log t through one complete relaxation. G_U and G_R are the unrelaxed and relaxed moduli, respectively.



4.6 Isochronals taken at t_a (see Figure 4.5) after the initiation of the stress relaxation experiment, $\sigma(t_a)$, and at t_b, $\sigma(t_b)$. The diagram illustrates the transition from linear to non-linear behaviour. Note that this σ–γ plot cannot be obtained in a conventional stress–strain experiment: it must be obtained by a series of stress relaxation experiments, as illustrated in Figure 4.5.



4.7 Stress relaxation modulus observed in tension $E(t)$ of polyisobutylene at different temperatures in the region of the glass–rubber relaxation ($T_g = -80°C$). At −83°C at short time, $E(t)$ approaches asymptotically the modulus of the glass; at −40°C at long time, $E(t)$ approaches asymptotically the modulus of the rubber. The relaxation is centred in the region of −66°C. Note the immense reduction in $E(t)$ of over 3 decades in a temperature rise of 43°C: this behaviour is typical of amorphous polymers at the glass–rubber relaxation

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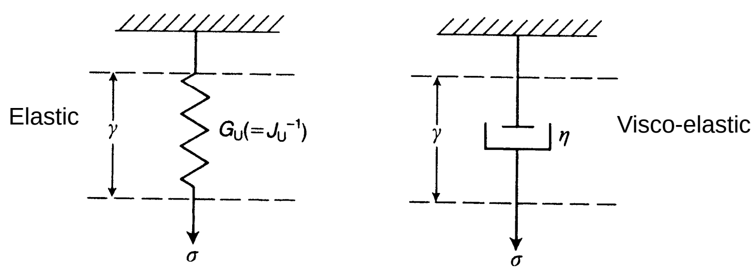
Time & Frequency Responses



4.13 The spring and dashpot. The strain in the spring is $\gamma = J_U \sigma$; the strain rate $(d\gamma/dt) = J_U(d\sigma/dt)$. The strain in the dashpot cannot be related simply to the stress (it depends on the stress history). The strain rate in the dashpot is proportional to the stress and is $(d\gamma/dt) = \sigma/\eta$.

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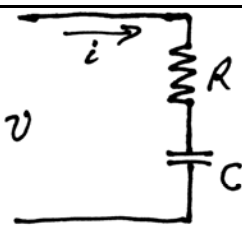
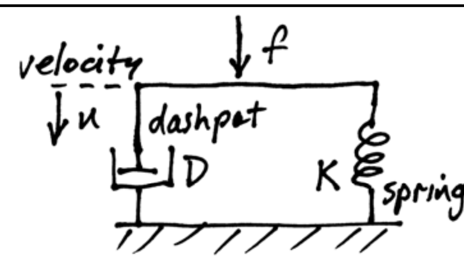
ANALOGS

Mechanical	M	D	K	f	u	x
Electrical	L	R	C	v	i	q
Rotational	J	D_r	K_r	τ	ω	θ
Thermal		R_T	C_p	τ	q_T	w_T
Electrical II	C	G	L	i	v	λ^*

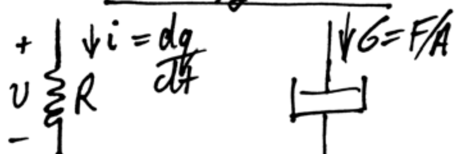
$^*\lambda$ – flux linkage $= \int v\,dt$



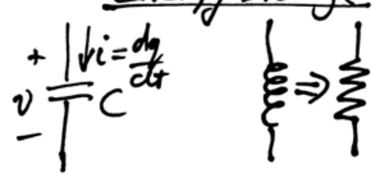


$$v = iR + \frac{1}{C}\int i\,dt \qquad f = Du + \frac{1}{K}\int u\,dt$$

Energy Loss



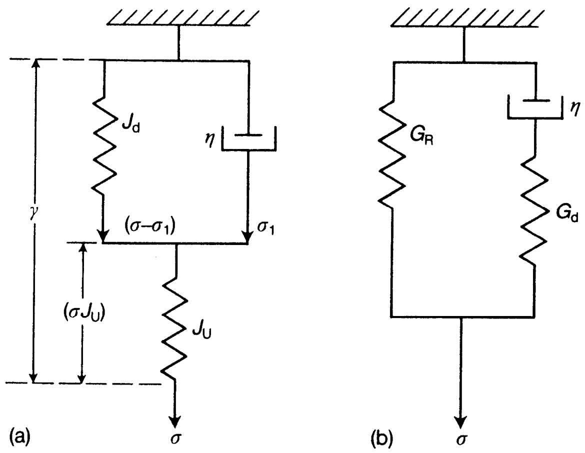
Energy storage



$$v = R\frac{dq}{dt} \qquad \sigma = \eta\frac{d\gamma}{dt} \qquad q = Cv \qquad \gamma = \sigma/G_u = \sigma J_u$$

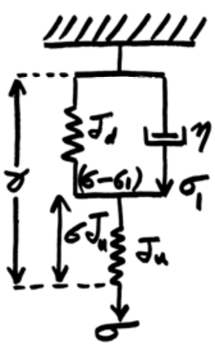
Modulus G_u

Compliance J_u



4.14 The Zener model (or standard linear solid). The model may be represented as a spring in series with a Kelvin model, as in (a), or as a spring in parallel with a Maxwell model, as in (b). The significant properties inherent in the Zener model include: (i) two time constants, one for constant stress τ_σ and one for constant strain τ_γ; (ii) an instantaneous strain at $t = 0$ when subject to a step-function stress; and (iii) full recovery following removal of the stress. For the Kelvin and Maxwell models, see Problem 4.8.

TIME RESPONSES



Overall Stress $\sigma \longrightarrow$ Strain γ

Dashpot stress σ_1, then elastic stress $\sigma_d = \sigma - \sigma_1$

$$\gamma_u = J_u \sigma \quad \& \quad \gamma - \sigma J_u = J_d (\sigma - \sigma_1)$$

$$\& \quad \sigma_1 = \eta \frac{d}{dt} (\gamma - \sigma J_u)$$

$$\therefore \gamma - \sigma J_u = J_d \sigma - J_d \eta \frac{d}{dt} (\gamma - \sigma J_u)$$

Note: Zener model includes 2 characteristic time constants

Constant stress $\rightarrow \tau_\sigma$

Constant strain $\rightarrow \tau_\gamma$

$J\eta$ units: Compare $J \rightarrow C$ & $\eta \rightarrow R$
so $J\eta \rightarrow$ time constant τ

Applied stress $\sigma \rightarrow$ write $J_d \eta = \tau_\sigma$

so $\gamma = \sigma(J_u + J_d) - \tau_\sigma \frac{d\gamma}{dt} + \tau_\sigma J_u \frac{d\sigma}{dt}$

Write $J_u + J_d = J_R$ $\therefore \gamma + \tau_\sigma \frac{d\gamma}{dt} = \sigma J_R + \tau_\sigma J_u \frac{d\sigma}{dt}$

$\rightarrow \frac{1}{J_R}\left[\gamma + \tau_\sigma \frac{d\gamma}{dt}\right] = \sigma + \tau_\sigma \left(\frac{J_u}{J_R}\right)\frac{d\sigma}{dt}$

$= \sigma + \tau_\gamma \frac{d\sigma}{dt}$

since $\frac{\tau_\sigma}{\tau_\gamma} =$ [redacted] $= \frac{J_R}{J_u} = \frac{G_u}{G_R}$ $=(J_d+J_u)/J_u=J_d\eta/\eta J_d J_u/(J_d+J_u)$

CREEP Apply constant stress σ_0 at $t=0$

$\therefore \frac{d\sigma}{dt} = 0$ and $\gamma + \tau_\sigma \frac{d\gamma}{dt} = J_R \sigma_0$

$\longrightarrow \gamma(t) = J_R \sigma_0 - \sigma_0 [J_R - J_u] \exp - t/\tau_\sigma$

$= J_R \sigma_0 [1 - \exp - t/\tau_\sigma] + J_u \exp - t/\tau_\sigma$

OR $J_u \sigma_0 + (J_R - J_u)(1 - \exp - \frac{t}{\tau_\sigma})\sigma_0$

$\div \sigma_0$ to get $J(t) = J_u + (J_R - J_u)(1 - \exp - \frac{t}{\tau_\sigma})$

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STRESS RELAXATION

Apply constant strain γ_0 at $t=0$

$$\therefore \frac{d\gamma}{dt} = 0 \text{ and } G_R \gamma = \sigma + \tau_\gamma \frac{d\sigma}{dt}$$

$$\longrightarrow \sigma(t) = G_R \gamma_0 + [\sigma(0) - G_R \gamma_0] \exp -t/\tau_\gamma$$

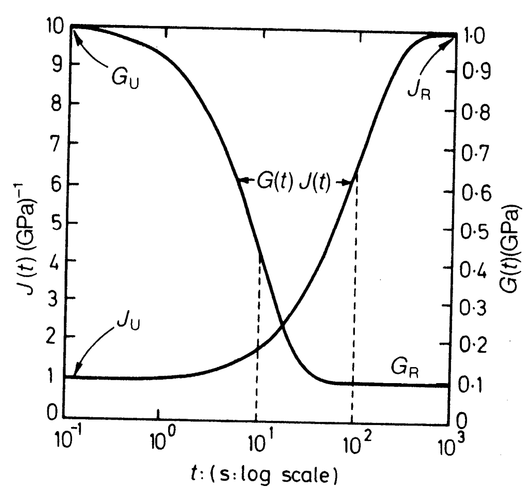
where $\sigma(0) = G_U \gamma_0$

$$\therefore \sigma(t) = G_R \gamma_0 + \gamma_0 [G_U - G_R] \exp -t/\tau_\gamma$$

OR

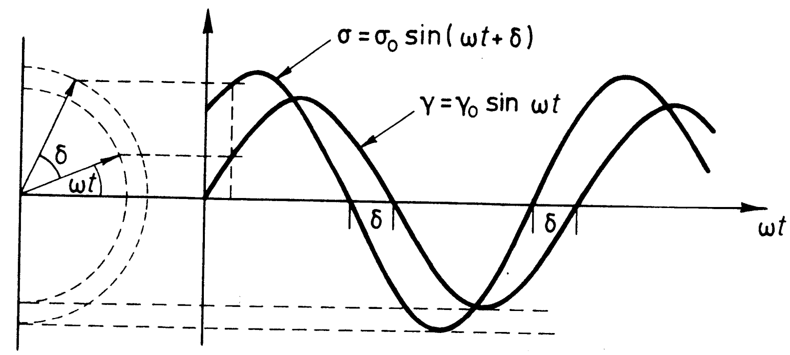
$$G(t) = G_R + (G_U - G_R) \exp -t/\tau_\gamma$$

$$= G_U - (G_U - G_R)(1 - \exp \frac{-t}{\tau_\gamma})$$



4.15 Solution to the Zener model for a creep experiment ($J(t)$) or for a stress-relaxation experiment ($G(t)$). Model with $J_U = 1$ GPa^{-1}, $J_R/J_U = 10$, and $\tau_\sigma = 100$ s. Note that from eqn 4.46 $\tau_\gamma = 10$ s.

Frequency Response



4.8 Vector representation of an alternating stress leading an alternating strain by phase angle δ.

FREQUENCY RESPONSE

$$\frac{1}{J_R}\left[\gamma + \tau_\sigma \frac{d\gamma}{dt}\right] = \sigma + \tau_\gamma \frac{d\sigma}{dt}$$

Write

$$\gamma^* = \gamma_0 \exp j\omega t$$
$$\sigma^* = \sigma_0 \exp j(\omega t + \delta)$$

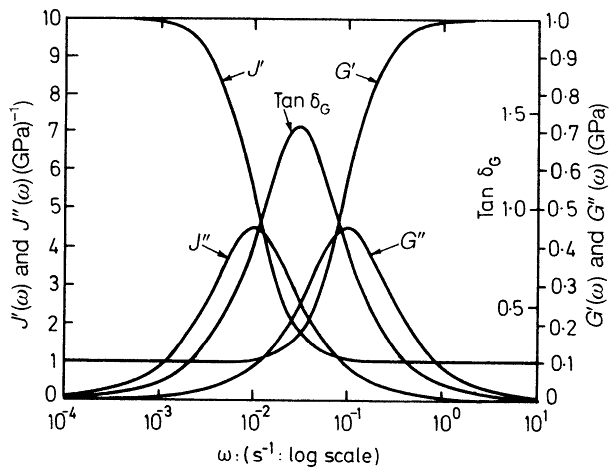
$$J^* = \gamma^* / \sigma^* = \frac{\gamma_0}{\sigma_0} \exp j\delta = J_0 (\cos\delta - j\sin\delta) = J' - j J''$$

$$\rightarrow J' = J_U + \frac{J_R - J_U}{1 + \omega^2 \tau_\sigma^2}$$

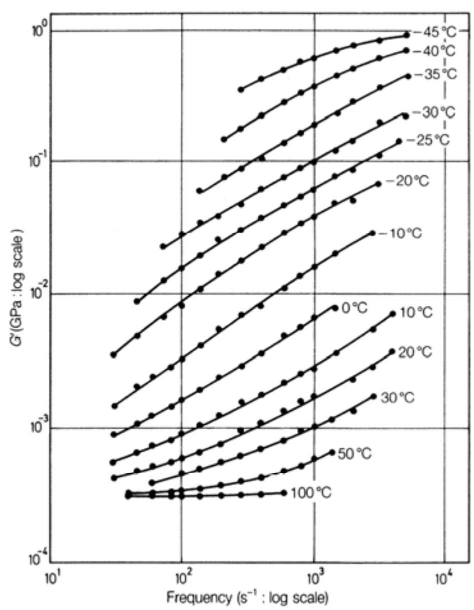
$$J'' = \frac{(J_R - J_U)\omega\tau_\sigma}{1 + \omega^2\tau_\sigma^2}$$

Similarly $G^* = G_U - \dfrac{G_U - G_R}{1 + j\omega\tau_\gamma}$

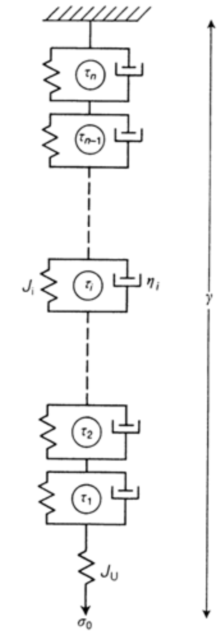
$$G' = G_R + \frac{(G_U - G_R)\omega^2\tau_\gamma^2}{1 + \omega^2\tau_\gamma^2} \qquad G'' = \frac{(G_U - G_R)\omega\tau_\gamma}{1 + \omega^2\tau_\gamma^2}$$



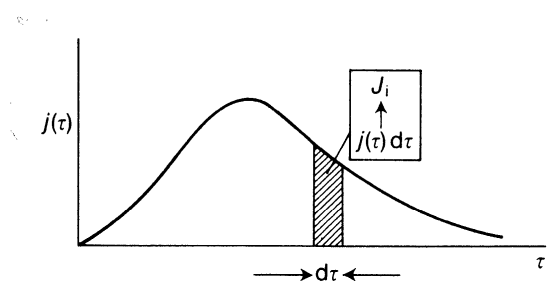
4.16 Solution to the Zener model for a dynamic experiment. Model with $J_U = 1$ GPa^{-1}, $J_R/J_U = 10$ and $\tau_\sigma = 100$ s. Note that maxima occur: in J'' at $\omega = \tau_\sigma^{-1} = 0.01$ rad s^{-1}; in G'' at $\omega = \tau_\gamma^{-1} = 0.1$ rad s^{-1}; and in tan δ at $\omega = (\tau_\sigma \tau_\gamma)^{-\frac{1}{2}} = 0.0316$ rad s^{-1}.



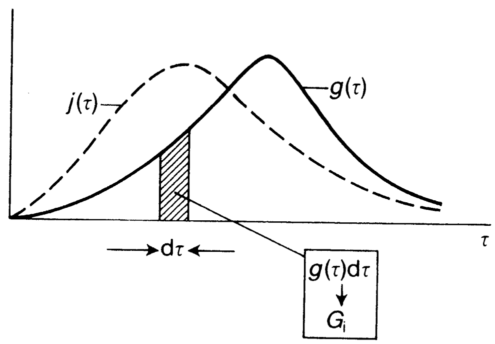
4.10 Frequency dependence of the dynamic shear modulus $G'(\omega)$ of polyisobutylene at different temperatures in the range −45°C to +100°C. This relaxation is the glass-to-rubber relaxation: it is observed here centred in the region of −10°C, well above the glass transition (−80°C) because of the high frequency of observation. The measurements were by forced oscillation (after Fitzgerald, Grandine, and Ferry).



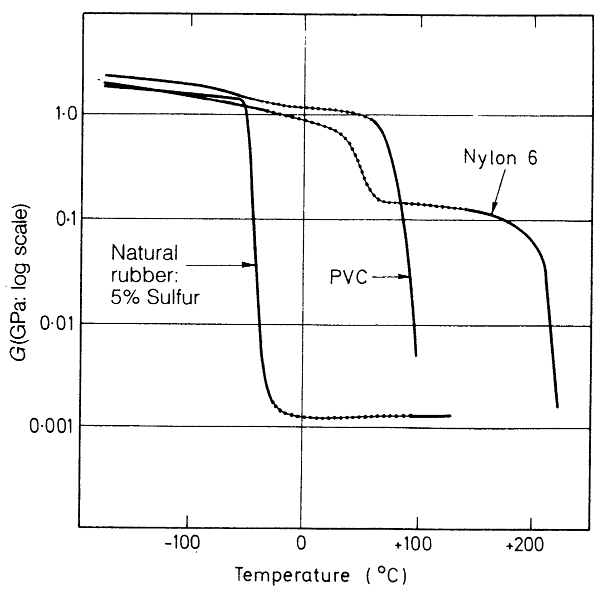
4.17 Distribution of n Kelvin relaxation elements $\tau_1, \tau_2, \ldots, \tau_i, \ldots, \tau_{n-1}, \tau_n$. Each element is stressed by the stress σ_0 which also acts on the instantaneous compliance J_U. The strains in the elements all add to give the total strain.



4.18 The distribution of relaxation times plotted against τ from the model of Figure 4.17. The element of area at τ, $j(\tau)\,d\tau$, represents the strength of the relaxation at τ. The integrated area (from eqn 4.64) equals $J_R - J_U$.



4.19 The distribution of relaxation times $g(\tau)$ for the model of Problem 4.12; a continuous distribution of Maxwell elements. The element of area at τ, $g(\tau)\,d\tau$, represents the strength of the relaxation at τ. The integrated area (from eqn 4.67) equals $G_U - G_R$.



4.21 Dependence of the shear modulus on temperature for three representative engineering polymers: natural rubber (cross-linked); PVC (essentially amorphous and not cross-linked); and nylon 6 (crystalline). The temperatures at which these polymers are used in technology are indicated (•–•–•–•–•) (after Wolf).

Temperature Dependence

$$\tau_T = a_T \, \tau_{T_0}$$

where $a_T = \exp \frac{\Delta H}{R}\left[\frac{1}{T} - \frac{1}{T_0}\right]$

at t, T_0

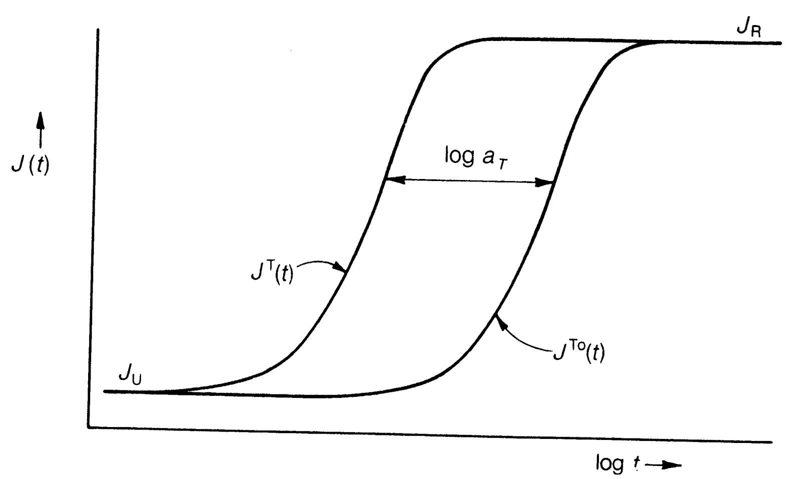
$$J^{T_0}(t) = J_u + (J_R - J_u)\left[1 - \exp -t/\tau_{T_0}\right]$$

at $a_T t, T$

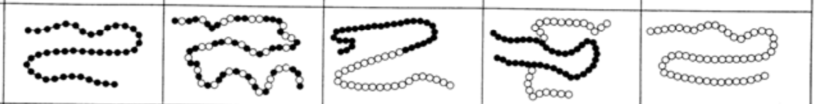
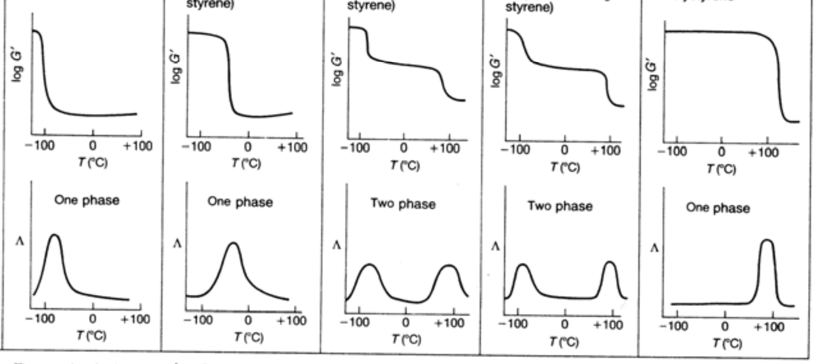
$$J^{T}(a_T t) = J_u + (J_R - J_u)\left[1 - \exp - \frac{a_T t}{a_T \tau_{T_0}}\right]$$

so $J^{T_0}(t) = J^{T}(a_T t)$

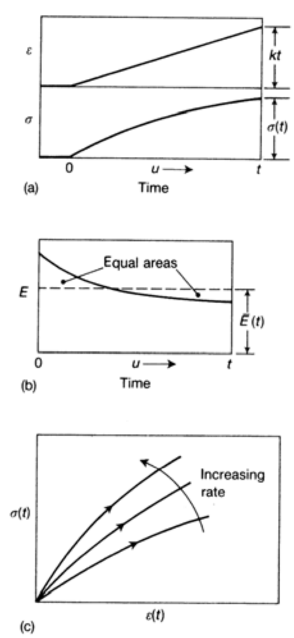
Plots vs Log t separated by Log a_T (shift factor)



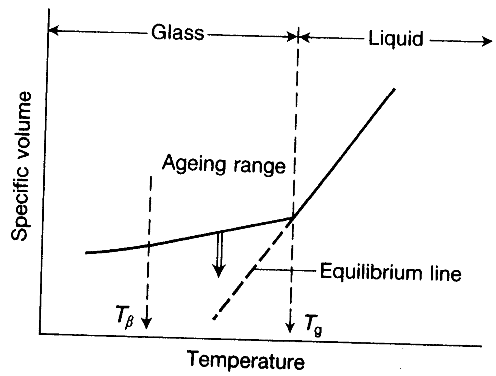
4.20 Illustration of the time–temperature shift (eqn 4.72). The shear compliance curves at T and T_0 when plotted against log t are simply displaced horizontally by log a_T: a_T is the shift factor (eqn 4.68). The small temperature dependence of J_U and J_R is neglected here.

Monomer(s)	A	A and B			B
Class of polymer	Homopolymer	Random copolymer	Block copolymer	Graft copolymer	Homopolymer
Chemical Name	Poly A	Poly (A-co-B)	Poly (A-b-B)	Poly (A-g-B)	Poly B
Schematic chemical structure					
Example	Polybutadiene	Poly (butadiene-co-styrene)	Poly (butadiene-b-styrene)	Poly (butadiene-g-styrene)	Polystyrene
Variation of shear modulus G' and log dec Λ with temperature					
	One phase	One phase	Two phase	Two phase	One phase

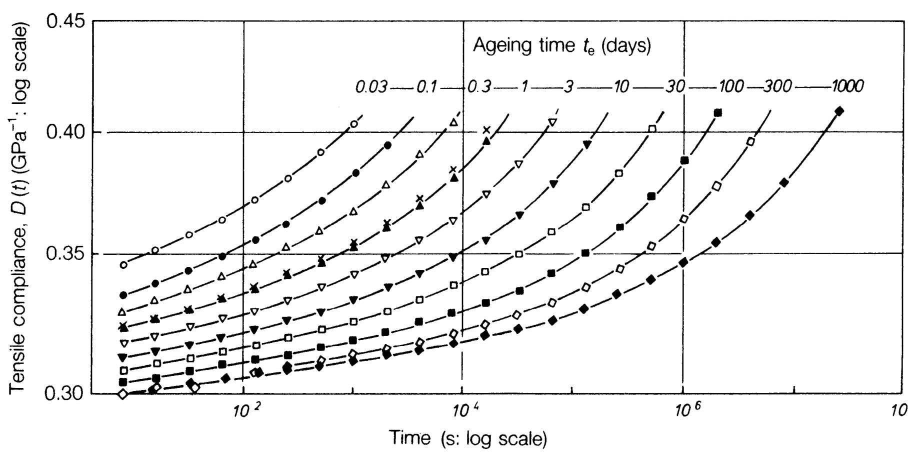
4.35 The mechanical spectra (log G' and Λ at ~1 Hz versus temperature) for copolymers (random, block, and graft).



4.32 (a) A constant strain rate leads to a non-linear stress; (b) the definition of $\bar{E}(t)$, the mean modulus; and (c) increasing the strain rate (k) increases the slope of the stress–strain curve.



4.28 The origin of physical ageing in an amorphous polymer. After quenching to a temperature below T_g (but above T_β, the temperature of the highest secondary relaxation), the volume slowly contracts: the movement is towards the equilibrium, which is the extrapolated $v - T$ line for the liquid (after Struik).



4.29 Tensile compliance of PVC at 20°C. The specimen was quenched from 90°C (20°C above T_g) to 20°C, and kept at 20°C for 4 years. At specific times (t_e) after the quench: 0.03, 0.1, 0.3 days, etc., the compliance was determined. Note the systematic shift of the creep curves. This effect is termed **physical ageing** (see also Figure 4.28) (after Struik).

Assignment #6

1. A straight rod of solid polymer of length 1m and diameter 10mm is linearly viscoelastic with tensile creep compliance

$$D = 2 - \exp(-0.1t)\ \text{GPa}^{-1}$$

where *t* is in hours. The rod is suspended vertically and a 10kg mass is hung from it for 10 hours. Find the change in length of the rod.

2. After 10 hours, the 10kg mass is removed. Find the strain remaining in the rod after a further 10 hours.
3. Dally et al: Problem 12.1
4. Dally et al: Problem 12.9
5. Dally et al: Problem 12.11
6. Dally et al: Problem 12.26